Morphological stability of interfaces during phase transitions stimulated by Joule heating

E. A. Brener

Institute of Solid-State Physics, Russian Academy of Sciences, 142432 Chernogolovka, Moscow Region, Russia

D. E. Temkin

Institute of Physical Metallurgy and the Physics of Metals, I. P. Bardin Central Scientific-Research Institute of Ferrous Metallurgy, 107005 Moscow, Russia

(Submitted 6 June 1995)
Zh. Éksp. Teor. Fiz. **109**, 1349–1358 (April 1996)

The morphological stability of a moving transformation front under the conditions of Joule heating is considered. The treatment is restricted to the geometrically simplest situation, i.e., two parallel electrodes and a planar front between them. Two situations are considered under these conditions: longitudinal steady motion of the front, in which the direction of motion is parallel to the direction of the applied electric field, and transverse motion, in which these two directions are mutually perpendicular. Both fixed-current regime and fixed-voltage regimes are considered in each of these geometries. Either steadily moving heat waves or two-phase periodic structures are realized, depending on the conditions. In a specific material with a fixed ratio between the conductivities of the low-temperature (σ_1) and high-temperature (σ_2) phases, the longitudinal wave geometry ($\sigma_1 > \sigma_2$) or the transverse geometry ($\sigma_1 < \sigma_2$) is morphologically stable. © *1996 American Institute of Physics.* [S1063-7761(96)02104-X]

1. INTRODUCTION

When a sample consisting of a material which undergoes a phase transformation is heated by an electric current, several phenomena, such as thermal waves, two-phase structures, etc., can appear due to the difference between the conductivities of the phases and the amounts of Joule heat evolved in them. A review of such effects was given in Ref. 1 in the case of the normal-metal-superconductor phase transition. A review of the analogous effects observed in the macroscopic kinetics of chemical reactions accompanied by heat and mass transfer was given in Ref. 2.

When an electric current passes, the uniform temperature of a sample is determined by the balance between the Joule heat evolved and the heat dissipated. This temperature differs for the different possible phases of the material due to the difference between their conductivities. This raises several questions. For example, which of the phases is stably realized under given conditions of Joule heating, how does the transition from one state to another occur, and under what conditions is the existence of two-phase structures possible? The transition from one state to another involves the formation of nuclei of the new phase, their passage through the critical state, and their subsequent growth. The growth of these nuclei results from the motion of the thermal wave, which is simultaneously the phase-transformation wave. The motion of a planar thermal wave in a pure material was investigated in Refs. 3–7, and its motion in an alloy was considered in Ref. 7. Along with the homogeneous state of a sample, the existence of two-phase periodic structures is possible under certain conditions.[3,4,8] The drift of such structures caused by the Peltier effect was considered in Ref. 4. The experimental possibilities of investigating phase transitions stimulated by Joule heating are quite broad. This is because the geometry of the samples and the electrodes can be varied, as well as because the experiment can be carried out under the conditions of a fixed current or a fixed voltage. The theoretical aspects of the problem are accordingly diverse and complicated.

In this paper we wish to consider the morphological stability of a moving transformation front, i.e., the stability of the smooth form of the front toward perturbations. As far as we know, this problem has not been considered in the literature in reference to phase transformations stimulated by Joule heating. We shall restrict ourselves to the geometrically simplest situation, i.e., two parallel electrodes and a planar front between them. Two situations will be considered under these conditions: longitudinal steady motion of the front [Fig. 1(a)], in which the direction of motion is parallel to the direction of the applied electric field, and transverse motion [Fig. 1(b)], in which these two directions are mutually perpendicular. We shall consider both a fixed-current regime and a fixed-voltage regime in each of these geometries.

2. FORMULATION OF THE PROBLEM

In this paper we consider the steady motion and stability of a planar thermal wave which is simultaneously a phase boundary. The sample is a thin film cooled in an external medium with a temperature T_c. It is heated by passing an electric current between the two parallel electrodes. In the simplest situations the planar phase boundary is assumed to be oriented parallel or perpendicularly to the current. For simplicity, the thermal characteristics of both phases are as-

 1063-7761/96/040727-06$10.00

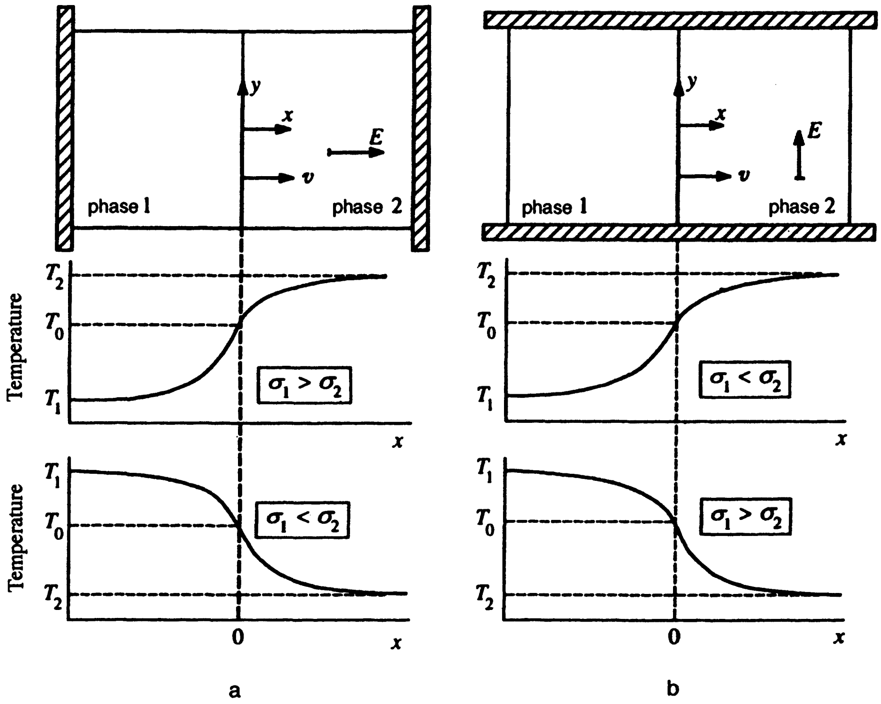

FIG. 1. Planar thermal wave in a two-dimensional sample heated by an electric current: a) longitudinal motion; b) transverse motion.

sumed to be identical. The phases have different conductivities, which, for simplicity, we assume to be independent of the temperature.

The current density **j** is related to the electric field **E** by Ohm's law

$$\mathbf{j}=\sigma\mathbf{E}=-\sigma\nabla U, \tag{1}$$

where σ is the conductivity and U is the electric potential, which satisfies the Laplace equation

$$\nabla^2 U=0. \tag{2}$$

We note that in writing the expression (1) for the current we have neglected the contribution to the current which is proportional to the temperature gradient and appears in a nonuniformly heated metal. This approximation is usually used[1] to simplify the problem and because the cross effects are small.

The density of the Joule heat evolved is given by the relation

$$Q=\mathbf{j}\cdot\mathbf{E}=j^2/\sigma=\sigma E^2=\sigma(\nabla U)^2. \tag{3}$$

The electric potential U and the normal component of the current j_n are continuous on the phase boundary:

$$U_1=U_2 \tag{4}$$

and

$$\sigma_1(\mathbf{n}\cdot\nabla U_1)=\sigma_2(\mathbf{n}\cdot\nabla U_2). \tag{5}$$

Here the subscripts 1 and 2 refer to the first and second phases, respectively, and **n** is the vector of a normal to the boundary. To be specific, we understand that phase 1 is the low-temperature phase, i.e., the phase which is thermodynamically stable at low temperatures ($T<T_0$) and metastable at high temperatures ($T>T_0$, where T_0 is the equilibrium temperature of phases 1 and 2).

The thermal fields (in the coordinate system associated with the moving transformation front) are described by the equation

$$\frac{1}{D}\frac{\partial T_i}{\partial t}=\nabla^2 T_i+\frac{v}{D}\frac{\partial T_i}{\partial x}+\frac{Q_i}{\kappa}-\frac{1}{h^2}(T_1-T_c),$$

$$i=1,2. \tag{6}$$

Here v is the steady-state velocity of the front; κ is the thermal conductivity; $D=\kappa/c$ is the thermal diffusivity; c is the specific heat; h is a coefficient with the dimensions of length, which characterizes the rate of heat transfer between the sample and the surrounding medium. This heat transfer is assumed to be linear, i.e., proportional to the temperature difference between the sample and the medium. The Gibbs–Thomson condition holds on the phase boundary:

$$T_1|_{\text{int}}=T_2|_{\text{int}}=T_0+(\gamma T_0/L)K. \tag{7}$$

Here K is the local curvature of the boundary, which we assume to be negative for a front bulging toward phase 2; γ is the surface energy of the interface, which is assumed to be isotropic; L is the latent heat of the transformation of phase 2 into phase 1. In addition, a heat balance is maintained on the interface, and with consideration of the Peltier effect the condition defining it has the form

$$\frac{L}{c}v_n+\frac{\Pi}{c}j_n=D\left(\frac{\partial T_1}{\partial n}-\frac{\partial T_2}{\partial n}\right). \tag{8}$$

Here Π is the Peltier coefficient for the boundary between phases 1 and 2. The sign in front of the term with Π in (8) was chosen so that when a current passes from phase 1 to phase 2, i.e., when $j_n>0$ holds, Peltier heat is evolved for $\Pi>0$ and is absorbed for $\Pi<0$.

Below we shall consider the steady motion of a planar front and its stability in two situations: when the motion of the front is parallel to the current [longitudinal motion, Fig. 1(a)] and when the motion of the front is perpendicular to the electric field (transverse motion, Fig. 1(b).

3. LONGITUDINAL MOTION OF THE THERMAL WAVE

In the geometry described in Fig. 1(a) the current density far from the interface in both phases is the same, j. Also, the temperatures far from the front are determined by the balance between the last two terms in Eq. (6), which, with consideration of (3), gives

$$T_1=T_c+j^2h^2/\kappa\sigma_1, \qquad T_2=T_c+j^2h^2/\kappa\sigma_2. \tag{9}$$

In the case of a steadily moving front with velocity v, the distribution of the temperature $T_{i,0}$ which satisfies the steady-state equation (6) and the condition (7) is described by the expressions

$$T_{1,0}(x)=T_1+(T_0-T_1)\exp(s_1x),$$
$$s_1=(V^2+1)^{1/2}-V, \tag{10}$$

$$T_{2,0}(x)=T_2+(T_0-T_2)\exp(-s_2x),$$
$$s_2=(V^2+1)^{1/2}+V. \tag{11}$$

Here and in what follows, all the lengths are measured in units of h, and $V=vh/2D$ is the dimensionless velocity. In deriving (10) and (11) we took into account that in the case of a planar front the current density j is constant across the sample. When the expressions (10) and (11) are substituted into the heat-balance condition (8), we obtain the equation for the velocity V of the front:

$$2V+P=(\Delta_1-\Delta_2)\sqrt{V^2+1}-V(\Delta_1+\Delta_2). \tag{12}$$

Here we have introduced the dimensionless temperature drops

$$\Delta_1=(T_0-T_1)c/L, \qquad \Delta_2=(T_2-T_0)c/L \tag{13}$$

and the dimensionless Peltier heat

$$P=\Pi hj/LD. \tag{14}$$

Since Δ_1 and Δ_2 depend on j^2 [see (9)], the rate at which V increases without consideration of the Peltier effect does not depend on j. Consideration of the Peltier effect introduces a dependence of the velocity on the direction in which the current flows. The situation in which each of the phases in the thermal wave is in the temperature range where it is thermodynamically stable, i.e., in which $T_1<T_0<T_2$, is most stable. For the longitudinal motion of the thermal wave under consideration, this is possible only for $\sigma_1>\sigma_2$ $(T_1<T_2)$ and for currents such that

$$\sigma_2<j^2h^2/\kappa(T_0-T_c)<\sigma_1.$$

The reverse situation, $T_1>T_0>T_2$ is most unstable in the thermodynamic sense. The investigation of the morphological stability, to which we now proceed, also demonstrates the existence of instability for $\sigma_1<\sigma_2$ when $T_1>T_2$.

To investigate the morphological stability of the steady-state solution, we represent the shape of the front in the form

$$x=\xi(y,\tau)=\delta\exp(\Omega\tau+iky). \tag{15}$$

We recall that all the lengths are measured in units of h, and the dimensionless time $\tau=Dt/h^2$. In (15) k is the wave number, and Ω is the growth rate of the perturbation. In the linear approximation the curvature of the front is given by the expression

$$K\simeq\partial^2\xi/\partial y^2=-k^2\xi. \tag{16}$$

The potential field satisfying the Laplace equation (2) is represented in the form

$$U_1=-jhx/\sigma_1+A_1\delta\exp(\Omega\tau+iky+|k|x), \tag{17}$$

$$U_2=-jhx/\sigma_2+A_2\delta\exp(\Omega\tau+iky-|k|x). \tag{18}$$

From the linearized conditions (4) and (5) we find the constants A_1 and A_2:

$$A_1=-\frac{jh}{\sigma_1}\frac{(\sigma_1-\sigma_2)}{(\sigma_1+\sigma_2)}, \qquad A_2=\frac{jh}{\sigma_2}\frac{(\sigma_1-\sigma_2)}{(\sigma_1+\sigma_2)}. \tag{19}$$

We substitute the fields (17) and (18) into the expression (3) for the Joule heat, $Q_i=\sigma_i(\nabla U_i)^2$. After linearization with respect to ξ we have

$$Q_1(x,y,\tau)=\frac{j^2}{\sigma_1}+\frac{2j^2}{\sigma_1}\frac{(\sigma_1-\sigma_2)}{(\sigma_1+\sigma_2)}|k|\exp(|k|x)\xi(y,\tau,), \tag{20}$$

$$Q_2(x,y,\tau)=\frac{j^2}{\sigma_2}+\frac{2j^2}{\sigma_2}\frac{(\sigma_1-\sigma_2)}{(\sigma_1+\sigma_2)}|k|\exp(-|k|x)\xi(y,\tau,). \tag{21}$$

Representing the temperature field in the form

$$T_i(x,y,\tau)=T_{i,0}+F_i(x)\xi(y,\tau) \tag{22}$$

and substituting (20)–(22) into Eq. (6), we obtain

$$F_1''+2VF_1'-(k^2+\Omega+1)F_1=\frac{2j^2h^2}{\kappa\sigma_1}\frac{(\sigma_1-\sigma_2)}{(\sigma_1+\sigma_2)}|k|\exp(|k|x), \tag{23}$$

$$F_2''+2VF_2'-(k^2+\Omega+1)F_2=\frac{2j^2h^2}{\kappa\sigma_2}\frac{(\sigma_1-\sigma_2)}{(\sigma_1+\sigma_2)}|k|\times\exp(-|k|x). \tag{24}$$

The solution of Eqs. (23) and (24) has the form

$$F_1=B_1\exp(q_1x)+C_1\exp(|k|x), \tag{25}$$

$$F_2=B_2\exp(-q_2x)+C_2\exp(-|k|x), \tag{26}$$

where the coefficients B_1 and B_2 are still arbitrary in this stage, and C_1 and C_2 are found from the inhomogeneous equations (23) and (24). In Eqs. (25) and (26)

$$q_1=\sqrt{V^2+k^2+\Omega+1}-V,$$
$$q_2=\sqrt{V^2+k^2+\Omega+1}+V. \tag{27}$$

The solutions (25)–(27) are valid provided $1+k^2+\Omega>0$. Otherwise, there is a continuous stable $(\Omega<0)$ spectrum. Next, we linearize Eqs. (7) and (8), and from the three equations obtained we find B_1, B_2, and the equation for the spectrum $\Omega(k)$. In the linear approximation the quantities v_n and j_n, appearing in Eq. (8) have the form

$$v_n \simeq v + \frac{D}{h}\frac{\partial \xi}{\partial \tau} = v + \frac{D\Omega}{h}\xi(y,\tau), \tag{28}$$

$$j_n \simeq -\frac{\sigma_i}{h}\frac{\partial U_i}{\partial x}\bigg|_{x=0} = j + j\frac{\sigma_1-\sigma_2}{\sigma_1+\sigma_2}|k|\xi(y,\tau). \tag{29}$$

After some transformations we obtain the dispersion relation sought

$$-\Omega - P\frac{\alpha-1}{\alpha+1}|k| = \Delta_2 s_2(q_2-s_2) + \Delta_1 s_1(q_1-s_1) + d_0k^2(q_1+q_2) + (q_1-|k|) \times \frac{2\alpha(\Delta_1+\Delta_2)|k|}{(1+\alpha)(1+\Omega-2|k|V)} + (q_2 - |k|)\frac{2(\Delta_1+\Delta_2)|k|}{(1+\alpha)(1+\Omega+2|k|V)}. \tag{30}$$

Here we have set $\alpha=\sigma_1/\sigma_2$, and $d_0=\gamma T_0 c/L^2 h$ is the dimensionless capillary length. We recall that s_1 and s_2 are defined in Eqs. (10) and (11), Δ_1 and Δ_2 are defined in (13), q_1 and q_2 are defined in (27), and the velocity V is found from (12).

In the case of small velocities, $V \ll 1$, $\Omega \ll 1$, and not excessively large k ($|k|V \ll 1$), the relation (30) is simplified significantly:

$$\Omega = -(\sqrt{1+k^2}-1)[(\Delta_1+\Delta_2) - V(\Delta_1-\Delta_2)] - 2(\Delta_1 + \Delta_2)[|k|\sqrt{1+k^2}-k^2] - 2d_0k^2\sqrt{1+k^2} - P\frac{\alpha-1}{\alpha+1}|k|. \tag{31}$$

In this limit ($V \ll 1$) the velocity is assigned by the expression

$$V = \frac{\Delta_1-\Delta_2-P}{2+\Delta_1+\Delta_2}. \tag{32}$$

In this case, for $\sigma_1>\sigma_2$ (when $\alpha>1$, $T_1<T_2$, and $\Delta_1+\Delta_2=(T_2-T_1)c/L>0$ hold), all the terms on the right-hand side of Eq. (31) are negative (except for the term with P and the small term with V) and thus correspond to morphological stability. At the same time, the influence of the Peltier effect on the morphological stability depends on the direction of the current. Thus, instability can appear due to the Peltier effect. This is easily seen from the behavior of Ω at small k:

$$\Omega \simeq -2|k|(\Delta_1+\Delta_2)\left[1+\frac{\Pi\sigma_1\sigma_2}{2jh(\sigma_1+\sigma_2)}\right]. \tag{33}$$

The condition $\Omega>0$ corresponds to instability. This a situation arises at sufficiently small currents for one of the directions of the current. We note that there is, in fact, no divergence when $j=0$ holds, since the multiplier $(\Delta_1+\Delta_2)$ in (33) is proportional to j^2. When $\sigma_1<\sigma_2$ holds, instability appears at small k even in the absence of the Peltier effect. This is seen from Eq. (33), since in this case we have $T_1>T_2$ and, therefore, $(\Delta_1+\Delta_2)<0$. When k increases, the spectrum (31) becomes stable for $k>k_0$ due to the stabilizing influence of the surface tension. We find k_0 from the condition $\Omega(k_0)=0$ without consideration of the Peltier effect:

$$|k_0| \simeq |\Delta_1+\Delta_2|/d_0, \qquad d_0/|\Delta_1+\Delta_2| \gg 1, \tag{34}$$
$$|k_0| \simeq \sqrt{|\Delta_1+\Delta_2|/(2d_0)}, \qquad d_0/|\Delta_1+\Delta_2| \ll 1.$$

Thus, in the case of a longitudinal thermal wave, the greater conductivity of the low-temperature phase ($\sigma_1>\sigma_2$) promotes both thermodynamic stability (under which $T_1<T_0$ and $T_2>T_0$ hold) and morphological stability of the phase boundary.

In the fixed-current regime the thermal wave moves with a constant finite velocity, which depends on the current. At the same time, a two-phase periodic structure appears in the longitudinal geometry in the fixed-voltage regime. When the period of the structure is large enough to neglect the thermal interaction between the individual phase boundaries in the structure, the condition for steady motion, as can easily be shown, has the form

$$\Delta_1=\Delta_2. \tag{35}$$

Then the two-phase structure moves with a velocity proportional to the Peltier coefficient

$$V=-P/(2+\Delta_1+\Delta_2), \tag{36}$$

and in a direction which depends on the direction of the current. The condition (35) determines the fractions of the phases in the two-phase structure under consideration. In fact, when the potential difference $U=-El$, where l is the length of the sample [Fig. 1(a)], is fixed, the current j, on which Δ_1 and Δ_2 depend, is determined by the fractions of the phases η_2 and $\eta_1=1-\eta_2$, according to the relation

$$j=-\frac{E\sigma_1\sigma_2}{\eta_1\sigma_2+\eta_2\sigma_1}.$$

Taking into account this relation, as well as (9) and (13), from Eq. (35) we find the fraction η_2 for a fixed value of E

$$\eta_2=\left[\sigma_2-\left[\frac{E^2h^2\sigma_1\sigma_2(\sigma_1+\sigma_2)}{2\kappa(T_0-T_c)}\right]^{1/2}\right](\sigma_2-\sigma_1)^{-1}. \tag{37}$$

A two-phase periodic structure exists in the range of values of E in which $0<\eta_2<1$. We recall that Eq. (37) is valid in the limit of large distances between the phase boundaries, which are significantly larger than the characteristic length h. The morphological stability of each of the fronts in this two-phase structure is determined by the results obtained above when the additional conditions (35) and (36) are satisfied.

4. TRANSVERSE MOTION OF THE THERMAL WAVE

When the geometry shown in Fig. 1(b) is studied, the field E far from the interface in both phases is the same. The corresponding temperatures far from the front are specified by the relations

$$T_1=T_c+\sigma_1E^2h^2/\kappa, \qquad T_2=T_c+\sigma_2E^2h^2/\kappa. \tag{38}$$

The steady-state velocity of an unperturbed planar thermal wave is determined precisely as in the preceding section [see Eq. (12)] with the one difference that there is no Peltier effect, since the current flows parallel to the front:

$$2V=(\Delta_1-\Delta_2)\sqrt{V^2+1}-V(\Delta_1+\Delta_2). \tag{39}$$

We shall not repeat the lengthy analysis of the stability, which is completely analogous to the analysis performed in the preceding section. We note only the following circumstance. In the zeroth approximation, i.e., when the motion of the planar front is steady, the current j_n through the interface is equal to zero. When this current is calculated in the linear approximation with respect to the perturbation we have

$$-j_n=\sigma_i\frac{\partial U_i}{\partial n}\simeq\sigma_i\left(\frac{\partial U_i}{\partial x}-\frac{\partial\xi}{\partial y}\frac{\partial U_i}{\partial y}\right)_{x=0}, \qquad i=1,2. \tag{40}$$

Finally, the expression analogous to (30) for the spectrum in this case has the form

$$-\Omega-i\tilde{P}\frac{2\sqrt{\alpha}}{\alpha+1}k=\Delta_2 s_2(q_2-s_2)+\Delta_1 s_1(q_1-s_1)+d_0k^2(q_1+q_2)+(q_1-|k|)\frac{2\alpha(\Delta_1+\Delta_2)|k|}{(1+\alpha)(1+\Omega-2|k|V)}+(q_2-|k|)\frac{2(\Delta_1+\Delta_2)|k|}{(1+\alpha)(1+\Omega+2|k|V)}, \tag{41}$$

and instead of (31), in the same approximation we have

$$\Omega=-(\sqrt{1+k^2}-1)[(\Delta_1+\Delta_2)-V(\Delta_1-\Delta_2)]-2(\Delta_1+\Delta_2)[|k|\sqrt{1+k^2}-k^2]-2d_0k^2\sqrt{1+k^2}-i\tilde{P}\frac{2\sqrt{\alpha}}{\alpha+1}k. \tag{42}$$

Here all the notation is the same as in Sec. 3, except

$$\tilde{P}=\frac{\Pi\sqrt{\sigma_1\sigma_2}Eh}{LD}. \tag{43}$$

Comparing (41) and (42) with (30) and (31), respectively, we see a difference in the influence of the Peltier effect. In the case of a longitudinal wave this effect changed only the real part of Ω, signifying enhancement or weakening of the stability. In the case of a transverse wave the Peltier effect causes the appearance of an imaginary contribution proportional to k in the spectrum $\Omega(k)$. This means that the perturbation not only grows (in the case of instability) or decays (in the case of stability), but also moves parallel to the front with a velocity V_t equal to

$$V_t=\frac{\mathrm{Im}\Omega}{k}=-\tilde{P}\frac{2\sqrt{\alpha}}{\alpha+1}. \tag{44}$$

As we have already noted, the stability or instability of a thermal wave is determined mainly by the sign of the quantity $\Delta_1+\Delta_2$. For example, for small k it follows from (42) that

$$\mathrm{Re}\Omega\simeq-2(\Delta_1+\Delta_2)|k|. \tag{45}$$

Therefore, stability corresponds to

$$(\Delta_1+\Delta_2)=E^2h^2(\sigma_2-\sigma_1)/DL>0.$$

In contrast to the case of a longitudinal wave, this situation is realized when $\sigma_1<\sigma_2$. Thus, in the case of a transverse wave, a smaller conductivity for the low-temperature phase ($\sigma_1<\sigma_2$) promotes both the thermodynamic stability of the phases and the morphological stability of the phase boundaries. In the opposite case ($\sigma_1>\sigma_2$), tendencies for both thermodynamic instability of the phases and morphological instability of the interface are displayed.

In the fixed-voltage regime a transverse thermal wave undergoes steady motion with a finite velocity, which depends on the voltage [see (39)]. In the fixed-current regime a stationary periodic structure appears. The Peltier effect, which caused the two-phase structure to move in the longitudinal geometry, is not manifested here, since there is no current component normal to the phase boundary. In the case of a fixed current with a mean density j, the field intensity E is assigned by the expression

$$E=j/(\eta_1\sigma_1+\eta_2\sigma_2).$$

The steady motion of the two-phase structure observed for sufficiently large distances between the fronts, which greatly exceed h, is specified, as before, by the condition (35) ($\Delta_1=\Delta_2$). Then for the fraction of phase 2, η_2, we find

$$\eta_2=\left[\left[\frac{j^2h^2(\sigma_1+\sigma_2)}{2\kappa(T_0-T_c)}\right]^{1/2}-\sigma_1\right](\sigma_2-\sigma_1)^{-1}. \tag{46}$$

This two-phase structure exists in the range of values of j in which $0<\eta_2<1$.

5. CONCLUSIONS

We have considered the morphological stability of a planar front in longitudinal and transverse geometries both in the fixed-current regime and in the fixed-voltage regime.

1. Longitudinal motion; fixed-current regime

In this case there is a regime of steady motion of the thermal wave, and the temperature in each of the phases far from the wave front is higher, the smaller is its conductivity ($T_1<T_2$ when $\sigma_1>\sigma_2$). Such a planar front is stable when the conductivity of the low-temperature phase σ_1 is greater than the conductivity of the high-temperature phase σ_2, and for $\sigma_1<\sigma_2$ it is accordingly unstable against perturbations with a wavelength exceeding a certain critical value.

2. Longitudinal motion; fixed-voltage regime

In this case there are solutions corresponding to drifting periodic structures in a certain range of applied voltages. The drift velocity is proportional to the Peltier coefficient, and the drift direction depends on the applied field intensity. The morphological stability was considered in the limit where the distances between the fronts in the two-phase structure is much greater than the thermal length h. As in case 1, the fronts are then stable for $\sigma_1>\sigma_2$ and unstable for $\sigma_1<\sigma_2$.

3. Transverse motion; fixed-voltage regime

In this case there is a regime in which the thermal wave moves at a constant rate, and the temperature in each of the phases far from the wave front is higher, the greater is its conductivity ($T_1 < T_2$ when $\sigma_1 < \sigma_2$). The planar front is stable for $\sigma_1 < \sigma_2$ and unstable for $\sigma_1 > \sigma_2$ in the long-wavelength part of the spectrum. The influence of the Peltier effect causes the perturbation not only to decay or grow (in the case of instability), but also to drift parallel to the front with a velocity proportional to the Peltier coefficient in a direction which depends on the direction of the applied field.

4. Transverse motion; fixed-current regime

In this case there are solutions corresponding to stationary periodic structures in a definite range of current densities. The morphological stability of the individual phase boundaries in this case is the same as in case 3, i.e., the boundaries are stable for $\sigma_1 < \sigma_2$ and unstable for $\sigma_1 > \sigma_2$.

In a specific material with a fixed relationship between the conductivities of the low-temperature (σ_1) and high-temperature (σ_2) phases, the longitudinal wave geometry (for $\sigma_1 > \sigma_2$) or the transverse geometry (for $\sigma_1 < \sigma_2$) is morphologically stable.

This work was performed with partial support from the Russian Fund for Fundamental Research (Project 95-5-2a/41), the International Science Foundation (Grant No. J3F100), and the Russian Government.

[1] A. V. Gurevich and R. G. Mints, Rev. Mod. Phys. **59**, 941 (1987).
[2] A. G. Merzhanov and É. N. Rumanov, Usp. Fiz. Nauk **151**, 553 (1987) [Sov. Phys. Usp. **30**, 293 (1987)].
[3] B. Fisher, J. Phys. C: Solid State Phys. **8**, 2072 (1975).
[4] B. Fisher, J. Phys. C: Solid State Phys. **9**, 1201 (1976).
[5] Yu. D. Kalafati, I. A. Serbinov, and L. A. Ryabova, JETP Lett. **29**, 583 (1979).
[6] A. G. Merzhanov, V. A. Raduchev, and É. N. Rumanov, Dokl. Akad. Nauk SSSR **253**, 330 (1980) [Sov. Phys. Dokl. **25**, 565 (1980)].
[7] D. E. Temkin, Kristallografiya **34**, 807 (1989) [Sov. Phys. Crystallogr. **34**, 483 (1989)].
[8] D. E. Temkin, *Abstracts of the 7th All-Union Conference on Crystal Growth* [in Russian], Moscow, (1988), Vol. 3, p. 69.

Translated by P. Shelnitz