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ADIABATIC PROPAGATION OF QUANTIZED LIGHT PULSES IN AN ATOMIC MEDIUM WITH THE TRIPOD LEVEL CONFIGURATION

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We consider adiabatic propagation of a pair of quantized light pulses in a coherently prepared atomic medium with the tripod level configuration. We find that under the conditions of the electromagnetically induced transparency, two distinct polariton modes are simultaneously formed in the medium. These polaritons, represented by certain coherent superpositions of the quantized fields, have different group velocities; the fast one propagates at essentially the speed of light, while the group velocity of the slow polariton can be dynamically reduced to zero. The state mapping between the electromagnetic field and atomic ensemble is also demonstrated.

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Shape-preserving adiabatic propagation of electromagnetic pulses, often termed adiabatons, in three-level atomic media has been studied over the last decade [1–3]. The underlying effect is the coherent population trapping phenomenon [4] that is the accumulation of atoms in a coherent superposition of states, which is immune to excitation by the given frequency-split laser radiation. Its extension to atomic or molecular systems containing more than just three levels, such as four-level atoms with the tripod level configuration, is becoming an active topic of current research [5].

The related effect of electromagnetically induced transparency (the manifestation of coherent population trapping in optically dense media) [6] is the basis for several groundbreaking recent achievements, such as reduction of the group velocity of weak light pulses to remarkably low values [7] or even down to complete stop [8, 9], single-photon pulse generation [10] and reversible quantum memories [11], which may eventually be employed to realize deterministic quantum computation with single-photon qubits [12].

In a recent paper [13], we have studied the adiabatic pulse propagation in a medium of atoms with the tripod level scheme (hereafter called a tripod medium). We there considered strong coherent pulses of large amplitudes, describable by the semiclassical approach. Weak quantum pulse propagation in such a system was studied in Ref. [14], where the possibility of achieving a quantum phase gate between a pair of single-photon pulses was demonstrated. Parametric generation of light in a medium of tripod atoms prepared in a certain coherent superposition of ground states was recently discussed in Ref. [15].

The main result of the semiclassical analysis in Ref. [13] is that a classical three-component light pulse propagating in a tripod medium under adiabaticity conditions asymptotically (at large propagation times or distances) evolves into a pair of nonlinear, shape-preserving pulses propagating at the different group velocities. The fast pulse propagates at the speed of light c, whereas the group velocity of the slow pulse is dynamically reduced with respect to c, in accordance with the standard formula for the slow-light velocity [1]. Remarkably, such an adiabatic propagation is essentially governed by the off-diagonal geometric phase that develops in the atomic state in a self-consistent way under the action of the light pulse. Before generalizing the treatment based on geometric-phase effects to a system where all the electromagnetic fields are quantized, it is reasonable to first extend the polaritonic theory of slow light propagation [8] to the case of a tripod medium.

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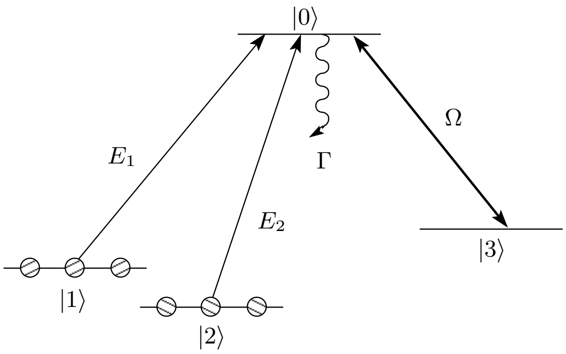

Level scheme of tripod atoms interacting with two weak fields $E_{1,2}$ and a strong driving field of a Rabi frequency Ω

In the present paper, we consider the interaction of two quantized optical fields and a strong classical driving field with a medium of atoms having a tripod configuration of levels (see the Figure). The lower states $|1\rangle$, $|2\rangle$, and $|3\rangle$ are the relevant Zeeman sublevels of the electronic ground state of the atoms. The optically excited state of the tripod scheme is denoted by $|0\rangle$. The transition $|3\rangle \to |0\rangle$ is driven by the classical, time-dependent in the general case, electromagnetic field with a Rabi frequency Ω. The quantized fields $\hat{E}_j$, $j = 1, 2$, excite the corresponding atomic transitions $|j\rangle \to |0\rangle$. All the fields are tuned exactly to resonance with the corresponding atomic transitions.

The interaction Hamiltonian is given by

$$\hat{V} = -\hbar \frac{N}{L} \int_0^L dz \left[\Omega(t)\hat{\sigma}_{03} + \sum_{j=1}^{2} g_j \hat{E}_j \hat{\sigma}_{0j} \right] + \text{H.c}, \quad (1)$$

where N is the total number of atoms in the quantization volume AL, A being the cross-section area and L being the medium length,

$$\sigma_{\mu\nu} \equiv |\mu\rangle\langle\nu|$$

are the atomic operators, and

$$g_j = d_{0j} \sqrt{\frac{ck_j}{2\hbar\varepsilon_0 AL}}$$

are the atom–field coupling constants, with d_{0j} being the optical transition dipole moments. We neglect thermal motion of atoms, i.e., set the atomic velocity to zero. The field operators $\hat{E}_j$ admit the mode decomposition

$$\hat{E}_j = \sum_k \hat{a}_j^{(k)}(t) \exp[ik(z - ct)],$$

where $\hat{a}_j^{(k)}(t)$ is the photon annihilation operator obeying the usual bosonic commutation rules. In the slowly varying envelope approximation, the propagation equations for the quantum field operators are given by

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z} \right) \hat{E}_j = i g_j N \hat{\sigma}_{j0}, \quad j = 1, 2. \quad (2)$$

On the other hand, the atomic operators $\sigma_{\mu\nu}$ in the interaction representation satisfy the evolution equation

$$\frac{\partial}{\partial t}\hat{\sigma}_{\mu\nu} = -\gamma_{\mu\nu}\hat{\sigma}_{\mu\nu} + \frac{i}{\hbar}[\hat{V}, \hat{\sigma}_{\mu\nu}] + \hat{F}_{\mu\nu}, \quad (3)$$

where $\gamma_{\mu\nu}$ are the relaxation constants and $\hat{F}_{\mu\nu}$ are the corresponding δ-correlated Langevin noise operators. In particular, Eqs. (3) explicitly yield

$$\frac{\partial}{\partial t}\hat{\sigma}_{10} = -\gamma_{10}\hat{\sigma}_{10} + i g_1 \hat{E}_1 (\hat{\sigma}_{11} - \hat{\sigma}_{00}) + \\ + i g_2 \hat{E}_2 \hat{\sigma}_{12} + i\Omega\hat{\sigma}_{13} + \hat{F}_{10}, \quad (4)$$

$$\frac{\partial}{\partial t}\hat{\sigma}_{20} = -\gamma_{20}\hat{\sigma}_{20} + i g_2 \hat{E}_2 (\hat{\sigma}_{22} - \hat{\sigma}_{00}) + \\ + i g_1 \hat{E}_1 \hat{\sigma}_{21} + i\Omega\hat{\sigma}_{23} + \hat{F}_{20}, \quad (5)$$

$$\frac{\partial}{\partial t}\hat{\sigma}_{j3} = -\gamma_{j3}\hat{\sigma}_{j3} - i g_j \hat{E}_j \hat{\sigma}_{03} + i\Omega^* \hat{\sigma}_{j0} + \hat{F}_{j3}, \quad (6)$$
$$j = 1, 2.$$

Let $|\Omega|$ be the largest frequency in the system. Assuming that it changes slowly enough,

$$\left| \frac{\partial \Omega}{\partial t} \right| \ll |\Omega|^2,$$

we can use the standard approximations commonly used in the adiabatic pulse propagation analysis [1, 8]. We can then neglect the noise terms for atomic coherences ($\mu \neq \nu$) and use Eqs. (4)–(6) to obtain

$$\hat{\sigma}_{j0} = -\frac{i}{\Omega^*} \frac{\partial}{\partial t}\hat{\sigma}_{j3}, \quad \hat{\sigma}_{j3} = -\frac{1}{\Omega} \sum_{l=1}^{2} g_l \hat{E}_l \hat{\sigma}_{jl}. \quad (7)$$

For simplicity, we assume $\Omega(t)$ and σ_{12} to be real; we recall that the number of photons in the weak fields $\hat{E}_j$ is so small that the populations of the atomic levels $|1\rangle$ and $|2\rangle$ and the coherence between them remain practically unchanged throughout the evolution. We can therefore replace the operators $\hat{\sigma}_{lj}$, $l, j = 1, 2$, by constant c-numbers σ_{lj}. Then propagation equations (2) for the quantized fields are reduced to

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z} \right) \hat{E}_j = -\frac{g_j N}{\Omega(t)} \frac{\partial}{\partial t} \sum_{l=1}^{2} \frac{g_l \hat{E}_l \sigma_{jl}}{\Omega(t)}, \quad (8)$$
$$j = 1, 2.$$

We consider two linear combinations of the operator variables $\hat{E}_j$, having the form

$$\hat{\Psi}_\pm = \frac{1}{\cos\vartheta_\pm(t)}\left(\sin\alpha_\pm \hat{E}_1 + \cos\alpha_\pm \hat{E}_2\right), \quad (9)$$

where

$$\cos\vartheta_\pm(t) = \frac{\Omega(t)}{\sqrt{\Omega^2(t) + P_\pm^2}} \quad (10)$$

are the mixing angles, and $\alpha_\pm$ and $P_\pm$ are some constants to be determined. We now prove that the above polariton operators $\hat{\Psi}_\pm$ satisfy the corresponding propagation equations

$$\left(\frac{\partial}{\partial t} + c\cos^2\vartheta_\pm(t)\frac{\partial}{\partial z}\right)\hat{\Psi}_\pm = 0, \quad (11)$$

where

$$c\cos^2\vartheta_\pm(t) \equiv v_\pm$$

evidently play the role of group velocities. Through the direct substitution, using Eq. (8), and temporarily dropping the subscripts «±», we obtain

$$\sin^2\vartheta\frac{\partial}{\partial t}\hat{\Psi} + \cos^2\vartheta\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right)\hat{\Psi} = 0,$$

or, in other words,

$$\sin^2\vartheta\frac{\partial}{\partial t}\left(\frac{\sin\alpha\,\hat{E}_1 + \cos\alpha\,\hat{E}_2}{\cos\vartheta}\right) - (\sin\alpha\,\hat{E}_1 + \cos\alpha\,\hat{E}_2)\frac{\partial}{\partial t}\cos\vartheta - \cos\vartheta\left[\sin\alpha\,\frac{g_1 N}{\Omega}\,\frac{\partial}{\partial t}\left(\frac{g_1\sigma_{11}\hat{E}_1 + g_2\sigma_{12}\hat{E}_2}{\Omega}\right) + \cos\alpha\,\frac{g_2 N}{\Omega}\,\frac{\partial}{\partial t}\left(\frac{g_1\sigma_{21}\hat{E}_1 + g_2\sigma_{22}\hat{E}_2}{\Omega}\right)\right] = 0. \quad (12)$$

Recalling Eq. (10), we transform Eq. (12) into

$$(P^2\sin\alpha - g_1^2\sigma_{11}N\sin\alpha - g_1 g_2\sigma_{21}N\cos\alpha)\frac{\partial}{\partial t}\left(\frac{\hat{E}_1}{\Omega}\right) + (P^2\cos\alpha - g_2^2\sigma_{22}N\cos\alpha - g_1 g_2\sigma_{12}N\sin\alpha)\times\frac{\partial}{\partial t}\left(\frac{\hat{E}_2}{\Omega}\right) = 0. \quad (13)$$

Because $\hat{E}_1$ and $\hat{E}_2$ are arbitrary and linearly independent, Eq. (13) results in the following set of equations for the unknown variables $\sin\alpha$ and $\cos\alpha$:

$$(P^2 - g_1^2\sigma_{11}N)\sin\alpha - g_1 g_2\sigma_{21}N\cos\alpha = 0, \quad (14)$$

$$-g_1 g_2 N\sigma_{12}\sin\alpha + (P^2 - g_2^2\sigma_{22}N)\cos\alpha = 0. \quad (15)$$

The solvability condition for the set of equations (14), (15),

$$(P^2 - g_1^2\sigma_{11}N)(P^2 - g_2^2\sigma_{22}N) - g_1^2 g_2^2\left|\sigma_{12}\right|^2 N^2 = 0,$$

yields the eigenvalues of P, given by

$$P_\pm^2 = \frac{N}{2}\left[g_1^2\sigma_{11} + g_2^2\sigma_{22} \pm \sqrt{(g_1^2\sigma_{11} + g_2^2\sigma_{22})^2 - 4g_1^2 g_2^2(\sigma_{11}\sigma_{22} - \sigma_{12}^2)}\right]. \quad (16)$$

Recalling that

$$\operatorname{Im}\sigma_{12} = 0$$

by assumption, we obtain

$$\operatorname{tg}\alpha_\pm = \frac{2g_1 g_2\sigma_{12}}{g_2^2\sigma_{22} - g_1^2\sigma_{11} \pm \sqrt{(g_1^2\sigma_{11} + g_2^2\sigma_{22})^2 - 4g_1^2 g_2^2(\sigma_{11}\sigma_{22} - \sigma_{12}^2)}}. \quad (17)$$

We note that

$$\operatorname{tg}\alpha_+\,\operatorname{tg}\alpha_- = -1. \quad (18)$$

In what follows, we assume that the medium is prepared in a pure state [13, 15], such that

$$\sigma_{12} = \sqrt{\sigma_{11}\sigma_{22}}. \quad (19)$$

Also, we neglect the spatial dependence of the Rabi frequency, assuming that it is a function of the time variable only, $\Omega = \Omega(t)$. By setting $\Omega = \Omega(t)$, we neglect the propagation effects linear in the ratio of the pulse propagation velocity to c. This ratio is small for the $\hat{\Psi}_+(z,t)$ polariton, but approaches 1 for the $\hat{\Psi}_-(z,t)$ polariton. However, in analogy with the classical pulse propagation in a tripod medium studied in Ref. [13], we find that in the quantum case, the fast mode also propagates at c, provided the medium is prepared in a pure state. Alternatively, we can assume that the classical driving field propagates in the direction perpendicular to the propagation direction of the quantized fields $\hat{E}_j$. We then have

$$\mathrm{tg}\,\alpha_- = -\frac{g_2\sqrt{\sigma_{22}}}{g_1\sqrt{\sigma_{11}}}, \quad P_-^2 = 0, \quad \cos\vartheta_-(t) = 1, \tag{20}$$

meaning that the group velocity of the $\hat{\Psi}_-(z,t)$ polariton is equal to the speed of light,

$$\hat{\Psi}_-(z,t) = \hat{\Psi}(z-ct,0) = \hat{\Psi}(0,t-z/c).$$

For the $\hat{\Psi}_+(z,t)$ polariton, we obtain

$$\mathrm{tg}\,\alpha_+ = \frac{g_1\sqrt{\sigma_{11}}}{g_2\sqrt{\sigma_{22}}}, \quad P_+^2 = N(g_1^2\sigma_{11} + g_2^2\sigma_{22}),$$
$$\cos\vartheta_+(t) = \frac{\Omega(t)}{\sqrt{\Omega^2(t) + P_+^2}}. \tag{21}$$

The group velocity of the $\hat{\Psi}_+(z,t)$ polariton can therefore become much smaller than c if the driving field Rabi frequency satisfies

$$\Omega \ll P_+.$$

The corresponding solution is then given by

$$\hat{\Psi}_+(z,t) = \hat{\Psi}_+\left(z - \int\limits_0^t v_+(t')dt', 0\right),$$

where

$$v_+(t) = c\cos^2\vartheta_+(t)$$

is the time-dependent group velocity. Thus, once the $\hat{\Psi}_+$ polariton has been fully accommodated in the medium, one can stop it completely by rotating the mixing angle ϑ_+ defined in Eq. (10) from its initial value $0 \le \vartheta_+ < \pi/2$ to $\vartheta_+ = \pi/2$, which amounts to switching off the Rabi frequency Ω. In the case of a constant Rabi frequency Ω, the above solution can be rewritten as

$$\hat{\Psi}_+(z,t) = \hat{\Psi}_+(z - v_+t, 0) = \hat{\Psi}_+(0, t - z/v_+).$$

From Eq. (9), using Eq. (18), we obtain

$$\hat{E}_1 = \sin\alpha_-\cos\theta_-\hat{\Psi}_- + \cos\alpha_-\cos\theta_+\hat{\Psi}_+, \tag{22}$$

$$\hat{E}_2 = \cos\alpha_-\cos\theta_-\hat{\Psi}_- - \sin\alpha_-\cos\theta_+\hat{\Psi}_+. \tag{23}$$

At large times $t > L/c$, when the fast polariton runs away from the medium and only the slow polariton remains inside the medium, the quantized field operators are expressed as

$$\hat{E}_1(z,t) = \cos\alpha_-\cos\vartheta_+(t) \times \times \left.\left(\frac{\cos\alpha_-\hat{E}_1 - \sin\alpha_-\hat{E}_2}{\cos\vartheta_+}\right)\right|_{0,t-z/v_+}, \tag{24}$$

$$\hat{E}_2(z,t) = -\sin\alpha_-\cos\vartheta_+(t) \times \times \left.\left(\frac{\cos\alpha_-\hat{E}_1 - \sin\alpha_-\hat{E}_2}{\cos\vartheta_+}\right)\right|_{0,t-z/v_+}, \tag{25}$$

We introduce the field operator for atoms in the state $|3\rangle$ via

$$\hat{\phi}_3(z,t) = \sqrt{\frac{N}{\sigma_{11}}}\hat{\sigma}_{13}(z,t). \tag{26}$$

Their plane-wave decomposition is given by

$$\hat{\phi}_3(z,t) = \sum_k \hat{\phi}_3^{(k)}(t)e^{ikz},$$

$$\hat{\phi}_3^{(k)}(t) = \frac{1}{L}\int\limits_0^L \hat{\sigma}_{13}(z,t)e^{-ikz}.$$

From Eq. (7), we obtain for large t that

$$\hat{\phi}_3 = -\frac{\sqrt{N}}{\Omega}\left(g_1\sqrt{\sigma_{11}}\hat{E}_1 + g_2\sqrt{\sigma_{22}}\hat{E}_2\right) = = -\frac{\sqrt{N}\cos\vartheta_+(t)}{\Omega(t)}\left(g_1\sqrt{\sigma_{11}}\cos\alpha_- - g_2\sqrt{\sigma_{22}}\sin\alpha_-\right) \times \times \hat{\Psi}_+(z,t), \tag{27}$$

which finally yields,

$$\hat{\phi}_3(z,t) = -\sin\vartheta_+(t)\hat{\Psi}_+(0,t-z/v_+). \tag{28}$$

The plane-wave decomposition operator coefficients of Eq. (28) are expressed through the photon annihilation operators $\hat{a}_j^{(k)}$ as

$$\hat{\phi}_3^{(k)} = \mathcal{F}(z,t) \times \times \left(\cos\alpha_-\hat{a}_1^{(k)}(t-z/v_+) - \sin\alpha_-\hat{a}_2^{(k)}(t-z/v_+)\right), \tag{29}$$

$$\mathcal{F}(z,t) = -\frac{\sin\vartheta_+(t)}{\cos\vartheta_+(t-z/v_+)}.$$

We now assume that

$$\cos\alpha_- = \frac{1}{\sqrt{2}}, \quad \sin\alpha_- = -\frac{1}{\sqrt{2}}. \tag{30}$$

Then

$$\hat{\phi}_3^{(k)} = \mathcal{F}(z,t)\hat{a}_+^{(k)},$$

where

$$\hat{a}_\pm^{(k)} = \frac{\hat{a}_1^{(k)} \pm \hat{a}_2^{(k)}}{\sqrt{2}}.$$

Thus, if a pair of incident quantized fields is generated by the down-conversion process and therefore contains equal numbers of the 1 and 2 photons,

$$\sum_{n=0}^{\infty}\sum_{k}\mathcal{C}_n\left(\hat{a}_1^{(k)\,\dagger}\hat{a}_2^{(k)\,\dagger}\right)^n = \\ = \sum_{n=0}^{\infty}\sum_{k}\mathcal{C}_n\left(\frac{1}{2}\,\hat{a}_+^{(k)\,\dagger}\hat{a}_+^{(k)\,\dagger}-\frac{1}{2}\,\hat{a}_-^{(k)\,\dagger}\hat{a}_-^{(k)\,\dagger}\right)^n,$$

then stopping the $\hat{\Psi}_+$ polariton makes the number of atoms in the state $|3\rangle$ even.

To summarize, we have considered adiabatic propagation of a pair of quantized light pulses in an atomic medium with the tripod level configuration, under the conditions of the electromagnetically induced transparency. We have identified the fast propagating and slowly propagating polariton modes of the system, and showed that the latter can be used for a state mapping between the electromagnetic field produced in a nondegenerate parametric down-conversion process and the coherently prepared atomic ensemble. A remarkable consequence of the large difference between the group velocity of the two polaritons is the possibility to create atomic subensembles with the definite (even) parity of the particle numbers.

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