Nonlinear zero sound in a normal Fermi liquid

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A nonlinear correction (proportional to the wave amplitude) to the zero-sound velocity is calculated. The growth rate of the amplitude of the second harmonic is also calculated. The result is expressed in terms of the derivative of a Landau function with respect to energy and in terms of its variational derivative with respect to the quasiparticle distribution function. Identities found in this paper make it possible to express the result in terms of the Landau function and its derivative with respect to the pressure, both of which are well known from experiments, in the approximation of the first two spherical harmonics.

1. INTRODUCTION

The nonlinear effects which accompany the propagation of zero sound in liquid He^3 have been studied in detail both theoretically and experimentally[1] at temperatures below the superfluid transition temperature T_c, but not in the region $T>T_c$, of the normal Fermi liquid. The reason is that the nonlinearity is anomalously pronounced in the superfluid phases: the nonlinear corrections to the velocity and attenuation of sound are governed by the dimensionless parameter $(\delta\rho/\rho)(E_F/T_c)$, where ρ is the density of the liquid, E_F is the Fermi energy, and $\delta\rho$ is the amplitude of the density oscillations in the wave. In the region $T>T_c$, on the other hand, there is no factor of E_F/T_c. Nevertheless, nonlinear effects accompanying the propagation of zero sound in a normal Fermi liquid can probably be studied experimentally. In this paper we take a theoretical look at this question.

We begin with the range of applicability of the approach described below. In the linear approximation in the wave amplitude, the sound absorption coefficient γ (the reciprocal of the wave lifetime) is given by the sum[2] $\gamma=\gamma_\omega+\gamma_T$, where $\hbar\gamma_\omega\sim(\hbar\omega)^2/E_F$ is the absorption coefficient at absolute zero, which depends on the sound frequency ω, and we have $\hbar\gamma_T\sim T^2/E_F$. For sound of finite amplitude, we calculate the correction $\Delta\omega\sim\omega(\delta\rho/\rho)$ for the frequency below (for a given wavelength). It is important to note, however, that a nonlinear correction to the absorption, $\Delta\gamma$, arises at the same time. In estimating $\Delta\gamma$ we note that zero sound consists of oscillations in the shape of the Fermi surface. We denote by Δp_F the amplitude of the deviation of the shape of the surface from spherical. Quasiparticles within a distance on the order of δp_F from the boundary of the Fermi surface have a finite mean free path, because there is the possibility of collisions of quasiparticles with each other even in the limit $\omega\to 0$, $T\to 0$. As usual, the typical collision rate differs from $E_F/\hbar$ by a small factor, on the order of the ratio of the square of the volume of momentum space in which collisions are possible (it is $p_F^2\delta p_F$ in the case at hand) to the square of the total volume (p_F^3) inside the Fermi surface. The nonlinear correction to the absorption which we are seeking, $\Delta\gamma$, is of the same order of magnitude as the rate of such collisions. As a result we have $\hbar\Delta\gamma\sim E_F(\delta p_F/p_F)^2\sim E_F(\delta\rho/\rho)^2$.

As long as the linear absorption is slight, it does not distort the wave (it does not give rise to higher harmonics, etc.). Shape distortions stem from the nonlinear effects $\Delta\omega$ and $\Delta\gamma$. Below we will take $\Delta\omega$ into account, ignoring $\Delta\gamma$; this approach is legitimate under the condition $\Delta\omega\gg\Delta\gamma$, i.e., for sufficiently small sound amplitudes, which satisfy the condition

$$\frac{\delta\rho}{\rho}\ll\frac{\hbar\omega}{E_F}. \tag{1}$$

In this case we have $\Delta\omega/\gamma<\Delta\omega/\gamma_\omega\sim(\delta\rho/\rho)(E_F/\hbar\omega)\ll 1$, so nonlinear distortions do not develop to any great extent over the lifetime of the wave, but they can reach a level sufficient for experimental observation. At the limits of applicability as defined by (1), the relative correction to the frequency (or, equivalently, the relative correction to the sound velocity c, i.e., $\Delta c/c\sim 10^{-3}$) is reached at $T\sim\hbar\omega\sim T_c\sim 1$ mK at wave quality factors $Q\sim\omega/\gamma\sim(E_F/\hbar\omega)\sim 10^3$.

2. NONLINEAR BOLTZMANN EQUATION

Under condition (1), we can (as mentioned above) ignore collisions in the Boltzmann equation,

$$\frac{\partial n}{\partial t}+\frac{\partial n}{\partial \mathbf{r}}\frac{\partial\varepsilon}{\partial\mathbf{p}}-\frac{\partial n}{\partial\mathbf{p}}\frac{\partial\varepsilon}{\partial\mathbf{r}}=0, \tag{2}$$

but we can incorporate terms quadratic in the deviation $\delta n(\mathbf{p})$ of the distribution function from its equilibrium value $n_0(p)=\theta(p_F-p)$ on the left side of the equation; here $\theta(x)=0$ at $x<0$ and $\theta(x)=1$ at $x>0$. We first introduce some new parameters of Fermi-liquid theory, which have seldom been considered in applications of this theory. We will also derive several relations among these parameters.

We expand the quasiparticle energy $\varepsilon(\mathbf{p})$ in terms of $\delta n(\mathbf{p})$, retaining quadratic terms:

$$\varepsilon(\mathbf{p})=\varepsilon_0(p)+\int f(\mathbf{p},\mathbf{p}')\delta n(\mathbf{p}')d\tau' +\frac{1}{2}\int \phi(\mathbf{p},\mathbf{p}',\mathbf{p}'')\delta n(\mathbf{p}')\delta n(\mathbf{p}'')d\tau' d\tau'', \tag{3}$$

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where $d\tau=2d^3\mathbf{p}/(2\pi\hbar)^3$, f is the ordinary Landau function, and ϕ is the third functional derivative of the total energy of the liquid with respect to $n(\mathbf{p})$, taken at $n(\mathbf{p})=n_0(p)$. Here and below, we omit the spin dependence. In addition to the ordinary effective mass m^*, which is defined in terms of the first derivative of the energy $\varepsilon(\mathbf{p})$ with respect to the momentum,

$$v_F=\frac{p_F}{m^*}=\left.\frac{\partial\varepsilon_0}{\partial p}\right|_{p=p_F}, \tag{4}$$

we also introduce a characteristic mass M, defined in terms of the second derivative:

$$\left.\frac{\partial^2\varepsilon_0(p)}{\partial p^2}\right|_{p=p_F}=\frac{1}{M}. \tag{5}$$

The function $f=f(p,p',\cos\alpha)$ actually depends on the absolute values of the momenta $\mathbf{p}$ and $\mathbf{p}'$ and on the angle α between them. In addition to the usual parameters F_l, $l=0,1,\ldots$, which determine an expansion of the function in Legendre polynomials $P_l(\cos\alpha)$,

$$\frac{m^*p_F}{\pi^2\hbar^3}f(p_F,p_F,\cos\alpha)=F(\alpha)=\sum_{l=0}^{\infty}F_lP_l(\cos\alpha), \tag{6}$$

we introduce some new dimensionless parameters F_l^1, $l=0,1,\ldots$, which are defined by a corresponding expansion for the derivatives (which are symmetric with respect to p and p') of the function f with respect to the absolute values of the momenta:

$$\frac{\partial f}{\partial p}(p_F,p_F,\cos\alpha)=\frac{\partial f}{\partial p'}(p_F,p_F,\cos\alpha)=\frac{\pi^2\hbar^3}{m^*p_F^2}F^1(\alpha),$$

$$F^1(\alpha)=\sum_{l=0}^{\infty}F_l^1P_l(\cos\alpha). \tag{7}$$

It is sufficient to consider the function ϕ on the Fermi surface, i.e., at $|\mathbf{p}|=|\mathbf{p}'|=|\mathbf{p}''|=p_F$. This function is then completely symmetric under interchange of the arguments of the function $\phi(\alpha,\beta,\gamma)$, i.e., the three angles α,β,γ, between the vectors $\mathbf{p}$ and $\mathbf{p}'$, $\mathbf{p}$ and $\mathbf{p}''$, $\mathbf{p}'$ and $\mathbf{p}''$, respectively. It is convenient to expand this function in functions corresponding to the addition of three angular momenta, i.e.,

$$\frac{m^*p_FN}{\pi^2\hbar^3}\phi(\alpha,\beta,\gamma)=\Phi(\alpha,\beta,\gamma)=\sum_{l_1l_2l_3}\Phi_{l_1l_2l_3}\sum_{m_1m_2m_3}\begin{pmatrix}l_1 & l_2 & l_3\\ m_1 & m_2 & m_3\end{pmatrix}\times Y_{l_1m_1}(\theta,\varphi)Y_{l_2m_2}(\theta',\varphi')Y_{l_3m_3}(\theta'',\varphi''). \tag{8}$$

Here

$$\begin{pmatrix}l_1 & l_2 & l_3\\ m_1 & m_2 & m_3\end{pmatrix}$$

are the $3j$ symbols; θ and φ are the angles which specify the orientation of the vector $\mathbf{p}$; θ', θ'', φ', and φ'' are the corresponding angles for $\mathbf{p}'$ and $\mathbf{p}''$; we have $\cos\alpha=\cos\theta\cos\theta'+\sin\theta\sin\theta'\cos(\varphi-\varphi')$ and corresponding relations for β and γ; $Y_{lm}(\theta,\varphi)$ are the spherical harmonics; and $N=p_F^3/3\pi^2\hbar^3$ is the number of particles per unit volume of the equilibrium liquid. The parameters $\Phi_{l_1l_2l_3}$ given by (8) are dimensionless. By virtue of the symmetry properties of the $3j$ symbols, these parameters are zero for a symmetric function ϕ if the sum $l_1+l_2+l_3$ is odd. The first two terms of series (8) are

$$\Phi(\alpha,\beta,\gamma)\approx\Phi_{000}+\Phi_{011}\sqrt{3}(\cos\alpha+\cos\beta+\cos\gamma). \tag{9}$$

We now wish to derive some relations among these new parameters. The familiar equality which follows from the Galilean principle of relativity takes the form

$$\int\mathbf{p}n(\mathbf{p})d\tau=m\int\frac{\partial\varepsilon}{\partial\mathbf{p}}n(\mathbf{p})d\tau, \tag{10}$$

where m is the mass of the He^3 atom. Varying (10) twice with respect to δn, and taking the result for $n=n_0$, we find

$$\frac{\partial f}{\partial\mathbf{p}}+\frac{\partial f}{\partial\mathbf{p}'}=-\int\phi\frac{\partial n_0(p'')}{\partial\mathbf{p}''}d\tau''.$$

It is then a simple matter to find the following result by setting $p=p'=p_F$:

$$\cos\frac{\alpha}{2}\left[F^1(\alpha)+\frac{\partial F(\alpha)}{\partial\cos\alpha}(1-\cos\alpha)\right]=-\frac{3}{4}\int\Phi(\alpha,\beta,\gamma)\cos\theta''d\Omega'',$$

$$d\Omega=\frac{\sin\theta d\theta d\varphi}{4\pi}. \tag{11}$$

Here the integration axis is directed along $\mathbf{p}_F+\mathbf{p}_F'$. Retaining from $F(\alpha)$ and $F^1(\alpha)$ the first two harmonics in an expansion in Legendre polynomials, and retaining series (9) from Φ, we also find relations of the following type from (11):

$$F_0^1\approx-F_1-\sqrt{3}\Phi_{011},\quad F_1^1\approx F_1. \tag{12}$$

Again using the Galilean principle, we can calculate $1/M$. To do this we need to vary expression (10) with respect to δn, differentiate with respect to p, and take the result at $n=n_0$, $p=p_F$. We find

$$\frac{m^*}{M}=1+\frac{F_1}{3}-\frac{F_1^1}{3}. \tag{13}$$

The parameters F_0 and F_1 depend on p_F, and their derivatives with respect to p_F are related to F_0^1, F_1^1 and Φ_{000}, Φ_{011} by

$$2F_0^1+3\Phi_{000}=\frac{\partial F_0}{\partial p_F}p_F-F_0-\frac{\partial F_1}{\partial p_F}\frac{p_FF_0}{3+F_1}, \tag{14}$$

$$2F_1^1+3\sqrt{3}\Phi_{011}=\frac{\partial F_1}{\partial p_F}\frac{3p_F}{3+F_1}-F_1. \tag{15}$$

These relations can easily be derived by writing the change in $f(p_F)$ in the case of an infinitesimal change δp_F in p_F:

$$\delta f(p_F)=\int \phi\delta n''d\tau''+2\delta p_F\frac{\partial f}{\partial p}(p_F),$$

$$\delta n(p)=\theta(p_F'-p)-\theta(p_F-p),\quad \delta p_F=p_F'-p_F.$$

Again using approximate relations (12), we find

$$\Phi_{000}\approx\frac{\partial F_0}{\partial p_F}\frac{p_F}{3}-\frac{F_0}{3}+\frac{\partial F_1}{\partial p_F}\frac{p_F}{3}\frac{2-F_0}{3+F_1},\tag{16}$$

$$\sqrt{3}\Phi_{011}\approx\frac{\partial F_1}{\partial p_F}\frac{p_F}{3+F_1}-F_1.\tag{17}$$

When the functions $F(\alpha)$, $F^1(\alpha)$, and $\Phi(\alpha,\beta,\gamma)$ are approximated by the first two terms of the expansions in (6), (7), and (8), respectively, the parameters F_0^1, F_1^1, Φ_{000}, and Φ_{011} are thus expressed exclusively in terms of F_0, F_1, and their derivatives with respect to p_F.

As we mentioned earlier, zero sound consists of oscillations in the shape of the Fermi surface. Consequently, the perturbed distribution function in (2) is

$$n(\mathbf{p},\mathbf{r},t)=\theta[p_1(\mathbf{n},\mathbf{r},t)-p],\tag{18}$$

where $\mathbf{n}=\mathbf{p}/p$, and

$$\nu(\mathbf{n},\mathbf{r},t)=\frac{p_1(\mathbf{n},\mathbf{r},t)-p_F}{p_F}\tag{19}$$

is a small parameter.

Using (3) and (19), we can rewrite kinetic equation (2) to quadratic accuracy in ν in the axisymmetric 1D case; i.e., ν depends on θ, x, and t, where θ is the angle between $\mathbf{p}$ and the x axis:

$$\frac{\partial\nu}{\partial t}+\frac{\partial\nu}{\partial x}v_F\cos\theta+v_F\cos\theta\int F(\alpha)\frac{\partial\nu(\theta')}{\partial x}d\Omega'$$
$$+v_FL(\theta,x,t)=0,\tag{20}$$

$$L(\theta,x,t)=3\cos\theta\int\Phi(\alpha,\beta,\gamma)\frac{\partial\nu'}{\partial x}\nu''d\Omega'd\Omega''$$
$$+\frac{m^*}{M}\cos\theta\nu\frac{\partial\nu}{\partial x}$$
$$+2\cos\theta\int F(\alpha)\nu'\frac{\partial\nu'}{\partial x}d\Omega'$$
$$+\cos\theta\int F^1(\alpha)\left[\frac{\partial\nu}{\partial x}\nu'+\frac{\partial\nu'}{\partial x}\nu+\frac{\partial\nu'}{\partial x}\nu'\right]d\Omega'$$
$$-\sin^2\theta\int\left[F(\alpha)\frac{\partial\nu'}{\partial x}\frac{\partial\nu}{\partial\cos\theta}-\frac{\partial F(\alpha)}{\partial\cos\theta}\frac{\partial\nu}{\partial x}\nu'\right]d\Omega',$$

$$\nu=\nu(\theta),\quad \nu'=\nu(\theta'),\quad \nu''=\nu(\theta'').$$

3. NONLINEAR OSCILLATIONS

We consider small axisymmetric oscillations of the normal Fermi surface, which are described by Eq. (20). In the collisionless case, in Landau's paper,[3] it is convenient to solve the problem with given initial conditions. We therefore solve Eq. (20) under the initial condition

$$\nu(\theta,x,t)|_{t=0}=g(\theta,x)\tag{21}$$

by the standard method of successive approximations in the amplitude ($\nu=\nu_1+\nu_2+...$). For the first approximation, ν_1, we use initial condition (21); for the successive approximations ($\nu_2,...$), we use homogeneous initial conditions.

Taking Fourier transforms,

$$\mathscr{F}[\nu]=\nu_k(\theta,t)=\int_{-\infty}^{+\infty}\nu(\theta,x,t)e^{-ikx}dx,$$

and Laplace transforms,

$$\mathscr{L}[\nu]=\nu_\omega(\theta,x)=\int_0^{+\infty}\nu(\theta,x,t)e^{i\omega t}dt,$$

of Eq. (20), and imposing initial condition (21), we find, for the first approximation,

$$-i\omega\nu_1^{(+)}(\theta)+ikv_F\cos\theta\nu_1^{(+)}(\theta)-g_k(\theta)$$
$$+ikv_F\cos\theta\int F(\alpha)\nu_1^{(+)}(\theta')d\Omega'=0,\tag{22}$$

where

$$\mathscr{L}\mathscr{F}[\nu_1]=\nu_1^{(+)}(\theta).$$

Approximating the Landau function by the first two harmonics,

$$F=F_0+F_1\cos\alpha,$$

we find

$$\nu_1^{(+)}(\theta)=-\frac{g_k(\theta)}{i\omega-ikv_F\cos\theta}$$
$$-\cos\theta\frac{F_0C_1+F_1C_2\cos\theta}{(s-\cos\theta)y(s)},$$

where

$$C_1=\int\frac{g_k(\theta)d\Omega}{i\omega-ikv_F\cos\theta}\left(1-F_1\left[s^2w-\frac{1}{3}\right]\right)$$
$$+\int\frac{g_k(\theta)\cos\theta d\Omega}{i\omega-ikv_F\cos\theta}F_1sw,$$

$$C_2=\int\frac{g_k(\theta)\cos\theta d\Omega}{i\omega-ikv_F\cos\theta}(1-F_0w)$$
$$+\int\frac{g_k(\theta)d\Omega}{i\omega-ikv_F\cos\theta}F_0sw,$$

$$y(s)=(1-F_0w)(1+F_1/3)-F_1s^2w,\quad s=\frac{\omega}{kv_F},$$

$$w=-1+\frac{s}{2}\ln\frac{s+1}{s-1}.$$

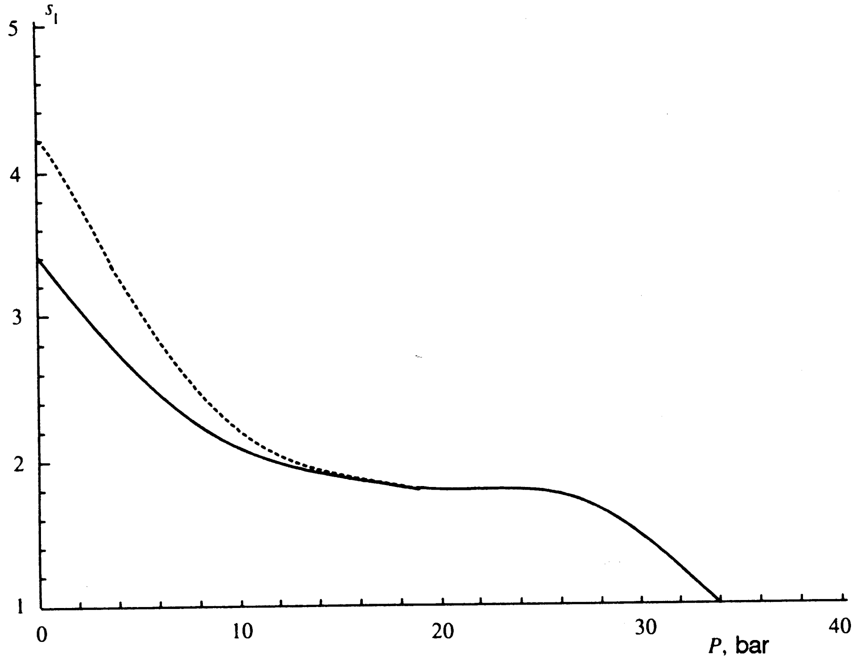

FIG. 1. Pressure dependence of s_1.

Now taking inverse Fourier transforms,

$$\mathscr{F}^{-1}[\nu_k]=\nu(\theta,x,t)=\int_{-\infty}^{+\infty}\nu_k(\theta,t)e^{ikx}\frac{dk}{2\pi},$$

and inverse Laplace transforms,

$$\mathscr{L}^{-1}[\nu_\omega]=\nu(\theta,x,t)=\int_{-\infty+i\alpha}^{+\infty+i\alpha}\nu_\omega(\theta,x)e^{-i\omega t}\frac{d\omega}{2\pi},$$

$\alpha>0$,

we obtain the inverse transform:

$$\nu_1(\theta,x,t)=\mathscr{L}^{-1}\mathscr{F}^{-1}[\nu_1^{(+)}].$$

The nondecaying singularities $\nu_1^{(+)}$ in the complex ω plane are as follows: a cut from $-kv_F$ to kv_F and the band $\omega=kv_F\cos\theta$, both of which correspond to one-particle excitations; and the two bands $\omega=c_0k, -c_0k$, which arise from $y(s_0)=0$, where $s_0=c_0/v_F, -c_0/v_F$, which correspond to longitudinal zero sound propagating rightward and leftward, respectively. Equating $y(s)$ to zero, we find a known relation for the velocity of longitudinal zero sound,

$$w_0=-1+\frac{s_0}{2}\ln\frac{s_0+1}{s_0-1}=\frac{1+F_1/3}{F_0+F_1s_0^2+F_0F_1/3}. \tag{23}$$

Retaining only the contribution from the band $\omega=c_0k$ $(s_0=c_0/v_F)$, we find

$$\nu_1(\theta,x,t)=A(\theta)\frac{\int g(\theta,x-c_0t)A(\theta)\frac{d\Omega}{\cos\theta}}{\int A^2(\theta)\frac{d\Omega}{\cos\theta}}$$

$$=A(\theta)K(x-c_0t), \tag{24}$$

$$A(\theta)=\frac{\cos\theta+\frac{1-F_0w_0}{F_0s_0w_0}\cos^2\theta}{s_0-\cos\theta}.$$

As expected, we have obtained a wave of constant profile traveling to the right.

For the function second approximation ν_2, we again have Eq. (22), with ν_1 replaced by ν_2, and $g_k(\theta)$ by

$$-\mathscr{L}\mathscr{F}[v_FL(\theta,x,t)],$$

where we have substituted the exact first approximation into $L(\theta,x,t)$ [the nonlinear part of Eq. (20)]. When we find the inverse transform of the second approximation, a secular term $\sim t$ arises, because of the simultaneous use of the band $\omega=c_0k$ (which corresponds to zero sound) in the first and second approximations:

$$-A(\theta)K(x-c_0t)\frac{\partial K(x-c_0t)}{\partial x}c_1t. \tag{25}$$

As a result there is a correction to expression (24). Making use of the property

$$\mathscr{L}^{-1}[\mathscr{L}[f_1(t)]\mathscr{L}[f_2(t)]]=\int_0^t f_1(y)f_2(t-y)dy$$

of Laplace transforms, we easily find the following result for c_1:

$$\frac{c_1}{v_F}=\frac{\int L_1(\theta)A(\theta)\frac{d\Omega}{\cos\theta}}{\int A^2(\theta)\frac{d\Omega}{\cos\theta}},$$

where $L_1(\theta)K(\partial K/\partial x)=L(\theta,x,t)$, and we have substituted (24) into $L(\theta,x,t)$. After some simple integrations, we find

$$\frac{c_1}{v_F}=\frac{[3w(s^2-1)-1][(s^2-1)F_0w(F_0w-2)-1]}{F_0sw[(s^2-1)(2F_0w-3)w+1]}$$

$$+\frac{\int A^3(\theta)\, d\Omega\, \frac{m^*}{M}+3\int F^1(\alpha)A(\theta)A^2(\theta')d\Omega d\Omega'+I_1}{\int A^2(\theta)\, \frac{d\Omega}{\cos\theta}},$$

$$I_1=3\int \Phi(\alpha,\beta,\gamma)A(\theta)A(\theta')A(\theta'')d\Omega d\Omega' d\Omega''. \quad (26)$$

Here and below, we omit the subscript 0 from s and w. The secular term in (25) leads to a correction $\delta c=c_1(F_0/3)(\delta N/N)$, where δN is the deviation of the concentration from the equilibrium value N, to the first-approximation velocity c_0 in (24). As a result, a traveling wave of constant profile, (24), corresponding to longitudinal zero sound, transforms into a simple traveling wave with a varying profile (different velocities correspond to different points on the profile), which is analogous to Riemann solutions in hydrodynamics.

If $K(x)\propto\cos kx$, and if we use (25), we see that oscillations $\delta N/N\approx A_1\cos k(x-c_0t)+A_2\sin 2k(x-c_0t)$ correspond to a zero-sound mode propagating to the right, where A_1 and A_2 are the amplitudes of the first and second harmonics, respectively. The growth rate of amplitude A_2 is

$$\frac{dA_2}{dt}=\frac{2c_1F_0kU}{(1+F_0)p_Fv_FN}, \quad (27)$$

where $U=(1+F_0)p_Fv_FNA_1^2/12$ is the energy density in the zero-sound mode.[4]

Approximating the functions $\Phi(\alpha,\beta,\gamma)$ and $F^1(\alpha)$ by the first two terms of expansions (7) and (8), and using relations (12), (16), and (17), we can carry out a numerical calculation of the pressure dependence of the quantity $s_1=c_1/v_F$ given by (26). The results of some calculations carried out on the basis of some experimental data of Ref. 5 are shown in Fig. 1. Also shown by the dotted line in this figure is the pressure dependence of s_1 for hydrodynamic sound, calculated from

$$s_1=\frac{3s}{F_0}\left(1+c'\rho\,\frac{dc'}{dP}\right)=\frac{s(9+11F_0+m^*/M+3F_0^1+3\Phi_{000})}{2F_0(1+F_0)},$$

where c' is the velocity of sound, P is the pressure, and ρ is the density.

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