Level-band and level-continuum quantum systems in an external field

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The dynamics of level-band and level-continuum quantum systems in a modulated strong quasiresonant external field are discussed. An analytical solution of the corresponding initial-value problems has been obtained for the case in which the Rabi parameter of the external field greatly exceeds the effective width of the continuum (or band). It is shown that the atom is stabilized and population trapping of the level occurs for a level-continuum system in a field with weak modulation. © *1995 American Institute of Physics.*

1. INTRODUCTION

The interaction of level-band and level-continuum quantum systems with a quasiresonant external field has been actively studied in recent years. These physical phenomena have numerous applications in quantum optics, quantum electrons, and some high-tech areas.[1–3] However, only the case of a monochromatic external field has been thoroughly discussed, largely due to mathematical difficulties in the analysis of such physical systems. In this paper we consider the behavior of level-band and level-continuum systems in a modulated external field having a spectral composition which is arbitrary in a certain sense. We recall that the level-band model has been used in quantum optics to describe the evolution of an atom in a field which is quasiresonant with the transitions between an isolated level and a set of closely arranged levels. If the amplitude of the field (in dimensionless units) greatly exceeds the distance between the levels in the set and we are interested in not very long time intervals, it may be assumed that the energies of the levels in the set densely fill the corresponding range of values. The level-band model can also be used in the physics of semiconductors to describe the transition from an acceptor level to an impurity band under the action of a quasiresonant field.[4] (The quasimomentum was not determined for the states in the impurity band, and thus there were no selection rules for this parameter.) The assumption that the upper limit of the band is infinite gives rise to the level-continuum model, which is used in quantum optics to describe resonant ionization processes. The main purpose of the present work was to describe the dynamics of level-band and level-continuum systems under the action of a modulated strong quasiresonant field in analytical terms. In particular, we shall describe the process of decay of a level into a continuum. We note that the case of a strong field acting on a level-continuum system has not been adequately investigated analytically even for a monochromatic external field.

The Schrödinger equation describing a level-band (level-continuum) system in a quasiresonant external field reduces to an infinite system of first-order ordinary differential equations.[1–3] It is apparently impossible to solve such problems analytically in the general case. However, we can identify situations in which the relationship between the physical parameters permits the reasonable introduction of a large (or small) dimensionless parameter and development of a suitable asymptotic procedure to solve the corresponding initial-value problem. Here we consider the following case. Let ρ be the properly introduced Rabi parameter of the external field, and let D be the width of the band or the effective width of the continuum (the exact definitions are presented below). The Rabi parameter specifies how high the trapping region of the external field is or, in other words, which states in the band (in the continuum) effectively interact with the level through the external field. We call the external field strong, if $\rho \gg D$ holds. We also assume that the carrier frequency of the external field $\Omega \approx E_1 - E_0$ (E_1 is the lower limit of the continuum or the band and E_0 is the energy of the level measured in frequency units) significantly exceeds all the other frequency parameters of the problem and that the rotating-wave approximation can be employed. We then neglect any continuum-continuum transitions and the possible presence of other discrete levels. In this situation the problem reduces to the construction of the solution of a singularly perturbed integrodifferential equation, the role of the small parameter being played by the ratio D/ρ. We construct its solution using asymptotic methods. The model of the atom (level-continuum system) investigated here, which provides a very schematic description of a real atom, retains the property which is most important for an ionization process, i.e., it has an infinite flat continuum.

The plan of this paper is as follows. In the next section we introduce the fundamental equations. In Sec. 3 we describe the dynamics of a level-band system, and in Sec. 4 we describe that of a level-continuum system. In the conclusion we summarize the results obtained. In particular, we shall show how a new physical mechanism, which accounts for the stabilization of an atom in a strong field, can be proposed in terms of them.

2. DESCRIPTION OF THE BASIC FORMALISM

Let us consider a level-band system under the influence of a quasiresonant external field of complex spectral composition. Let $|0\rangle$ be the wave function of the level, let $|E\rangle$ be the wave function of the states in the band, and let $E_1 \leqslant E \leqslant E_2$. If $E_2 = \infty$ holds, we have a level-continuum system. Then the decay of the level into the continuum corresponds to ioniza-

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tion of the atom by the external field. We represent the wave function of the atom in the form of a superposition

$$\psi(t)=A(t)|0\rangle+\int_{E_1}^{E_2}B(E,t)|E\rangle dE.$$

We write out the Schrödinger equation for our system in terms of the amplitudes $A(t)$ and $B(E,t)$, neglecting the continuum-continuum transitions:[1-3]

$$\frac{dA}{dt}=-iE_0A-iL(t)\int_{E_1}^{E_2}g(E)B(E,t)dE,$$

$$\frac{dB}{dt}=-iEB-iL(t)g(E)A. \tag{1}$$

Here

$$L(t)=\int_{\omega_1}^{\omega_2}d\omega\, f(\omega)\cos[\phi(\omega)+t(\Omega+\omega)],$$

Ω is the optical frequency, $f(\omega)$ describes the modulation of the external field, and the function $\phi(\omega)$ describes the initial phases of the individual harmonics. The function $g(E)$ is the matrix element of the dipole moment operator. It describes the interaction of the states in the band and the level with the external field. We assume that the relations $|\omega_1|,|\omega_2|\ll\Omega$ hold. We are interested in the solution of the system of equations (1) with the following initial conditions, which correspond to the problem of decay of the level into the band (continuum):

$$A(0)=1,\quad B(E,0)=0. \tag{2}$$

We introduce $\tilde{E}$, the "mean" value of the energy of the states in the band (continuum):

$$\tilde{E}\int_{E_1}^{E_2}g^2(E)dE=\int_{E_1}^{E_2}Eg^2(E)dE \tag{3}$$

(in the case of a continuum, we assume that the two integrals converge). We define optically slow variables using the replacements

$$B(E,t)=\exp[-i\tilde{E}t]b(E,t),$$

$$A(t)=\exp[-i(\tilde{E}-\Omega)t]a(t). \tag{4}$$

We henceforth use the rotating-wave approximation. This means that the optical frequency Ω (the carrier frequency of the external field, which is close to the value of E_1-E_0) is significantly greater than the other frequency parameters of the problem and that we disregard any multiphoton processes. Then for the variables $a(t)$ and $b(E,t)$ we have the system of equations

$$\frac{da}{dt}=i\Delta a-iH(t)\int_{s_1}^{s_2}g(s)b(s,t)ds,$$

$$\frac{db}{dt}=-isb-iH^*(t)g(s)a, \tag{5}$$

where

$$\Delta=\tilde{E}-E_0-\Omega,\quad s=E-\tilde{E},\quad s_k=E_k-\tilde{E}.$$

The function

$$H(t)=\int_{\omega_1}^{\omega_2}d\omega\, f(\omega)\exp[i(\phi(\omega)+\omega t)]$$

describes the modulation of the external field. Here and below an asterisk denotes complex conjugation. From (2) it follows that

$$a(0)=1,\quad b(s,0)=0. \tag{6}$$

We set $H(t)=\rho h(t)$, where ρ is the dimensional amplitude of the external field and $h(t)$ is a dimensionless function, which describes the modulation of the external field. In Eq. (5) ρ and g appear only in the form of the product ρg. Therefore, the dimensional multipliers can be arbitrarily distributed between them without altering the dimensions of the product. It is convenient to distribute the dimensional multipliers in ρg so that $[g]=E^{-1/2}$. Then $[b]=E^{-1/2}$ and $[H]=E$. We set

$$D=\int_{s_1}^{s_2}|s|g^2(s)ds,\quad \tau=Dt,\quad s=D\zeta,$$

$$\zeta_{1,2}=s_{1,2}/D,\quad R=\rho/D,\quad \delta=\Delta/D.$$

We make the replacements

$$h(\tau/D)=m(\tau),\quad b(D\zeta,\tau/D)=c(\zeta,\tau)D^{-1/2},$$

$$g(D\zeta)=v(\zeta)D^{-1/2}.$$

Equation (5) can be written in these terms in the form:

$$a'=i\delta a-iRm(\tau)\int_{\zeta_1}^{\zeta_2}v(\zeta)c(\zeta,\tau)d\zeta,$$

$$c'=-i\zeta c-iRm^*(\tau)v(\zeta)a,\quad a'(\tau)\equiv\frac{da}{d\tau}. \tag{7}$$

We seek a solution of this system of equations which satisfies the initial conditions

$$a(0)=1, \tag{8}$$

$$c(\zeta,0)=0. \tag{9}$$

Integrating the second equation in (7) together with (9) and plugging the result into the first equation, we obtain

$$a'=i\delta a-R^2m(\tau)\int_0^{\tau}m^*(x)a(x)Q(\tau-x)dx, \tag{10}$$

$$Q(y)=\int_{\zeta_1}^{\zeta_2}v^2(\zeta)e^{-i\zeta y}d\zeta. \tag{11}$$

Thus, the solution of our initial-value problem reduces to the construction of a solution of the integrodifferential equation (10) which satisfies the initial condition (8). We note the following fact. There was some arbitrariness in the introduction of D, and we could multiply ρ by an arbitrary constant ϑ, if we simultaneously multiplied g by ϑ^{-1}. To be specific, we assume that in the integral on the right-hand side of (10) the multipliers are grouped in the product so that $\max|m(\tau)|=1$ and $\max|Q(\tau)|=1$. Then Eq. (10) is essentially independent of the choice of ϑ, and the remaining arbitrariness in $\vartheta=O(1)$ is arbitrariness in the scale of the time τ. Thus, we shall call the fixed quantity R the dimensionless

Rabi parameter. The corresponding quantity ρ is the dimensional Rabi parameter. We seek a solution of (10) with the initial condition (8) under the assumption that

$$\rho \gg D(R \gg 1), \quad \delta = O(1). \tag{12}$$

Here the dynamics of the atom can be divided into two parts, viz., "fast" and "slow." In this case the entire band (or the entire continuum, in some effective sense) falls within the trapping region of the external field, and the behavior of the system, as we shall see below, is largely similar to the behavior of a two-level atom (TLA) placed in an external field. This condition permits the application of asymptotic methods to solve (10) with initial condition (8), similar to those employed in Refs. 5 and 6 to describe the dynamics of a TLA in a strong polyharmonic external field. The procedure for constructing the solution of our initial-value problem for Eq. (10) depends on whether $m(\tau)$ takes the value zero in the time interval of interest to us. In the present paper we assume that the external field has weak modulation in terms of Refs. 5 and 6, i.e.,

$$p(\tau) \equiv |m(\tau)| > 0. \tag{13}$$

3. LEVEL-BAND SYSTEM IN A STRONG FIELD

In the case of a level-band system, both limits of the integral in the definition of $Q(y)$ in (11) are finite. We assume $\zeta_1, \zeta_2 = O(1)$. The form of Eq. (10) leads to the hypothesis that the dominant term in the asymptotic form of the solution of (10) with the initial condition (8) is a "fast" function, i.e., its derivative has a higher asymptotic order than the function itself. At the same time, according to (11), $Q(y)$ is a "slow" function, i.e., its derivative has the same asymptotic order as the function. We shall also assume below that $m(\tau)$ is a "slow" function, i.e., the modulation spectrum of the external field is $O(1)$. The integral of the product of the "fast" and "slow" functions can be expanded into a series in decreasing powers of a large parameter by integrating by parts, transferring the derivative from the "fast" cofactor to the "slow" one. These arguments make it possible to propose an asymptotic procedure for solving (10) with the initial condition (8). More specifically, we set

$$a(\tau) = F(\tau) + S(\tau), \tag{14}$$

where $F(\tau)$ is the "fast" function and $S(\tau)$ is the "slow" function. On the right-hand side of (10) we integrate the integral containing the "fast" function by parts. Since we intend to obtain an expansion up to terms $O(1)$, we make the substitution

$$F(\tau) = w'''(\tau)/m^*(\tau). \tag{15}$$

Plugging (14) and (15) into (10), we obtain

$$S'(\tau) - i\delta S(\tau) + R^2 m(\tau) \int_0^\tau dx \quad m^*(x) S(x) Q(\tau - x)$$

$$= -\frac{1}{m^*(\tau)} \{ w^{\mathrm{IV}}(\tau) - (i\delta + [\ln m^*(\tau)]') w'''(\tau)$$

$$+ R^2 p^2(\tau) [Q_0 w''(\tau) - Q_2 w(\tau) - Q(\tau) w''(0)$$

$$- Q'(\tau) w'(0) - Q''(\tau) w(0) \}. \tag{16}$$

Here and in the following we use the notation

$$Q_k = \int_{\zeta_1}^{\zeta_2} d\zeta \; v^2(\zeta) \zeta^k = i^k Q^{(k)}(0).$$

For a level-band system all the $Q_k = O(1)$, and, according to (3) and (4), $Q_1 = 0$. Distinguishing the "fast" and "slow" parts of Eq. (16), we arrive at the system of equations

$$w^{\mathrm{IV}} - (i\delta + [\ln m^*(\tau)]') w'''$$

$$+ R^2 p^2(\tau) [Q_0 w'' - Q_2 w] = 0, \tag{17}$$

$$S' - i\delta S + R^2 m(\tau) \int_0^\tau dx \quad m^*(x) S(x) Q(\tau - x)$$

$$= R^2 m(\tau) \{ Q(\tau) w''(0) + Q'(\tau) w'(0)$$

$$+ Q''(\tau) w(0) \}. \tag{18}$$

The fourth-order linear equation (17) has only two rapidly oscillating solutions, whose asymptotic forms can easily be written out using the standard WKB approximation:[8]

$$w_{1,2}(\tau) = \exp\left\{ \int_0^\tau [i\kappa R p(s) + \Phi(s)] ds \right\},$$

$$\kappa = \gamma \sqrt{Q_0}, \quad \gamma = \pm 1, \quad \Phi(s) = \Phi_0(s) + \Phi_1(s)/R + \ldots,$$

$$\Phi_0(s) = i\delta/2 - 5p'(s)/(2p(s)) + [\ln m^*(s)]'/2,$$

$$\Phi_1(s) = \frac{i}{2\kappa p^3(s)} \{ Q_2 p^2(s) \kappa^{-2} + 5p^2(s) \Phi_0^2(s)$$

$$+ 5p^2(s) \Phi_0'(s) + 4p(s) p''(s) + 3(p'(s))^2$$

$$- [2\Phi_0(s) + 5p'(s)/p(s)][3\Phi_0(s)$$

$$+ 3p'(s)/p(s)] p^2(s) \}. \tag{19}$$

Therefore, $w(\tau)$ is a linear combination of two linearly independent solutions $w_{1,2}(\tau)$. Hence we can specify only two initial conditions to solve Eq. (17). We set

$$w'''(0) = m^*(0), \quad w''(0) = 0. \tag{20}$$

This choice leads to the lowest asymptotic order of the right-hand side of Eq. (18).

As a whole, the procedure for constructing the solution of the initial-value problem consisting of (10) and (8) has the following form. In the first step we construct rapidly oscillating solutions of Eq. (17), and we take a linear combination which satisfies conditions (20) as the function $w(\tau)$. Then we can calculate the right-hand side of Eq. (18) and construct the solution of that equation. This permits determination of the value of $S(0)$. In the next step of the procedure we write out a more accurate equation than (17) for the rapidly oscillating part of $a(\tau)$ and use the value of $S(0)$ and (14) to refine the initial conditions (20). This simultaneously leads to a more accurate equation for the "slow" part, which includes terms outside the integral that do not appear in the analog of Eq. (17) for the "fast" part of the solution. In this paper we restrict ourselves to an analysis of the terms up to $O(R^{-1})$, for which it is sufficient to perform the first step of the procedure.

It follows from (18)–(20) that there are quantities of order unity on the right-hand side of Eq. (18), and thus $S(\tau)=O(R^{-2})$. We call the functions

$$F_k(\tau)=\frac{w_k'''(\tau)}{R^3 m^*(\tau)},k=1,2,$$

generalized Rabi harmonics. We arrive at the conclusion that the dominant term in the asymptotic form of the solution of our initial-value problem is a linear combination of two generalized Rabi harmonics and that the coefficients with which they appear in this linear combination are of the order unity. We omit their explicit expressions. According to these evaluations,[14] the "slow" term in O (1) gives only a correction $O(R^{-2})$.

As follows from (19), the generalized Rabi harmonics do not exhibit decay. For a monochromatic external field [i.e., for $m(\tau)$=const], this fact follows in the present asymptotic case from the results in Ref. 7. From the physical standpoint this is attributable to the analogy between a level-band system in a strong field and a TLA in an external field. If the Rabi parameter of the field significantly exceeds the width of the band and the width of the modulation spectrum of the external field, the band is similar to a level "as seen by the external field." A formal transition to the TLA model can be accomplished by setting $Q_2=0$ in (17). Then we arrive at the equation to which the Schrödinger equation for a TLA reduces following the use of the rotating-wave approximation (see, for example, Ref. 5). The latter system has a constant of motion, i.e., the norm of the wave function of the two-level atom is maintained (this is an exact analytical, rather than asymptotic, result). In the case of a strong field, a level-band system behaves similarly in the dominant orders of the asymptote [up to $O(R^{-1})$]; and the level does not decay into the band. The population of the level oscillates between values which are asymptotically close to 0 and 1.

Remark 1. The fact that the generalized Rabi harmonics do not decay follows from relations (19). The following argument is useful here. For $Q_2=0$ Eq. (17) transforms into the equation for a TLA, for which neither of the generalized Rabi harmonics decays, so the decay in (19) is attributable only to terms which depend on Q_2. However, it is not difficult to show that the corresponding, purely imaginary term causes only displacement of the effective frequency of a generalized Rabi harmonic.

Remark 2. The construction of the asymptotes of solutions of singularly perturbed integrodifferential equations was discussed in Ref. 9. As follows from the results in that book, in the case of a level-band system, the asymptotic form of arbitrary order for the solution of the initial-value problem can be constructed by continuing the above procedure. In this case it is significant that all the derivatives of $Q(y)$ are bounded functions.

4. LEVEL-CONTINUUM SYSTEM IN A STRONG FIELD

Let us now discuss how the above approach should be modified in the case of a level-continuum system. From the formal standpoint level-band and level-continuum systems differ for two reasons. First, if we assume that for large arguments $v(\zeta)$ goes algebraically to zero, the Q_k for a level-continuum system become infinite after a certain k. Thus, the recurrence relation for expanding the integral on the right-hand side of (11) into a series runs into obstacles, and the methods in Ref. 9 are not directly applicable in the present situation. Second, the spectrum of the function $Q(\tau)$ for a level-continuum system is unbounded; therefore, the scheme for constructing Eq. (18) must be modified: functions with an unbounded spectrum can appear on its right-hand side. We note, nevertheless, that if $v(\zeta)$ decreases rapidly as $\zeta\to\infty$, $Q(\tau)$ is basically "slow," and thus, as in the case of a level-band system, integration by parts can be employed to construct the asymptotic expansions. We conclude that the behavior of $v(\zeta)$ as $\zeta\to\infty$ must be accurately taken into account for a level-continuum system. If we specify this behavior in some manner, we can construct the asymptotic form of the solution of the initial-value problem consisting of (10) and (8) with consideration of the corresponding singularities in Eqs. (17) and (18). We shall henceforth assume that

$$\zeta^2 v^2(\zeta)=\theta(1+\zeta)^{-1}+n(\zeta), \tag{21}$$

where $n(\zeta)=O(\zeta^{-2})$ when $\zeta\to\infty$ and θ=const>0. On the one hand, this modification does not significantly complicate the mathematical machinery, and, on the other hand, it makes it possible to describe decay in a level-continuum system. In this case $Q_2=-\infty$, and thus the transition from (10) to (16) is impossible. However, as is easily shown,

$$D=\int_{\zeta_1}^{\infty}|\zeta|v^2(\zeta)d\zeta,$$

is finite, and, therefore, we can use the former notation.

Before preceding to a description of the solution of the initial-value problem in this case, let us discuss the physical status of our model. The function $v(\zeta)$, one of the functional parameters of our model, contains implicit information on the effective potential of the atom and is arbitrary over a very broad range.

For the further mathematical manipulations to be correct, it is sufficient that θ and $n(\zeta)$ $=O(1)$. We express these relations in the original terms. If, following (21), we assume $s^2g^2(s)=\theta_1 s^{-1}+n_1(s)$ and $n_1(s)=O(s^{-2})$ in the limit $s\to\infty$, these relations can be rewritten in the form

$$\rho_0\gg D_1,\quad \rho_0\gg D,$$

where

$$\rho_0=\rho\left(\int_{s_1}^{\infty}g^2(s)ds\right)^{1/2},\quad D_1=D^{-1}\int_{s_1}^{\infty}|s^2g^2(s)-\theta_1 s^{-1}|ds.$$

Here ρ_0 is the effective Rabi parameter of the external field. We note that D_1, unlike D, does not depend on the distribution of the factors between ρ and g. If the first inequality holds, satisfaction of the relation adopted above

$\max|Q(\tau)|=1$ can be ensured with the aid of the dimensionless multiplier ϑ. Here we assume that $D=O(1)$, i.e., the second inequality is satisfied. Equation (10) was derived in the rotating-wave approximation, and its corollaries hold when the following two conditions hold: $E_1-E_0 \gg \rho_0 \gg D_1$. It follows from our definitions that ρ_0 is linearly dependent on the amplitude of the external field. For $E_1-E_0 \gg D_1$ the external field can be selected so that both conditions hold. Thus, from the formal standpoint there is a possibility for satisfying both these conditions, and in this sense our model is consistent. We note that the condition $E_1-E_0 \gg D_1$ justifies the neglect of transitions to the continuum. Here the question of whether there is a range of values of the parameters for real atoms where the latter condition holds remains open. However, in any case the present problem [the investigation of the behavior of a level-continuum system in a strong quasiresonant external field in the rotating-wave approximation when condition (12) holds] is still one of the new problems in quantum optics for which the dynamics of a level-continuum system can be systematically described in analytical terms. Our approach makes it possible to explicitly describe the process of population decay of the level and to calculate the ionization rate of the atom. Our methods can also be applied to the solution of more complicated problems which more accurately describe the ionization process of a real atom.

Instead of (16), in the case of a level-continuum system, in the same terms we have

$$S'(\tau)-i\delta S(\tau)+R^2m(\tau)\int_0^\tau dx\;\; m^*(x)S(x)Q(\tau-x)$$

$$=-\frac{1}{m^*(\tau)}\Bigg\{w^{\mathrm{IV}}(\tau)-[i\delta+(\ln m^*(\tau))']w'''(\tau)$$

$$+R^2p^2(\tau)\bigg[Q_0w''(\tau)+\int_0^\tau Q''(\tau-x)w'(x)dx$$

$$-Q(\tau)w''(0)-Q'(\tau)w'(0)\bigg]\Bigg\}. \tag{22}$$

In this equation we must separate the "fast" and "slow" parts. It follows from (21) that

$$Q''(y)=\theta\ln y+i\theta\beta(y)+N(y), \tag{23}$$

where

$$\beta(y)=\int_y^\infty \mu^{-1}\sin\mu\;d\mu,\quad \beta(0)=\pi/2.$$

$N(y)$ is a real function, whose derivative is a function that can be integrated at zero. We seek its explicit form. Substituting (23) into (22), we can isolate the singularity associated with the presence of $\ln(\tau-x)$, or, in the final analysis, with the asymptotic form (21).

On the right-hand side of (22) we leave only the term proportional to $\ln(\tau-x)$ in the integrand, and we integrate the remaining terms by parts (possible owing to the assumptions that we have made). Ultimately, we arrive at the relation

$$S'(\tau)-i\delta S(\tau)+R^2m(\tau)\int_0^\tau dx\;\; m^*(x)S(x)Q(\tau-x)$$

$$=-\frac{1}{m^*(\tau)}\Bigg\{w^{(iv)}(\tau)-[i\delta+(\ln m^*(\tau))']w'''(\tau)$$

$$+R^2p^2(\tau)\bigg[Q_0w''(\tau)+\theta\int_0^\tau \ln(\tau-x)w'(x)dx$$

$$+[i(\pi\theta/2)+N(0)]w(\tau)-Q'(\tau)w'(0)$$

$$-[i\theta\beta(\tau)+N(\tau)]w(0)+\ldots\bigg]\Bigg\}. \tag{24}$$

As before, in the case of a level-continuum system $w(\tau)=\alpha_1w_1(\tau)+\alpha_2w_2(\tau)$, and $w_{1,2}(\tau)$ are rapidly oscillating Rabi harmonics. The dominant terms in the asymptotic form of the Rabi harmonics are already known, and we can use this information to calculate the integral on the right-hand side of (24). We shall also use the following analog of Erdélyi's lemma:[10]

$$\int_0^\tau \ln x\,\exp\bigg[iR\int_0^x f(s)ds\bigg]dx=-\frac{1}{f(0)}\bigg[\frac{\pi}{2R}+i(\ln R$$

$$-C)/R\bigg]-\frac{i\ln\tau}{Rf(\tau)}\exp\bigg[iR\int_0^\tau f(s)ds\bigg]$$

$$+O(\ln R/R^2), \tag{25}$$

where C is Euler's constant.

There is some difficulty in separating the "fast" terms in (24) from the "slow" terms due to the terms outside the integral in (24) which were obtained, in particular, with the use of (25). These terms include harmonics of arbitrarily high frequency, in contrast to the case of a level-band system. These terms are too complicated to give explicitly. Let their sum be equal to $M(\tau)$. We divide this function into two parts, viz., a "slow" part $\hat{P}M(\tau)$ and a "fast" part. If

$$M(\tau)=\int_{-\infty}^{\infty}\chi(\omega)e^{i\omega\tau}d\omega,$$

let

$$\hat{P}M(\tau)=\int_{\sqrt{R}}^{\sqrt{R}}\chi(\omega)e^{i\omega\tau}d\omega.$$

We ultimately arrive at the following equations for the "fast" and "slow" functions:

$$w^{\mathrm{IV}}(\tau)-[i\delta+(\ln m^*(\tau))']w'''(\tau)+R^2p^2(\tau)\{Q_0w''(\tau)$$

$$+[i\pi\theta(1-\gamma)/2+N(0)+\theta(\ln R-C)]w(\tau)$$

$$+(1-\hat{P})M(\tau)\}=0, \tag{26}$$

$$S'(\tau)-i\delta S(\tau)+R^2m(\tau)\int_0^\tau m^*(x)S(x)Q(\tau-x)dx$$

$$=-R^2m(\tau)PM(\tau). \tag{27}$$

As in the level-band case, the first equation has only two rapidly oscillating, linearly independent solutions, i.e., Rabi harmonics. Therefore, we seek a solution of (26) which satisfies the two initial conditions (20). Our assumptions regarding the rate of decay of $v(\zeta)$ as $\zeta\to\infty$, the choice of the operator $\hat{P}$, and conditions (20) imply $w'(0)=O(R^{-2})$ and $w(0)=O(R^{-3})$ and, accordingly, $\hat{P}M(\tau)=O(R^{-3/2})$ and $(1-\hat{P})M(\tau)=O(R^{-5/2})$. Thus we have $S(\tau)=O(R^{-3/2})$, and the Fourier components of the dominant term in the asymptotic form of this function are nonzero only in the interval $[-R^{1/2},R^{1/2}]$. Therefore, in this case, as before, the "slow" term in (14) acts as a small correction. If we restrict ourselves to calculating the asymptotic forms of the generalized Rabi harmonics only to terms $O(R^{-1})$, the discontinuity in Eq. (26) can be neglected. Equation (26) differs from (17) only in the presence of an imaginary quantity in the coefficient in front of $w(\tau)$ (if the differences in the real part of this coefficient are disregarded). Therefore, relations (19), appropriately modified, are valid in this case, too. Using arguments similar to those offered in *Remark 1*, we can conclude that decay of the generalized Rabi harmonics can be caused only by the terms associated with $Q''(\tau-x)$, i.e., with θ in the present case. We ultimately find that in the order of the asymptotes under consideration [up to $O(R^{-1})$] the decay of the generalized Rabi harmonics is described by the relation

$$|F_{1,2}(\tau)|=\exp\left[\pi\theta(1-\gamma)\int_0^\tau\frac{ds}{4R\kappa^3p(s)}\right],\quad \gamma=\pm1. \tag{28}$$

This relation is the main result of the present work.

Using initial conditions, we can calculate the explicit forms of α_1 and α_2 and construct the solution of the initial-value problem. However, as follows from (14), analysis of (28) is sufficient to describe the population decay of the level. It follows from this relation, in particular, that the Rabi harmonic corresponding to $\gamma=-1$ decays in a strong field with weak modulation. The second Rabi harmonic does not decay in this order of the asymptotic expression.

We first point out the formal reasons for the presence of the factor $1-\gamma$ in (28). The term which does not depend on γ is associated with the presence of the term $\pi/(2R)$ on the right-hand side of (25). The term proportional to γ is associated with the presence of the imaginary part $i\pi\theta/2$ of the multiplier in front of $w(\tau)$ on the right-hand side of (24). Let us discuss the physical meaning of this result. As already mentioned, in the strong field case with our assumptions regarding the parameters, a level-continuum system is largely similar to a TLA in an external field. For a TLA in a strong external field (here the term "strong field" signifies that the Rabi parameter of the field significantly exceeds the width of its modulation spectrum) one of the Rabi harmonics has an effective frequency which is smaller than the frequency of the transition between the levels, and the other harmonic has an effective frequency which is greater, regardless of the detuning of the carrier frequency from the transition frequency. As noted above, the generalized Rabi harmonics for a TLA in an external field do not decay, this being a consequence of the constant of motion for this system.

A similar finding is also observed for a level-continuum system under our assumption, i.e., up to terms of order unity there is no decay, and the structure of the solution of the initial-value problem is similar to the structure of the solution for a TLA. However, the presence of the continuum makes decay possible. In this case only one Rabi harmonic, the one that "enters" the continuum, decays into the continuum, which lies entirely above the effective transition frequency. Thus, a phenomenon of population trapping of the level appears in a strong field with weak modulation. Of course, the analogy between a level-continuum system and a TLA (like any analogy) does not signify complete similarity between the dynamics of these system: in a level-continuum system the level decays, while in the case of a TLA the population of the level oscillates. Nevertheless, this analogy, as was shown above, helps us to understand some important features of the behavior of the ionization process in our problem.

The results which we have obtained enable us to describe the dynamics of a level-continuum system as a whole. The solution of our initial-value problem in the dominant term of the asymptote is a linear combination of two generalized Rabi harmonics, the "slow" term giving a correction $O(R^{-3/2})$. On the time scale of order unity the system undergoes oscillations similar to the Rabi oscillations of a TLA in an external field. Here the population of the level oscillates between values which are asymptotically close to 0 and 1. On the time scale $O(R)$ one generalized Rabi harmonic decays, and thus the solution has the form of the sum of one Rabi harmonic (which corresponds to $\gamma=+1$) and a small correction. The population of the level tends to a value close to 1/4.

5. CONCLUSIONS

We have examined the dynamics of level-band and level-continuum systems in an external field under fairly broad assumptions regarding its modulation. The dynamics of such systems in a monochromatic field is usually investigated using the Laplace transformation. In the case of a modulated external field, this approach is inapplicable, since there is no separation of the variables in the problem; therefore, we have employed other analytical tools. We note that a similar technique can be developed to describe the dynamics of a band-band system (for example, if a transition between impurity bands in a semiconductor is being studied).

We have discussed the case of a strong field, in which the Rabi parameter of the external field greatly exceeds the effective width of the band (continuum) and the width of the modulation spectrum of the external field, using the rotating-wave approximation. It has been shown in a level-band system that decay of the level does not occur and that its population oscillates between values close to 0 and 1. The solution of the initial-value problem in the predominant term of the asymptote is a sum of two generalized Rabi harmonics, the slow term making only a small correction. In a level-continuum system the behavior of the solution of our initial-value problem is more complicated. Of the two generalized Rabi harmonics which together form the dominant term in the asymptotic form of the solution of the initial-value problem, only the harmonic which "enters" the continuum decays. The population of the level initially oscillates between 0 and 1 and then stabilizes near a value of 1/4. The decay rate of the harmonic is $O(R^{-1})$ and decreases as R increases, i.e., as the intensity of the external field increases. Thus, there is a phenomenon of stabilization of an atom in a strong external field. This stabilization mechanism, which is associated with the analogy between a level-continuum system and a TLA in a strong external field differs from the mechanisms discussed in Refs. 11–15.

We describe once again the basic assumptions regarding the parameters of the system, which lead to this effect. First, we assumed that the carrier frequency of the external field Ω significantly exceeds the effective width of the continuum D_1 and that the width of the modulation spectrum ω_0 is of the same order as D_1, i.e., that $\Omega \gg D_1$, ω_0. This permits the use of the rotating-wave approximation and the neglect of multiphoton processes. For a Rabi parameter ρ of the external field such that $\Omega \gg \rho \gg D_1$ $(R=\rho/D \gg 1)$, the system becomes similar to a TLA in a strong external field. Then the greater the Rabi parameter, the "closer" the level-continuum system to a TLA. A TLA does not decay in an external field, and the norm of its wave function is maintained (this is also true after application of the rotating-wave approximation). This property is also inherited by the level-continuum system, i.e., the ionization rate of the atom decreases as the Rabi parameter increases.

In this paper we have discussed the situation in which the asymptotic form of $v(\zeta)$ at $\zeta \to \infty$ was assigned by relation (21). Our technique can also be used to examine the case $v(\zeta)=O(\zeta^{-1-\alpha})$ when $\zeta \to \infty$ and $\alpha > 0$. Calculations similar to those performed above lead to the conclusion that in this case the ionization rate of the atom (the decay rate of the Rabi harmonic) is $O(R^{-2\alpha})$. Thus, by measuring the ionization rate of an atom in a strong field, we can obtain information on the behavior of the matrix element of the dipole moment operator of the atom.

We note one more consequence of our results. As was noted above, the study of the dynamics of a level-continuum system in a strong field led to the conclusion that the population of the level stabilizes near a value of 1/4 on a time scale $O(R)$. Then population trapping of the level occurs, associated with the unique coherence between the strong external field and the induced oscillations of the level-continuum system. For more effective ionization, the external field should be turned off to destroy this coherence and then turned back on. Afterwards the population of the level can be lowered to a value near 1/16, etc. Thus, in a strong external field the ionization of an atom is more effective when a series of pulses is employed than when a field of constant amplitude is used.

Above we discussed a strong external field which satisfies (13), i.e., we assumed that the modulation of the external field is weak. If the modulation of the external field is deep, so that $m(\tau)$ has a large number of zeros in the time interval of interest to us, the problem of describing the dynamics of level-band and level-continuum systems becomes significantly more complicated. If, for example, $m(\tau)$ is a periodic function and has zeros in each period, the external field is strong in part of the period and may be considered weak in an asymptotically small part of the period. In this case decay of the level can probably be observed even for a level-band system. It follows from qualitative arguments that no population trapping of the level occurs in this case.

The analogy with a TLA can possibly be useful in examining the limit $\rho \gg \Omega$, D. In this case a phenomenon which has attracted considerable attention in recent years, viz., barrier-suppression ionization,[2,16] is observed. It is known that stabilization of the atom is observed under these conditions, too. When the dynamics of a level-continuum system is studied in this case, multiphoton processes should, of course, be taken into account, and the rotating-wave approximation is unsuitable. However, in this case, too, the level-continuum system is similar to a TLA "as seen by the external field." The equations derived in this case are similar to those presented above. Here it becomes necessary to consider a deeply modulated field. For example, in the case of a monochromatic external field we have $h(t)=\cos\Omega t$.

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