Resonant Coulomb excitation of atomic nuclei propagating through a crystal in the channeling mode

A. V. Stepanov

Institute for Nuclear Research, Russian Academy of Sciences, 117312 Moscow, Russia
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The Coulomb-excitation total cross section and the distribution of decay products originating from a resonant state of a nucleus interacting with a crystal lattice has been calculated for the case of a single inelastic collision (with respect to internal degrees of freedom in a nucleus). These observables have been expressed in terms of time-dependent correlators which describe thermal oscillations of lattice nuclei and the motion of the center of mass of a nucleus propagating across a crystal target in the channelling mode. An expression generalizing the spectrum of equivalent photons calculated by the Weizsäcker–Williams method is given.
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1. INTRODUCTION

In order to describe the Coulomb excitation and fragmentation of atomic nuclei, and creation of particles in the strong Coulomb field of colliding heavy ions, the method of equivalent photons is widely used, in which the total cross section $\sigma_t(E)$ of the reaction is presented in a concise and physically explicit form:

$$\sigma_t(E)=\int d\omega \frac{dN_\gamma(\omega,E)}{d\omega}\sigma_\gamma(\omega). \quad (1)$$

Here $\sigma_\gamma(\omega)$ is the absorption cross section of a real photon with the energy $\hbar\omega$, and $dN_\gamma(\omega,E)/d\omega$ is the spectrum of equivalent photons generated by electromagnetic fields due to nuclei colliding with energy E.[1–4] A detailed investigation of this method, however, demonstrated that its domain of applicability has well defined boundaries. One of its limitations, for example, is the assumption that in the center-of-mass reference frame the colliding nuclei move along a straight-line trajectory, which allows one to save calculation time. The method of equivalent photons should be also modified to analyze the interaction between nuclear beams and lattice nuclei.[5] In this case the efficiency of coherent excitation in a propagating nucleus strongly depends on the synchronization of the electromagnetic fields due to thermally oscillating lattice nuclei.[5] The effect of the thermal motion on the averaged potential, which defines trajectories of propagating nuclei in the lattice, was thoroughly investigated by Kagan and Kononets, who studied the effect of channelling.[6] But their model does not fully take account the effect of lattice thermal oscillations in the case of Coulomb resonant excitation of nuclei. The direct unification of the results by Okorokov[5] and by Kagan and Kononets,[6] and the analogy with the Mössbauer effect[7,8] must be justified. A consistent approach to the problem can be formulated in terms of the two-potential model,[9] in which one component of the potential affects the wave incident on the target and the other excites internal degrees of freedom of the colliding nuclei. This paper gives quantum-mechanical expressions for the total cross section for resonant Coulomb excitation and the distributions of fragmentation products of nuclei propagating across a crystal in a resonant state in the approximation of a single inelastic collision (with respect to internal nuclear degrees of freedom). By comparing the formula for the total cross section of the Coulomb interaction with Eq. (1), we can derive a generalized spectrum for the equivalent photons. This result is useful since it can be applied to a variety of problems ranging from the Coulomb dissociation of neutron-rich nuclei, such as ^{11}Li,[10] to the efficiency of the Coulomb excitation of nuclei with a view to designing a γ-laser.[11,12] Section 2 gives general formulas needed for the solution of the problem. Section 3 describes the calculation of the total cross section for the Coulomb excitation of nuclei propagating across a crystal lattice. Section 4 deals with expressions for the distributions of decay products of an intermediate resonant state generated in an incident nucleus. The results are discussed and summarized in the Conclusion. Our model is limited to the nonrelativistic approximation with only one relativistic correction, namely, the rest mass of a particle is replaced with its relativistic value.

2. PROBLEM STATEMENT. SOME DEFINITIONS AND GENERAL RELATIONSHIPS

We represent the Hamiltonian of the projectile nucleus (P) plus crystalline target (T) as a sum

$$\hat{H}=\hat{H}_P^{(0)}(\mathbf{R}_P,\{\xi_P\})+\hat{H}_T(\mathbf{R}_T,\{\xi_T\}) + V(\mathbf{R}_P,\{\xi_P\};\{\mathbf{R}_T\},\{\xi_T\}). \quad (2)$$

Here $\hat{H}_P^{(0)}(\mathbf{R}_P,\{\xi_P\})$ is the free-nucleus Hamiltonian, $\mathbf{R}_P$ is the radius-vector of its center of mass, $\{\xi_P\}$ is the set of parameters which describe the internal motion in the projectile nucleus, $\hat{H}_T(\{\mathbf{R}_T\},\{\xi_T\})$ is the crystal-target Hamiltonian, $\{\mathbf{R}_T\}$ are center-of-mass coordinates of lattice nuclei, $\{\xi_T\}$ are parameters which describe internal motion of nuclei and electrons in the lattice, and V is the potential of the interaction between the traveling nucleus P and target T.

First let us restrict our consideration to the processes in which the nuclei and electron shells of the crystal atoms are not excited. In this approximation, lattice nuclei act only as sources of a screened Coulomb field, undergoing thermal motion at a temperature T_0:

$$\hat{H}_T(\{\mathbf{R}_T\},\{\xi\})=E_T^{\rm GS}+\hat{H}_T(\{\mathbf{R}_T\}). \tag{3}$$

Here $E_T^{\rm GS}$ is the sum of the ground-state energies of nuclei and electron shells of the lattice atoms. Assuming that electric charge distributions of interacting nuclei do not overlap, we can separate from the potential energy $V(\mathbf{R}_P,\{\xi_P\};\{\mathbf{R}_T\})$ the monopole component $V^e(\mathbf{R}_P,\{\mathbf{R}_T\})$, which does not affect the internal coordinates $\{\xi_P\}$ of the nucleus P, i.e.,

$$V(\mathbf{R}_P,\{\xi_P\};\{\mathbf{R}_T\})=V^e(\mathbf{R}_P,\{\mathbf{R}_T\})+\hat{H}^{\rm I}_{\rm int}(\mathbf{R}_P,\{\xi_P\};\{\mathbf{R}_T\}). \tag{4}$$

In what follows, we will ignore, unless otherwise stated, the effect of exciting of internal degrees of freedom in the nucleus P on its center-of-mass motion. In this case, the fine structure of the shift and deformation of a resonance line[13] are excluded from our consideration. In this approximation, the center-of-mass motion of the nucleus P across the lattice is described by the Hamiltonian

$$\hat{H}^e_P=\hat{T}_P+V^e(\mathbf{R}_P,\{\mathbf{R}_T\}), \tag{5}$$

where $\hat{T}_P$ is the kinetic energy of the nucleus P.

This complicated problem of the motion of a nucleus in the ground state through a channel in a single crystal was solved by means of the theory of the channelling effect.[6,14–16] For simplicity, we will take the Hamiltonian in Eq. (5) instead of an effective one-particle Hamiltonian

$$\hat{H}^{\rm eff}_P=\hat{T}_P+V_{\rm eff}(\mathbf{R}_P), \tag{6}$$

where $V_{\rm eff}(\mathbf{R}_P)$ is the "optical" potential, which takes into account the effect of thermally oscillating lattice nuclei on the center-of-mass motion of the nucleus P. This Hermitian potential is derived in most simply by averaging $V^e(\mathbf{R}_P,\{\mathbf{R}_T\})$ with respect to the thermal oscillations of lattice nuclei and then over the crystal axes (or planes) of the target.[14–16]

In order to save effort, it is convenient to separate the variables in the term $\hat{H}^{\rm I}_{\rm int}(\mathbf{R}_P,\{\xi_P\},\{\mathbf{R}_T\})$ in Eq. (4). To this end, let us use the Fourier transform. We express the internal-motion Hamiltonian as

$$\begin{aligned}\hat{H}^{\rm I}_{\rm int}&(\mathbf{R}_P,\{\xi_P\},\{\mathbf{R}_P\})\\ &\equiv\hat{H}^{\rm I}_{\rm int}(\mathbf{R}_P,\{\mathbf{r}_j\},\{\mathbf{R}_l\})\\ &=\left[\sum_{j=1}^{Z_P}\sum_{l=1}^{N}\hat{h}(\mathbf{R}_P+\mathbf{r}_j-\mathbf{R}_l)\right]'\\ &=\int\frac{d\mathbf{q}}{(2\pi)^3}\tilde{v}_0(\mathbf{q})\exp(-i\mathbf{q}\mathbf{R}_P)\sum_{l=1}^{N}\exp(i\mathbf{q}\mathbf{R}_l)\\ &\quad\times\left[\sum_{j=1}^{Z_P}(e^{-i\mathbf{q}\mathbf{r}_j}-1)\right].\end{aligned} \tag{7}$$

Here $\mathbf{r}_j$ is the coordinate of the j-th proton in the nucleus P with respect to its center of mass, Z_P is the total number of protons in the nucleus, $\mathbf{R}_l$ is the coordinate of the l-th nucleus of the target lattice, N is the total number of nuclei in the lattice, and

$$\tilde{v}_0(\mathbf{q})=\int d\mathbf{r}e^{i\mathbf{q}\mathbf{r}}\hat{h}(\mathbf{r}).$$

The prime in Eq. (7) means that the monopole component of interaction between the nucleus P and l-th lattice nucleus is omitted. Let us introduce the following notation:

$$\hat{\rho}_T(\mathbf{r})=\sum_{l=1}^{N}\delta(\mathbf{r}-\mathbf{R}_l)$$

is the density operator of the lattice nuclei, and $\hat{\rho}_P(\mathbf{r})=\delta(\mathbf{r}-\mathbf{R}_P)$ is the density operator of the projectile nucleus P. Since

$$\sum_{l=1}^{N}\exp(i\mathbf{q}\mathbf{R}_l)=\int d\mathbf{r}\hat{\rho}(\mathbf{r})\exp(i\mathbf{q}\mathbf{r})\equiv\tilde{\rho}_T(\mathbf{q}),$$

$$\begin{aligned}\exp(-i\mathbf{q}\mathbf{R}_P)=\tilde{\rho}_P(-\mathbf{q}),\quad &\sum_{j=1}^{Z_P}\exp(-i\mathbf{q}\mathbf{r}_j)=4\pi\\ &\times\sum_{L,\mu}(-i)^LY_{L\mu}(\hat{\mathbf{q}})\sum_{j=1}^{Z_P}j_L(qr_j)\\ &\times Y^*_{L\mu}(\theta_j,\varphi_j),\end{aligned}$$

we can derive the following expression for $\hat{H}^{\rm I}_{\rm int}$:

$$\begin{aligned}\hat{H}^{\rm I}_{\rm int}&(\mathbf{R}_P,\{\mathbf{r}_j\};\{\mathbf{R}_l\})\\ &=\int\frac{d\mathbf{q}}{(2\pi)^3}\tilde{v}_0(\mathbf{q})\tilde{\rho}_P(-\mathbf{q})\tilde{\rho}_T(\mathbf{q})\\ &\quad\times\Bigg\{4\pi\sum_{L,\mu}(-i)^LY_{L\mu}(\hat{\mathbf{q}})\\ &\quad\times\sum_{j=1}^{Z_P}j_L(qr_j)Y^*_{L\mu}(\theta_j,\varphi_j)-Z_P\Bigg\},\end{aligned} \tag{8}$$

where $\hat{\mathbf{q}}=\mathbf{q}/q$.

If we use the simplest form of the screened potential in $\hat{h}(\mathbf{r})$:

$$\hat{h}(\mathbf{r})=Z_Te^2\exp(-\beta r)/r, \tag{9}$$

where $\beta^{-1}\approx0.8853a_0(Z_P^{1/2}+Z_T^{1/2})^{-2/3}\approx0.8853a_0(Z_P^{2/3}+Z_T^{2/3})^{-1/2}$ is the screening radius due to electrons, and a_0 is the first Bohr radius, then

$$\tilde{v}_0(q)=4\pi Z_Te^2/(\beta^2+q^2). \tag{10}$$

In the simplest case of the $E1$ electric-dipole transition,

$$\begin{aligned}\hat{H}^{\rm I}_{\rm int}&(\mathbf{R}_P,\{\mathbf{r}_j\},\{\mathbf{R}_l\})\\ &=\int\frac{d\mathbf{q}}{(2\pi)^3}\frac{\tilde{v}_0(\mathbf{q})}{e}\tilde{\rho}_P(-\mathbf{q})\tilde{\rho}_T(\mathbf{q})\\ &\quad\times\frac{4\pi}{3i}q\sum_{\mu=-1}^{1}Y_{1\mu}(\hat{\mathbf{q}})\sum_{j=1}^{Z_P}er_jY^*_{1\mu}(\theta_j,\varphi_j).\end{aligned} \tag{11}$$

3 TOTAL CROSS SECTION OF RESONANT COULOMB EXCITATION

The probability of a two-step transition from the state $|i\rangle$ with energy E_i to the state $|f\rangle$ with energy E_f via the intermediate resonant state $|\lambda\rangle$ with energy E_λ, where the full width of the resonance is $\Gamma_\lambda(E_i)$, is determined by the squared absolute value of the appropriate t-matrix element:[17]

$$\langle f|t|i\rangle=\sum_\lambda \frac{\langle f|\hat{H}^{\mathrm{II}}_{\mathrm{int}}|\lambda\rangle\langle\lambda|\hat{H}^{\mathrm{I}}_{\mathrm{int}}|i\rangle}{E_i-E_\lambda+i\Gamma_\lambda(E_i)/2}. \quad (12)$$

Here $\hat{H}^{\mathrm{I}}_{\mathrm{int}}$ is the Coulomb potential of the interaction between the nucleus P and target lattice nuclei (see the previous section), and $\hat{H}^{\mathrm{II}}_{\mathrm{int}}$ is the operator of the interaction between nucleons of the nucleus P (or between the nucleus P and electromagnetic field), which leads to the transition $|\lambda\rangle\rightarrow|f\rangle$.

According to the optical theorem, the total cross section of the Coulomb excitation is determined by the formula

$$\sigma_t(E_i)=-\frac{2}{\hbar}\left(\frac{v_L}{\Omega}\right)^{-1}\mathrm{Im}\langle i|t|i\rangle, \quad (13)$$

where v_L is the velocity of incident particles in the laboratory reference frame and Ω is the normalizing volume.

From Eqs. (12) and (13) we derive

$$\mathrm{Im}\langle i|t|i\rangle=-\frac{1}{2}\sum_\lambda \Gamma_\lambda(E_i)\frac{|\langle\lambda|\hat{H}^{\mathrm{I}}_{\mathrm{int}}|i\rangle|^2}{(E_i-E_\lambda)^2+\Gamma^2_\lambda(E_i)/4}, \quad (14)$$

$$\sigma_t(E_i)=\frac{\Omega}{v_L}\sum_\lambda \frac{\Gamma_\lambda(E_i)}{\hbar}\frac{|\langle\lambda|\hat{H}^{\mathrm{I}}_{\mathrm{int}}|i\rangle|^2}{(E_i-E_\lambda)^2+\Gamma^2_\lambda(E_i)/4}. \quad (15)$$

We introduce the following notation: E_{m_0} and $|m_0\rangle$ ($E_{m'}$ and $|m'\rangle$) are the energy and wave function of the center-of-mass motion of lattice nuclei in the initial (intermediate) state; ε_a, $|a\rangle$ and ε_B, $|b\rangle$ are similar parameters for the motion of the nucleus P as a whole; and $|s\rangle$ and $|r\rangle$ are the wave functions which describe the internal motion of the nucleus P in the ground and intermediate states. We assume that the nucleus P has only one excited state with an energy $\mathscr{E}_r$. In the case of the $E1$ transition, to which our discussion will be limited, we obtain the following quantity in calculating the matrix element with respect to the wave functions of the internal motion of the nucleus P:

$$M(E1,\mu)=\langle r|d_P Y_{1\mu}(\hat{\mathbf{d}}_P)|s\rangle,$$

where

$$\mathbf{d}_P=\sum_{j=1}^{Z_P} e\mathbf{r}_j$$

is the electric dipole moment of the nucleus P. Assuming that the width $\Gamma(E_i)$ is independent of the center-of-mass motion of the nucleus P, we derive from Eq. (15)

$$\sigma_t(E_i)=\frac{\Omega}{v_L}\frac{\Gamma(E_i)}{\hbar}$$
$$\times\sum_{m',b}\frac{|\langle m',b,r|\hat{H}^{\mathrm{I}}_{\mathrm{int},E1}|s,a,m_0\rangle|^2}{(\mathscr{E}_s+\varepsilon_a+E_{m_0}-\mathscr{E}_r-\varepsilon_b-E_{m'})^2+\Gamma^2(E_i)/4},$$

$$E_i=E_{m_0}+\varepsilon_a+\mathscr{E}_s. \quad (16)$$

We substitute Eq. (11) into Eq. (16) and transform the energy denominator in Eq. (16) in accordance with the formula

$$\left(a^2+\frac{\Gamma^2}{4}\right)^{-1}=\frac{2}{\hbar\Gamma}\mathrm{Re}\int_0^\infty dt\,\exp\left(\frac{iat}{\hbar}-\frac{\Gamma t}{2\hbar}\right).$$

We take the operators in the Heisenberg representation,

$$\hat{A}(t)=\exp\left(\frac{i\hat{H}t}{\hbar}\right)\hat{A}(0)\exp\left(-\frac{i\hat{H}t}{\hbar}\right),$$

and calculate the sum over the entire system of functions $|m'\rangle$ and $|b\rangle$. After averaging the resulting expression over the distribution of initial states $|m_0\rangle$ and $|a\rangle$, we obtain

$$\sigma_t=\frac{8\pi}{3}\frac{\mathscr{B}(E1)}{\hbar^2 v_L}\mathrm{Re}\int_0^\infty dt\,\exp\left(-\frac{\Gamma t}{2\hbar}\right)\exp\left(i\frac{\mathscr{E}_s-\mathscr{E}_r}{\hbar}t\right)$$
$$\times\int\frac{d\mathbf{q}}{(2\pi)^3}\left|\frac{\tilde{v}_0(\mathbf{q})}{e}\right|^2 q^2\tilde{K}_P(\mathbf{q},t)\tilde{K}_T(-\mathbf{q},t). \quad (17)$$

Here $\mathscr{B}(E1)$ is the reduced probability of the $E1$-transition resulting from averaging $M(E1,\mu)M^*(E1,\mu')$ over projections of the spin of the nucleus P,[18] and

$$\tilde{K}_T(\mathbf{q},t)=\left\langle\left\langle\sum_j^N\sum_{j'}^N\exp[i\mathbf{q}\hat{\mathbf{R}}_j(t)]\exp[-i\mathbf{q}\hat{\mathbf{R}}_{j'}(0)]\right\rangle\right\rangle_{T_0}$$

is the Fourier transform of the time-dependent correlator of target nuclear density,

$$\hat{\mathbf{R}}_j(t)=\exp\left(\frac{i\hat{H}_T t}{\hbar}\right)\hat{\mathbf{R}}_j(0)\exp\left(-\frac{iH_T t}{\hbar}\right).$$

In Eq. (17)

$$\tilde{K}_P(\mathbf{q},t)=\langle\langle\exp[i\mathbf{q}\hat{\mathbf{R}}_P(t)]\exp[-i\mathbf{q}\hat{\mathbf{R}}_P(0)]\rangle\rangle_P$$

is the Fourier transform of the time-dependent correlator of the density of the projectile nucleus P. This correlator contains information about the center-of-mass motion of this nucleus in the target:

$$\hat{\mathbf{R}}_P(t)=\exp\left(i\frac{\hat{H}^{\mathrm{eff}}_P t}{\hbar}\right)\hat{\mathbf{R}}_P(0)\exp\left(-\frac{i\hat{H}^{\mathrm{eff}}_P t}{\hbar}\right).$$

The Fourier transform is performed with respect to the spatial coordinates. In deriving Eq. (17), we assumed that the beam of incident nuclei is uniform in space, so it contains a factor $[(2\pi)^3/\Omega]\delta(\mathbf{q}-\mathbf{q}')$ that allows us to integrate with respect to $d\mathbf{q}'/(2\pi)^3$ ... The notation $\langle\langle\ldots\rangle\rangle_{T_0}$ means averaging over states of the target nuclei at the temperature T_0. Similarly, $\langle\langle\ldots\rangle\rangle_P$ means averaging over states of the projectile nucleus P in terms of the center-of-mass motion across the target. The correlator $\tilde{K}_T(\mathbf{q},t)$ was investigated in detail in studies of the inelastic scattering of neutrons in matter and the Mössbauer effect. Generalized expressions for $\tilde{K}_T(\mathbf{q},t)$ for an arbitrary structure of a crystalline target are given in Ref. 19.

Let us consider lattice vibrations in the harmonic approximation and write an analytical expression for the correlation function $\widetilde{K}_T(\mathbf{q},t)$. The position of the j-th atom in the lattice is represented as the sum of vectors $\mathbf{R}_j=\mathbf{n}+\boldsymbol{\rho}_\alpha+\mathbf{u}_{\mathbf{n}\alpha}$, where $\mathbf{n}$ is the crystal-cell vector, $\boldsymbol{\rho}_\alpha$ is the equilibrium position of the nucleus with respect to a certain point in the cell, and $\mathbf{u}_{\mathbf{n}\alpha}$ is the displacement of the nucleus from its equilibrium position. Then[19]

$$\widetilde{K}_T(\mathbf{q},t)=\sum_{\alpha=1}^{w}\sum_{\alpha'=1}^{w}\exp[i\mathbf{q}(\boldsymbol{\rho}_\alpha-\boldsymbol{\rho}_{\alpha'})]\sum_{\mathbf{n}}^{N_1}\sum_{\mathbf{n}'}^{N_1}\exp[i\mathbf{q}(\mathbf{n}-\mathbf{n}')]\chi_{\mathbf{n}\alpha,\mathbf{n}'\alpha'}(\mathbf{q},t). \tag{18}$$

Here w is the number of atoms in the crystal cell, N_1 is the number of elementary cells in the crystal, and

$$\begin{aligned}\chi_{\mathbf{n}\alpha,\mathbf{n}'\alpha'}(\mathbf{q},t)&=\langle\langle\exp[i\mathbf{q}\hat{\mathbf{u}}_{\mathbf{n}\alpha}(t)]\exp[-i\mathbf{q}\hat{\mathbf{u}}_{\mathbf{n}'\alpha'}(0)]\rangle\rangle_{T_0}\\&=\exp[-(W_\alpha+W_{\alpha'})]\exp\left\{\frac{1}{N_1}\sum_\lambda^{3N}\frac{\hbar G(\omega_\lambda)}{2\sqrt{M_\alpha M_{\alpha'}}}\right.\\&\quad\times\left[(\mathbf{q}\mathbf{e}_\alpha^\lambda)(\mathbf{q}\mathbf{e}_{\alpha'}^\lambda)^*\exp[i\boldsymbol{\kappa}(\mathbf{n}-\mathbf{n}')]\right.\\&\quad\times\exp\left(-i\omega_\lambda t+\frac{\hbar\omega_\lambda}{2k_BT_0}\right)+(\mathbf{q}\mathbf{e}_\alpha^\lambda)^*(\mathbf{q}\mathbf{e}_{\alpha'}^\lambda)\\&\quad\left.\left.\times\exp\left(i\omega_\lambda t-\frac{\hbar\omega_\lambda}{2k_BT_0}\right)\right]\right\},\end{aligned} \tag{19}$$

where

$$G(\omega_\lambda)=\frac{\exp(-\hbar\omega_\lambda/2k_BT_0)}{\omega_\lambda[1-\exp(-\hbar\omega_\lambda/k_BT_0)]},$$
$$W_\alpha=\frac{1}{N_1}\sum_\lambda^{3N}\frac{\hbar|\mathbf{q}\mathbf{e}_\alpha^\lambda|^2}{4M_\alpha\omega_\lambda}\coth\frac{\hbar\omega_\lambda}{2k_BT_0}. \tag{20}$$

The summation with respect to λ incorporates all phonon modes with frequencies ω_λ, polarization vectors $\mathbf{e}_\alpha^\lambda$, and quasimomenta $\hbar\boldsymbol{\kappa}$; $N=N_1w$ is the total number of atoms in the crystal.

Equation (19) is easy to simplify in the limiting cases $t\to\infty$ and $t\to 0$. In the first case

$$\lim_{t\to\infty}\chi_{\mathbf{n}\alpha,\mathbf{n}'\alpha'}(t)=\exp[-W_\alpha-W_{\alpha'}] \tag{21}$$

and, correspondingly,

$$\lim_{t\to\infty}\widetilde{K}_T(\mathbf{q},t)=\sum_{\alpha=1}^{w}\sum_{\alpha'=1}^{w}\exp[i\mathbf{q}(\boldsymbol{\rho}_\alpha-\boldsymbol{\rho}_{\alpha'})]\times\sum_{n,n'}\exp[i\mathbf{q}(\mathbf{n}-\mathbf{n}')]\exp(-W_\alpha-W_{\alpha'}). \tag{22}$$

The last factor on the right of Eq. (22) is the traditional Debye–Waller factor

$$\exp\left[-\frac{1}{2}\langle\langle|\mathbf{q}\mathbf{u}_\alpha|^2\rangle\rangle_{T_0}-\frac{1}{2}\langle\langle|\mathbf{q}\mathbf{u}_{\alpha'}|^2\rangle\rangle_{T_0}\right].$$

Equation (22) also holds at arbitrary t if oscillations of different atoms $j\neq j'$ are not correlated, whereas the component with $j=j'$ ("incoherent component") is a complicated function of t (see below). One can easily prove that for $t\to 0$

$$\begin{aligned}\lim_{t\to 0}\chi_{\mathbf{n}\alpha,\mathbf{n}'\alpha'}(t)&=\exp\left\{-\frac{1}{N_1}\sum_\lambda^{3N}\frac{\hbar}{4\omega_\lambda}\coth\frac{\hbar\omega_\lambda}{k_BT_0}\right.\\&\quad\left.\times\left|\frac{(\mathbf{q}\mathbf{e}_\alpha^\lambda)}{\sqrt{M_\alpha}}e^{i\boldsymbol{\kappa}\mathbf{n}}-\frac{(\mathbf{q}\mathbf{e}_{\alpha'}^\lambda)}{\sqrt{M_{\alpha'}}}e^{i\boldsymbol{\kappa}\mathbf{n}'}\right|^2\right\}\\&\quad\times\exp\left\{-it\frac{1}{N_1}\sum_\lambda^{3N}\frac{\hbar}{4\sqrt{M_\alpha M_{\alpha'}}}\right.\\&\quad\times[(\mathbf{q}\mathbf{e}_\alpha^\lambda)(\mathbf{q}\mathbf{e}_{\alpha'}^\lambda)^*e^{i\boldsymbol{\kappa}(\mathbf{n}-\mathbf{n}')}\\&\quad\left.+(\mathbf{q}\mathbf{e}_\alpha^\lambda)^*(\mathbf{q}\mathbf{e}_{\alpha'}^\lambda)e^{-i\boldsymbol{\kappa}(\mathbf{n}-\mathbf{n}')}]\right\}\\&\quad\times\exp\left\{-\frac{t^2}{2}\frac{1}{N_1}\sum_\lambda^{3N}\frac{\hbar\omega_\lambda}{4\sqrt{M_\alpha M_{\alpha'}}}\coth\frac{\hbar\omega_\lambda}{2k_BT_0}\right.\\&\quad\times[(\mathbf{q}\mathbf{e}_\alpha^\lambda)(\mathbf{q}\mathbf{e}_{\alpha'}^\lambda)^*e^{i\boldsymbol{\kappa}(\mathbf{n}-\mathbf{n}')}\\&\quad\left.+(\mathbf{q}\mathbf{e}_\alpha^\lambda)^*(\mathbf{q}\mathbf{e}_{\alpha'}^\lambda)e^{-i\boldsymbol{\kappa}(\mathbf{n}-\mathbf{n}')}]+\dots\right\}.\end{aligned} \tag{23}$$

The first exponent in Eq. (23) is

$$\exp\left[-\frac{1}{2}\langle\langle|\mathbf{q}(\mathbf{u}_{\alpha\mathbf{n}}(0)-\mathbf{u}_{\alpha'\mathbf{n}'}(0))|^2\rangle\rangle_{T_0}\right].$$

In the case of a monatomic lattice ($M_\alpha=M_{\alpha'}=M$, $N_1=N$) the right-hand side of Eq. (23) can be simplified:

$$\begin{aligned}\lim_{t\to 0}\chi_{\mathbf{n}\alpha,\mathbf{n}'\alpha'}(t)&=\exp\left\{-\frac{1}{N}\sum_\lambda^{3N}\frac{\hbar}{2\omega_\lambda}\frac{|\mathbf{q}\mathbf{e}^\lambda|^2}{M}\coth\frac{\hbar\omega_\lambda}{2k_BT_0}[1\right.\\&\quad\left.-\cos(\boldsymbol{\kappa}(\mathbf{n}-\mathbf{n}'))]\right\}\exp\left\{-it\frac{1}{N}\sum_\lambda^{3N}\frac{\hbar|\mathbf{q}\mathbf{e}^\lambda|^2}{2M}\cos(\boldsymbol{\kappa}(\mathbf{n}\right.\\&\quad\left.-\mathbf{n}'))\right\}\exp\left\{-\frac{t^2}{2N}\sum_\lambda^{3N}\frac{\hbar\omega_\lambda}{2M}|\mathbf{q}\mathbf{e}^\lambda|^2\coth\frac{\hbar\omega_\lambda}{2k_BT_0}\right.\\&\quad\left.\times\cos(\boldsymbol{\kappa}(\mathbf{n}-\mathbf{n}'))+\dots\right\}.\end{aligned} \tag{24}$$

For the incoherent component ($\mathbf{n}=\mathbf{n}'$) we have

$$\lim_{t\to 0} \chi_{\mathbf{n}\alpha,\mathbf{n}\alpha}(t)$$

$$=\exp\left\{ -it\frac{1}{N}\sum_{\lambda}^{3N} \frac{\hbar|\mathbf{q}\mathbf{e}^{\lambda}|^2}{2M} - \frac{t^2}{2M} \times\sum_{\lambda}^{3N} \frac{\hbar\omega_{\lambda}}{2M}|\mathbf{q}\mathbf{e}^{\lambda}|^2 \coth\frac{\hbar\omega_{\lambda}}{2k_BT_0} + \dots \right\}. \tag{25}$$

The comparison of Eqs. (23)–(25) with Eqs. (19)–(22) indicates that in the limit $t\to 0$ the Debye–Waller factor reduces only the interference term with $j\neq j'$. Furthermore, even for $j\neq j'$ this reduction is partly cancelled by the factor

$$\exp[\langle\langle(\mathbf{q}\mathbf{u}_j(0))(\mathbf{q}\mathbf{u}_{j'}(0))\rangle\rangle_{T_0}],$$

i.e., the expression for the cross section of the coherent Coulomb excitation contains a term which is not accounted for in the traditional approach (see Introduction). This term is similar to the expression for the cross section of diffuse coherent scattering of X-rays in crystals.[20] It may partly compensate for the decrease in contributions from higher harmonics when their order increases.[5,8] In the case of a cubic Bravais lattice, the parameter

$$\exp\left[-\frac{1}{2}\langle\langle(\mathbf{q}(\mathbf{u}_j(0)-\mathbf{u}_{j'}(0)))^2\rangle\rangle\right]_{T_0}$$

can be calculated using the Debye model. We have a series of obvious identities:

$$\frac{1}{2}\langle\langle(\mathbf{q}(\mathbf{u}_j(0)-\mathbf{u}_{j'}(0)))^2\rangle\rangle_{T_0}$$

$$=\frac{1}{N}\sum_{\lambda}^{3N} \frac{\hbar}{2M\omega_{\lambda}}|\mathbf{q}\mathbf{e}^{\lambda}|^2 \coth\frac{\hbar\omega_{\lambda}}{2k_BT_0}[1-\cos(\boldsymbol{\kappa}(\mathbf{n}-\mathbf{n}'))]$$

$$=\frac{q^2}{3}\sum_{\lambda}^{3N} \frac{\hbar}{2M\omega_{\lambda}} \coth\frac{\hbar\omega_{\lambda}}{2k_BT_0}[1-\cos(\boldsymbol{\kappa}(\mathbf{n}-\mathbf{n}'))]$$

$$=\frac{q^2}{3}\int_0^{\omega_{\max}} g(\omega)d\omega\frac{\hbar}{2MN\omega} \coth\frac{\hbar\omega}{2k_BT_0}[1 -\cos(\boldsymbol{\kappa}(\mathbf{n}-\mathbf{n}'))]. \tag{26}$$

Here $g(\omega)$ is the distribution function of the phonon frequencies. In the Debye model

$$g(\omega)=\begin{cases} 9N\omega^2/\omega_{\max}^3, & \omega\leqslant\omega_{\max}, \\ 0, & \omega>\omega_{\max}, \end{cases}$$

and $\hbar\boldsymbol{\kappa}$ is the phonon quasi-momentum. In this case, after simple transformations we have

$$\frac{1}{N}\sum_{\lambda}^{3N} \frac{\hbar}{2M\omega_{\lambda}}|\mathbf{q}\mathbf{e}^{\lambda}|^2 \coth\frac{\hbar\omega_{\lambda}}{2k_BT_0}[1-\cos(\boldsymbol{\kappa}(\mathbf{n}-\mathbf{n}'))]$$

$$=\frac{3\hbar q^2}{M\omega_{\max}}\left\{\left(\frac{T_0}{\Theta}\right)^2\int_0^{\Theta/T_0}\frac{x\,dx}{e^x-1}+\frac{1}{4} -\frac{1-\cos(\kappa_D|\mathbf{n}-\mathbf{n}'|)}{2(\kappa_D|\mathbf{n}-\mathbf{n}'|)^2}-\left(\frac{T_0}{\Theta}\right)^2\sum_{l=0}^{\infty}\frac{(-1)^l}{(2l+1)!} \times\left(\frac{\kappa_D|\mathbf{n}-\mathbf{n}'|}{\Theta}T_0\right)^{2l}\int_0^{\Theta/T_0}\frac{x^{2l+1}dx}{e^x-1}\right\}. \tag{27}$$

Here $\Theta=\hbar\omega_{\max}/k_B$ is the Debye temperature, $\kappa_D=\omega_{\max}/c_0$,

$$\frac{\sqrt[3]{3}}{c_0}=\left(\frac{1}{c_{\parallel}^3}+\frac{2}{c_{\perp}^3}\right)^{1/3},$$

and $c_{\parallel}$ and $c_{\perp}$ are the group velocities of longitudinal and transverse phonons.

In the case of a simple cubic lattice, we have the following compact expression for the "incoherent" component $\tilde{K}_T(\mathbf{q},t)$ in the Debye model:

$$\tilde{K}_T(\mathbf{q},t)=\exp\{\gamma(\mathbf{q},\mathbf{q},t)-\gamma(\mathbf{q},\mathbf{q},0)\}. \tag{28}$$

Here we have introduced the notation

$$\gamma(\mathbf{p},\mathbf{p}',t)=\langle\langle(\mathbf{p}\hat{\mathbf{R}}(t))(\mathbf{p}\hat{\mathbf{R}}(0))\rangle\rangle_{T_0} =\frac{\hbar(\mathbf{p}\mathbf{p}')}{6MN}\int_0^{\infty}\frac{g(\omega)d\omega}{\omega}[\langle n(\omega)\rangle e^{i\omega t}+(\langle n(\omega)\rangle +1)e^{-i\omega t}], \tag{29}$$

$$\langle n(\omega)\rangle=[\exp(\hbar\omega/k_BT_0)-1]^{-1}. \tag{30}$$

Now let us calculate the correlator $\tilde{K}_P(\mathbf{q},t)$ for the projectile nucleus P. We will consider only those penetration depths of the nucleus into the target at which the transient processes to the channelling regime are terminated and the fast longitudinal motion along a crystal axis or plane is decoupled from the slow transverse motion. With this assumption, the function $\tilde{K}_P(\mathbf{q},t)$ is factorized:

$$\tilde{K}_P(\mathbf{q},t)=\tilde{K}_{Pz}(q_z,t)\tilde{K}_{P\perp}(\mathbf{q}_{\perp},t), \tag{31}$$

where

$$\tilde{K}_{Pz}(q_z,t)=\langle\langle\exp[iq_z\hat{Z}_P(t)]\exp[-iq_z\hat{Z}_P(0)]\rangle\rangle_P \tag{32}$$

is the correlation function of the longitudinal motion of the nucleus P, and z-axis is aligned with the channelling direction of the nucleus P. Hence

$$\tilde{K}_{P\perp}(\mathbf{q}_{\perp},t)=\langle\langle\exp[i\mathbf{q}\hat{\mathbf{B}}_P(t)]\exp[-i\mathbf{q}\hat{\mathbf{B}}_P(0)]\rangle\rangle, \quad \mathbf{R}_P=\{\mathbf{B}_P,Z_P\}. \tag{33}$$

The longitudinal motion can be described as free propagation (see the comment about Eq. (6)). In this case, given that

$$\hat{Z}_P(t)=\hat{Z}_P(0)+\frac{1}{M_P}\hbar\hat{p}_z(0)t,$$

where $\hbar \hat{p}_z(0)$ is the operator of z-component of the nucleus P momentum, we have

$$\tilde{K}_{Pz}(q_z,t)=\exp\left(-it\frac{\hbar q_z^2}{2M_P}\right)\left\langle\left\langle \exp\left(\frac{iq_z\hat{p}_z(0)\hbar}{M_P}t\right)\right\rangle\right\rangle_P$$

$$=\exp\left(-it\frac{\hbar q_z^2}{2M_P}\right)\exp\left(it\frac{\hbar q_z p_{0z}}{M_P}\right)$$

$$\times\exp\left(-t^2\frac{\hbar^2 q_z^2}{4M_P^2}\frac{1}{\zeta_z}\right). \quad (34)$$

The last factor on the right of Eq. (34) accounts for the spread of the z-component of the P nucleus momentum around its average $\hbar p_{0z}$ since the beam of incident nuclei is not monochromatic: $(2\zeta_z)^{-1}=\langle(\Delta p_z)^2\rangle$.

The effect of inelastic collisions on the propagation of nuclei in the channel (motion at a constant energy) can be taken into account using the multiple-scattering theory through the imaginary component of the effective potential

$$\text{Im } V_{\text{eff}}(\mathbf{r})=-\frac{p_{0z}\hbar^2}{2M_P l_{\text{in}}(\mathbf{r})}=-\frac{\hbar W_{\text{in}}(\mathbf{r})}{2},$$

where $l_{\text{in}}(\mathbf{r})$ is the free path with respect to inelastic collisions, and $W_{\text{in}}(\mathbf{r})$ is the probability per unit time of an inelastic transition. The loss of the incident flux due to $\text{Im}V_{\text{eff}}(\mathbf{r})$ mainly tends to diminish the value of the resonance excitation cross section. This approach, however, does not take into account excitations due to several inelastic collisions. In order to describe the total contribution of all processes leading to the resonance excitation, we should use an expression for the reaction cross section averaged with respect to the energy distribution of beam particles, penetration depth $z-z_0$ in the target, where z_0 is the coordinate of the exposed target surface, and the impact parameter B_P:

$$\langle\sigma(E_0)\rangle=\frac{\int\Phi(z,B_P,E,E_0)\sigma(E,z,B_P)dE\, dz\, d\mathbf{B}_P}{\int\Phi(z,B_P,E,E_0)dE\, dz\, d\mathbf{B}_P}.$$

Here $\Phi(z,B_P,E,E_0)$ is the flux of particles, which satisfies the kinetic equation.[6] If the target is sufficiently thin so that the redistribution of particles over the energy $E_\perp$ of transverse motion can be neglected, then

$$\Phi(z,B_P,E_\parallel,E_{0\parallel})$$

$$=\frac{1}{\kappa(E_\parallel,B_P)}\exp\left(-\int_{E_\parallel}^{E_{0\parallel}}\frac{dE'}{\kappa(E',B_P)}\frac{1}{l_{\text{in}}(E',B_P)}\right)$$

$$\times\delta\left(z-z_0-\int_{E_\parallel}^{E_{0\parallel}}\frac{dE'}{\kappa(E',B_P)}\right).$$

Here $\kappa(E_\parallel,B_P)=-(dE/dz)$ is the deceleration efficiency of the target and $E_\parallel$ is the energy of the longitudinal motion of a beam particle. In the limit $l_{\text{in}}^{-1}\to 0$, we have a simple equation for a uniform target[11]

$$\langle\sigma(E_0)\rangle=\int_{E_{0\parallel}-\Delta E}^{E_{0\parallel}}\frac{dE'}{\kappa(E')}\sigma(E')\Bigg/\int_{E_{0\parallel}-\Delta E}^{E_{0\parallel}}\frac{dE'}{\kappa(E')}.$$

Here $\Delta E=\kappa(E)h$ and h is the target thickness. The resonance broadening due to the beam deceleration can be estimated assuming that

$$\Phi(z,B_P,E_\parallel,E_{0\parallel})=\frac{1}{2}[\Phi(z,B_P,E_{0\parallel},E_{0\parallel})+\Phi(z_0+h,B_P,E_{0\parallel}-\Delta E,E_{0\parallel})].$$

As a result, we have two overlapping resonances shifted with respect to each other by ΔE. For example, in the case of ^{11}Li ions with an energy $E_{0\parallel}=1.6$ GeV/nucleon and $h=0.1$ mm, we have $\Delta E\approx 0.37$ MeV (see also Conclusion).

In order to describe the transverse motion of particles propagating in the channelling mode across a target, we can use the model of a harmonic oscillator with frequency ω_0. Then $\tilde{K}_{P\perp}(\mathbf{q},t)$ in Eq. (33) can be expressed as (see Eqs. (28)–(30))

$$\tilde{K}_P(\mathbf{q}_\perp,t)=\exp\left\{\frac{q_\perp^2\hbar}{2M_P\omega_0}[\langle n(\omega_0)\rangle e^{i\omega_0 t}+(\langle n(\omega_0)\rangle+1)e^{-i\omega_0 t}]\right\}$$

$$\times\exp\left\{-\frac{q_\perp^2\hbar}{2M_P\omega_0}[2\langle n(\omega_0)\rangle+1]\right\}, \quad (35)$$

or

$$\tilde{K}_{P\perp}(\mathbf{q}_\perp,t)=\exp\left\{-\frac{q_\perp^2\hbar}{2M_P\omega_0}\coth\frac{\hbar\omega_0}{2k_BT_0}\right\}\sum_{n=-\infty}^{\infty}\left[e^{-i\omega_0\tau}\right.$$

$$\times\exp\left(-\frac{\hbar\omega_0}{2k_B\tau_0}\right)\Bigg]^n I_n\left(\frac{\hbar q_\perp^2}{M_P\omega_0}\left[2\right.\right.$$

$$\left.\left.\times\sinh\left(\frac{\hbar\omega_0}{2k_B\tau_0}\right)\right]^{-1}\right). \quad (36)$$

Here τ_0 is the quasi-temperature of the channeled beam.[21]

Approximate estimates can be derived by neglecting the transverse motion and using a certain distribution function $\Phi(B_P)$ of impact parameters. In the case of a flat distribution of B_P over an interval $(B_{\min},B_{\max})$ and channelling along an axis, we have

$$\Phi(B_P)=\frac{1}{\pi(B_{\max}^2-B_{\min}^2)}\theta(B_{\max}-B_P)\theta(B_P-B_{\min}), \quad (37)$$

where $\theta(x)$ is the step function.

Unbounded transverse motion of particles with energies above the barrier is described in two limiting cases by the following functions $\tilde{K}_{P\perp}(\mathbf{q},t)$:

A. Free motion:

$$\tilde{K}_{P\perp}(\mathbf{q}_\perp,t)=\exp\left(-it\frac{\hbar q_\perp^2}{2M_P}\right)\exp\left(it\frac{\hbar\mathbf{q}_\perp\mathbf{p}_{0\perp}}{M_P}\right)$$

$$\times\exp\left(-t^2\frac{\hbar^2 q_\perp^2}{8M_P}\frac{1}{\zeta_\perp}\right),$$

$$(2\zeta_\perp)^{-1}=\langle(\Delta P_\perp)^2\rangle. \quad (38)$$

B. Classical (not quantum) diffusion in the transverse plane:

$$\tilde{K}_{P\perp}(\mathbf{q}_\perp,t)=\exp[-q_\perp^2 D_P|t|],\qquad (39)$$

where D_P is the diffusion coefficient of the nucleus P.

Equation (17) for the total cross section σ_t for Coulomb excitation can be simplified in limiting cases of long-lived and short-lived (with respect to the characteristic times t_P^* and t_T^* of correlators $\tilde{K}_P$ and $\tilde{K}_T$) excited states of the nucleus P. In the former case ($\Gamma\to 0$, $\mathscr{E}_s\simeq\mathscr{E}_r$), expressions for the correlators $\tilde{K}_P(\mathbf{q},t)$ and $\tilde{K}_T(\mathbf{q},t)$ at $t\to\infty$ can be substituted into Eq. (17). Then

$$\sigma_t=\frac{8\pi}{3}\frac{\mathscr{B}(E1)}{\hbar v_L}\frac{\Gamma/2}{(\mathscr{E}_s-\mathscr{E}_r)^2+\Gamma^2/4}\int\frac{d\mathbf{q}_\perp}{(2\pi)^2}\tilde{K}_{P\perp}(\mathbf{q}_\perp,\infty)\times\int_{-\infty}^{\infty}\frac{dq_z}{2\pi}\tilde{K}_{Pz}(q_z,\infty)\tilde{K}_T(-\mathbf{q}_\perp,-q_z,\infty)(q_\perp^2+q_z^2)\times\left|\frac{\tilde{v}_0(\mathbf{q}_\perp,q_z)}{e}\right|^2.\qquad (40)$$

In the case of the diffusion model (Eq. (39)), Eq. (40) should be modified:

$$\sigma_t=\frac{8\pi}{3}\frac{\mathscr{B}(E1)}{\hbar^2 v_L}\times\int\frac{d\mathbf{q}_\perp}{(2\pi)^2}\frac{\Gamma/2\hbar+q_\perp^2 D_P}{(\mathscr{E}_s-\mathscr{E}_r)^2\hbar^{-2}+(\Gamma/2\hbar+q_\perp^2D_P)^2}\times\int_{-\infty}^{\infty}\frac{dq_z}{2\pi}\tilde{K}_{Pz}(q_z,\infty)\tilde{K}_T(-\mathbf{q}_\perp,-q_z,\infty)(q_\perp^2+q_z^2)\times\left|\frac{\tilde{v}_0(\mathbf{q}_\perp,q_z)}{e}\right|^2.\qquad (41)$$

Thus the diffusive motion of the nucleus P in the transverse direction leads to an additional broadening of the resonance.

In this case of either short-lived states ($\Gamma\to\infty$) or $|\mathscr{E}_s-\mathscr{E}_r|\gg(\hbar/t_P^*,\hbar/t_T^*)$, we substitute into Eq. (17) the expressions for the correlators $\tilde{K}_P(t)$ and (for the incoherent component) $\tilde{K}_T(t)$ derived from Eqs. (28), (29), (35), and (36) at $t\to 0$:

$$\tilde{K}_P(\mathbf{q},t)\underset{t\to 0}{=}\exp\left(it\frac{\hbar q_z p_{0z}}{M_P}\right)\exp\left(it\frac{\hbar\mathbf{q}_\perp\mathbf{p}_{0\perp}}{M_P}\right)\times\exp\left(i\frac{\Delta_P^z}{\hbar}t\right)\exp\left(i\frac{\Delta_P^\perp}{\hbar}t\right),$$

$$\tilde{K}_T(\mathbf{q},t)\underset{t\to 0}{=}\exp\left(i\frac{\Delta_T}{\hbar}t\right).\qquad (42)$$

Here

$$\Delta_P^z=-\frac{\hbar^2q_z^2}{2M_P},\quad \Delta_P^\perp=-\frac{\hbar^2q_\perp^2}{2M_P},\quad \Delta_T=-\frac{\hbar^2q^2}{2M_T}.$$

Then we have

$$\sigma_t=\frac{8\pi}{3}\frac{\mathscr{B}(E1)}{\hbar v_L}\int\frac{d\mathbf{q}_\perp}{(2\pi)^2}\int_{-\infty}^{\infty}\frac{dq_z}{2\pi}(q_\perp^2+q_z^2)\left|\frac{\tilde{v}_0(\mathbf{q}_\perp,q_z)}{e}\right|^2\times\frac{\Gamma/2}{\left(\mathscr{E}_s-\mathscr{E}_r+\frac{q_zp_{0z}\hbar^2}{M_P}+\frac{\mathbf{q}_\perp\mathbf{p}_{0\perp}\hbar^2}{M_P}+\Delta_T+\Delta_P^z+\Delta_P^\perp\right)^2+\Gamma^2/4}.\qquad (43)$$

Thus the position of the resonance in the integrand on the right of Eq. (43) is shifted with respect to $\mathscr{E}_r$. Terms proportional to t^2 in the exponents of Eq. (42) should, evidently, cause broadening of the resonance described by Eq. (43).

To conclude this section, let us try to convert our result expressed by Eq. (17) to a form similar to Eq. (1). To this end, we use the expression for the total cross section for absorption of a real photon with an energy $E_\gamma=\hbar p_\gamma c$ in a lattice nucleus.[22,23] We obtain

$$\sigma_t=\frac{8\pi}{3}\frac{\mathscr{B}(E1)}{\hbar^2 v_L}\frac{2\hbar}{\sigma_0\Gamma}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int\frac{d\mathbf{q}}{(2\pi)^3}\times\left|\frac{\tilde{v}_0(\mathbf{q})}{e}\right|^2q^2\tilde{k}_P(\mathbf{q},-\omega)\hat{\sigma}_a(\hbar\omega,\mathbf{q}).\qquad (44)$$

Here

$$\hat{\sigma}_a(E_\gamma,\mathbf{p}_\gamma)=\frac{\sigma_0\Gamma}{2\hbar}\mathrm{Re}\int_0^\infty dt\,\exp\left[\frac{it(E_\gamma-\mathscr{E}_r+\mathscr{E}_s)}{\hbar}-\frac{\Gamma t}{2\hbar}\right]\tilde{K}_T(-\mathbf{p}_\gamma,t),\qquad (45)$$

$\sigma_0=\hat{\sigma}_a(E_\gamma=\mathscr{E}_r-\mathscr{E}_s)$ in the case of an isolated nucleus P, $\hat{\sigma}_a(E_\gamma,\mathbf{p}_\gamma)$ is the total cross section for resonant absorption of a virtual photon ($E_\gamma\neq\hbar p_\gamma c$) in a target whose nuclei have the same resonances as P, but dynamic lattice parameters are the same as in the real target. Here we have introduced the notation

$$\tilde{k}_P(\mathbf{q},\omega)=\int_{-\infty}^{\infty}dt\,e^{i\omega t}\tilde{K}_P(\mathbf{q},t)=\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\tilde{k}_{Pz}(q_z,\omega')\tilde{k}_{P\perp}(\mathbf{q}_\perp,\omega-\omega').\qquad (46)$$

The latter equality in Eq. (46) is valid when the longitudinal and transverse motion of the nucleus P are decoupled. Assuming that the longitudinal motion is free, let us totally ignore its transverse motion. Given Eq. (34) and that $\tilde{K}_{P\perp}(\mathbf{q},\omega-\omega')=2\pi\delta(\omega-\omega')$ in this case, we derive from Eq. (44)

$$\sigma_t=\frac{16\pi}{3}\frac{\mathscr{B}(E1)}{\hbar v_L\sigma_0\Gamma}\frac{2M_P}{\hbar}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int\frac{d\mathbf{q}_\perp}{(2\pi)^2}\times\sum_{\nu=1,2}\left|\frac{\tilde{v}_0(\mathbf{q}_\perp,q_z^{(\nu)})}{e}\right|^2\frac{q_\perp^2+(q_z^{(\nu)})^2}{2|q_z^{(\nu)}-p_{0z}|}\hat{\sigma}_a(\hbar\omega;\mathbf{q}_\perp,q_z^{(\nu)}).\qquad (47)$$

Here

$$q_z^{(1,2)}=p_{0z}\pm\sqrt{p_{0z}^2-\frac{2M_P\omega}{\hbar}}=\frac{v_LM_P}{\hbar}\left[1\pm\sqrt{1-\frac{\hbar\omega}{E_P}}\right]$$

are the roots of the equation

$$q_z^2 - 2q_z p_{0z} + \frac{2M_P\omega}{\hbar} = 0.$$

Usually the longitudinal motion of a fast particle is described in terms of classical mechanics. In this case we should retain only one root, i.e., $q_z^{(2)} \cong \omega/v_L$. In the limiting cases $\Gamma \to 0$ and $\Gamma \to \infty$ we have

$$\hat{\sigma}_a(E,\mathbf{q}) = \widetilde{K}_T(-\mathbf{q},\infty)\frac{\sigma_0(\Gamma/2\hbar)^2}{(E-\mathscr{E}_r+\mathscr{E}_s)^2/\hbar^2+(\Gamma/2\hbar)^2},$$

$$\hat{\sigma}_a(E,\mathbf{q}) = \widetilde{K}_T(-\mathbf{q},0)\frac{\sigma_0(\Gamma/2\hbar)^2}{(E-\mathscr{E}_r+\mathscr{E}_s+\Delta_T)^2/\hbar^2+(\Gamma/2\hbar)^2}, \tag{48}$$

respectively. Given Eqs. (47) and (48), we can write the final result as

$$\sigma_t = \frac{16\pi}{3}\frac{\mathscr{B}(E1)}{\hbar v_L \Gamma\sigma_0}\frac{M_P}{\hbar}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int\frac{d\mathbf{q}_\perp}{(2\pi)^2}$$
$$\times\sum_{\nu=1,2}\left|\frac{\widetilde{v}_0(\mathbf{q}_\perp,q_z^{(\nu)})}{e}\right|^2\frac{q_\perp^2+(q_z^{(\nu)})^2}{|q_z^{(\nu)}-p_{0z}|}$$
$$\times\begin{cases} K_T(-\mathbf{q}_\perp,-q_z^{(\nu)};\infty)\dfrac{\sigma_0(\Gamma/2\hbar)^2}{(\mathscr{E}_s-\mathscr{E}_r+\hbar\omega)^2/\hbar^2+(\Gamma/2\hbar)^2}, \\ K_T(-\mathbf{q}_\perp,-q_z^{(\nu)};0) \\ \dfrac{\sigma_0(\Gamma/2\hbar)^2}{[\mathscr{E}_s-\mathscr{E}_r+\Delta_T(-\mathbf{q}_\perp,-q_z^{(\nu)})+\hbar\omega]^2/\hbar^2+(\Gamma/2\hbar)^2}\end{cases}. \tag{49}$$

Even in the limiting cases $\Gamma \to 0$ and $\Gamma \to \infty$ the expression for σ_t cannot be written in a form similar to Eq. (1), which is used in the method of equivalent photons. To achieve this end, we must impose additional conditions $\Delta_T(\mathbf{q},q_z^{(\nu)})=0$ and $q_z^{(\nu)} \approx \omega/v_L$. In this case

$$\sigma_t = \frac{16\pi}{3}\frac{\mathscr{B}(E1)}{\hbar^2\Gamma\sigma_0}M_P\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{1}{\omega}$$
$$\times\frac{\sigma_0(\Gamma/2\hbar)^2}{(\mathscr{E}_s-\mathscr{E}_r+\hbar\omega)^2/\hbar^2+(\Gamma/2\hbar)^2}$$
$$\times\left|1-\frac{p_{0z}v_L}{\omega}\right|^{-1}\int\frac{d\mathbf{q}_\perp}{(2\pi)^2}\left|\frac{\widetilde{v}_0(\mathbf{q}_\perp,\omega/v_L)^2}{e}\right|^2$$
$$\times\left[q_\perp^2+\left(\frac{\omega}{v_L}\right)^2\right]\widetilde{K}_T\left(-\mathbf{q}_\perp,-\frac{\omega}{v_L};t^*\right). \tag{50}$$

Here t^* should be extrapolated either to infinity or to zero in order to obtain equations for the two limiting cases, $\Gamma \to \infty$ and $\Gamma \to 0$. From the comparison between Eqs. (50) and (1), we derive the following expression for the equivalent photon spectrum:

$$\frac{dN_\gamma(\omega)}{d\omega} = \frac{8}{3}\frac{\mathscr{B}(E1)}{\hbar^2\Gamma\sigma_0}\frac{M_P}{\omega}\int\frac{d\mathbf{q}_\perp}{(2\pi)^2}\left|\frac{\widetilde{v}_0(\mathbf{q}_\perp,\omega/v_L)}{e}\right|^2\left[q_\perp^2\right.$$
$$\left.+\left(\frac{\omega}{v_L}\right)^2\right]\left|1-\frac{p_{0z}v_L}{\omega}\right|^{-1}\widetilde{K}_T\left(-\mathbf{q}_\perp,-\frac{\omega}{v_L};t^*\right). \tag{51}$$

Thus, our analysis has demonstrated that the equivalent photon method can be applied to resonant Coulomb excitation of nuclei propagating across a crystal target only if several conditions are satisfied, namely,

1. The transverse and longitudinal motions of a nucleus are decoupled. This condition holds when the target thickness is sufficiently large that the transverse motion in the channel should become quasi-equilibrium.[6,21]

2. The motion along the channel should be free. This means, primarily, that we neglect both the energy loss of the particle propagating along the channel and the spread of its energy (see the discussion of the approximation concerning energy loss in Eq. (34) and Conclusion). Besides, this condition implies that we use the approximation that the continuous potential V_{eff} does not vary as a function of z along the channel axis. In the case of short-lived resonances, the error of this approximation can be estimated as follows. We write the operator $\hat{\mathbf{R}}_P(t)$ as a power series in t:

$$\hat{\mathbf{R}}_P(t) = \hat{\mathbf{R}}_P(0) + t\frac{\hat{\mathbf{P}}}{M_P} - \frac{t^2}{2M_P}\nabla V + \dots .$$

The last term contributes to the correlator of the propagating particles in Eq. (34) an additional factor

$$\left\langle \exp\left(-it^2\frac{q_z^2}{2M_P}\frac{dV}{dz}\right)\right\rangle.$$

In our estimate we take

$$\frac{dV}{dz} = \frac{Z_P Z_T}{B_P^2}e^2, \quad t_0 = \frac{a_z^2}{v_P} \gtrsim \frac{\hbar}{\Gamma},$$

where a_z is the lattice constant along the z-axis and Γ is the resonance width. Then we obtain at $Z_T=80$, $Z_P=3$, $B_P=10^{-9}$ cm, $q_z=10^9$ cm^{-1}, $M_P=2\cdot 10^{-23}$ g, and $t_0=0.3\cdot 10^{-18}$ s an estimate of the exponent in the additional exponential function:

$$\frac{q_z}{2M_P}\left|\frac{dV}{dz}\right|t_0^2 \lesssim 10^{-6}.$$

In the case of long-lived intermediate states, the model of a continuous potential V_{eff} which does not vary with z means that a small correction to V_{eff} is ignored:

$$\Delta V \approx \frac{1}{2M_P}\left(\frac{dV}{dz}\right)^2\left(\frac{t_0}{2\pi}\right)^2 \ll V_{\text{eff}};$$

ΔV_{eff} is also constant with z. At the parameters given above, $\Delta V/V_{\text{eff}} \lesssim 10^{-4}$.

Thus in the two limiting cases of short-lived ($t_0 = a_z/v_P \gg \hbar/\Gamma$) and long-lived ($t_0 \ll \hbar/\Gamma$) resonances, the model of the continuous potential V_{eff} independent of the coordinate z yields an adequate description of the channelled particle longitudinal motion.

3. Quantum effects can be ignored in the description of the fast ion longitudinal motion. This condition is not essential for $v_L \gtrsim 0.1c$, when the contribution due to the root $q^{(1)} \approx 2M_P v_L/\hbar$ can be neglected because this parameter is larger than any other characteristic inverse length of the problem; as a result, the term with $\nu=1$ in Eq. (49) is very small. But the classical description of the longitudinal mo-

tion lacks the resonance shift due to the momentum imparted by a virtual photon. In the formalism used to solve our problem, this effect is purely quantum-mechanical. Even in the case of a broad resonance the parameter $\Delta_{Pz}=(\hbar q_z)^2/2M_P$ may be comparable to the resonance width Γ. For example, at $\hbar P_{0z}=1$ GeV/c, $M_P=2\cdot 10^{-23}$ g, $\mathscr{E}_r-\mathscr{E}_s=100$ keV, and $\Gamma=0.1$ keV we have $(\hbar q_z)^2/2M_P\approx 0.4\Gamma$.

4. The method of equivalent photons does not describe shifts of the resonance and deformations of the Lorentzian curve of the photon absorption cross section $\sigma_\gamma(\omega)$ caused by thermal motion of lattice nuclei and oscillations of the channeled ion in the direction perpendicular to the channel (see Eqs. (43) and (49)).

4. DIFFERENTIAL CROSS SECTIONS FOR RESONANT COULOMB EXCITATION

The differential cross section of the reaction

$$P+T\to T+a+b+c+\ldots,$$

when the final state $|f\rangle$ includes fragments of the nucleus P with momenta $\mathbf{p}_1,\ldots,\mathbf{p}_n$ has the form

$$d\sigma_{t,i}=\frac{(2\pi)^4}{v_L\hbar}\frac{\Omega}{(2\pi)^3}\frac{\Omega}{(2\pi\hbar)^3}d\mathbf{p}_1\ldots \times\frac{\Omega}{(2\pi\hbar)^3}d\mathbf{p}_n|\langle f|t|i\rangle|^2\delta(E_i-E_f). \quad (52)$$

The matrix element of the transition $\langle f|t|i\rangle$ via an intermediate resonant state was determined above (see Eq. (12)). The transformation to the time representation of the inclusive transition probability, described in the preceding section, allows us to express the target cross section $\langle d\sigma_{f,i}\rangle$ summed over all initial and final states of the target through an integral formula with four-time correlation functions for the target nuclear coordinates. This result is a natural generalization of multiple time correlators previously used to describe resonant scattering of photons and slow neutrons.[13,24] Such complicated formulas necessarily emerge when center-of-mass coordinates of target nuclei are involved in both the first and second stages of the transition $|i\rangle\to|\lambda\rangle\to|f\rangle$. One example of such transitions is elastic scattering of the P nucleus in a Coulomb field due to target nuclei (the process is elastic in a sense that the state of internal degrees of freedom remains unchanged). In those cases when the reaction $|\lambda\rangle\to|f\rangle$ in the lattice proceeds as in free space (for example, fragmentation of the nucleus P), the expression for the differential cross section is somewhat simpler. Consider as an example a disintegration of the nucleus P into two fragments. Then for a fixed center-of-mass momentum $\mathbf{P}_f$ and a fixed momentum of the relative motion of the fragments $\mathbf{p}_f$, the differential cross section can be expressed as follows:

$$\langle d\sigma_{t,i}\rangle=\frac{(2\pi)^4}{v_L\hbar}\frac{\Omega}{(2\pi)^3}\frac{\Omega}{(2\pi\hbar)^3}d\mathbf{P}_f\frac{\Omega}{(2\pi\hbar)^3}d\mathbf{p}_f\frac{1}{\Omega}\frac{1}{2\pi\hbar}\times\int_{-\infty}^{\infty}d\mu\,\exp\left(i\mu\frac{\mathscr{E}_s-\mathscr{E}_u}{\hbar}\right)\exp\left(i\mu\frac{\varepsilon_a-\varepsilon_c}{\hbar}\right)\frac{1}{\hbar^2}$$
$$\times\int_0^\infty dt\int_0^\infty dt'\exp\left[-\frac{\Gamma}{2\pi}(t+t')\right]\exp\left[i\frac{\varepsilon_a-\varepsilon_c}{\hbar}(t-t')\right]\exp\left[i\frac{\mathscr{E}_s-\mathscr{E}_r}{\hbar}(t-t')\right]\int\frac{d\mathbf{q}}{(2\pi)^3}|\tilde{v}_0^{\mathrm{I}}(\mathbf{q})|^2\tilde{K}_T$$
$$\times(-\mathbf{q},t-t'+\mu)|\langle c|\tilde{\rho}_P(\mathbf{q})|a\rangle|^2\times|\langle r|\hat{h}^{\mathrm{I}}_{\mathrm{int}}(\mathbf{q})|s\rangle|^2|u|\hat{H}^{\mathrm{II}}_{\mathrm{int}}|r\rangle|^2. \quad (53)$$

This formula includes mostly parameters introduced in Secs. 2 and 3. The additional notations are: $|u\rangle$ and $\mathscr{E}_u$ are the wave function and energy of the relative motion of the nucleus P fragments, $|c\rangle$ and ε_c are the wave function and energy of the center-of-mass motion in the final state, and

$$\hat{h}^{\mathrm{I}}_{\mathrm{int}}(\mathbf{q})=4\pi\sum_{L\mu}(-i)^L Y^*_{L\mu}(\hat{\mathbf{q}})\sum_{j=1}^{Z_P}j_L(\mathbf{q}\mathbf{r}_j)Y^*_{L\mu}(\theta_j,\varphi_j)-Z_P.$$

Equation (53) takes into account that in the final stage of the process the lattice state and the center-of-mass motion of the nucleus P do not change. If the final state of the center of mass is not fixed, the summation over $|c\rangle$ and averaging over $|a\rangle$ in Eq. (53) yields

$$\langle\langle d\sigma_{t,i}\rangle\rangle=\frac{(2\pi)^4}{v_L\hbar}\frac{\Omega}{(2\pi)^3}\frac{\Omega}{(2\pi\hbar)^3}d\mathbf{p}_f\frac{1}{\Omega}\frac{1}{2\pi\hbar}\times\int_{-\infty}^{\infty}d\mu\,\exp\left(i\mu\frac{\mathscr{E}_s-\mathscr{E}_u}{\hbar}\right)\frac{1}{\hbar^2}\int_0^\infty dt\int_0^\infty dt'$$
$$\times\exp\left[-\frac{\Gamma}{2\hbar}(t+t')\right]\exp\left[i\frac{\mathscr{E}_s-\mathscr{E}_r}{\hbar}(t-t')\right]\times\int\frac{d\mathbf{q}}{(2\pi)^3}|\tilde{v}_0^{\mathrm{I}}(\mathbf{q})|^2\tilde{K}_T(-\mathbf{q},t-t'+\mu)\tilde{K}_P$$
$$\times(\mathbf{q},t-t'+\mu)|\langle r|\hat{h}^{\mathrm{I}}_{\mathrm{int}}(\mathbf{q})|s\rangle|^2|\langle u|\hat{H}^{\mathrm{II}}_{\mathrm{int}}|r\rangle|^2. \quad (54)$$

The correlation function $\tilde{K}_P(\mathbf{q},t)$ of the nucleus P was determined in the previous section. It can be modified by taking into account the difference between the center-of-mass Hamiltonian $\hat{H}^{e'}_P$ of the excited nucleus P and the similar Hamiltonian $\hat{H}^e_P$ of the nucleus in the ground state:

$$\tilde{K}_P(\mathbf{q},t)=\left\langle\left\langle\exp(i\mathbf{q}\hat{\mathbf{R}}_P(t))\exp\left(\frac{i\hat{H}^e_Pt}{\hbar}\right)\exp\left(\frac{-i\hat{H}^{e'}_Pt}{\hbar}\right)\times\exp(-i\mathbf{q}\hat{\mathbf{R}}_P(0))\right\rangle\right\rangle_P$$
$$\approx\left\langle\left\langle\exp\left(i\frac{\hat{H}^e_Pt}{\hbar}\right)\exp\left(-i\frac{\hat{H}^{e'}_Pt}{\hbar}\right)\right\rangle\right\rangle_P\times\langle\langle\exp(i\mathbf{q}\hat{\mathbf{R}}_P(t))\exp(-i\mathbf{q}\hat{\mathbf{R}}_P(0))\rangle\rangle_P$$
$$\approx\exp\left[i\left\langle\left\langle\frac{\hat{H}^e_P-\hat{H}^{e'}_P}{\hbar}\right\rangle\right\rangle_P t\right]\times\langle\langle\exp(i\mathbf{q}\hat{\mathbf{R}}_P(t))\exp(-i\mathbf{q}\hat{\mathbf{R}}_P(0))\rangle\rangle_P. \quad (55)$$

The evolution of the operators $\hat{\mathbf{R}}_P(t)$ in the last factor on the right of Eq. (55) is determined by the Hamiltonian $\hat{H}^e_P$.

Integrating Eq. (54) with respect to $\mathbf{p}_f$ and taking into account the relations

$$\int \frac{d\mathscr{E}_u}{2\pi\hbar}\exp\left(-i\frac{\mu\mathscr{E}_u}{\hbar}\right)=\delta(\mu)$$

and[25]

$$\Gamma=2\pi\frac{\Omega}{(2\pi\hbar)^3}\int\frac{p_f^2dp_f}{d\mathscr{E}_u}d\Omega_{p_f}|\langle u|\hat{H}^{\mathrm{II}}_{\mathrm{int}}|r\rangle|^2,$$

we obtain an equation for the total cross section of the Coulomb excitation due to interaction of all electric moments with electric field:

$$\sigma_t=\frac{\Gamma}{\hbar^3}\frac{1}{v_L}\int_0^\infty dt\int_0^\infty dt'\exp\left[-\frac{\Gamma(t+t')}{2}\right]\exp\left[i\frac{\mathscr{E}_s-\mathscr{E}_r}{\hbar}\times(t-t')\right]\int\frac{d\mathbf{q}}{(2\pi)^3}|\tilde{v}_0^{\mathrm{I}}(\mathbf{q})|^2\tilde{K}_T(-\mathbf{q},t-t')\times\tilde{K}_P(\mathbf{q},t-t')|\langle r|\hat{h}^{\mathrm{I}}_{\mathrm{int}}|s\rangle|^2. \quad (56)$$

Noting that

$$\frac{1}{\hbar^2}\int_0^\infty dt\int_0^\infty dt'\exp\left[-\frac{\Gamma}{2\hbar}(t+t')\right]F(t-t')=\frac{1}{\hbar\Gamma}\int_{-\infty}^\infty d\tau\,F(\tau)\exp\left(-\frac{\Gamma}{2\hbar}|\tau|\right)=\frac{2}{\hbar\Gamma}\mathrm{Re}\int_0^\infty d\tau\,F(\tau)\exp\left(-\frac{\Gamma}{2\hbar}\tau\right),$$

$$F(-\tau)=F^*(\tau),$$

we derive from Eq. (56)

$$\sigma_t=\frac{2}{\hbar^2v_L}\mathrm{Re}\int_0^\infty dt\,\exp\left(-\frac{\Gamma t}{2\hbar}\right)\exp\left(i\frac{\mathscr{E}_s-\mathscr{E}_r}{\hbar}t\right)\times\int_0^\infty\frac{d\mathbf{q}}{(2\pi)^3}|\tilde{v}_0^{\mathrm{I}}(\mathbf{q})|^2\tilde{K}_T\times(-\mathbf{q},t)\tilde{K}_P(\mathbf{q},t)|\langle r|\hat{h}^{\mathrm{I}}_{\mathrm{int}}(\mathbf{q})|s\rangle|^2. \quad (57)$$

In the case of the dipole $E1$-transition, when

$$|\langle r|\hat{h}^{\mathrm{I}}_{\mathrm{int}}(\mathbf{q})|s\rangle|^2=\frac{4\pi}{3}q^2\frac{\mathscr{B}(E1)}{e^2},$$

Eq. (17) directly follows from Eq. (57).

Now let us consider Eq. (53). In the previous section we demonstrated that for $t\rightarrow\infty$

$$\tilde{K}_T(\mathbf{q},t)\rightarrow\tilde{K}_T(\mathbf{q},\infty)=\mathrm{const},$$

and for $t\rightarrow 0$

$$\tilde{K}_T(\mathbf{q},t)\rightarrow\tilde{K}_T(\mathbf{q},0)\exp(i\Delta_Tt).$$

Substituting these expressions into Eq. (53), we obtain

$$\langle d\sigma_{t,i}\rangle=\frac{2\pi}{v_L\hbar}\frac{\Omega d\mathbf{P}_F}{(2\pi\hbar)^3}\frac{\Omega d\mathbf{p}_f}{(2\pi\hbar)^3}\int\frac{d\mathbf{q}}{(2\pi)^3}|\tilde{v}_0(\mathbf{q})|^2\times|\langle c|\tilde{\rho}_P(\mathbf{q})|a\rangle|^2|\langle r|\hat{h}^{\mathrm{I}}_{\mathrm{int}}(\mathbf{q})|s\rangle|^2|\langle u|\hat{H}^{\mathrm{II}}_{\mathrm{int}}(\mathbf{q})|r\rangle|^2\times\left\{\begin{array}{l}\tilde{K}_T(-\mathbf{q},\infty)\dfrac{\delta(\varepsilon_a-\varepsilon_c+\mathscr{E}_s-\mathscr{E}_u)}{\Gamma^2/4+(\varepsilon_a+\mathscr{E}_s-\mathscr{E}_r-\varepsilon_c)^2}\\ \tilde{K}_T(-\mathbf{q},0)\dfrac{\delta(\varepsilon_a-\varepsilon_c+\mathscr{E}_s-\mathscr{E}_u+\Delta_T)}{\Gamma^2/4+(\varepsilon_a+\mathscr{E}_s-\mathscr{E}_r-\varepsilon_c+\Delta_T)^2}\end{array}\right.. \quad (58)$$

If the P nucleus propagates along a crystal axis ($p_{\perp a}\equiv p_{\perp c}\equiv 0$) and the range of the impact parameters $B_{\min}\leqslant B_P\leqslant B_{\max}$ is bounded, then we should substitute into Eq. (58)

$$|\langle c|\tilde{\rho}_P(\mathbf{q})|a\rangle|^2=\frac{(2\pi)^3}{\Omega}\delta\left(\frac{p_{z0}-p_{zc}}{\hbar}-q_z\right)\times\frac{N}{sq_\perp^4}[(B_{\max}q_\perp)J_1(B_{\max}q_\perp)-(B_{\min}q_\perp)J_1(B_{\min}q_\perp)]^2. \quad (59)$$

Here N is the total number of channels in the target, $s=\pi(B^2_{\max}-B^2_{\min})$, and $J_1(x)$ is the Bessel function.

To conclude this section, note that in the equations for the distribution functions of the fragmentation products of the intermediate resonant state, the width of this state can be expressed through respective vertex functions that would allow us to go beyond the perturbation theory.[25]

5. CONCLUSION

This paper gives a more or less detailed analysis of equations for the total cross section σ_t for the Coulomb excitation of intermediate resonant states and for distribution functions of products of the fragmentation of these excited states when a nucleus propagates through a crystalline target. When the final states of the center-of-mass motion of the incident nucleus are not fixed, the proposed technique allows us to express the total cross section σ_t and differential cross sections of inclusive reactions in terms of integrals of correlation functions which describe the motion of the target nuclei, $\tilde{K}_T$, and the center of mass of the projectile nucleus, $\tilde{K}_P$. The functions $\tilde{K}_T$ have been studied in detail both theoretically and experimentally because the interaction of photons and slow neutrons with condensed media has been investigated for many years. The correlator $\tilde{K}_P$ of a nucleus propagating along a channel in a crystalline target can be calculated using simple models, or expressed through the density matrix or the quantum Green's function, which can be derived using powerful techniques.[6,21] Note that electronic degrees of freedom and resulting dynamic screening of Coulomb interaction between colliding nuclei can be similarly included in the scheme. Let us write the operator for the Coulomb interaction of the nucleus P with nuclei and electrons of the crystal target:

$$V(\mathbf{R}_P,\{\mathbf{r}_j\},\{\mathbf{R}_l\},\{\boldsymbol{\rho}_{lk}\})=\sum_l\sum_{j=1}^{Z_P}\frac{Z_Te^2}{|\mathbf{R}_P+\mathbf{r}_j-\mathbf{R}_c|}-\sum_l\sum_{j=1}^{Z_P}\sum_{k=1}^{Z_T}\frac{e^2}{|\mathbf{R}_P+\mathbf{r}_j-\mathbf{R}_l-\boldsymbol{\rho}_{kl}|}. \quad (60)$$

Here $\mathbf{R}_P$ are center-of-mass coordinates of the nucleus P, $\mathbf{r}_j$ are coordinates of the jth proton in the nucleus P with respect to its center of mass, $\mathbf{R}_l$ are coordinates of the center of mass of the lth lattice nucleus, and $\boldsymbol{\rho}_{kl}$ are coordinates of the kth electron in the lth lattice atom with respect to the center of mass of the lth nucleus. By performing the Fourier transform with respect to the spatial coordinates, one can separate the variables of the electronic subsystem of the crystal. Assuming that in the crystal Hamiltonian the contributions of the phonon and electron subsystems have already been separated, we repeat the calculations discussed in Sec. 3. As a result, we find out that the integrand in Eq. (17) should be multiplied by the factor

$$\left[1+\left(\frac{|\tilde{v}_0(\mathbf{q})|}{|\tilde{v}_0(\mathbf{q})|}\right)^2\tilde{K}_T^e(-\mathbf{q},t)\right],$$

where $\tilde{v}_0(\mathbf{q})$ and $\tilde{v}_0(\mathbf{q})$ are Fourier transforms of the screened and "bare" Coulomb potentials,

$$\tilde{K}_T^e(\mathbf{q},t)=\langle 0|\tilde{\rho}_e(\mathbf{q},t)\tilde{\rho}_e(\mathbf{q},0)|0\rangle-|\langle 0|\tilde{\rho}_e(\mathbf{q})|0\rangle|^2$$

is the Fourier transform of the correlator of electron density fluctuations in the crystal, $\tilde{\rho}_e(\mathbf{q})$ is the Fourier transform of the electron density operator, $|0\rangle$ is the wave function of the ground state of the electron subsystem in the target. In calculating this correlator, it is convenient to separate the contributions of valence electrons and electrons of inner shells.[26,27]

Thus, by introducing these two correlators within the framework of the proposed technique, we have separated two effects on the motion of the propagating nucleus caused by the lattice nuclei: a) shaping of the center-of-mass motion described by the function $\tilde{K}_P$; b) the effect on the shape of the resonance line described by the function $\tilde{K}_T$. We have demonstrated that conventional simple equations based on the method of equivalent photons and on the assumption that thermal motion of lattice nuclei is uncorrelated have a very limited domain of application. Namely, they yield correct results only when excited states have a long lifetime (in comparison to the typical times $t_T\sim\hbar/k_B\Theta\sim 10^{-13}-10^{-14}$ s and $t^*\sim\hbar/\Delta E\sim 10^{-16}-10^{-17}$ s, where ΔE is the energy difference between the levels of transverse motion in the channel). These equations may be useful in analyzing excitation of isomer states which are interesting as a possible route toward building γ-lasers.[11,12]

Let us briefly discuss limitations on experiments with resonant Coulomb excitation of nuclei due to energy losses of a charged particle to a target. First of all, note that in the channelling mode the average energy loss and its spread are several times lower than in an amorphous target.[27,28] In order to estimate whether the parameters of a resonance in $\sigma_t(E)$ can be derived from experimental data, let us assume that this is possible when the width of a resonance is comparable to the spread of the energy loss of a particle propagating through a target. Using the data by Esbensen *et al.*,[29] we have the following distribution of the loss rate for the ^{11}Li nucleus with an energy of 1.6 GeV/nucleon: from 2.4 MeV/mm on the channel axis to 5.0 MeV/mm near the channel wall in a germanium target. For a target thickness of 0.1 mm, the spread of the energy loss is less than 250 keV. Therefore, energy losses of particles propagating in the target do not contribute significant uncertainties to the width of the expected resonance in the spectrum of the neutron-rich ^{11}Li nucleus derived from experimental data (an estimate of the width of this resonance yields 100–200 keV).[30,31] Thinner targets and more intense beams of monochromatic charged particles are required for studies of narrow resonances through their direct excitation. When isomer states are populated by preliminary excitation to broader levels of higher energies,[12] the effect of energy losses from beam particles is not important. A consistent theoretical analysis of this multistage process, however, demands a more general formalism than that proposed in this paper.

The case of excited states with short lifetimes in comparison to t_T and t^* deserves a separate consideration. Investigation of the Coulomb dissociation of exotic nuclei like ^{11}Li, ^{11}Be, ^{14}Be, etc. interacting with lattice nuclei in the channelling mode allows one to exclude almost completely the purely nuclear mechanism with its inherent uncertainties.[10] Even when heavy nuclei collide and the Coulomb mechanism of excitation dominates, additional complications emerge since contributions due to multiquantum excitations of intermediate states should be separated.[32] Experiments with neutron-rich nuclei propagating in channels of crystalline targets are apparently free from these complications. On the other hand, some estimates[33] suggest that the efficiency of the Coulomb excitation in the channelling mode is a factor of 0.1–0.01 lower than in traditional experiments, i.e., the excitation cross section may be several tens of millibarn, whereas in traditional experiment these cross sections are 1–2 b. Investigation of structure and interactions of such exotic objects as neutron-rich nuclei also present some interest from a more general viewpoint as an example of a weakly bound system in a strong external field.[34–36]

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