Quasiparticle currents in one-dimensional correlated models

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We consider current carrying states in a one-dimensional system of interacting electrons. New exact results for the Hubbard model and the bosonization method are exploited. We find that away from half-filling, both spin and charge excitations carry currents which are proportional to their momenta in most cases. Being in qualitative agreement with a single particle picture of the noninteracting 1-*d* Fermi gas, this result contradicts the spin-charge separation concept as it is usually derived from bosonization concepts and strong coupling arguments. We show that the bosonization procedure can be reconciled with the exact one (in the weak interaction limit) if spectrum parabolicity is taken into account. Both the current and the Hamiltonian acquire nonlinear cross-terms between the two channels. Finally, we study current carrying excited states for the Hubbard model in the presence of a magnetic field flux. In addition to the known diamagnetic contribution, we find a temperature-dependent orbital paramagnetic contribution from both singlet and triplet excited states. This paper extends and details the recent statements formulated by P. Nozieres and the present authors.

1. INTRODUCTION

One of the most-discussed properties of quasi-one-dimensional correlated electronic systems (see, e.g., Refs. 1 and 2 for reviews) is the separation of spin and charge degrees of freedom. Related phenomena are described in the context of two substantionally different approaches: bosonization methods,[3,4] and exact Bethe ansatz solutions.[5–7] Being related to the linear spectrum approximation, the bosonization method has advantages in calculations of physical properties and the system descriptions in terms of phenomenological hydrodynamic parameters corresponding to charge-and spin-density sounds, providing an explicit spin–charge separation: typical operators are additive, while correlation functions become multiplicative. Exactly solvable models, primarily the Hubbard model,[7] describe the same system in terms of certain particles at a deeper level (nowadays sometimes called holons and spinons) which are also assumed to correspond to charge and spin degrees of freedom independently.

In this article, we will show that the concept of spin–charge separation should be revised in the following sense. All elementary excitations of the Hubbard model are charged as long as the band is not half-filled. The spin excitations, both singlet and triplet, carry similar current to charge excitations. In the bosonization approach, the Fermi velocity dispersion (the spectrum parabolicity) should be taken into account, whenever both charge and spin excitations are found to carry a charge current consistent with the Hubbard model. The actual charge–spin separation emerges only at the level of a macroscopic current-carrying ground state, e.g., in an applied magnetic field, where the number of charged bosons is not conserved and the current appears as their condensate. These statements have recently been formulated in Ref. 8 and the present publication provides the necessary details.

The currents have been already calculated[9] in the strong-interaction limit: $u=U/t\gg 1$, where U is an on-site repulsion energy and t is a nearest-sites hopping amplitude. An unexpected result was obtained: both charge and spin excitations carry electric currents, which are typically proportional to their momentum p as $p\to 0$. On the other hand, in the weak-coupling limit ($u\ll 1$) at least, a continuum model with a linearized electron spectrum is expected to be adequate when the bosonization method results in spin–charge separation. Spin excitations are then not presumed to carry a current. To elucidate this inconsistency, we now study the excitation currents in the Hubbard model for the case of weak interactions, where the bosonization technique is applied and the two methods should be reconciled. The following types of excitations will be considered: spin triplet and singlet pairs, hole and particle states, added particles, gap states at half filling ($\rho=1$). We will find that for both charge and spin states, the currents are proportional to the momenta, which confirms the disagreement with the usual bosonization results. The conflict will be resolved by studying effects of spectrum parabolicity and reexamining the structure of the current operator.

This article is arranged as follows. In Sec. 2 we consider currents of spin and charge excitations for the Hubbard model. We summarize and refine earlier results[9] for a simple case of the strong repulsion, and we present new calculations for weak repulsion. In Sec. 3 we discuss results of the bosonization method. Agreement with exact results is achieved by taking into account the electron spectrum parabolicity near the Fermi surface (the Fermi velocity dispersion). We point out an important distinction between the ground-state current and the currents due to excitations. In Sec. 4 we discuss the solution for a Hubbard ring with excitations in a magnetic field. We show that all properties are periodic as a function of the magnetic flux through the ring with a period of the magnetic flux quantum $\Phi_0=ch/e$, while a separate excitation does not demonstrate local gauge invariance. We calculate the total orbital magnetic susceptibility of the Hubbard ring, consisting of a large diamagnetic part due to the ground-state current, and a paramagnetic part due to both spin and spinless excitations. Sec. 5 is devoted to conclusions. A detailed analysis of spin singlet excitations and new results at arbitrary ρ are given in the Appendix.

2. CURRENTS OF EXCITATIONS FOR THE HUBBARD MODEL

The Hamiltonian for the Hubbard model is

$$H=-t\sum_{n=1}^{N_a}(c^{\dagger}_{n,\sigma}c_{n+1,\sigma}+\text{h.c.})+U\sum_n c^{\dagger}_{n,\uparrow}c_{n,\uparrow}c_{n,\downarrow}c_{n,\downarrow}, \tag{1}$$

where N_a is the number of sites, $c^{\dagger}_{n,\sigma},c_{n,\sigma}$ are the creation and annihilation operators of electrons with spins $\sigma=\uparrow,\downarrow$, and $U>0$ is the on-site repulsion amplitude of particles with opposite spins.

The Bethe ansatz equations[7] are

$$N_a k_j=2\pi I_j+\sum_{\beta=1}^{M}\theta(2\sin k_j-2\lambda_\beta),\quad j=1,...,N, \tag{2}$$

$$\sum_{j=1}^{N}\theta(2\sin k_j-2\lambda_\alpha)=2\pi J_\alpha-\sum_{\beta=1}^{M}\theta(\lambda_\alpha-\lambda_\beta),$$
$$\alpha=1,...,M, \tag{2a}$$

where

$$\theta(x)=-2\tan^{-1}(2x/u),\quad u=U/t,$$

$N=\rho N_a$ is the number of particles, and M is the number of spins "down".

In the ground state, N is even, $M=N/2$, $S_z=N/2-M=0$. The values I_j,J_α,k_j are distributed symmetrically about zero:

$$I^0_{j+1}-I^0_j=1,\quad J^0_{\alpha+1}-J^0_\alpha=1,\quad \sum I_j+\sum J_\alpha=0,$$

$$-Q=k_1<k_2<...<k_N=Q. \tag{3}$$

The momentum p, the energy E, and the current j are given by[7,9]

$$p=\sum k_j,\quad E=-2\sum\cos k_j,\quad j=2\sum\sin k_j. \tag{4}$$

The electric current follows either from the charge conservation law

$$\frac{d\rho_n}{dt}+j_{n+1}-j_n=0,\quad \rho_n=\sum_\sigma c^{\dagger}_{n,\sigma}c_{n,\sigma},\quad \frac{d\rho_n}{dt}=i[\rho,H]$$

or from the general definition $j=-\delta H/\delta A$, where A is the vector potential introduced into the Hamiltonian (1) by the transformation $t\to t\exp(ieA/\hbar c)$.

We now consider relevant results for various basic excitations of the Hubbard model.

2.1. Spin triplet excitations

Spin excitations were studied basically in Refs. 10–13. As in the 1-d Heisenberg model, spin degrees of freedom of the 1-d Hubbard model are spanned by an even number of doublets spinons ($s=1/2$). Two spinons can form a spin singlet or a spin triplet excitation.

The spin triplet excitations can be obtained by the following choice of numbers I_j,J_α:

$$I_j=I^0_j,\quad J_{\alpha+1}-J_\alpha=1+\delta_{\beta,\beta_1}+\delta_{\beta,\beta_2}.$$

Excited states are conveniently described[9] by the function

$$\tilde\rho(k_j)=N_a\rho_0(k_j)\delta k_j,\quad \rho_0(k_j)=1/N_a(k^0_{j+1}-k^0_j),$$

where ρ_0 is a known function for the ground state,[7] and δk_j is the shift in the wave number k_j due to the excitation. The function $\tilde\rho$ must be determined from the equation

$$\tilde\rho(k)=f(\sin k),$$

$$f(t)=\sum_{i=1,2}\frac{1}{\pi}\tan^{-1}\left[\exp\left(\frac{2\pi}{u}(t-\lambda_i)\right)\right]+\int_{-\sin Q}^{\sin Q}K(t-t')f(t')dt', \tag{5}$$

where

$$K(t)=\frac{1}{\pi u}\int_{-\infty}^{\infty}\frac{\exp(2ity/u)}{\exp|y|+1}dy,\quad t=\sin k.$$

The interval Q in (3), (5) is defined self-consistently via the normalization

$$\int_Q^Q\rho_0(k)dk=N/N_a=\rho.$$

The momentum, energy, and current (4) are expressed in terms of $\tilde\rho(k)$ as follows:

$$p=\int_{-Q}^{Q}\tilde\rho(k)\,dk,$$

$$E-E_0=2\int_{-Q}^{Q}\tilde\rho(k)\sin k\,dk,$$

$$j=\int_{-Q}^{Q}\tilde\rho(k)\cos k\,dk, \tag{6}$$

where E_0 is the ground-state energy. The interval value Q was known in the limit $u\gg1$ to be[14]

$$Q\approx\pi\rho-4\rho\ln 2\frac{\sin\pi\rho}{u}+O\left(\frac{1}{u^2}\right),\quad u\gg1. \tag{7}$$

In the limit $u\ll\sin\pi\rho$, we can similarly obtain

$$Q\approx\frac{\pi\rho}{2}+\frac{2}{\pi^3}\frac{u}{\sin(\pi\rho)},\quad u\ll1. \tag{8}$$

In the limit $u\ll1$, we find from (5) that

$$\tilde\rho(k)\approx\theta(\sin k-\lambda_1)+\theta(\sin k-\lambda_2), \tag{9}$$

and

$$E(p_i)\approx-2\left[\cos\frac{\pi\rho}{2}-\cos\left(\frac{\pi\rho}{2}-p_i\right)\right],$$

$$j(p_i)=2\left[\sin\frac{\pi\rho}{2}-\sin\left(\frac{\pi\rho}{2}-p_i\right)\right], \tag{10}$$

$$p=p_1+p_2, \quad E=E_1+E_2, \quad j=j_1+j_2.$$

In the limit $u \gg 1$, the solution of (5) was found in Ref. 9. At small momentum $p \ll 1$, we find for both limits the currents:

$$u \ll 1:\ j \approx 2p\cos\frac{\pi\rho}{2}; \quad u \gg 1:\ j \approx 2p\frac{\sin\pi\rho}{\pi\rho}. \tag{11}$$

We conclude that at $\rho \neq 1$, spin waves carry an electric current proportional to the momentum.

2.2. Spin singlet excitations

The spin singlet states have been shown[12] to be described by an additional pair of complex numbers $\lambda_0=\Lambda \pm i\Gamma$. They are described by significantly different and more complicated Bethe-ansatz equations, which will be described in Appendix. Nevertheless, we have found their properties to be identical to properties of spin triplet states, see Sec. 2.1.

2.3. Hole and particle states

These states determine the lowest branches of charge excitations.[7,9] Consider the hole states, which are described by a hole in the k distribution:

$$I_{j+1}-I_j=1+\delta_{j,j_0}, \quad J_\alpha=J_\alpha^0.$$

The following equation was derived in Ref. 9 for this case:

$$\tilde{\rho}(k)=\theta(k-k_0)+\int_{-Q}^{Q}dk'\cos k'\tilde{\rho}(k')\times K(\sin k-\sin k'), \tag{12}$$

where $k_0=k_{j0}$. In the limit $u \gg 1$, the energy[10] and the current[9] were found to be

$$\epsilon \equiv E-E_0 \approx -2[\cos(\pi\rho)-\cos(\pi\rho-p)], \tag{13}$$

$$j \approx 2[\sin(\pi\rho)-\sin(\pi\rho-p)]+O(1/u) \Rightarrow 2p\cos(\pi\rho)+O(1/u).$$

Note that at quarter filling ($\rho=1/2$), the current (13) vanishes to zeroth order in $1/u$. The first-order expression $j \sim p/u$ can be extracted from Eq. (13) of the Ref. 9. In the opposite limit $u \ll 1$, we find from (6) and (12) that

$$\varepsilon \approx -4\left[\cos\frac{\pi\rho}{2}-\cos\left(\frac{\pi\rho}{2}-\frac{p}{2}\right)\right], \tag{14}$$

$$j \approx 4\left[\sin\frac{\pi\rho}{2}-\sin\left(\frac{\pi\rho}{2}-\frac{p}{2}\right)\right] \Rightarrow 2p\cos\frac{\pi\rho}{2}.$$

The same expressions can be obtained for particle states. The results are similar to results for spin excitations (10). Thus, we see that in the weak-coupling limit, both spin and particle–hole excitations have similar spectra and carry a current $j \propto p$ at small p.

2.4. Excitations with a gap at $\rho=1$

These states are described by a pair of complex quasimomenta $k_{j0}=\kappa \pm i\chi$,[12] and by two holes in the k distribution $I_l \Leftrightarrow k_l$, $I_m \Leftrightarrow k_m$. The energy is known at large u to be[12]

$$\epsilon \approx u+2(\cos k_l+\cos k_m-2\cos Q).$$

For those excitations, the following equation was derived in Ref. 9:

$$\tilde{\rho}(k)=\theta(k-k_l)+\theta(k-k_m)-1-\frac{1}{4}\times\operatorname{sign}(2\sin k-\sin k_l-\sin k_m)+\int_{-Q}^{Q}dk'\cos k'\tilde{\rho}(k')K(\sin k-\sin k'). \tag{15}$$

It follows from (15) and (6) that $j \equiv 0$, so that these states carry no current for any value of u.

2.5. States with one added particle

These states are described by the equation[9]

$$\tilde{\rho}(k)=\int_k^{k_0}dk' K\cos k'(\sin k-\sin k')+\int_{-Q}^{Q}dk'\tilde{\rho}(k')K\cos k'(\sin k-\sin k').$$

We find that for both limits, $u \gg 1$[9] and $u \ll 1$, the energy and current are given by

$$\varepsilon \approx -2\cos p, \quad j \approx \sin p, \quad |p|>Q.$$

The conclusion of this section is that not only charge excitations (hole and particle states, states with added particles) but also spin states (spin singlet and triplet excitations) carry electric charge current. In the next Section, we consider this problem in the framework of the bosonization approach.

3. THE BOSONIZATION TECHNIQUE

The bosonization procedure[3,4,15] relies upon a decomposition of the Fermi operator into right- and left-moving parts, and on linearization of the spectrum in the vicinity of $\pm k_F$:

$$\Psi_\sigma(x)=\Psi_{\sigma+}(x)\exp(ik_Fx)+\Psi_{\sigma-}(x)\exp(-ik_Fx), \tag{16}$$

$$H=H_0+H_{int},$$

$$H_0=v_F\left[\Psi_{\sigma+}^\dagger\left(-i\frac{\partial}{\partial x}\right)\Psi_{\sigma+}-\Psi_{\sigma-}^\dagger\left(-i\frac{\partial}{\partial x}\right)\Psi_{\sigma-}\right]+\text{h.c.}$$

The interaction part H_{int} of the Hamiltonian is proportional to u, or in the more general case it has the form of "g-ology".[16,17]

In the notation of Ref. 18, one introduces the Bose field φ_σ and the conjugate momentum π_σ:[18]

$$\Psi_{\sigma,\pm} \propto \exp[\pm i\sqrt{4\pi}\varphi_{\sigma,\pm}(x)], \tag{17}$$

$$\varphi_{\sigma\pm}=\frac{1}{2}\left(\varphi_\sigma \mp \int_{-\infty}^{x}\pi_\sigma(x')dx'\right).$$

Being related to exact gauge invariance and approximate chiral invariance, these transformations are assumed to be asymptotically valid[8] for all coupling strengths within a gapless sector of the phase diagram. While in relativistic field theory[4] the bosonization is introduced axiomatically, we must follow the primary[3] explicit formulation with a smooth momentum cutoff which allows for a direct substitution of (17) into the original Hamiltonian. In the new variables, the Hamiltonian takes the form

$$H \Rightarrow H(\varphi)+H(\sigma),$$

where $\varphi=(\varphi_\uparrow+\varphi_\downarrow)/2$ and $\sigma=(\varphi_\uparrow-\varphi_\downarrow)/2$ are the charge and spin polarization fields. For the forward scattering case (the Tomonaga–Luttinger model), or asymptotically for the repulsive Hubbard model at $\rho\neq 1$, the Hamiltonian take the sound forms[19]

$$H(\varphi)\approx\frac{a}{2}(\partial_x\varphi)^2+\frac{b}{2}\pi_\varphi^2,\quad H(\sigma)\approx\frac{c}{2}(\partial_x\sigma)^2+\frac{d}{2}\pi_\sigma^2. \tag{18}$$

For the weak-coupling model, the coefficients a, b, c, and d are close to 1, e.g., for the Hubbard model, $b=d=1$ and $a,c=1\pm u/\pi$. Within the present discussion the difference between them is not important, and we will set all of them to unity as for a noninteracting Fermi gas.

The electric current expression is obtained from the definition

$$j=-\frac{\delta H}{\delta A}=\Psi^\dagger\sigma_z\Psi=\sqrt{\frac{\pi}{2}}\,\pi_\varphi\propto\frac{\partial\varphi}{\partial t}, \tag{19}$$

so that it contains the charge field operators only. Consequently, the eigenstates of the spin Hamiltonian $H(\sigma)$ will carry no current j, and they will not interact with the electric field. (A similar problem was addressed in Ref. 20, where was shown that phonon-assisted processes mix the charge and the spin degrees of freedom, which, for example, makes it possible to observe optical absorption across the spin gap for the attractive model, or to excite the spin wave continuum for the allowed absorptions in the repulsive case.) This common conclusion is in apparent disagreement with our exact results for the Hubbard model, as we have discussed above.

In order to resolve the disagreement, we take into account the spectrum parabolicity (the Fermi velocity dispersion). Then the Hamiltonian (16) becomes

$$H\to H+\delta H,\quad \delta H=-\Gamma\Psi^\dagger\frac{\partial^2}{\partial x^2}\Psi, \tag{20}$$

so that for the Hubbard model

$$\Gamma=\frac{1}{2}\frac{\partial^2\epsilon}{\partial k^2}\bigg|_{k_F}\approx\cos\frac{\pi\rho}{2}.$$

The electric current becomes

$$j\Rightarrow j+\delta j,\quad \delta j=\Gamma\Psi^\dagger\left(-i\frac{\partial}{\partial x}\right)\Psi$$
$$\sim-\Gamma:\left(\frac{\partial\varphi}{\partial x}\pi_\varphi+\frac{\partial\sigma}{\partial x}\pi_\sigma\right):, \tag{21}$$

where : (...): denotes the normal ordering of operators in (...). The charge density operator remains unchanged: $\rho\propto(1/\pi)(\partial\varphi/\partial x)$, so that the current (21) gives the expression for the time derivative $j\propto-(1/\pi)(\partial\varphi/\partial t)$. The conservation law $(\partial j/\partial x)=-(\partial\rho/\partial t)=i[\rho,H]$ is preserved due to an additional term δH_0 in the Hamiltonian coming from (20). In the representation (17), we find it to be

$$\delta H=\sqrt{\frac{\pi}{2}}\,\Gamma:\left\{\left(\frac{\partial\varphi}{\partial x}\right)^3+3\frac{\partial\varphi}{\partial x}\left[\left(\frac{\partial\sigma}{\partial x}\right)^2+\pi_\varphi^2+\pi_\sigma^2\right]\right.$$
$$\left.+6\pi_\varphi\pi_\sigma\left(\frac{\partial\sigma}{\partial x}\right)\right\}: .$$

The expressions for δj and δH may be easily obtained with the help of the following identities, which can be derived as in Ref. 18:

$$\Psi_\pm^\dagger(y)\Psi_\pm(x)=:\exp\{\mp 2i\sqrt{\pi}[\varphi_\pm(y)$$
$$-\varphi_\pm(x)]\}:\frac{\mp i/2\pi}{x-y\mp ia_0}.$$

Here a_0 is a cutoff parameter.[3]

Consider now the effects of these corrections.

a) Spin excitation currents

The lowest excitations (magnons) of the spin Hamiltonian can be obtained by the quantization of H_σ:

$$H_\sigma\propto\pi_\sigma^2+\left(\frac{\partial\sigma}{\partial x}\right)^2=\sqrt{cd}\sum_k|k|a_k^\dagger a_k, \tag{22}$$

where $a_k^\dagger,a_k$ are the magnon creation and annihilation operators. Then for the state with one spin excitation $\Omega=a_k^\dagger|0\rangle$, we find from (21) the average current

$$\langle\Omega|j|\Omega\rangle\approx\Gamma k \tag{23}$$

to be proportional to the momentum of the spin wave, which is in the agreement with the exact results. Quantitative agreement on the dependence of the interactions can be achieved by taking account of the coefficients in (18) and (23).

b) Charge excitation currents

For excitations of the charge Hamiltonian $H(\varphi)$, we consider as in (22) the state with one charge density sound quantum $\Omega=b_k^\dagger|0\rangle$, where $b_k^\dagger$ is the charge sound creation operator.

Unlike the spin case, the current operator now has two contributions, (19) and (21). Nevertheless, due to the nondiagonality of $\pi_\varphi\propto(b_k^\dagger-b_k)/\sqrt{k}$, we have $\langle\Omega|\pi_\varphi|\Omega\rangle=0$, so that the average value of the current remains the same as for the spin excitation:

$$\langle\Omega|j|\Omega\rangle=\langle\Omega|\pi_\varphi|\Omega\rangle+\Gamma\left\langle\Omega\left|\pi_\varphi\frac{\partial\varphi}{\partial x}\right|\Omega\right\rangle\approx\Gamma k. \tag{24}$$

This observation is quite natural, since

$$\langle\pi_\varphi\rangle=\langle\Psi^\dagger\sigma_z\Psi\rangle=N_+-N_-=0,$$

where $N_\pm$ are the particle numbers in the right- and the left-hand branches of the spectrum. For an arbitrary linear superposition of charge sound bosons, the numbers N_+, N_- then remain equal, and the current is zero until the velocity dispersion Γ is taken into account.

The mean value $\langle\pi_\varphi\rangle$ can be nonzero, and the contribution linear in the boson operators will appear only for those states which are not eigenstates of the charge sound Hamiltonian. In other words, these states should change the numbers N_+, N_-. There are several important examples.

1. The simplest umklapp $2k_F$ state, which may correspond to the case 2.3 of the Hubbard model eigenstates. Then

$$\Omega_1=\Psi_+^\dagger\Psi_-|0\rangle\sim e^{i\varphi}|0\rangle,\quad j\propto\langle\pi_\varphi\rangle\propto v_F.$$

2. The macroscopic current-carrying ground state, e.g., for a ring with a magnetic flux, when the number of sound bosons is not conserved due to the presence of the term $jA\propto\pi_\varphi$ in the Hamiltonian (A is the vector potential).

The orbital effects of the magnetic field will be studied in detail in the next section in the context of the Hubbard model.

4. THE HUBBARD RING IN A MAGNETIC FIELD. DIAMAGNETISM AND PARAMAGNETISM

Consider the Hubbard model for a ring with magnetic flux through it. The Bethe ansatz equations have the form[21–23]

$$e^{i(k_j-\nu)N_a}=\prod_{\beta=1}^{M}\frac{\lambda_\beta-\sin k_j+2iu}{\lambda_\beta-\sin k_j-2iu},\quad j=1,\dots,N,$$
$$\prod_{j=1}^{N}\frac{\lambda_\alpha-\sin k_j+iu}{\lambda_\alpha-\sin k_j-iu}=\prod_{\beta\neq\alpha}\frac{\lambda_\alpha-\lambda_\beta+2iu}{\lambda_\alpha-\lambda_\beta-2iu},\tag{25}$$
$$\alpha=1,\dots,M,$$

where $\nu=(2\pi/N_a)(\Phi/\Phi_0)$, $\Phi_0=hc/e$ is the magnetic flux quantum, and Φ is the magnetic flux through the ring. Equations (25) are valid both for the ground[21,22] and excited states, so that all properties are periodic as a function of Φ, with period Φ_0. In this way, the system is manifestly gauge-invariant, although it does not seem to hold for a single exitation,[9,24] since the ratio of the current to the group velocity is not universal.

Suppose that there is a spin singlet or spin triplet excitation on the ring. By analogy with (5), we can obtain from (25) the following equation for the function $\tilde{\rho}(t)$:

$$\tilde{\rho}(t)=\sum_{i=1,2}\frac{1}{\pi}\tan^{-1}\left[\exp\left(\frac{2\pi}{u}(t-\lambda_i)\right)\right]+\frac{\nu N_a}{\pi}+\int_{-\sin Q}^{\sin Q}\tilde{\rho}(t')dt'K(t-t'),\tag{26}$$

where $t=\sin k$. Expressions (6) for the momentum and energy can be generalized to

$$p=\int_{-Q+\nu}^{Q+\nu}\tilde{\rho}(k)dk,\quad \epsilon=2\int_{-Q+\nu}^{Q+\nu}\tilde{\rho}(k)\sin k\,dk,\tag{27}$$

while the current should be obtained afterwards via $j=-\delta H/\delta A$. There are two contributions due to the magnetic field: the new term ν/π in Eq. (26), and the offsets in the integration limits in (27). For $u\gg1$, the solution of (27) yields

$$\epsilon=v_s|p_s|+2\frac{\sin\pi\rho}{\pi}\nu^2N_a+2p_s\frac{\sin\pi\rho}{\pi\rho}\nu,\tag{28}$$

where the spin velocity[10] and momentum are

$$v_s=\frac{2\pi}{\rho u}\left(1-\frac{\sin 2\pi\rho}{2\pi\rho}\right),\quad p_s=p+2\pi\frac{\Phi}{\Phi_0}\rho.$$

The current is found by variation of (28) over A, were $A=\Phi/L$ is the vector potential:

$$j=-\frac{2\pi}{\Phi_0}\left(8\frac{\Phi}{\Phi_0}\sin\pi\rho+2p_s\frac{\sin\pi\rho}{\pi\rho}\right).\tag{29}$$

The first term in (29) is the ground state contribution, and the second is the spin-wave current (11). [Unlike (11), expressions (29) etc. are given in dimensional units.]

With the help of these results we obtain the orbital susceptibility

$$\chi_{\rm orb}=-\frac{\partial^2\Omega}{\partial\mathscr{H}^2}\approx-\frac{16}{\pi}\sin(\pi\rho)\frac{t}{\Phi_0^2}N_a^3+\frac{2T^2}{\pi v_s^3t}\left(\frac{\sin(\pi\rho)}{\pi\rho}\right)^2\frac{1}{\Phi_0^2}N_a^3,\tag{30}$$

where Ω is the thermodynamic potential, $\mathscr{H}$ is the magnetic field, T is the temperature.

The first term in (30) is the ground state contribution, and the second is due to spin waves. Note that spin-wave currents contribute to the paramagnetic susceptibility.

For particle–hole excitations, we easily obtain a similar expression, but with $v_s\Rightarrow v_c\propto\sin\pi\rho$.

The total susceptibility consists of the orbital part (30) and a spin-paramagnetic part:

$$\chi=\chi_{\rm orb}+\chi_{\rm spin},\quad \chi_{\rm spin}=\frac{2N_a\mu_B^2}{\pi v_s}.$$

5. CONCLUSIONS

1. We have studied the current-carrying states of the Hubbard model. We found that both spin (singlet and triplet) and charge (hole and particle) excitations carry current $j\propto p$ (for $p\ll1$), except for the half-filled case, where only states with one added particle carry the current. We also found that for $\rho\neq1$, the singlet excitation spectrum is terminated at some end point.

2. In the bosonization approach for the linearized bare electron spectrum, neither spin nor charge sound excitations carry the current. The current arises only for macroscopic coherent states in which the number of charge sound bosons is not conserved, as in the presence of magnetic flux.

Taking account of the spectrum parabolicity leads to nonzero currents ($j \propto p$) for both charge and spin boson excitations, which is in agreement with exact results for the Hubbard model.

3. The gauge invariance of the theory shows itself via the periodicity of all properties with respect to the magnetic flux, with period Φ_0. The currents that result from the magnetic field lead to the orbital susceptibility, which for small Φ/Φ_0 has a diamagnetic ground state part and a temperature-dependent paramagnetic contribution due to both singlet and triplet excitations.

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APPENDIX. SPIN SINGLET EXCITATIONS

Spin singlet states are characterized[12] by an additional pair of complex parameters $\lambda_0=\Lambda\pm i\Gamma$. We omitted in Eq. (2) two equations with J_{β_1} and J_{β_2}, and added two additional equations with an integer J and corresponding complex values λ_0 and λ_0^*. The set (3) of numbers changes as follows: $I_j=I_j^0$, numbers $J_{\beta 1}\Leftrightarrow\lambda_1$, $J_{\beta 2}\Leftrightarrow\lambda_2$ are omitted, numbers $J\Leftrightarrow\lambda_0=\Lambda\pm i\Gamma$ are added.

Equations (2) become

$$N_a k_j=2\pi I_j+\sum_{\beta=1}^{M-2}\theta(2\sin k_j-2\lambda_\beta)+\theta(2\sin k_j-2\lambda_0)+\theta(2\sin k_j-2\lambda_0^*),\quad j=1,...N,$$

$$\sum_{j=1}^{N}\theta(2\sin k_j-2\lambda_\alpha)=2\pi J_\alpha-\sum_{\beta=1}^{M-2}\theta(\lambda_\alpha-\lambda_\beta)-\theta(\lambda_\alpha-\lambda_0)-\theta(\lambda_\alpha-\lambda_0^*),$$
$$\alpha\neq\beta_1,\beta_2, \tag{A1}$$

$$\sum_{j=1}^{N}\theta(2\sin k_j-2(\Lambda\pm i\Gamma))=2\pi J-\sum_{\beta=1}^{M-2}\theta(\Lambda\pm i\Gamma-\lambda_\beta)-\theta(\pm 2i\Gamma).$$

By comparing imaginary parts in Eq. (A1) and taking Eq. (2a) into account, we find as in Ref. 12 that $\Gamma=u/4$. Then by using identities

$$\mathrm{Re}[2\tan^{-1}(A\pm i)]=\frac{\pi}{2}\,\mathrm{sign}\,A+\tan^{-1}\left(\frac{A}{2}\right),$$

$$\mathrm{Re}[2\tan^{-1}(A\pm i/2)]=\tan^{-1}(2A)+\tan^{-1}(2A/3),$$

we arrive at

$$N_a k_j=2\pi I_j+\sum_{\beta=1}^{M-2}\theta(2\sin k_j-2\lambda_\beta)-\pi\,\mathrm{sign}(\sin k_j-\Lambda)-2\tan^{-1}\frac{2(\sin k_j-\Lambda)}{u},\quad j=1,...,N, \tag{A2}$$

$$\sum_{j=1}^{N}\theta(2\sin k_j-2\lambda_\alpha)=2\pi J_\alpha-\sum_{\beta=1}^{M-2}\theta(\lambda_\alpha-\lambda_\beta)+2\tan^{-1}\frac{4(\lambda_\alpha-\Lambda)}{u}+2\tan^{-1}\frac{4(\lambda_\alpha-\Lambda)}{3u},\quad \alpha\neq\beta_1,\beta_2, \tag{A3}$$

$$\sum_{j=1}^{N}\left[-\frac{\pi}{2}\,\mathrm{sign}(\sin k_j-\Lambda)-\tan^{-1}\frac{2(\sin k_j-\Lambda)}{u}\right]=2\pi J+\sum_{\beta=1}^{M-2}\left[\tan^{-1}\frac{4(\Lambda-\lambda_\beta)}{u}+\tan^{-1}\frac{4(\Lambda-\lambda_\beta)}{3u}\right]. \tag{A4}$$

From Eqs. (A2)—(A4) we can derive the coupled integral equations for functions $\tilde{\rho}(k)$ and $\tilde{\sigma}(\lambda)$, where

$$\tilde{\sigma}(\lambda_a)=N_a\sigma_0(\lambda_a)\delta\lambda_\alpha,\quad \sigma_0(\lambda_\alpha)=\frac{1}{N_a(\lambda_{\alpha+1}-\lambda_\alpha)},$$

and $\sigma_0(\lambda_\alpha)$ is the known ground-state function.[7] The functions ρ and σ allow[9] the sums over j and α to be transformed to integrals over k and λ:

$$\sum_{j=1}^{N}f(k_j)=N_a\int f(k)\rho_0(k)dk+\int\frac{\partial f}{\partial k}\tilde{\rho}(k)dk,$$

$$\sum_{\alpha=1}^{M}g(\lambda_\alpha)=N_a\int g(\lambda)\sigma_0(\lambda)d\lambda+\int\frac{\partial g}{\partial\lambda}\tilde{\sigma}(\lambda)d\lambda.$$

We have

$$2\pi\tilde{\rho}(k)=\int_{-\infty}^{\infty}\frac{8u\tilde{\sigma}(\lambda)d\lambda}{u^2+16(\sin k-\lambda)^2}-\pi\,\mathrm{sign}\times(\sin k-\Lambda)-2\tan^{-1}\frac{2(\sin k-\Lambda)}{u}+\sum_{i=1}^{2}2\tan^{-1}\frac{4(\sin k-\lambda_{\beta i})}{u}, \tag{A5}$$

$$\int_{-Q}^{Q}\frac{8u\tilde{\rho}(k)\cos k\,dk}{u^2+16(\sin k-\lambda)^2}=2\pi\tilde{\sigma}(\lambda)+\int_{-\infty}^{\infty}\frac{4u\tilde{\sigma}(\lambda')d\lambda'}{u^2+4(\lambda-\lambda')^2}-2\tan^{-1}\frac{4(\lambda-\Lambda)}{u}-2\tan^{-1}\frac{4(\lambda-\Lambda)}{3u}$$

$$+\sum_{i=1}^{2} 2\tan^{-1}\frac{2(\sin k-\lambda_{\beta i})}{u}. \quad (A6)$$

From a requirement of the unique correspondence between sets of I_j and of k_j, we can derive the condition

$$|\Lambda|\geqslant \sin k_j, \quad j=1,...,N. \quad (A7)$$

Indeed, if (A7) does not hold, then for some j we have $\sin k_j<\Lambda<\sin k_{j+1}$, $I_{j+1}=I_j+1$ and $k_{j+1}=k_j$ as follows from (A2). But this equality is prohibited.

Taking advantage of the infinite integration limits over λ, we can apply the Fourier transform to Eqs. (A5), (A6). In this way we can exclude $\tilde{\sigma}(\lambda)$ from (A5), (A6) to arrive at a separate equation for $\tilde{\rho}(k)$:

$$\tilde{\rho}(t)=\sum_{i=1,2}\frac{1}{\pi}\tan^{-1}\left[\exp\left(\frac{2\pi}{u}(t-\lambda_i)\right)\right]-\theta(t-\Lambda)$$

$$+\int_{-\sin Q}^{\sin Q}K(t-t')\tilde{\rho}(t')dt', \quad (A8)$$

where $t=\sin \mathbf{k}$ and the kernel K was defined in (5).

The equation to determine Λ is found from (A4) with the help of (A5)—(A7). After some cancellations we obtain

$$\pm N+4J-\frac{2}{\pi}\left[\left(\tan^{-1}\frac{4(\Lambda-\lambda_1)}{u}+\tan^{-1}\frac{4(\Lambda-\lambda_2)}{u}\right]=0. \quad (A9)$$

The sign "±" in (A9) depends on the sign of $(\sin Q-\Lambda)$. Since the first and the second terms on the left-hand side of (A9) are integers, the expression in square parentheses must be an integer of $\pi/2$. Consequently, we have from (A9) solutions for the parameter Λ:

$$\Lambda=(\lambda_1+\lambda_2)/2, \quad \Lambda=\pm\infty. \quad (A10)$$

The Eq. (A8) differs from the similar Eq. (5) for the triplet states only by the term $\theta(t-\Lambda)$. Provided the inequality (A7), this term is constant (0 for $\Lambda<0$, or 1 for $\Lambda>0$), and we arrive at the same expression for the energy and the momentum (for $\Lambda<0$ the momentum is shifted by the period $\pi\rho$). The currents of singlet excitations are given by the same Eq. (10) as for triplet states.

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