Quadratic effective characteristics of transport in two-component materials. Computer simulation of a three-dimensional disordered lattice

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Averaged quadratic characteristics of electric transport in a disordered, two-component simple cubic lattice have been calculated in the context of the conductivity problem. Average squares of electric field strengths in each component have been found, thus we could derive not only the effective conductivity, but also its derivative with respect to one argument within the same accuracy. We have studied longitudinal and transverse quadratic characteristics as well as a more general bilinear form of electric field, which has allowed us to tabulate several functions of two parameters which determine the Hall coefficient and magnetoresistance in a weak magnetic field. We have demonstrated that calculations of average quadratic characteristics around the metal–dielectric transition yield detailed information about the critical behavior of kinetic parameters and permit a comprehensive verification of the similarity hypothesis.

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1. INTRODUCTION

The conductivity of disordered two-component lattices has been studied using computer simulations by many authors (see Refs. 1–4 and references therein). In standard computer simulations, only the effective conductivity \( \sigma_\text{eff} = \sigma_1 f(\rho, \lambda) \) is usually calculated, where \( \rho \) is the concentration of the first component and \( \lambda = \sigma_\text{eff} / \sigma_1 \) is the ratio of the conductivities of the components. In this case, attention is usually focused on \( \sigma_1 \) (or the function \( f \), which is the same) around the metal–dielectric transition and the respective critical exponents. Nonetheless, not only the function \( f \) (throughout the domain of its arguments), but its derivative \( f' = \partial f / \partial \lambda \) is also important for studies of transport in two-component materials, namely for calculations of low-frequency dispersion of conductivity,\(^5\) thermoelectric power,\(^6\) and galvanomagnetic effects in low magnetic fields.\(^7\) Other parameters, which are usually termed as effective quadratic characteristics,\(^8\) must be also calculated.

This has led us to a conclusion that even in studies of conductivity the set of calculated parameters must be extended to include quadratic characteristics in addition to \( \sigma_\text{eff} \). In the two-dimensional configuration, a generalized computer simulation was performed on a disordered quadratic lattice.\(^9\) This simulation not only reproduced and improved the results by other authors, but also yielded new data. For example, not only the function \( f \) and its derivative \( f' \) were tabulated, but also the other functions included in the expressions for the Hall coefficient and magnetoresistance. The behavior of all these parameters was also studied in the critical region.

This paper describes a computer simulation on a three-dimensional, two-component lattice with randomly distributed bonds between lattice sites characterized by the conductivities \( \sigma_1 \) and \( \sigma_2 \). The simulation has been performed to comprehensively study the lattice’s electric characteristics. Given a specific configuration of bonds of two sorts, a system of Kirchhoff’s equations was solved on a computer, and potentials \( V_\text{eff} \) were calculated at all lattice sites. They were used to calculate various linear and quadratic effective characteristics, specifically the function \( f \) and its derivative \( f' \). Besides, we have calculated and tabulated (in a graphic form) the longitudinal and transverse functions \( \phi_\text{L} \) and \( \phi_\text{T} \), included in the expression for the magnetoresistance.\(^5\) We solved concurrently a similar problem of the conductivity in an auxiliary lattice, which was a generalization of the two-dimensional dual lattice\(^10\) to the three-dimensional case. This has allowed us to calculate the function \( \psi \) in the expression for the effective Hall coefficient.\(^11\) The behavior of each function has been studied around the metal–dielectric transition, and not only their critical exponents, but also the coefficients of the respective expansions have been determined.

The calculations have been performed on a Convex C210 vector computer by simulating a 50X50X50 lattice at six values of the parameter \( \lambda \), namely \( \lambda = 10^{-m} \), where \( m = 1, \ldots, 6 \). The bond problem of the simple cubic lattice has been studied, in this case the critical concentration (percolation threshold) is \( p_c = 0.247 \). The effective parameters were determined by averaging over twelve configurations. In the critical region, which is more difficult for calculation and...
FIG. 1. Basic and auxiliary (dashed lines) lattices.

where the spread of parameters in different configurations is larger, the data were averaged over twenty configurations.

2. ORGANIZATION OF THE COMPUTER SIMULATION

The problem of the conductivity in an inhomogeneous lattice has been formulated in the conventional manner.\textsuperscript{1,10} We have considered a simple cubic lattice that was generated,\textsuperscript{5} where the spread of parameters in different configurations is larger, the data were averaged over twenty configurations.

We have considered a simple cubic lattice that was generated,\textsuperscript{5} where the spread of parameters in different configurations is larger, the data were averaged over twenty configurations.

For each configuration, the calculations were performed by six values of the parameter $h$: $h = 10^{-m}$, where $m = 1, \ldots, 6$. The potentials calculated at a fixed $m$ were used as initial values in the calculation at $m + 1$.

In order to check the calculations and estimate their accuracy, we calculated at all stages the total current across each section of the lattice, i.e., the total current in the links perpendicular to the $x$-axis connecting the planes $j$ and $j + 1$, where $j$ is a number ranging between 1 and $N - 1$. The iteration process was interrupted when the spread of the integral current in the cross sections was within 0.1%. The resulting potentials $V_j$ were used to calculate linear and quadratic parameters of the lattice. In order to reduce the effect of fluctuations in the distribution of defect bonds owing to the finite lattice dimensions, we performed calculations for several random configurations and averaged the characteristics. The uncertainty in the effective parameters was estimated as the rms deviation from the average. The
calculations of the effective characteristics are given in Figs. 2-6. Outside the critical region, the averaging was performed over twenty configurations, and the spread was within 1%. In studying the critical behavior of the parameters (around the percolation threshold at $p_c=0.247$), which takes place in the concentration range $0.25-0.3$, we detected large fluctuations of parameters calculated in different configurations (at $h=10^{-3}$, and especially at $h=10^{-5}$). Therefore, the number of configurations in the critical region was increased to twenty, but even under these conditions the spread of the effective parameters was up to 15–25%.

2. LINEAR AND QUADRATIC CHARACTERISTICS

1. Let the average electric field be directed along the $x$-axis. Then the effective conductivity $\sigma_\varepsilon$ of a material to which Ohm's law applies, $j=\sigma(r)E$, is defined conventionally:

$$\sigma_\varepsilon=(j_x)(E_x).$$

Here $j$ is the current density, $E$ is the electric field strength, and $(\ldots)$ denotes averaging over the sample volume $V$:

$$(\ldots) = \frac{1}{V} \int (\ldots) \, dr. \quad (4)$$

In this definition $V\to\infty$. The parameters $(j_x)$ and $(E_x)$ can be expressed in terms of the total current $I$ and potential difference $U$. As a result, for a cubic sample with an edge length $L$ we have

$$\sigma_\varepsilon = L^{-1} I / U, \quad (5)$$

where $L=N-1$ if the bond length equals unity.

In a two-component lattice, the conductivity $\sigma(r)$ may have two constant values $\sigma_1$ and $\sigma_2$ in the first and second components, respectively. The effective conductivity $\sigma_\varepsilon = \sigma_\varepsilon(p;\sigma_1,\sigma_2)$ of this system can be expressed as

$$\sigma_\varepsilon = \sigma(p;\sigma_1,\sigma_2), \quad (6)$$

where $p$ is the concentration (volume fraction) of the first component. The function $f$—the dimensionless effective conductivity—is a fundamental parameter in the theory of transport in two-component media, and its calculation is the main goal of the computer simulation.

The parameter $\sigma_\varepsilon$ is a response function to an external field characterizing the medium as a whole. One may introduce "partial" response functions $\xi_i$ ($i=1,\ldots,n$) for an $n$-component material ($n \geq 2$):

$$(E^{(i)})^{(1)} = \xi_i(E), \quad (7)$$

where

$$(\ldots)^{(i)} = \frac{1}{V_{i}} \int (\ldots) \, dr \quad (8)$$

means averaging over the volume $V_{i}$ of the $i$-th component. The parameters $\xi_i$ are also effective characteristics of a medium when $V\to\infty$ and $V_{i}\to\infty$.

Since

$$\sum_i (E^{(i)})^{(1)} = (E), \quad (9)$$

the parameters $\xi_i$ should obey the sum rule:

$$\sum_i \xi_i = 1. \quad (10)$$

The average current density $(\bar{j})$ is expressed in terms of $\xi_i$ as follows:

$$(\bar{j}) = \sum_i \sigma_i(E)^{(1)} \sum_i \xi_i(E), \quad (11)$$

hence

$$\sigma_\varepsilon = \sum_i \sigma_i \xi_i, \quad (12)$$

where $\sigma_i$ is the conductivity of the $i$-th component. In the case of a two-component system ($i=1,2$), the parameters $\xi_1$ and $\xi_2$ can be expressed in terms of $\sigma_\varepsilon$. We derive from Eqs. (10) and (12)

$$\xi_1 = \frac{\sigma_2 - \sigma_\varepsilon}{\sigma_2 - \sigma_1}, \quad \xi_2 = \frac{\sigma_\varepsilon - \sigma_1}{\sigma_2 - \sigma_1}, \quad (13)$$

which are identical to the expressions given in Ref. 8. We define $\sigma_\varepsilon$ and $\xi_i$ as linear effective characteristics.

2. Various quadratic effective characteristics are also specific response functions. They include the functions $\phi_i$ which determine electric field strength squared averaged over the volume of the $i$-th component:

$$(E^{(i)})^{(2)} = \phi_i(E_i), \quad (14)$$

It is convenient to introduce the dimensionless electric field strength in the medium:

$$e(r) = \frac{E(r)}{E}, \quad (15)$$

Then the functions $\phi_i$ are expressed in terms of $e(r)$:

$$\phi_i = \phi_i(e), \quad (16)$$

In a two-component system, the functions $\phi_i$ are expressed in terms of the dimensionless effective conductivity $f$:

$$\phi_1 = (e^{(1)})_1 = f - h f', \quad (17)$$

$$\phi_2 = (e^{(2)})_2 = f' \frac{\partial f(p,h)}{\partial h}. \quad (18)$$

According to Eq. (18), the derivative $f'$ can be numerically calculated without differentiating $f$. The function $f$ can be expressed in terms of the averaged parameters squared using Eqs. (17) and (18):

$$f = \phi_1 + h \phi_2 = (e^{(1)})_1 + h (e^{(2)})_2, \quad (19)$$

Equation (19) also derives from the well-known identity $(E) = (j)(E)$.

It is known that in a random inhomogeneous medium the simultaneous change of variables $\sigma_1 \rightarrow \sigma_2$ and $p \rightarrow 1-p$ does not change the macroscopic properties of the material hence $\sigma_\varepsilon(p;\sigma_1,\sigma_2) = \sigma_\varepsilon(1-p;\sigma_2,\sigma_1)$ and the following relation applies to the function $f$ defined by Eq. (6):

$$f(p,h) = h f(l - p; l/h) \to \infty. \quad (20)$$

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Similarly, we have the following relations for the functions $\phi_i$:

$$\phi_1(p, h) = \phi_0(1 - p, 1/h), \quad \phi_2(p, h) = \phi_1(1 - p, 1/h).$$  

(21)

The validity of Eq. (21) is proved by substituting Eq. (20) into Eqs. (17) and (18).

In a calculation of lattice parameters, the electric field strength $E_j(x) = E_j(j, k, l)$ is defined as a difference between potentials at neighboring sites (along the $x$-axis):

$$E_j(j, k, l) = V_{j,k,l} - V_{j+1,k,l}.$$  

(22)

Therefore, we can express the total current as

$$I = \sum_{j=1}^{N-1} \sum_{k=1}^{N-1} \sigma(j, k, j+1, k, l)(V_{j,k,l} - V_{j+1,k,l}).$$  

(23)

where $j$ is an arbitrary number ranging between 1 and $N-1$. In the problem statement given in the previous section ($\sigma = 1$, $U = 1$), the function $f$ is expressed in terms of the total current as $f = I/(N-1)$, where $f$ is defined by Eq. (23). In the case of a discrete lattice, the functions $\phi_i$ are defined by the equation

$$\phi_i = \frac{1}{L} \sum_{\Omega} (V_{j,k,l} - V_{j+1,k,l})^2 + \frac{1}{L} \sum_{\Omega} (V_{j,k,l} - V_{j,k+1,l})^2$$

$$+ \frac{1}{L} \sum_{\Omega} (V_{j,k,l} - V_{j,k,l+1})^2,$$

(24)

where the sums are taken over all bonds with the conductivity $\sigma_i$ and $L = N - 1$.

In this work we calculate numerically the function $f(p, h)$ using both Eq. (5) and Eq. (19). The two results coincide within the calculation uncertainty. Figure 2a shows the common logarithm of the dimensionless effective conductivity versus concentration $p$ at six values of the second argument $h$. Figure 2b shows the parameters $\psi_1$ and $\psi_2$ as functions of $p$ at fixed $h$. Although the functions $f$, $\phi_1$, and $\phi_2$ have been calculated only at $h < 1$, they can be easily extrapolated to $h > 1$ using Eqs. (20) and (21). Note that at all stages of the calculations Eq. (13) was used as an additional criterion of correctness of results.

4. FUNCTIONS $f$ AND $\phi_i$ IN THE CRITICAL REGION

In a system undergoing the metal–dielectric phase transition, the function $f$ can be expanded, according to the similarity hypothesis, in the critical region ($h < 1$, $|\tau| < 1$, where $\tau = (p - p_c)/p_c$, $p_c$ is the critical concentration) as follows:  

$$f = \tau \left[A_0 + A_1 \frac{h}{\tau} + A_2 \left(\frac{h}{\tau}\right)^2 + \ldots\right], \quad \tau > 0,$$

$$\Delta_0 < \tau < 1,$$

(25a)

$$f = h \left[B_0 + B_1 \frac{\tau}{h} + B_2 \left(\frac{\tau}{h}\right)^2 + \ldots\right], \quad |\tau| < \Delta_0,$$

(25b)

$$f = \left(\frac{h}{\tau}\right)^{1/2} [C_1 + C_2 \ln \left(\frac{h}{\tau}\right) + \ldots], \quad \tau < 0,$$

$$\Delta_0 < |\tau| < 1,$$

(25c)

FIG. 2. (a) Common logarithm of the dimensionless effective conductivity, (b) $\phi_1 = (\psi_1^{(1)})^{1/2} = f - \phi_2$, and (c) $\phi_2 = (\psi_2^{(2)})^{1/2}$ as functions of the concentration $p$ at several values of the parameter $h$: (1) $h = 10^{-1}$; (2) $h = 10^{-2}$; (3) $h = 10^{-3}$; (4) $h = 10^{-4}$; (5) $h = 10^{-5}$; (6) $h = 10^{-6}$. The critical concentration $p_c = 0.247$.  

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where
\[ q = j - t, \]
where
\[ \Delta_0 = \frac{h}{\tau^{1/2}} \]
is the transition width. The critical exponents \( t, s, \) and \( q \) are related by Eq. (26). The numerical factors \( A_k, a_k, \) and \( B_k \) are equal to unity within one order of magnitude, and it is obvious that \( A_0, a_0, \) and \( B \) are positive. As was proved in Ref. 5, \( A_1 > 0 \) and \( B < 0. \) One can easily prove that \( a_1 > 0. \)

The following expansion of the function \( \phi_1 \) can be derived from Eqs. (17) and (25):
\[ \psi_1 = \tau \left[ A_0 - A_2 \left( \frac{h}{\tau^{1/2}} \right)^2 + \cdots \right], \quad \tau > 0, \quad \Delta_0 \ll \tau \ll 1, \quad (28a) \]
Note that, unlike Eq. (25a), the correction in Eq. (28a) is quadratic in the small parameter \( h/\tau^{1/2}. \) In deriving Eq. (28c) we have taken into account Eq. (26).
The expansion of the function \( \phi_2 \) is derived from Eqs. (18) and (25):
\[ \psi_2 = \frac{1}{\tau^2} \left[ A_1 + 2A_2 \left( \frac{h}{\tau^{1/2}} \right)^2 + \cdots \right], \quad \tau > 0, \quad \Delta_0 \ll \tau \ll 1, \quad (29a) \]
FIG. 5. Function $\psi$ calculated by Eq. (41).

$$\psi_z = \frac{1}{K^{1/2}} \left[ z_0 \phi_z + \frac{t-1}{t} \frac{\tau}{z_0^{3/2}} \right]^{1/2}, \quad |\tau|<\Delta_0,$$

(29b)

$$\psi_z = \frac{1}{(1-\tau)^2} \left[ 1 + 2B_1 \frac{h}{(1-\tau)^2} + \ldots \right], \quad \tau<0, \quad \Delta_0<|\tau|<1,$$

(29c)

$$q^t = \frac{1}{2} t,$$

(30)

According to Eq. (29), $\psi_2$ as a function of the concentration has a sharp peak around $p=p_c$ if $h<1$ (Fig. 2c). If Eqs. (26) and (30) are valid, $q^t=q$, i.e., the critical exponents of $\psi_2$ at $r>0$ and $r<0$ coincide provided that the similarity hypothesis holds. As was noted above, the coefficient $a_1$ is positive, therefore the maximum of $\psi_2$ plotted against $p$ is centered at $p>p_c$ (Fig. 2c) and is shifted with respect to the critical point $p_c$, by the value $-h^{1/4}$.

Since the functions $\phi_1$ and $\phi_2$ are positive by definition, it follows from Eqs. (17) and (18) that $f>f^*$. After substituting this into Eq. (25), we obtain the condition $s<1$ and the inequalities $A_1>0$ and $B_2<0$ derived in Ref. 5 from physical considerations.

In processing the results of computer simulation at $p>p_c$ (outside the transition region), it is more convenient to derive the exponent $t$ not from the function $f$, but $\phi_2$ because its correction is smaller than that of $f$ owing to the smallness of $h$ [compare Eq. (25a) with Eq. (28a)]. The factor $A_2$ was determined simultaneously with $t$. Then the exponent $q$, as well as the factors $A_1$ and $B_1$, can be derived by processing the calculations of $\psi_2$ using Eqs. (29a) and (29c) (the equality $q^t=q$ can be also checked). Finally, the critical exponent $s$ and the factor $a_2$ can be calculated by three methods, namely, from $f$, $\phi_1$, and $\phi_2$ as functions of $h$ at $p=p_c$. The parameters of the critical region derived from computer simulations are listed in Table I.

5. LONGITUDINAL AND TRANSVERSE QUADRATIC CHARACTERISTICS

Beside the functions $\phi_1$, it is also interesting to investigate the functions $\phi_2 = (\phi_2)^{(i)}$, $\phi_2 = (\phi_2)^{(j)}$ ($i=1, 2$),

(31)

where $\phi_i$ and $\phi_j$ are the components of the vector $e$ defined by Eq. (15) and oriented parallel and perpendicular to the average electric field $\langle E \rangle$, where $\langle \ldots \rangle^{(i)}$ has the same sense as in Eq. (8). The functions in Eq. (31) are related to $\phi_1$ in Eq. (10) by an obvious equation

$$\phi_i = \phi_1 + \phi_2.$$

(32)

In the case of a discrete lattice (problem statement in Sec. 2), the function $\phi_2$ is defined by the first sum in Eq. (24), and $\phi_1$ by the other two sums.

In the case of a random inhomogeneous medium, we have relations similar to Eq. (21) (see, for example, Sec. 7):

$$\phi_i(p, h) = \phi_i(1-p, 1/h), \quad \phi_i(p, h) = \phi_i(1-p, 1/h),$$

(33)

and two more relations derived from Eq. (33) by exchanging the indices 1 and 2. Equation (33) allows us to determine the functions $\phi_2(p, h)$ and $\phi_2(p, h)$ at $h>1$ if their values at $h<1$ are known.

No relations like Eqs. (17) and (18) are known for the functions $\phi_2$ and $\phi_2$, but their basic properties can be derived from Eqs. (31) and (32). The shape of $\phi_2$ (for $h<1$) can be seen in Fig. 3a, which shows $\phi_2$ as a function of concentration $p$ at two values of the other argument, $h=10^{-2}$ and $h=10^{-4}$. Note that the shape of the longitudinal function $\phi_2$ is almost identical to that of $\phi_1$, as well as its amplitude. The shape of the transverse function $\phi_1$ (Fig. 3b) is similar to those of $\phi_1$ and $\phi_1$ at $p=0.4$, but is very different in the interval $0.4<p<1$ and turns to zero at
TABLE I. Summary table of parameters in the critical region.

<table>
<thead>
<tr>
<th>$p=p_c$</th>
<th>$p&gt;p_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_1$</td>
<td>$\phi_1$</td>
</tr>
<tr>
<td>$t_1=2.1\pm0.2$</td>
<td>$t_1=1.9\pm0.2$</td>
</tr>
<tr>
<td>$A_{11}=0.4\pm0.2$</td>
<td>$A_{11}=0.2\pm0.1$</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>$\phi_2$</td>
</tr>
<tr>
<td>$\theta=0.8\pm0.2$</td>
<td>$\theta=0.7\pm0.1$</td>
</tr>
<tr>
<td>$\Delta_{21}=3.0\pm0.9$</td>
<td>$\Delta_{21}=1.5\pm0.4$</td>
</tr>
<tr>
<td>$\psi$</td>
<td>$\psi$</td>
</tr>
<tr>
<td>$\ell=3.7\pm0.4$</td>
<td>$g=0.3\pm0.1$</td>
</tr>
<tr>
<td>$C=0.5\pm0.3$</td>
<td>$W=1.7\pm0.3$</td>
</tr>
<tr>
<td>$p=p_c$</td>
<td>$p&gt;p_c$</td>
</tr>
<tr>
<td>$\phi_3$</td>
<td>$\phi_3$</td>
</tr>
<tr>
<td>$\psi=0.7\pm0.1$</td>
<td>$\psi=0.8\pm0.1$</td>
</tr>
<tr>
<td>$\Delta_{33}=0.3\pm0.2$</td>
<td>$\Delta_{33}=0.2\pm0.1$</td>
</tr>
<tr>
<td>$\theta_3$</td>
<td>$\theta_3$</td>
</tr>
<tr>
<td>$\Delta_{31}=0.3\pm0.9$</td>
<td>$\Delta_{31}=1.5\pm0.4$</td>
</tr>
<tr>
<td>$\omega$</td>
<td>$\omega$</td>
</tr>
<tr>
<td>$\ell=3.7\pm0.4$</td>
<td>$g=0.3\pm0.1$</td>
</tr>
<tr>
<td>$C=0.5\pm0.3$</td>
<td>$W=1.7\pm0.3$</td>
</tr>
</tbody>
</table>

$p=1$. The drop of this function to zero at $p=1$ is quite natural because in a homogeneous medium the transverse component of the electric field should be zero. Finally, the shapes of the functions $\phi_{21}$ and $\phi_{22}$, which are similar to that of $\phi_2$, are shown in Fig. 4.

The critical behavior of the functions $\phi_{21}$ in system undergoing the metal–dielectric transition can be described similarly to Eq. (28) (only the expansion terms of the lowest order are given):

$$\phi_{21}=A_{21}^1\tau^{x_{21}}, \quad \tau>0, \quad \Delta_{21}=\ell^{1/2}.$$ (34a)

Here $\Delta_{21}=h^{1/10}$ is the peak width of the function $\phi_2$.

According to the similarity hypothesis, all critical phenomena at zero magnetic field are characterized by one scale, therefore the value of $\Delta_{21}$ should be comparable to $\Delta_1$ in Eq. (27) (compare to the discussion in Ref. 8). Hence, follows the second relations among the introduced critical exponents:

$$s_{11}/s_{11}=s/s.$$ (36)

Since both $\phi_{21}$ and $\phi_{22}$ are positive, it follows from Eq. (42) that $\phi_{11}$ cannot decay more slowly or increase faster than $\phi_1$. From this condition we derive the limitations on the respective exponents:

$$1>l_{11}, \quad s_{11}>s, \quad 2g_{11}+s_{11}<2g+s.$$ (37)

The critical behavior of the function $\phi_{21}$ is described identically. The same limitations and relations apply to $\phi_{31}$ as to $\phi_{11}$, only the label $I$ is substituted by $r$.

The behavior of the function $\phi_{31}$ in the critical region can be described similarly to Eq. (29) (here we also give only the expansion terms of the lowest order):

$$\phi_{31}=A_{31}^1\tau^{x_{31}}, \quad \tau>0, \quad \Delta_{31}=\ell^{1/2}.$$ (37a)

$$\phi_{32}=A_{32}^1\tau^{x_{32}}, \quad \tau<0, \quad \Delta_{32}=\ell^{1/2}.$$ (37b)

$$\phi_{32}=B_{32}^1\tau^{x_{32}}, \quad \tau<0, \quad \Delta_{32}=\ell^{1/2}.$$ (37c)

$$\lambda_{21}/\lambda_{22}=s/s.$$ (38)

Equation (38) derives from the condition $\Delta_{21}=\Delta_{22}$, where $\Delta_{21}=h^{1/10}$ is the peak width of the function $\phi_{21}$. Since the singularity of $\phi_{21}$ cannot be of a higher order than that of $\phi_2$, the following conditions should be satisfied:

$$q_{11}<q_{21}, \quad \lambda_{21}<\lambda_{22}$$ (39).

In the critical region, the same relations and conditions apply to $\phi_{22}$ as to $\phi_{21}$, only the label $l$ is substituted by $r$.

Thus only one new critical exponent is needed to describe the critical behavior of the functions $\phi_{11}, \phi_{12}, \phi_{21}, \phi_{22},$ and $\phi_{31},$ and the complete set of critical exponents can be composed of $\lambda_{11}, \lambda_{12}, \lambda_{21},$ and $\lambda_{22}$. The rest of the critical exponents can be expressed as

$$\lambda_{12}/\lambda_{11}=s/s.$$ (40)

The parameters derived by processing computer simulations of the functions $\phi_2$ and $\phi_3$ in the critical region are listed in Table I.
6. HALL COEFFICIENT

The conductivity of an isotropic material in the magnetic field \( H \) is described by the tensor
\[
\sigma = \begin{pmatrix}
\sigma_x & \sigma_y & 0 \\
-\sigma_y & \sigma_x & 0 \\
0 & 0 & \sigma_z
\end{pmatrix},
\]
in which we use the notations of Ref. 8. In a weak magnetic field \( (H \approx 0) \) the Hall component \( \sigma_z \) is linear in \( H \). In this approximation the Hall component of the effective conductivity tensor can be expressed as
\[
\sigma_{\text{eff}} = \sigma_z + (\sigma_x - \sigma_y) \varphi(p, H),
\]
where the galvanomagnetic parameters of the components are given in the explicit form. The function \( \varphi(p, H) \) in Eq. (40) is determined by the medium properties at \( H=0 \).

\[
\varphi(p, H) + \varphi(1-p, 1/H) = 1,
\]
hence the function \( \varphi(p, H) \) can be determined at \( h>1 \) if it is known at \( h<1 \).

Let us denote the electric field in the medium determined in solving the conductivity problem at \( H=0 \) for a given electric field \( \mathbf{E}(\mathbf{r}) \) as \( \mathbf{E}_0(\mathbf{r}) \), where the superscript \( v \) denotes that the average field is aligned with the axis \( v \). The function \( \varphi(p, H) \) can be expressed in terms of the electric field \( \mathbf{E}_0(\mathbf{r}) \) as follows:
\[
\varphi(p, H) = \frac{(E_0^x)^2 - (E_0^y)^2}{E_0^z}.
\]
In this paper \( E_0^x(\mathbf{r}) \) in Eq. (41) is the field in the basic lattice, and \( E_0^x(\mathbf{r}) \) is the field calculated in the auxiliary lattice (see Section 2). The field strength \( E_0^x(\mathbf{r}) \) and \( E_0^y(\mathbf{r}) \) are expressed in terms of the potentials \( V_x \) and \( V_y \) by equations like Eq. (22). The result of the computer simulation is shown in Fig. 5, where \( \varphi(p, H) \) is plotted against concentration \( p \) at two values of \( h \).

7. MAGNETORESISTANCE

Under a weak magnetic field \( (H \approx 0) \), the corrections to the diagonal components of the conductivity tensor, \( \sigma_{\text{eff}} \), are quadratic in \( H \):
\[
\begin{align*}
\sigma_{\text{eff}} &= \sigma_{\text{eff}} + a_{\text{eff}} H^2, \\
\end{align*}
\]
where \( \sigma_{\text{eff}} \) is the i-th component of conductivity at \( H=0 \). Let us express the effective parameters \( \sigma_{\text{eff}} \) and \( \sigma_{\text{eff}} \) in the form similar to Eq. (47):
\[
\begin{align*}
\sigma_{\text{eff}} &= \sigma_{\text{eff}} + \sigma_{\text{eff}} + \sigma_{\text{eff}} + \sigma_{\text{eff}}, \\
\end{align*}
\]
where \( \sigma_{\text{eff}} \) is the same as in Eq. (6). For a two-component system in the approximation quadratic in \( H \), we have the following expressions for \( \sigma_{\text{eff}} \) and \( \sigma_{\text{eff}} \):
\[
\begin{align*}
\sigma_{\text{eff}} &= \gamma_x \phi_x(\mathbf{r}) + \gamma_y \phi_y(\mathbf{r}) + \gamma_z \phi_z(\mathbf{r}), \\
\sigma_{\text{eff}} &= \gamma_x \phi_x(\mathbf{r}) + \gamma_y \phi_y(\mathbf{r}) + \gamma_z \phi_z(\mathbf{r}).
\end{align*}
\]

In a system undergoing the metal–dielectric transition, the functions \( \varphi \) and \( \varphi \) are the critical region \( |\mathbf{r}| < 1, h < 1 \) are given by the following expressions derived from the similarity hypothesis:
\[
\begin{align*}
\varphi &= C r^4, \quad \varphi = D r^2, \\
\varphi &= C r^4, \quad \varphi = D r^2,
\end{align*}
\]
in order to characterize the function \( \varphi \), we have introduced a new critical exponent labeled by \( f \) in addition to those of the function \( f \). The other two exponents can be expressed in terms of \( f, t, \) and \( q \):
The properties of the functions $\phi_1$ and $\phi_2$ (in particular, their critical behavior) were discussed in detail in Sec. 5. The results of Sec. 5 can be directly applied to the functions $\phi^{(1)}$ and $\phi^{(2)}$, taking into account Eqs. (51) and (52).

The functions $X_1$ and $X_2$ are expressed in terms of the electric field strength at $H=0$ in the medium in a more complex form; therefore their determination may be a subject of a dedicated study. In this paper we only note that the shapes of the curves of $X_1$ and $X_2$ versus $p$ (at fixed $h$) are similar to those of $\phi_1$, although each new function demands a new critical exponent.

In a random inhomogeneous medium the simultaneous substitutions $p \rightarrow 1-p$ and $\delta_1 \rightarrow \delta_2$ do not change its macroscopic properties:

$$\delta_1(p;\delta_1,\delta_2) = \delta_2(1-p;\delta_1,\delta_1).$$

Hence, follow Eq. (20) in the approximation of zeroth order in $H$, the expression for $\varphi$ from the previous section in the linear approximation, and Eqs. (53) and the relations for $X_1$ and $X_2$ in the quadratic approximation with due account of Eqs. (47)–(52):

$$X_1(p,h) = \frac{1}{K} X_1(1-p,1), \quad X_2(p,h) = \frac{1}{K} X_2(1-p,1).$$

Using these relations, the functions $X_1(p,h)$ and $X_2(p,h)$ can be calculated at $h>1$ if they are known at $h<1$.

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