Acousto-electron effects in layered conductors

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A theoretical study of elastic waves propagation in layered conductors under external magnetic field is presented. The quasi-two-dimensional nature of the electronic spectrum in such conductors leads to different attenuation lengths of acoustic waves with polarization parallel and perpendicular to the layers and to some specific effects, such as a magneto-acoustic resonance without a drift of charge carriers along the wave vector and an orientational effect, namely, sharp maxima on the curve of sound attenuation versus angle between the magnetic field and normal to the layers.

The interest in research of low-dimensional conductors is closely related to the search for superconducting materials with high critical parameters. Most of new superconductors synthesized over recent years are layered structures with a marked anisotropy in the electric conductivity in the normal (not superconducting) state, i.e., the in-plane conductivity is significantly higher than along the normal **n** to the layers. The discovery of Shubnikov–de-Haas oscillations of the magnetoresistance in organic superconductors (see Refs. 1 and 2 and citations therein) and metallic conductivity in most of them indicate that well-developed models of electric current in metals are also applicable to layered conductors. But the correctness of introducing quasi-particles similar to conduction electrons in metals and the lifetime of quasiparticles with a charge e and an energy ε close to the Fermi energy ε_F can be determined only by investigating the physical properties of layered conductors in the normal state, specifically, by solving an inverse problem to derive the shape of the Fermi surface $\varepsilon(\mathbf{p})=\varepsilon_F$ from experimental data. The achievements in studies of electronic spectra of metals are to a great extent due to experiments on magneto-acoustic effects in a strong magnetic field when the radius of curvature r of carrier trajectories is much smaller than their mean free path l.

The propagation of acoustic waves in layered conductors with a quasi-two-dimensional spectrum of carriers has some specific features, especially when the maximum electron drift velocity across the layers, $v_z=\mathbf{v}\cdot\mathbf{n}$, is comparable to or smaller than the sound velocity s.[3] It is also known that the temperature of the superconducting transition of one modification of tetrathiafulvalene, β-$(ET)_2IBr_2$, is about a factor of three higher under strain.[4] Therefore the response of the electron system to the crystal strain is undoubtedly very interesting.

We shall consider propagation of an acoustic wave with a frequency ω in conductors with a quasi-two-dimensional electron spectrum

$$\varepsilon(\mathbf{p})=\sum_{n=0}^{\infty}\varepsilon_n(p_x,p_y)\cos\frac{anp_z}{\hbar} \tag{1}$$

and analyze the damping of this wave due to the interaction between charge carriers and coherent phonons. In the general case, terms proportional to $\varepsilon_n'(p_x,p_y)\sin(anp_z/\hbar)$, where $\varepsilon_n'(-p_x,-p_y)=-\varepsilon_n'(p_x,p_y)$, should be added to the right-hand side of Eq. (1). However, these terms do not change radically the results presented below, although they complicate the calculations considerably. We assume that the anisotropy of the spectrum described by Eq. (1) is not very large and also that $A_1=\eta A_0\ll A_0$ and $A_{n+1}\ll A_n$, where A_n is the maximum of the function $\varepsilon_n(p_x,p_y)$ on the Fermi surface. Here $\mathbf{p}$ is the quasi-momentum of conduction electrons and a is the separation between adjacent layers.

The main cause of acoustic wave damping due to conduction electrons at a temperature below the Debye temperature is the resistive dissipation of energy of the electromagnetic wave generated by sound[5,6] and so-called deformational absorption due to renormalization of the carrier energy $\delta\varepsilon$ when the crystal is deformed.[7] When the length of a low-amplitude acoustic wave and the conductor dimension d are much smaller than the attenuation length $l_{\rm at}$, only terms linear in the strain tensor $u_{ij}=\partial u_i/\partial x_j$ need be retained, and the carrier energy renormalization under strain is

$$\delta\varepsilon=\lambda_{ij}(\mathbf{p})u_{ij}, \tag{2}$$

where the components of the deformation potential tensor λ_{ij} for electrons on the Fermi surface are of the same order as the Fermi energy.

The electric field $\mathbf{E}$ generated by the acoustic wave is derived from the Maxwell equation

$$\nabla\times\nabla\mathbf{E}-\frac{\omega^2}{c^2}\mathbf{E}=\frac{4\pi i\omega}{c^2}\mathbf{j}, \tag{3}$$

and from the condition of conductor neutrality, which is equivalent to the continuity of electric current, i.e.,

$$\nabla\cdot\mathbf{j}=0. \tag{4}$$

Equations (3) and (4), combined with the elasticity equation

$$-\omega^2\rho u_i=\lambda_{ijlm}\frac{\partial u_{em}}{\partial x_j}+F_i \tag{5}$$

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form the complete system of equations of the problem if the electric current density $\mathbf{j}$ and the force $\mathbf{F}$ applied to the lattice from the electron system driven by the acoustic wave are expressed in terms of the nonequilibrium term $-\psi\partial f_0/\partial\varepsilon$ added to the Fermi distribution function f_0:

$$\mathbf{j}=\frac{2}{(2\pi\hbar)^3}\frac{eH}{c}\int d\varepsilon\,\delta(\varepsilon-\varepsilon_F)\int dp_H\int dte\mathbf{v}\psi$$

$$\equiv\langle e\mathbf{v}\psi\rangle. \tag{6}$$

Here ρ and λ_{ijlm} are the density and elastic tensor of the crystal, $p_H=\mathbf{p}\cdot\mathbf{H}/H$, c is the speed of light, and t is the period of the electron gyation in the magnetic field according to the equation of motion

$$\frac{\partial\mathbf{p}}{\partial t}=\frac{e}{c}[\mathbf{vH}]. \tag{7}$$

If the lattice strain is small, the force due to electrons acting on the oscillating lattice is[1)]

$$F_i=\frac{1}{c}[\mathbf{jH}]_i+\frac{m}{e}i\omega j_i+\frac{\partial}{\partial x_k}\langle\Lambda_{ik}\psi\rangle, \tag{8}$$

where $\Lambda_{ik}(\mathbf{p})=\lambda_{ik}(\mathbf{p})-\langle\lambda_{ik}(\mathbf{p})\rangle/\langle l\rangle$, m is the electron mass.[8,9]

The function ψ satisfies the Boltzmann kinetic equation

$$\frac{\partial\psi}{\partial t}+\mathbf{v}\frac{\partial\psi}{\partial\mathbf{r}}+\left(\frac{1}{\tau}-i\omega\right)\psi=-i\omega\Lambda_{ij}u_{ij}+e\mathbf{v}\mathscr{E}\equiv G, \tag{9}$$

$$\mathscr{E}=\mathbf{E}-\frac{i\omega}{c}[\mathbf{uH}]+\frac{m\omega^2}{e}\mathbf{u}. \tag{10}$$

Equation (9) is linear in the small perturbation of the electron distribution function, and the relaxation time approximation is used in the collision integral: $w_{\mathrm{col}}=(f_0-f)/\tau$. Since the acoustic wave is harmonic, time differentiation is equivalent to multiplication by $-i\omega$.

For $d<l$ acousto-electron effects are very sensitive to the conditions of electron reflection from the crystal boundary,[11–13] which are included in the boundary conditions of Eq. (3). In bulk crystals ($l\ll d$), however, the boundary conditions are not essential. The condition $l\ll d\ll l_{\mathrm{at}}$ is quite feasible if the frequency of electron collisions in the volume, τ^{-1}, is much higher than the acoustic frequency. The solution of the kinetic equation (9) in a bulk conductor can be expressed as

$$\psi=\int_{-\infty}^{t}dt'\exp[\nu(t'-t)]G(t',p_H,\mathbf{r}+\mathbf{r}(t',p_H)-\mathbf{r}(t,p_H)), \tag{11}$$

where $\nu=-i\omega+1/\tau$.

Let us consider an acoustic wave propagating in plane in the x direction and, using the Fourier method, derive from Eqs. (3)–(5) a set of equations for the Fourier components of the electric field $\mathscr{E}_i(k)$ and ion displacements $u_i(k)$:

$$\frac{4\pi i\omega}{c^2}j_\alpha(k)=k^2E_\alpha(k)-\frac{\omega^2}{c^2}E_\alpha(k),\quad \alpha=y,z,$$

$$j_x(k)=0, \tag{12}$$

$$-\omega^2\rho u_i(k)=-\lambda_{ixlx}k^2u_l+\frac{im\omega}{e}j_i(k)+\frac{1}{c}[\mathbf{j}(k)\mathbf{H}]_i$$

$$+ik\langle\Lambda_{ix}\psi\rangle.$$

Using the kinetic equation solution (11), we can conveniently express the parameters $j_i(k)=\langle ev_i\psi(k)\rangle$ and $\langle\psi(k)\Lambda_{ix}\rangle$, which characterize the system response to the acoustic wave, in the form

$$j_i(k)=\sigma_{ij}(k)\mathscr{E}_j(k)+a_{ij}(k)k\omega u_j(k),$$

$$\langle\psi(k)\Lambda_{ix}\rangle=b_{ij}(k)\mathscr{E}_j(k)+c_{ij}(k)k\omega u_j(k), \tag{13}$$

where the Fourier components of the conductivity tensor and of acousto-electric coupling tensors are

$$\sigma_{ij}(k)=e^2\left\langle v_i(t)\int_{-\infty}^{t}dt'\mathscr{F}(t,t',k)v_j(t')\right\rangle,$$

$$a_{ij}(k)=e\left\langle v_i(t)\int_{-\infty}^{t}dt'\mathscr{F}(t,t',k)\Lambda_{jx}(t')\right\rangle,$$

$$b_{ij}(k)=e\left\langle \Lambda_{ix}(t)\int_{-\infty}^{t}dt'\mathscr{F}(t,t',k)v_j(t')\right\rangle,$$

$$c_{ij}(k)=\left\langle \Lambda_{ix}(t)\int_{-\infty}^{t}dt'\mathscr{F}(t,t',k)\Lambda_{jx}(t')\right\rangle, \tag{14}$$

where

$$\mathscr{F}(t,t',k)=\exp\{\nu(t'-t)+ik[x(t')-x(t)]\}.$$

By substituting the expressions in Eq. (13) into the equation system (12), we obtain a system of linear algebraic equations in $u_i(k)$ and $\mathscr{E}_i(k)$.

After the inverse Fourier transform of these solutions, the problem of the electric and strain fields in the conductor will be solved completely. The acoustic wave damping factor can be derived from the formula

$$\Gamma=\mathrm{Im}\,k, \tag{15}$$

where k is the root of the equation system determinant.

We are interested in the acoustic wave absorption as a function of the absolute value and direction of magnetic field $\mathbf{H}=(0,H\sin\theta,H\cos\theta)$ orthogonal to the acoustic wave vector.

Longitudinal acoustic wave

If the acoustic wave polarization is aligned with its wave vector ($\mathbf{u}=(u,0,0)$), the equation system (12) after the exclusion of $\mathscr{E}_x$ takes the form

$$\left(\tilde{a}_{yx}k\omega+\frac{k^2c^2-\omega^2}{4\pi c}H_z\right)u+\left(\tilde{\sigma}_{yy}-\frac{k^2c^2-\omega^2}{4\pi i\omega}\right)\mathscr{E}_y$$

$$+\tilde{\sigma}_{yz}\mathscr{E}_z=0,$$

$$\left(\tilde{a}_{zx}k\omega-\frac{k^2c^2-\omega^2}{4\pi c}H_y\right)u+\tilde{\sigma}_{zy}\mathscr{E}_y$$

$$+\left(\tilde{\sigma}_{zz}-\frac{k^2c^2-\omega^2}{4\pi i\omega}\right)\mathscr{E}_z=0, \tag{16}$$

$$(\omega^2-s_l^2k^2)\rho u+\left[ik\tilde{c}_{xx}+\frac{1}{c}(\tilde{a}_{yx}H_z-\tilde{a}_{zx}H_y)k\omega u\right]$$
$$+\left[ik\tilde{b}_{xy}+\frac{1}{c}(\tilde{\sigma}_{yy}H_z-\tilde{\sigma}_{zy}H_y)\right]\mathscr{E}_y$$
$$+\left[ik\tilde{b}_{xz}+\frac{1}{c}(\tilde{\sigma}_{yz}H_z-\tilde{\sigma}_{zz}H_y)\right]\mathscr{E}_z=0,$$

where

$$\rho s_l^2=\lambda_{xxxx},\quad \tilde{\sigma}_{\alpha\beta}=\sigma_{\alpha\beta}-\sigma_{\alpha x}\sigma_{x\beta}/\sigma_{xx},$$
$$\tilde{a}_{\alpha j}=a_{\alpha j}-a_{xj}\sigma_{\alpha x}/\sigma_{xx},\quad \tilde{b}_{i\beta}=b_{i\beta}-b_{ix}\sigma_{x\beta}/\sigma_{xx},\tag{17}$$
$$\tilde{c}_{ij}=c_{ij}-b_{ix}a_{xj}/\sigma_{xx},\alpha,\quad \beta=y,z.$$

The compatibility condition is derived by equating the determinant D_l of the system (16) to zero, and this condition describes the spectrum of resulting waves and the interaction between electromagnetic and acoustic waves.

For $\omega\tau\ll 1$ one root of the dispersion equation

$$D_l=0\tag{18}$$

is close to ω/s_l, so we seek one solution of Eq. (18) in the form

$$k=\frac{\omega}{s_l}+k_1,\tag{19}$$

where the imaginary component of k_1 is the acoustic wave damping factor, and the real component describes the sound velocity s_l renormalization. Other roots of Eq. (18) describe the velocity and damping of electromagnetic waves driven by the sound wave.

If the Fermi surface is slightly warped ($\eta\ll 1$), the asymptotic of k_1 has the form

$$k_1=\frac{ik^2}{2\rho s_l}\left.\frac{\tilde{\sigma}_{yy}\tilde{c}_{yy}-\tilde{a}_{yx}\tilde{b}_{xy}+\dfrac{\omega^2-k^2c^2}{4\pi i\omega}\left[\tilde{c}_{xx}-i(\tilde{a}_{yx}-\tilde{b}_{xy})\dfrac{H_z}{kc}+\tilde{\sigma}_{yy}\left(\dfrac{H_z}{kc}\right)^2\right]}{\tilde{\sigma}_{yy}+\dfrac{\omega^2-k^2c^2}{4\pi i\omega}}\right|_{k=\omega/s_l}.\tag{20}$$

In the range of moderate magnetic fields, where the electron orbit diameter, $2r$, is considerably larger than the acoustic wave length, but much smaller than the carrier mean free path ($kl\gg kr\gg 1$), the acousto-electric tensor components oscillate with the magnetic field (the Pippard effect[14]). In the limit $kr\eta\gg 1$ the amplitudes of the oscillations are smaller than the smooth components of these functions, because the oscillations are due to a small fraction of carriers of the order of $(kr\eta)^{-1/2}\ll 1$, whose orbit diameters are close to the maximum value. In a stronger magnetic field, when $1\ll kr\ll\eta^{-1}$ holds, the spread of electron orbit diameters, $\Delta D\simeq 2r\eta$, is much smaller than the acoustic wave length, and practically all the carriers on the Fermi surface contribute to the oscillations. The amplitude of the oscillations of the acousto-electrical tensor components may be comparable to the slowly varying parts of these functions.

For example, after integration with respect to t and t' by the method of stationary phases, the expression for σ_{yy} has the form

$$\sigma_{yy}=\frac{2e^2}{(2\pi\hbar)^3}\frac{2\pi eH}{c}\int\frac{dp_H}{1-e^{-2\pi\gamma}}\left\{\left(\frac{v_y^2(t_1)}{k|v_x'(t_1)|}+\frac{v_y^2(t_2)}{k|v_x'(t_2)|}\right)\frac{1+e^{-2\pi\gamma}}{2}+\frac{2v_y(t_1)v_y(t_2)e^{-\pi\gamma}}{k|v_x'(t_1)v_x'(t_2)|^{1/2}}\sin(kD)\right\}.\tag{21}$$

For simplicity we assume that there are only two points of stationary phase, t_1 and t_2, where $kv(t_{1,2})=\omega$, on an electron orbit. Here the prime means differentiation with respect to time, $D=x(t_2)-x(t_1)$, $\gamma=\nu/\Omega$, Ω is the frequency of charge motion along a closed trajectory in magnetic field. Taking into account the central symmetry of the electron spectrum, $\varepsilon(-\mathbf{p})=\varepsilon(\mathbf{p})$, we obtain the following expressions for σ_{yy} in a two-dimensional conductor ($\eta=0$) at $|\gamma|\ll 1$, $kr\gg 1$:

$$\sigma_{yy}=\frac{4e^3H}{c(2\pi\hbar)^3}\frac{2\pi\hbar}{a}\cos\theta\frac{v_y^2(t_1)}{\gamma k|v_x'(t_1)|}[1-\sin(kD)].\tag{22}$$

At $kD/2=\pi n+\pi/4$, terms of higher order with respect to the small parameters γ and $(kr)^{-1}$ should be retained. In a quasi-two-dimensional conductor ($\eta=0$), the p_H- dependence of the integrand in Eq. (21) determined by the electron spectrum of the material should be taken into account. It will be illustrated below by a specific example.

One can easily check that the parameter $\tilde{\sigma}_{yy}$ is largely controlled by the component σ_{yy}. Therefore the denominator on the right of Eq. (20) is considerably smaller at $kD/2=\pi n+\pi/4$. But in this case the numerator does not change a lot, although not only $\sigma_{\alpha\beta}(k)$, but all components of the acousto-electric tensors oscillate with large amplitudes for $1\ll kr\ll\eta-1$.

The quasi-two-dimensional spectrum of electrons results in sharp maxima of the acoustic absorption rate. These resonant maxima are periodic with $1/H$, so they can be used to derive the Fermi surface diameter from the oscillation period. Let us recall that the magneto-acoustic resonance can be observed in conductors without pronounced anisotropy of the Fermi surface only when charge carriers drift along the wave vector $\mathbf{k}$.[15]

The amplitude of resonant peaks drops with the magnetic field, and for $kr\lesssim 1$ the resonance is not observable.

Explicit expressions for Γ can be easily derived at any $kr\eta$. Consider as an example a layered conductor whose electron spectrum has the form

$$\varepsilon(\mathbf{p})=\frac{p_x^2+p_y^2}{2m^*}+\eta\frac{\hbar}{a}v_0\cos\frac{ap_z}{\hbar},\quad v_0=\frac{2\varepsilon_F}{m^*},\quad m^*=\text{const}, \tag{23}$$

and assume that the magnetic field is perpendicular to the layers. In this case, to lowest order in the small parameters γ and $(kr)^{-1}$, the conductivity component

$$\sigma_{yy}=\frac{Ne^2}{m^*\nu}\frac{2}{\pi kr_0}[1-J_0(kR\eta)\sin(2kr_0)], \tag{24}$$

where N is the charge carrier density, $r_0=v_0/\Omega$, $R=2\hbar c/eHa$, and J_0 is the Bessel function.

For $kR\eta\gg 1$, the Fermi surface warping is essential, and the acoustic absorption is similar to that in an ordinary (nearly isotropic) metal:

$$\Gamma=\frac{Nm^*\omega v_0}{4\pi\rho s_l^2}\Omega\tau\left[1+\sqrt{\frac{2}{\pi kR\eta}}\cos\left(kR\eta-\frac{\pi}{4}\right)\times\sin(2kr_0)\right]_{k=\omega/s_l}. \tag{25}$$

For $kR\eta\ll 1$, the quasi-two-dimensional nature of the conductor is essential, and Γ is described by the expression

$$\Gamma=\frac{Nm^*\omega v_0}{4\pi\rho s_l^2}\Omega\tau\,\mathrm{Re}\left\{\frac{(\pi\gamma)^2+(kR\eta)^2/2+i\mu[1+\sin(2kr_0)]}{1-\sin(2kr_0)+\frac{(\pi\gamma)^2}{2}+\frac{(kR\eta)^2}{2}+\frac{1}{2}\left(\frac{3}{4kr_0}\right)^2+i\mu}\right\}_{k=\omega/s_l}, \tag{26}$$

where $\mu=\pi v_0c^2\omega^2/2s_l^3\omega_0^2\Omega\tau$ and ω_0 is the plasma frequency. If it is comparable to that of common metals (10^{15}–10^{16} s^{-1}), the parameter μ in the range of ultrasonic frequencies is fairly small, and the function $\Gamma(1/H)$ has giant resonant oscillations. This shape of $\Gamma(1/H)$ is usual for any electron spectrum described by Eq. (1).

Transverse acoustic wave

In the case of transverse acoustic wave polarization, $\mathbf{u}=(0,u_y,u_z)$, the external magnetic field $\mathbf{H}=(0,H_y,H_z)$ is contained only in expressions for acousto-electric coefficients, hence

$$\mathscr{E}_\alpha=\frac{m\omega^2}{e}u_\alpha+\xi j_\alpha,\quad \xi=\frac{4\pi i\omega}{k^2c^2-\omega^2},\quad \alpha=y,z. \tag{27}$$

Having excluded $\mathscr{E}_\alpha$ using Eq. (13), we obtain

$$\begin{aligned} j_y(1-\xi\tilde{\sigma}_{yy})-j_z\xi\tilde{\sigma}_{yz}&=\left(k\omega\tilde{a}_{yy}+\frac{m\omega^2}{e}\tilde{\sigma}_{yy}\right)u_y+\left(k\omega\tilde{a}_{yz}+\frac{m\omega^2}{e}\tilde{\sigma}_{yz}\right)u_z,\\ -j_y\xi\tilde{\sigma}_{zy}+j_z(1-\xi\tilde{\sigma}_{zz})&=\left(k\omega\tilde{a}_{zy}+\frac{m\omega^2}{e}\tilde{\sigma}_{zy}\right)u_y+\left(k\omega\tilde{a}_{zz}+\frac{m\omega^2}{e}\tilde{\sigma}_{zz}\right)u_z. \end{aligned} \tag{28}$$

Taking a combination of these equations with elasticity equations (5), we obtain the equation system, whose self-consistency condition

$$\begin{vmatrix} 1-\xi\tilde{\sigma}_{yy} & -\xi\tilde{\sigma}_{yz} & \chi_{yy} & \chi_{yz}\\ -\xi\tilde{\sigma}_{zy} & 1-\xi\tilde{\sigma}_{zz} & \chi_{zy} & \chi_{zz}\\ \frac{i\omega m}{e}+ik\xi\tilde{b}_{yy} & ik\xi\tilde{b}_{yz} & (\omega^2-s_y^2k^2)\rho+\varphi_{yy} & \varphi_{yz}\\ ik\xi\tilde{b}_{zy} & \frac{i\omega m}{e}+ik\xi\tilde{b}_{zz} & \varphi_{zy} & (\omega^2-s_z^2k^2)\rho+\varphi_{zz} \end{vmatrix}=0 \tag{29}$$

yields damping parameters of the acoustic wave and the co-moving electromagnetic wave. Here $s_y=\sqrt{\lambda_{yzyx}/\rho}$ and $s_z=\sqrt{\lambda_{zxzx}/\rho}$ are the velocities of y- and z-polarized sound, respectively;

$$\chi_{\alpha\beta}=-k\omega\tilde{a}_{\alpha\beta}-\frac{m\omega^2}{e}\tilde{\sigma}_{\alpha\beta},$$

$$\varphi_{\alpha\beta}=ik\left(k\omega\tilde{c}_{\alpha\beta}+\frac{m\omega^2}{e}\tilde{b}_{\alpha\beta}\right), \quad (30)$$

and the components of the elastic tensor, λ_{yxzx} and λ_{zxyx}, are zero if, for example, the xy plane is a crystal symmetry plane.[16] Otherwise these components should be taken into account, but they do not essentially affect the result. This crystal symmetry is implied in Eq. (1).

The pronounced anisotropy of the electron spectrum in layered conductors leads to essentially different attenuation lengths of sound with polarizations perpendicular and parrallel to the layers.[2)] In the limit of small η, the displacement of ions along the normal to the layers decays over a length over a length a factor of η^{-2} larger than a wave with y-polarization. One can easily prove that expansions in powers of η of acousto-electric constants with one or two z indices start with terms of the second or higher order. Omitting in Eq. (29) terms of the order higher than two with respect to η, we obtain

$$\left\{[(\omega^2-k^2s_y^2)\rho+\varphi_{yy}](1-\xi\tilde{\sigma}_{yy})\chi_{yy}\left(\frac{i\omega m}{e}+ik\xi\tilde{b}_{yy}\right)\right\}$$
$$\times\left\{[(\omega^2-k^2s_z^2)\rho+\varphi_{zz}]\left(1-\xi\tilde{\sigma}_{zz}-\chi_{zz}\frac{i\omega m}{e}\right\}=0. \quad (31)$$

Since Eq. (31) is factored, acoustic waves with y- and z-polarizations do not interfer in this approximation. By equating to zero the first multiplicator in Eq. (31), we obtain the equation for $k=\omega/s_y+k_2$, from which follows

$$k_2=\frac{i}{2\rho s_y^2(1-\xi\sigma_{yy})}\left[\xi k\omega(\tilde{a}_{yy}\tilde{b}_{yy}-\tilde{\sigma}_{yy}\tilde{c}_{yy})+\frac{m\omega^2}{e}(\tilde{a}_{yy}+\tilde{b}_{yy})k\omega\tilde{c}_{yy}+\frac{m^2\omega^3}{ke^2}\tilde{\sigma}_{yy}\right]_{k=\omega/s_y}. \quad (32)$$

The denominator in this equation is similar to that in the equation for k_1, so the absorption of the y-polarized wave has the same resonances as the longitudinal wave. The deviation of the other root of Eq. (31) from ω/s_z is proportional to η^2 when $\eta\rightarrow 0$ and is described by the expression

$$k_3=\frac{i}{2\rho s_z^2}\left[\frac{m\omega^2}{e}\left(\frac{\tilde{a}_{zz}}{1-\xi\tilde{\sigma}_{zz}}+\tilde{b}_{zz}\right)+\left(\frac{m\omega}{e}\right)^2\frac{s_z\tilde{\sigma}_{zz}}{1-\xi\tilde{\sigma}_{zz}}+\frac{\omega^2}{s_z}\tilde{c}_{zz}\right]_{k=\omega/s_z}, \quad (33)$$

which indicates that the damping of a z-polarized wave at $\eta\rightarrow 0$ has no resonances.

In this case, the quasi-two-dimensional nature of the electron spectrum is manifested at a higher magnetic field, when $kr\ll 1$. Under this condition, electro-acoustic coefficients are very susceptible to the magnetic field alignment with respect to the layers. If in the expressions for Λ_{zz} and v_z, i.e.,

$$\Lambda_{zz}(\mathbf{p})=\sum_{n=1}^{\infty}\Lambda_n(p_x,p_y)\cos\frac{anp_z}{\hbar}, \quad (34)$$

$$v_z(\mathbf{p})=-\sum_{n=1}^{\infty}n\varepsilon_n(p_x,p_y)\frac{a}{\hbar}\sin\frac{anp_z}{\hbar}, \quad (35)$$

the functions $\Lambda_n(p_x,p_y)$ and $\varepsilon_n(p_x,p_y)$ decrease rapidly with n, the asymptotic forms of the acousto-electric coefficients are essentially different at some angles θ between the magnetic field and normal to the layers. These are the values $\theta=\theta_c$ at which the terms with η^2 in the expansion in powers of η equal zero. For $\tan\theta\gg 1$ these terms turn to zero repeatedly with a period $\Delta(\tan\theta)=2\pi\hbar/D_p$, where D_p is the Fermi surface diameter along the p_y axis.

One can easily find that the last term in Eq. (33) is a factor of $(v_F/s_z)^2$ larger than other term in brackets. If θ is essentially different from θ_c, the following expression can be relatively easily derived for $\Gamma=\mathrm{Im}\,k_3$ for $kr\ll 1$ and $\omega\tau\ll 1$:

$$\Gamma=\frac{N\varepsilon_F}{\rho s_z^3}\omega^2\tau\eta^2. \quad (36)$$

But at $\theta=\theta_c$ the acoustic attenuation length $l_{\mathrm{at}}=1/\Gamma$ is considerably larger because

$$\Gamma=\frac{N\varepsilon_F}{\rho s_z^3}\omega^2\tau\eta^2\left[\eta^2+(kr)^2+\left(\frac{s_z}{v_F}\right)^2 f(\eta)\right]. \quad (37)$$

The latter term in Eq. (37) is due to the mismatch between the roots of $\tilde{a}_{zz}(\theta)/\eta^2$ and $\tilde{b}_{zz}(\theta)/\eta^2$, on one side, and $\tilde{c}_{zz}(\theta)/\eta^2$, on the other side, at $\eta\rightarrow 0$.

For an electron spectrum of this form (Eqs. (1), (34), and (35)) the acousto-electric coefficients a_{zz} and b_{zz} tend to zero at $\eta\rightarrow 0$ faster than η^2, i.e., $f(\eta)$ also tends to zero at a small η. Strictly speaking, this is the main feature of the electron spectrum (Eq. (1)) selected in our analysis. For this reason we retained the parameters $\tilde{a}_{zz}$ and $\tilde{b}_{zz}$ in the final formulas for k_3, although this does not correspond to the actual accuracy of the formulas, given the electron spectrum described by Eq. (1).

If η is not infinitesimal, but satisfies the condition

$$\left(\frac{\omega c}{\omega_0 s_z}\right)(\omega\tau)^{-1/2}\ll\eta\ll 1, \quad (38)$$

the term $\xi\tilde{\sigma}_{zz}$ in the denominator of Eq. (33) cannot be omitted. For $kr\gg 1$ the damping rate of sound with z-polarization may have resonances if

$$\left(\frac{\omega c}{\omega_0 s_z}\right)(\omega\tau)^{-1/2}\ll\eta\ll(kr)^{-1}\ll 1. \quad (39)$$

For $\theta=0$ we obtain the following expression for $\tilde{\sigma}_{zz}$ by retaining only the terms proportional to η^2:

$$\tilde{\sigma}_{zz}=\frac{2eH}{(2\pi\hbar)^2 ca}\frac{v_z^2(t_1)}{|kv_x'(t_1)|}\left[1+\sin\frac{kD_p c}{eH}\right]. \quad (40)$$

For

$$H=H_n=\frac{2\pi(n-1/4)kcD_p}{e}$$

the terms of higher orders in η and γ should be retained in the expression for $\tilde{\sigma}_{zz}$, which results in sharp maxima of Γ versus $1/H$ at $H=H_n$.

If the magnetic field is tilted with respect to the normal to the layers at $kr \gg 1$, we have

$$\tilde{\sigma}_{yy}=\frac{eH\cos\theta}{(2\pi\hbar)^2 ca}\frac{v_y^2(t_1)}{|kv_x'(t_1)|}2\left(1-\sin\frac{kcD_p}{eH\cos\theta}\right), \tag{41}$$

$$\tilde{\sigma}_{zz}=\frac{eH\cos\theta}{(2\pi\hbar)^2 ca}\frac{v_z^2(t_1)}{|kv_x'(t_1)|}(2+\sin\kappa_+ +\sin\kappa_-), \tag{42}$$

where

$$\kappa_\pm = D_p\left|\frac{kc}{eH\cos\theta}\pm\frac{a}{\hbar}\ \mathrm{tg}\theta\right|.$$

In this case the magneto-acoustic resonance takes place only at some fixed magnetic field orientations, when $D_p(a/\hbar)\tan\theta=\pi n$, where n is integer. For even n positions of sharp maxima in Γ versus $1/H_z$ are the same as in the case of $\theta=0$, and for odd n the resonant curves for acoustic waves polarized parallel and perpendicular to the layers are inverted with respect to each other.

The condition of the magneto-acoustic resonance is rather strict for tetrathiafulvalene salts, which have been extensively investigated recently. In these compounds the mean free path is 10^{-3}–10^{-2} cm, and resonances can be observed at acoustic frequencies of the order of $10^9\ \mathrm{s}^{-1}$. But the effect of field alignment on the sound absorption can be observed in such layered materials at acoustic frequencies of the order of $10^8\ \mathrm{s}^{-1}$ because for $kr \ll 1$ the ratio of the electron mean free path to the acoustic wavelength is not essential, and only the condition $r \ll l$, which is fulfilled in a field of 10–20 T, is obligatory.

The specific behavior of damping of acoustic waves with different polarizations can be used in filters transmitting waves of a definite polarization, and the sound absorption may be a very accurate tool for studying electron spectra in layered conductors.

Shear wave propagating perpendicular to the layers.

Let us consider a shear wave propagating across the layers $[(\mathbf{k}=(0,0,k)$ and $\mathbf{u}=(u_x,u_y,0)]$. In this configuration the features of layered conductors are seen most explicitly at electron velocities below the sound velocity, $v_z<s$, i.e., when the Fermi surface cylinder is slightly warped. Therefore the most interesting case is the limit $\eta\to 0$. In reality this corresponds to the metal conductivity along the layers and a low jump conductivity along the z-axis, which takes place, for example, in intercalated dichalcogenides of transitional metals.[19] It follows from the equation of motion (7) that for $\eta\to 0$ the kinetic coefficients are susceptible only to the z-component of the magnetic field, hence the field can be aligned with $\mathbf{k}$ with no loss of generality. Besides, to make the final results more visual, we shall neglect the anisotropy of both electron spectrum and elastic constants in the xy plane.

Under these assumptions, the exact expressions for the conductivity tensor and electro-acoustic coefficients can be easily derived from Eq. (14). Taking the deformation potential in the form[9]

$$\Lambda_{\alpha z}=-v_\alpha p_z/2,$$

we have

$$\sigma_{\alpha\beta}=\frac{Ne^2/m}{\Omega^2+\nu^2}\begin{pmatrix}\nu & -\Omega\\ \Omega & \nu\end{pmatrix},\quad a_{\alpha\beta}=b_{\alpha\beta}=0,$$

$$c_{\alpha\beta}=\frac{1}{3}\left(\frac{\pi\hbar}{2ea}\right)^2\sigma_{\alpha\beta},\quad \alpha,\beta=x,y. \tag{43}$$

Introducing "circularly polarized" parameters

$$u_\pm\equiv u_x\pm iu_y,\quad \sigma_\pm\equiv\sigma_{xx}\pm i\sigma_{yx} \tag{44}$$

and excluding the electric field

$$E_\pm=\frac{\omega H}{c}\left(\frac{mc\omega}{eH}\mp 1\right)\frac{\sigma_\pm u_\pm}{k^2c^2/4\pi i\omega-\sigma_\pm} \tag{45}$$

from elastic equations, we obtain the acoustic problem solution in the k-representation. If a displacement $u(z=0)=(u_0,0,0)$ is defined on the boundary of a semi-infinite sample $(0\leqslant z<\infty)$, the exact solution for $u_\pm(k)$ takes the form

$$u_\pm(k)=-2iku_0\frac{1+\Delta_\pm}{k^2(1+\Delta_\pm)-\omega^2/s^2}, \tag{46}$$

where

$$\Delta_\pm(k)=\frac{B_1}{\beta_\pm}+\frac{B_2(1\mp\omega m/\Omega m^*)^2}{1+\beta_\pm k^2c^2/\omega_0^2},$$

$$\beta_\pm\equiv 1+\frac{i}{\omega\tau}\pm\frac{\Omega}{\omega},\quad B_1\equiv\frac{\pi}{12}\frac{\varepsilon_F}{\rho s^2a^3},\quad B_2\equiv\frac{H^2}{4\pi\rho s^2}. \tag{47}$$

At a sufficiently high frequency $(\omega\tau\gg 1)$ the first term in the expression for Δ_-, which describes the deformation component of the acousto-electric coupling, has a sharp resonance when the acoustic and Larmor frequencies are equal: $\Omega=\Omega_c\cong\omega$. In three-dimensional metals this corresponds to the so-called Doppler-shifted cyclotron resonance at frequencies $\Omega=kv_F\pm\omega$.[20] The conventional acoustic cyclotron resonance is possible only when $\mathbf{k}$ and $\mathbf{H}$ are orthogonal.[21]

The pole of the second term in the expression for $\Delta_-(k)$ (see also Eq. (45)) corresponds to the well known helicoidal electromagnetic wave[22,23] with a spectrum

$$\omega(k)=\left(\Omega-\frac{i}{\tau}\right)\frac{k^2c^2}{k^2c^2+\omega_0^2}. \tag{48}$$

In a conventional metal, where charge carriers moving at large velocities of order v_F are always present and hence the conductivity exhibits spatial dispersion, only the low-frequency branch of the helicon spectrum described by Eq. (48) limited by the condition $\Omega/\omega-1\gg v_F^2/v_A^2$ $(v_A^2\equiv H^2/4\pi Nm^*)$ is observed under realistic magnetic fields, in which $v_A<10^6$–10^7 cm/s. In the quasi-two-dimensional case this limitation is not critical because

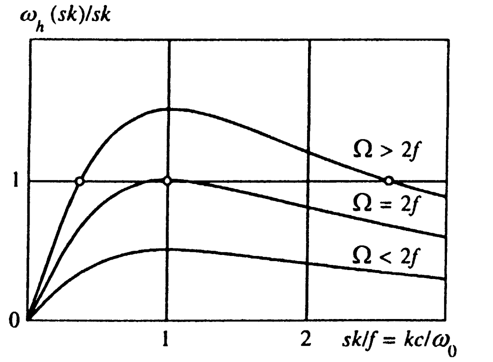

FIG. 1. Crossing between spectra of acoustic and helicoidal electromagnetic waves in a two dimensional conductor takes place at $2ks=\Omega \pm \sqrt{\Omega^2-4f^2}$ ($f=\omega_0 s/c$, ω_0, and Ω are the plasma and Larmor frequencies, respectively).

$v_z \simeq \eta v_F$, and in a purely two-dimensional configuration it is irrelevant since the maximum phase velocity of a helicon is achieved at $k=\omega/c$ and is equal to $v_A/2$.

The helicon-phonon resonance, when the curve described by Eq. (48) crosses the non-perturbed acoustic spectrum $\omega(k)=sk$, takes place at

$$\Omega=\omega_h \cong \omega + f^2/\omega, f \equiv \omega_0 s/c. \tag{49}$$

The resonance is possible when $v_A \geqslant 2s$ ($\Omega > 2f$), and the acoustic frequency falls in the interval $|\omega-\Omega/2| \leqslant \sqrt{\Omega^2/4-f^2}$ (Fig. 1). Thus the conditions of the helicon-phonon resonance in a layered conductor are less strict than in three-dimensional conductors, in which the respective conditions are[3)]

$$v_A^2 \gg s v_F, \quad f^2/\Omega < \omega \ll f\sqrt{s/v_F}.$$

From this it follows that features of layered conductors are seen most explicitly at high frequencies, when ω is comparable to the characteristic frequency of $f \approx 10^9\ \mathrm{s}^{-1}$ or even higher. A simple analysis of roots of the dispersion equation, i.e., zeros of the denominator in Eq. (46) for $u(k)$ reveals that the parameter of coupling between the acoustic and electromagnetic branches of the spectrum is

$$B_2(\omega_h)|f^2\tau^2+i\omega\tau| = \frac{Nm^*}{\rho}\left(1+\frac{f^2}{\omega^2}\right)^2 |f^2\tau^2+i\omega\tau|. \tag{50}$$

Since $Nm^*/\rho \simeq 10^{-6}-10^{-5}$, $\omega < 10^{12}\ \mathrm{s}^{-1}$, and the free-travel time of charge carriers even in purest metals is within 10^{-8} s, the coupling parameter in Eq. (50) is very small at $\omega \geqslant f$ and the spectrum can be studied quantitatively in the local approximation, i.e., the problem is reduced to a calculation of acoustic spectrum renormalizations. Assuming that they are small, we derive from Eqs. (46) and (47)

$$u_\pm(z) \cong u_0 \exp(ik_\pm z), \quad k_\pm \cong \frac{\omega}{s}\left(1-\frac{\Delta_\pm(\omega/s)}{2}\right). \tag{51}$$

Then from the definition in Eq. (44) it follows that

$$u(z)=\sqrt{u_+u_-}=\exp[iz(k_+ + k_-)/2]$$

and the sound attenuation (in terms of the displacement amplitude) is

$$\Gamma \cong -\frac{\omega}{4s}\,\mathrm{Im}(\Delta_+ + \Delta_-). \tag{52}$$

The difference between k_+ and k_- in magnetic field leads to rotation of the acoustic polarization plane as it propagates along z, the respective phase factor Q being determined by the formula

$$Q \cong \frac{\omega}{4s}\,\mathrm{Re}(\Delta_- - \Delta_+), \tag{53}$$

where the values of the functions $\Delta_\pm(k)$, as in Eq. (52), are taken at $k=\omega/s$.

After simple calculations, we obtain

$$\Gamma(\Omega)=\frac{B_1}{4s\tau}\sum_\pm \left\{\frac{\omega^2}{(\Omega\pm\omega)^2+\tau^{-2}} + C\frac{(\Omega\mp\omega m/m^*)^2}{(\Omega\pm\omega_h)^2+\tau^{-2}}\right\},$$

$$Q(\Omega)=-\frac{B_1}{4s}\sum_\pm \left\{\frac{\omega^2(Q\pm\omega)}{(\Omega\pm\omega)^2+\tau^{-2}} + C(\Omega\pm\omega_h)\frac{(\Omega\mp\omega m/m^*)^2}{(\Omega\pm\omega_h)^2+\tau^{-2}}\right\}, \tag{54}$$

where $C=(12/\pi^2)(am^*s/\hbar)^2$.

At typical values of the parameters [$a \simeq 10^{-7}$ cm, $m^* \simeq 10^{-27}$ g, $s^2 \simeq 10^{11}$ (cm/s)^2] the helicon-phonon resonance is much weaker than the cyclotron resonance:

$$\frac{\Gamma(\omega_h)}{\Gamma(\Omega_c)} \simeq C\left(1+\frac{m}{m^*}+\frac{f^2}{\omega^2}\right)^2 \ll 1.$$

But at a lower frequency or at a higher mass of charge carriers this relationship may change considerably so that both resonances may be observable.

Equation (54) presents properties of the resonances in a fairly graphic form, and for brevity we shall only consider the first terms in the braces responsible for the cyclotron resonance. These terms are plotted versus $\Omega^2/\omega^2 \sim H^2$ in Fig. 2. The curve $\Gamma(\Omega)$ has a maximum only if $\omega\tau > 1$ at

$$\Omega^2=\Omega_c^2=2\omega\sqrt{\omega^2-\tau^{-2}}-\omega^2-\tau^{-2}.$$

At the maximum

$$\Gamma(\Omega_c)=\frac{\pi}{96}\frac{\varepsilon_F}{\rho s^3 a^3}\frac{\omega}{\sqrt{1+\omega^2\tau^2}-\omega\tau}, \tag{55}$$

which is always higher than the attenuation factor at zero magnetic field (see Eq. (61) below).

For large $\omega\tau$ we have

$$\Omega_c \approx \omega, \quad \Gamma(\omega) \cong \frac{\omega}{4s}B_1\omega\tau = \frac{\pi}{48}\frac{\varepsilon_F\omega^2\tau}{\rho s^3 a^3}, \tag{56}$$

and the extrema of the function $Q(\Omega)$ are shifted by $\pm\tau^{-1}$ and equal to

$$Q(\omega\pm\tau^{-1}) \cong \frac{1}{2}\Gamma(\omega)\sqrt{1\pm\frac{2}{\omega\tau}}. \tag{57}$$

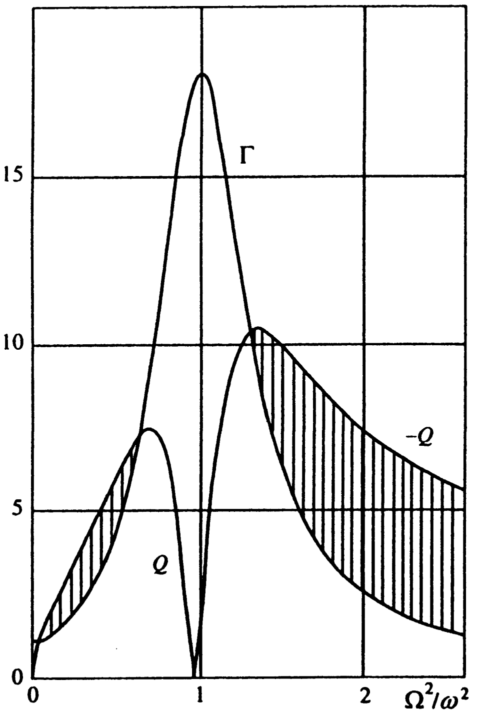

FIG. 2. Sound absorption factor Γ and the rate of polarization plane rotation Q versus square of magnetic field intensity around the acoustic cyclotron resonance; $\omega\tau=6$. In the hatched area $|Q|>\Gamma$.

On both sides of these maxima, the parameter $|Q|$ is larger than Γ, i.e., the rate of polarization plane rotation is larger than its attenuation (Fig. 2). An important point is that the effect takes place in a relatively weak magnetic field, when $\Omega<\tau^{-1}<\omega$.

It is noteworthy that the parameter $B_1=(\pi/12)\varepsilon_F/\rho s^2a^3$ is not very small: an estimate yields $B_1\simeq 10^{-2}$, and in high-quality samples (at low temperatures and hypersound frequencies) the condition $B_1\omega\tau\ll 1$ may not hold. Then the relative change in k is not small, and instead of Eq. (51) we have

$$k_\pm=\frac{\omega}{s}\sqrt{\frac{i\nu\pm\Omega}{B_1\omega+i\nu\pm\Omega}}. \tag{58}$$

You can see that the resonance occurs at $\Omega_c\cong\omega(1+B_1)$, which can be used to estimate B_1 from experimental data:

$$k_+(\Omega_c)\approx\frac{\omega}{s},\quad k_-(\Omega_c)\approx\frac{\omega}{s}\sqrt{1+iB_i\omega\tau}. \tag{59}$$

In the opposite case, $B_1\omega\tau\gg 1$, the parameters Γ and $|Q|$ at the resonance are approximately equal:

$$\Gamma(\Omega_c)\approx -Q(\Omega_c)\approx\frac{\omega v_0}{8s^2}\sqrt{\frac{\pi m^*}{3\rho a^3}}\,\omega\tau. \tag{60}$$

The distinction from a three-dimensional metal can be observed even at zero magnetic field. At $H=0$ we derive from Eqs. (46) and (47)

$$u_x(z)=u_0\exp(ik_0z),\quad k_0=\frac{\omega}{s}\sqrt{\frac{\omega\tau+i}{\omega\tau(1+B_1)+i}},$$

i.e., the acoustic attenuation factor is

$$\Gamma(0)\approx\frac{\pi}{24}\frac{\varepsilon_F}{\rho s^3a^3}\frac{\omega^2\tau}{1+\omega^2\tau^2}. \tag{61}$$

For $\omega\tau\ll 1$ the factor Γ is proportional to $\omega^2\tau$, as in a conventional metal when sound is attenuated owing to the electron viscosity.[7] But in a two-dimensional conductor the collisionless regime is not realized, and for $\omega\tau>1$ the attenuation is proportional to the scattering frequency of charge carriers.

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[1)]A complete equation system taking into account nonlinear effects was derived by Andreev and Pushkarov (see Ref. 10).

[2)]In the case of a large anisotropy of elastic constants of a layered crystal, a wave with a polarization parallel to **n** may have a nonlinear spectrum $\omega(k)$ (see Ref. 17). The attenuation of such waves by electrons was discussed in Ref. 18.

[3)]The theory of the helicon-phonon resonance and related phenomena in conventional metals and semimetals have been developed in details in 70th (see Refs. 24–27).

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