On the possibility of Bose condensation of pions in ultrarelativistic collisions of nuclei

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(Submitted 30 November 1993)
Zh. Eksp. Teor. Fiz. **105**, 1473–1497 (June 1994)

The properties of the pion subsystem that arises in the process of ultrarelativistic collisions of nuclei are studied. For this a model of the expanding pion fireball is used in which, right up to the time the fireball breaks up, it is assumed that local thermal quasiequilibrium is present but chemical equilibrium is absent. The properties of a pion system with a Bose condensate are studied in the approximation of an ideal relativistic pion gas. The value of the critical Bose-condensation temperature T_c of the pions is estimated, and it is shown that this value apparently exceeds the characteristic value that determines the slope of the experimental pion spectra in ultrarelativistic collisions of nuclei. The yield of pions is found, and is shown to be enhanced in the region of small momenta. It is shown that if there is a condensate at the initial time it is also preserved during subsequent uniform isentropic expansion. Certain particular solutions of the hydrodynamic equations are found in the case of uniform and nonuniform expansion for $T \leqslant T_c$. Specific properties possessed by a pion gas for $T < T_c$, analogous to the properties of a superfluid system or a superconducting system are formulated. Collisions of the pion gas is taken into account by means of the Lagrangian approach. The excitation spectrum in the presence of a π^0 condensate, and also π^+ and π^- condensates, is found. It is shown that the low-lying branch of the excitations in the electrically neutral system corresponds to Goldstone oscillations. It is demonstrated that the photon acquires mass as a consequence of the interaction of the π^+ and π^- condensates with the electromagnetic vacuum. It is shown that the ρ^0 meson acquires a large additional mass as a consequence of the analogous effect. Arguments are given in favor of a substantial decrease in the yield of dileptons in the presence of a Bose condensate of pions. The properties of a system with a pion condensate at finite temperatures are studied. The specifics of Bose condensation of pions in the σ-model are considered. It is shown that the nucleon mass decreases as the density of the pion condensate increases, and can even vanish for $\rho > \rho_c \sim \rho_{\rm nucl}$, where $\rho_{\rm nucl}$ is the density of the atomic nucleus. This decrease of the nucleon mass can manifest itself in an enhanced yield of $N\bar{N}$ pairs from the region of the pion fireball.

1. INTRODUCTION

In recent years the first experimental data on collisions of nuclei with ultrarelativistic energies have appeared.[1,2] They have shown that the rapidity characteristics of the pions differ substantially from those of the nucleons. Therefore, the pion subsystem can be considered independently of the nucleon subsystem (the so-called Bjorken model[3]). In addition, it has been found that the yield of pions is substantially greater than the yield of nucleons. For example, $\sim 10^3$ pions emerge in central Au+Au collisions at energy 10.6 GeV nucleon, and the ratio of the number of pions to the number of protons is $N_\pi/N_p \simeq 5$. In central O+Au and S+S collisions at 200 GeV/nucleon more than 300 pions emerge, corresponding to $N_\pi/N_p \simeq 10$. At the energies of the SPS, RHIC, and LHC accelerators we expect a yield of tens of thousands of pions.[4,5] Therefore, it is of central interest to describe the pion subsystem.

Experiments on $\pi\pi$ correlations[6] indicate that the source emitting the pions has a large characteristic radius ($R_b \simeq 4$–7 fm), are substantially greater than the characteristic radius of the region of the initial overlap of the nuclei. The spectra of the large-momentum pions (hard pions) are approximately exponential. This makes it possible to assume local thermodynamic equilibrium, to introduce the concept of a local temperature, and to use the hydrodynamic approach. In view of this, to describe the dynamics of the pion subsystem it is natural to develop an expanding-pion-fireball model analogous to the expanding-nuclear-fireball model used to describe the collisions of nuclei in the region of energies lower than a few GeV/nucleon.[7,8]

We shall assume that, in the characteristic collision time $\tau_{\rm col}$, a dense hot pion fireball, characterized by a certain initial pion-density distribution $\rho_m(t=0,r)$ and transverse-temperature distribution $T_m(t=0,r)$, is formed. It then expands into the vacuum until a certain stage ("breakup") is reached, at which local thermal equilibrium is violated and the momentum distributions of the pions become frozen and are characterized by the density $\rho_b[t_0(r)]$ and temperature $T_b[t_0(r)]$.

The available experimental data on ultrarelativistic collisions of nuclei indicate point to a substantial excess of small-momentum pions (soft pions) in comparison with the relative yield of soft pions in $pp \to \pi X$ reactions. To

 1063-7761/94/060793-13$10.00

explain this interesting fact, it was suggested in Refs. 9–11 that chemical equilibrium is absent in the pion fireball. An estimate of the mean free path of the pion[11] has shown that the pions have a large absorption mean free path, while they have a relatively small mean free path with respect to elastic collisions. This justifies assuming the presence of local thermal equilibrium and the absence of chemical equilibrium. Thus, it is assumed that the number of pions that were created in the initial stage of the collision of the nuclei, principally as a result of the decay of a large number of resonances, subsequently remains fixed until the fireball breaks up. Therefore, the pions can be characterized by a nonzero value of the chemical potential $\mu(t)$. The experimental pion spectra, including the enhanced yield of soft pions, are reasonably well described by a distribution of pions with chemical potential $\mu_b=\mu(t_0)\simeq 126$ MeV and temperature $T_b\simeq 167$ MeV for O+Au collisions, and with $\mu_b\simeq 118$ MeV and $T_b\simeq 164$ MeV for S+S collisions, at 200 GeV/nucleon.[10] The values of μ determined in this way are close to the value $\mu=m\simeq 140$ MeV at which Bose condensation of pions with momentum $k=0$ arises.

The existing experimental data are still too sparse for one to state with certainty that $\mu_\pi<m$. They could also be explained by assuming $\mu_\pi \gtrsim m$ with slightly higher values of T_b. In addition, as already stated, we should expect a still higher yield of pions at the energies of the SPS, RHIC, and LHC accelerators. But with a large number of pions created as a result of decay of resonances, and with a comparatively small initial volume and comparatively low temperature of the pion fireball, in the absence of dissipative processes some of the pions will be forced to go over to a Bose-condensate state. Therefore, in our view, it is of interest to admit the possibility of Bose condensation of pions and to study its consequences in the hope that specific distinctive features of the Bose condensation might be manifested in experiment in the future. In addition, the investigation of Bose condensation in relativistic dynamical systems is of interest in itself.

The hypothesis of possible Bose condensation of pions in ultrarelativistic collisions of nuclei has also been put forward recently in Ref. 12, in which it was shown that the system cools faster in the presence of a Bose condensate than in its absence. In the present paper we shall investigate other consequences of the Bose condensation of pions.

To avoid misunderstandings, we draw attention once again to the fact that we are concerned with a nonequilibrium system for the important times $t\lesssim t_0\ll\tau_{\rm abs}$, where t_0 is the time after which freezing of the momentum distributions of the pions occurs, and $\tau_{\rm abs}$ is the characteristic absorption time. Only in this stage can it be assumed that the number of pions is approximately fixed and the pion gas is characterized by a chemical potential $\mu_\pi\neq 0$. In the opposite limiting case $t_0\gg\tau_{\rm abs}$ the pion distribution would obviously be characterized by the value $\mu_\pi=0$.

To describe the pion subsystem with a Bose condensate we shall use the model of an expanding pion fireball, i.e., we assume that the hydrodynamic approximation is applicable. The kinetics of the formation of a Bose condensate in nonrelativistic systems was studied in Ref. 13. A rough estimate indicates that, as in the nonrelativistic case, in the relativistic case the Bose condensate is formed in a characteristic time $\tau_{\rm col}\sim 1/\sigma_{\pi\pi}\rho$, where $\sigma_{\pi\pi}\sim 1/m_\pi^2$ is the $\pi\pi$-scattering cross section. But the approximation $\tau_{\rm col}\ll t_0$ with $t_0\sim 10m_\pi$ (Ref. 11) appears to be fully applicable. For $\tau_{\rm col}\ll t<t_0$ the conditions for applicability of the hydrodynamic approach are also fulfilled. A detailed analysis of the kinetics of formation of a Bose condensate in relativistic systems is of interest in itself, but lies outside the scope of this paper.

The paper is organized as follows: In Sec. 2 we study the Bose–Einstein condensation in a model of an equilibrium ideal relativistic pion gas. In Sec. 3 we study the hydrodynamic expansion of a pion fireball at $T\leqslant T_c$. In Sec. 4, using the Lagrangian approach in the $\lambda\varphi^4$ model, we consider the condensation of a nonideal pion gas. In Sec. 5 we determine the excitation frequencies of the system in the presence of a pion condensate. The masses of the photon and ρ^0 meson are calculated. We discuss the possibility of suppressing the yield of dileptons from the π-condensate region. In Sec. 6 we study the specifics of the Bose condensation of pions at nonzero temperature. In Sec. 7 we consider pion condensation in the σ-model. In Sec. 8 we formulate possible experimental manifestations of the phenomenon considered, and make some concluding remarks.

2. BOSE CONDENSATION IN A RELATIVISTIC EQUILIBRIUM IDEAL PION GAS

For simplicity we shall consider first an ideal pion gas in thermal quasiequilibrium with temperature $T(t)$ and density $\rho(t)$, but in the absence of chemical equilibrium. The latter implies that the number N_π of particles in the gas is fixed and the pion gas is characterized by a nonzero chemical potential $\mu(t)$.

The pion density ρ, temperature T, and chemical potential μ are related by the expression

$$\rho=3\int\frac{d^3k}{(2\pi)^3}\frac{1}{\exp((\sqrt{k^2+m^2}-\mu)/T)-1},\qquad(2.1)$$

where, for simplicity, we have assumed that $N_{\pi^-}=N_{\pi^+}=N_{\pi^0}=N/3$ and $T>T_c$. The critical temperature of the Bose condensation is determined from the condition $\mu=m$. For $T<T_c$ we have for the condensate density $\rho(k=0)$:

$$\rho=\rho(k=0)+3\int\frac{d^3k}{(2\pi)^3}\frac{1}{\exp((\sqrt{k^2+m^2}-m)/T)-1}.\qquad(2.2)$$

The momentum distributions of the emerging pions are characterized by the values T_b and V_b of the temperature and volume at the time fireball breaks up. For pions of a given kind we obtain

$$\frac{d^3N}{[d^3k/(2\pi)^3]V_b}\equiv n_\pi(k)=\frac{1}{\exp((\sqrt{k^2+m^2}-\mu_b)/T_b)-1},$$

$$k>0,\qquad(2.3)$$

with the value $\mu_b=\mu(T_b)<m$ for $T>T_c$, and $\mu_b=m$ for $T_b<T_c$. The distribution of pions of a given kind with $k=0$ for $T_b\ll T_c$ has the form

$$n_\pi(k=0)=\tfrac{1}{3}\rho(k=0)\delta(k). \quad (2.4)$$

For the soft-pion distribution ($k\ll m$) at $T<T_c$ we have, according to (2.3),

$$n_\pi(k)=\frac{1}{\exp[(k^2/2m)/T]-1} \quad (2.5)$$

instead of the distribution

$$n_\pi(k)\simeq\frac{1}{\exp[(m+k^2/2m)/T]-1} \quad (2.6)$$

at zero chemical potential, corresponding to a nonfixed number of pions. It is obvious that the expression (2.5) corresponds to a substantially larger total number of pions and a larger relative number of low-momentum pions than does (2.6), and this agrees with the well known experimental data on ultrarelativistic collisions of nuclei.

It should be noted that in a realistic formulation of the problem, corresponding to the dynamical problem, instead of the distribution (2.4), (2.6) we have a distribution with a finite, albeit narrow, peak at $k=0$. This circumstance must be taken into account when the theoretical distributions (2.4) and (2.5) are compared with experimental data.

Knowing the pion distribution, we can find all the thermodynamic characteristics of the system. As is well known, the specific heat C_V, entropy S, and pressure P are determined by the particles with momenta $k>0$. The Bose-condensate particles do not make a contribution to these characteristics. In addition, we draw attention to the fact that the pressure does not depend on the volume, so that the isotherms are straight lines.

To represent the results in analytical form we turn to a consideration of the two opposite limiting cases of a nonrelativistic and an ultrarelativistic gas.

Nonrelativistic pion gas

In this case most of the pions have small characteristic momenta

$$k_{ch}\ll m \quad (2.7)$$

and small energies

$$\varepsilon=\sqrt{k^2+m^2}\simeq m+k^2/2m.$$

Substituting the latter value into (2.2), we can easily determine the critical temperature of the Bose–Einstein condensation:

$$T_c\simeq\frac{1.59}{m}\rho^{2/3}. \quad (2.8)$$

According to (2.2), $k_{ch}\sim\sqrt{2mT_c}$, and, taking (2.7) into account, we have the inequalities

$$T_c\ll m/2,\quad \rho\ll 0.3\rho_N, \quad (2.9)$$

where $\rho_N\simeq 0.5m^3$ is the density of the atomic nucleus, for which the nonrelativistic approximation is valid.

From (2.2) we determine the density of the Bose-condensate pions:

$$\rho(k=0)=\rho\left[1-\left(\frac{T}{T_c}\right)^{3/2}\right],\quad T<T_c. \quad (2.10)$$

For the thermodynamic characteristics when $T<T_c$ holds we have

$$E\equiv E_0+\Delta E=\rho(k=0)mV+c_0T^{5/2}V,$$

$$c_0\simeq 0.384m^{3/2},\quad C_V=\frac{5}{2}\frac{\Delta E}{T},\quad S=\frac{5}{3}\frac{\Delta E}{T}, \quad (2.11)$$

$$F=-\frac{2}{3}\Delta E+E_0,\quad P=\frac{2}{3}\frac{\Delta E}{V}.$$

It follows from these expressions, for $T\ll T_c$, that

$$P=\frac{2}{3}c_0T^{5/2}=\frac{2}{5}\left(\frac{3}{5}\right)^{2/3}\frac{(ms)^{5/3}}{c_0^{2/3}}\rho^{5/3},$$

$$s=\frac{S}{M}=\frac{5}{3}c_0T^{3/2}/m\rho, \quad (2.12)$$

corresponding to a polytrope with exponent $\gamma=5/3$.

In Ref. 14 it is argued that the initial distributions of the pions produced as a result of the decay of resonances are isotopically asymmetric. Therefore, in addition to the case considered above, with $N_{\pi^+}=N_{\pi^-}=N_{\pi^0}=N/3$, another limiting case, when there are only pions of one kind, is also of interest. In this case, the density of the Bose-condensate pions is related to the density ρ by the same expression (2.10), but the quantity T_c is

$$T_c\simeq\frac{3.31}{m}\rho^{2/3}. \quad (2.13)$$

The energy, pressure, free energy, and entropy differ by a factor of 1/3 from the corresponding expressions (2.11), (2.12).

Ultrarelativistic pion gas

In this case, for the characteristic values of the momenta of the pions we have

$$\varepsilon=\sqrt{m^2+k^2}\simeq k,\quad k_{ch}\gg m. \quad (2.14)$$

Taking this into account in (2.2), in the case $N_{\pi^+}=N_{\pi^-}=N_{\pi^0}=N/3$ we obtain

$$T_c\simeq 1.40\rho^{1/3}. \quad (2.15)$$

For pions of one kind, instead of (2.15) we have

$$T_c\simeq 2.02\rho^{1/3}. \quad (2.16)$$

The inequality $k_{ch}\sim T\gg m$ used in the derivation of (2.15) and (2.16) implies that

$$\rho\gg 0.7\rho_N \quad (2.17)$$

in the case when $N_{\pi^+}=N_{\pi^-}=N_{\pi^0}$, and

$$\rho\gg 0.2\rho_N \quad (2.18)$$

for one kind of pion. Both conditions are apparently fulfilled for ultrarelativistic collisions of nuclei, even in the stage when the pion fireball breaks up ($\rho \sim \rho_b$). In fact, the radius of the pion source, as estimated from $\pi\pi$ correlations, is $R_b \simeq 4$–7 fm (Ref. 6) when the number of emerging pions is $\simeq 300$, corresponding to a pion density $\rho_b \sim (1$–$6)\rho_N$. However, even such a wide interval of possible values of R_b and ρ_b is not guaranteed, since in the treatment of the correlation data certain theoretical assumptions were used.

We draw attention to the fact that the quantity T_c is not so sensitive to the value of ρ_b, since $T_c \propto \rho^{1/3} \propto R_b^{-1}$. In the case $N_{\pi^+} = N_{\pi^-} = N_{\pi^0}$, for $R_b \simeq 4$ fm ($\rho_b \simeq 6\rho_N$) we have $T_c \simeq 2m_\pi \simeq 280$ MeV, which is substantially greater than the characteristic experimental inverse slope coefficients ($T_0^{\exp} \simeq 170$ MeV), while for $R_b \simeq 7$ fm ($\rho_b \simeq 1.2\rho_N$) we have $T_c \simeq T_0^{\exp}$. In the case of pions of the same kind we obtain $T_c \simeq 580$ MeV for $R_b \simeq 4$ fm and $T_c \simeq 240$ MeV for $R_b \simeq 7$ fm.

Thus, the above rough estimates are wholly encouraging for the possibility of Bose condensation of pions in ultrarelativistic collisions of nuclei. Nevertheless, in order finally to confirm or refute the hypothesis of Bose condensation we must formulate its consequences and compare its results with experiment.

For a temperature in the interval $m \ll T \ll T_c$, according to (2.2) we have

$$\rho(k=0) = \rho\left[1 - \left(\frac{T}{T_c}\right)^3\right]. \tag{2.19}$$

Putting $T \simeq 0.8T_c$ in (2.19), we have $\rho(k=0)/\rho \simeq 0.7$, while from (2.10) we obtain $\rho(k=0)/\rho \simeq 0.3$. Thus, in the ultrarelativistic limit the Bose-condensate state fills up faster as the temperature decreases than in the nonrelativistic case. For the thermodynamic characteristics of the system when $T < T_c$ holds we obtain

$$E \equiv E_0 + \Delta E = 3\rho(k=0)mV + c_1 VT^4, \quad c_1 = \frac{7\pi^4}{120}, \tag{2.20}$$

$$C_V = \frac{4\Delta E}{T}, \quad S = \frac{4}{3}\frac{\Delta E}{T},$$

$$F = -\frac{1}{3}\Delta E + E_0, \quad P = \frac{1}{3}\frac{\Delta E}{V}.$$

From this we have

$$P = \frac{1}{3}c_1 T^4, \quad s = \frac{S}{M} = \frac{4}{3}c_1\frac{T^3}{m\rho}, \quad P = \frac{3^{1/3}(ms)^{4/3}}{4^{4/3}c_1^{1/3}}\rho^{4/3}, \tag{2.21}$$

which corresponds to a polytrope with exponent $\gamma = 4/3$.

3. HYDRODYNAMIC EXPANSION OF AN IDEAL PION GAS

At $T \gg T_c$ the ordered motion of an ideal pion gas with a fixed number of particles is described by the hydrodynamic equations of an ideal liquid, i.e., by the continuity equation

$$\partial_t \rho + \operatorname{div}(\rho \mathbf{u}) = 0, \tag{3.1}$$

the Euler equation

$$\partial_t \mathbf{u} + (\mathbf{u}\nabla)\mathbf{u} = -\frac{1}{\rho m}\nabla P \tag{3.2}$$

and the entropy-transport equation

$$\partial_t s + \mathbf{u}\nabla s = 0, \tag{3.3}$$

where u is the velocity of the ordered motion of the gas or liquid, and s is the entropy per unit mass of the substance: $s = S/M$, $M = mN$. Equations (3.1)–(3.3) hold for pions of each kind separately. If the initial profiles of the distributions of the π^+, π^-, and π^0 mesons are the same, the system of equations (3.1)–(3.3) describes the distribution of the total density and temperature of the pions.

Note that Eqs. (3.1)–(3.3) are applicable only when the velocity u of the ordered motion and the characteristic thermal velocity v_t of the particles in the gas are small in comparison with the speed of light. For $u \sim c$ or $v_T \sim c$, instead of the system (3.1)–(3.3) it is necessary to use the more complicated equations of relativistic hydrodynamics.[15]

It is also of interest to consider the limiting case of a relativistic gas undergoing slow ordered motion. In this case the matter-continuity equation and entropy-transport equation coincide fully with (3.1) and (3.3), but the Euler equation is represented in the form[15]

$$\partial_t \mathbf{u} + (\mathbf{u}\nabla)\mathbf{u} = -\frac{\nabla P}{w}. \tag{3.4}$$

Equation (3.4) differs from (3.2) in that the thermal function w, rather than the matter density ρ_m, appears in the right-hand side. The passage to (3.2) in the limit is implemented taking into account that $w = \rho_m + \tilde{w}$ for $v_T \ll c$, where $\tilde{w}$ is the nonrelativistic thermal function. For an ultrarelativistic ideal gas, $w \simeq 4P$.

For $T < T_c$ Eqs. (3.1)–(3.3) [or (3.4)] no longer describe the motion of a pion gas, since some of the pions fall into the Bose condensate, and for the pions of each kind we must use the equations of a two-component liquid. The Bose-condensate particles form one component (s), and the particles above the condensate (thermal excitations) form the other (n). In the case of an ideal gas for $T < T_c$ these equations can be derived starting from microscopic considerations. In the nonrelativistic approximation ($u \ll c, v_T \ll c$) they have the form[16]

$$\partial_t \rho_s + \nabla(\rho_s \mathbf{u}_s) = 0, \tag{3.5}$$

$$\partial_t \mathbf{u}_s + (\mathbf{u}_s\nabla)\mathbf{u}_s = -\frac{\nabla\mu}{m}, \tag{3.6}$$

$$\partial_t \rho_n + \nabla(\rho_n \mathbf{u}_n) = 0, \tag{3.7}$$

$$\partial_t \mathbf{u}_n + (\mathbf{u}_n\nabla)\mathbf{u}_n = -\frac{\nabla P}{\rho_n m}, \tag{3.8}$$

$$\partial_t s + \mathbf{u}_n \nabla s = \frac{s}{\rho}\nabla[(\mathbf{u}_s - \mathbf{u}_n)\rho_s], \tag{3.9}$$

where ρ_s and u_s are, respectively, the density and velocity of the Bose condensate, ρ_n and u_n are the density and velocity of the particles above the condensate, and the quantity μ plays the role of the chemical potential of the nonuniform nonrelativistic pion gas:

$$\mu=-\Delta\sqrt{\rho_s}/2m\sqrt{\rho_s}. \tag{3.10}$$

In a uniform nonrelativistic gas with a Bose condensate we have $\mu=0$ (the chemical potential is reckoned from the mass as origin).

Because of the presence of the term (3.10), the stresses in an ideal Bose gas at $T<T_c$ are anisotropic, and this leads to violation of Pascal's law.

We now proceed to solve the hydrodynamic equations in certain limiting cases.

Uniform expansion at $T<T_c$

We shall assume that the initial distribution of pions is characterized by uniform densities ρ_n and ρ_s, equal velocities u_n and u_s, and a spatially constant entropy. Then the dynamics of the system is also determined by constancy of the entropy. Using the expressions (2.11) and (2.20), we have

$$\frac{T(t)}{T_m}=\left(\frac{V_m}{V(t)}\right)^{2/3} \tag{3.11}$$

in the case of a nonrelativistic pion gas, and

$$\frac{T(t)}{T_m}=\left(\frac{V_m}{V(t)}\right)^{1/3} \tag{3.12}$$

for an ultrarelativistic gas. Here, in both cases,

$$\begin{aligned}&\rho[k=0,V(t)]=\rho(k=0,V=V_m)[V_m/V(t)],\\&N_\pi[k=0,V=V(t)]=N_\pi(k=0,V=V_m).\end{aligned} \tag{3.13}$$

Thus, in the course of the isentropic expansion, not only is the total number of pions conserved, but so too are the numbers of condensate pions and above-condensate pions separately. From this we can draw the important conclusion that if $T_c<T_m$ holds and there is already a condensate initially ($T=T_m$, $V=V_m$), it remains present throughout the whole period of the isentropic expansion.

In order to determine the explicit dependences $\rho_n(t)$, $\rho_s(t)$, $u_n(t)$, and $u_s(t)$ at $T<T_c$, we shall consider the case of spherically symmetric uniform nonrelativistic expansion of the gas. Setting $u_n=u_s=r\dot{R}/R$, from the system (3.5)–(3.19) we easily obtain

$$\begin{aligned}&\rho_n=\rho_{0n}R_0^3/R^3(t),\quad \rho_s=\rho_{0s}R_0^3/R^3(t),\\&R(t)=R_0+vt,\\&u_n(t)=u_s(t)=rv/(R_0+vt),\end{aligned} \tag{3.14}$$

where ρ_{0n}, ρ_{0s}, R_0, and v are constants determining the initial distributions. As can be seen from (3.14), uniform isentropic expansion corresponds to a finite initial velocity. The dependence $T(t)$ is determined by the constancy of the entropy. In the case of a nonrelativistic pion gas, setting $N_{\pi^+}=N_{\pi^-}=N_{\pi^0}$ we have

$$T(t)=\frac{T_0R_0^2}{R^2(t)},\quad T_0=\left(\frac{2S}{5c_0V_m}\right)^{2/3}. \tag{3.15}$$

In the ultrarelativistic case,

$$T(t)=\frac{T_0R_0}{R(t)},\quad T_0=\left(\frac{3S}{4c_1V_m}\right)^{1/3}. \tag{3.16}$$

Nonuniform expansion

In a system that is not too large, finite-size effects can play an important role and the initial density and temperature profiles can be substantially nonuniform. In this case, the solution of the equations of two-fluid (and even one-fluid) hydrodynamics is a complicated problem. Therefore, below, we shall find only certain particular solutions for initially nonuniform distributions. For simplicity we shall consider the case when the temperature is close to critical and the density of the Bose condensate is small. In this case we can assume that $\rho_s=0$, and the normal component for nonrelativistic expansion obeys Eqs. (3.1)–(3.3) [or (3.4)].

According to (2.12), a nonrelativistic Bose gas at $T<T_c$ corresponds to a polytrope with exponent $\gamma=5/3$. As is well known, a nondegenerate nonrelativistic Bose gas ($T\gg T_c$) also corresponds to a polytrope with the same exponent γ. For the equation of state corresponding to a polytrope with an arbitrary exponent γ there is a whole class of particular solutions of the system of hydrodynamic equations (3.1)–(3.3). For spherically symmetric expansion we have[17]

$$\begin{aligned}&\rho(t,r)=\frac{\rho_0R_0^3}{R^3(t)}\left[1-\frac{r^2}{R^2(t)}\right]^\alpha\theta[r-R(t)],\\&u(t,r)=r\dot{R}/R,\end{aligned} \tag{3.17}$$

where $\theta(x)$ is the Heaviside step function, α is an arbitrary constant, and ρ_0 and R_0 determine the initial distributions:

$$\int_{R_0}^{R}\frac{dR}{\sqrt{1-(R_0/R)^{3\gamma-3}}}=u_0t,\quad u_0=\left[\frac{u\rho_0^{\gamma-1}b(\alpha+1)}{3(\gamma-1)}\right]^{1/2}. \tag{3.18}$$

The constant b is found from the relation

$$P\rho^{-\gamma}=b\left(1-\frac{r^2}{R^2}\right)^{1-\alpha(\gamma-1)}\theta(r-R). \tag{3.19}$$

Using these expressions, for a nonrelativistic pion gas ($\gamma=5/3$) we obtain

$$\begin{aligned}&u(t,r)=\frac{rtu_0^2}{u_0^2t^2+R_0^2},\quad R(t)=\sqrt{u_0^2t^2+R_0^2},\\&P(t,r)=b\rho_0^{5/3}\left(\frac{R_0}{R}\right)^5\left(1-\frac{r^2}{R^2}\right)^{\alpha+1}\theta(r-R).\end{aligned} \tag{3.20}$$

Then, for a nondegenerate Bose gas ($T\gg T_c$), with equation of state $P=\rho T$, we have

$$T(t,r)=T_0\frac{R_0^2}{R^2}\left(1-\frac{r^2}{R^2}\right),\quad T_0=b\rho_0^{2/3}, \tag{3.21}$$

while for a Bose gas at $T<T_c$, for $N_{\pi^+}=N_{\pi^-}=N_{\pi^0}$, using (2.12) we obtain

$$T(t,r)=T_0\frac{R_0^2}{R^2}\left(1-\frac{r^2}{R^2}\right)^{2/5(\alpha+1)}\theta(r-R),\quad T_0\simeq\text{const},$$
$$b\simeq\frac{2}{3}\frac{c_0T_0^{5/2}}{\rho_0^{5/2}},\quad u_0=\sqrt{\frac{4}{3}c_0(\alpha+1)}\frac{T_0^{5/4}}{\rho_0^{1/2}}. \tag{3.22}$$

As already stated, for $T<T_c$, instead of the system of equations (3.1)–(3.3) we must use the system (3.5)–(3.9). Therefore, the solution (3.20) holds only for $T\simeq T_c$ ($T<T_c$), when the density of the Bose condensate is sufficiently small. Imposing the requirement $T(t,r)\simeq T_c(t,r)$ and using the expressions (2.8), (3.17), and (3.20), we find $\alpha=3/2$. After this, the final distributions acquire the form

$$\begin{aligned}
&T(t,r)\simeq T_c(t,r)=T_0\frac{R_0^2}{R^2}\left(1-\frac{r^2}{R^2}\right)\theta(r-R),\\
&\rho(t,r)\simeq\rho_n(t,r)=\rho_0\frac{R_0^3}{R^3}\left(1-\frac{r^2}{R^2}\right)^{3/2}\theta(r-R),\\
&\rho_s(t,r)\simeq 0,\quad u(t,r)=\frac{rtu_0^2}{R^2},\\
&R(t)=\sqrt{u_0^2t^2+R_0^2},\quad u(t\to 0)\to 0.
\end{aligned} \tag{3.23}$$

As can be seen from the expressions (3.17) and (3.20)–(3.22), the solution (3.23) holds not only for $T\simeq T_c$ but also for $T\gg T_c$. Therefore, it can be used as a good interpolation in the entire range of temperatures $T\gtrsim T_c$.

In the general case for $T<T_c$ we must use the system of equations (3.5)–(3.9) of two-fluid hydrodynamics. If at the initial time the s- and n-components of the Bose liquid have different velocities, the fountain effect should arise, as in ^{4}He.

A system described by the equations (3.5)–(3.9) of two-fluid hydrodynamics also possesses a number of other specific properties. For example, for $T>T_c$ there is only one type of acoustic excitations, corresponding principally to density oscillations, while for $T<T_c$ first- and second-sound waves can propagate. For motion with a velocity greater than the first-sound velocity a density discontinuity arises, while for motion with a velocity greater than the second-sound velocity a temperature discontinuity arises.

We should also draw attention to specific features of the hydrodynamics of an ideal gas. For an ideal gas the pressure and chemical potential are not related by the thermodynamic relation that holds in the general case of an interacting gas. For a nonideal gas the relationship (3.10) is not present in the equations of Landau two-fluid hydrodynamics,[15,16] and for $T<T_c$ the stresses remain isotropic and Pascal's law is obeyed. The excitation spectrum of the ideal Bose gas at $T<T_c$ is given by the relationship $\omega\simeq k^2/2m$ (for $k\to 0$), and does not satisfy the Landau criterion for superfluidity. At the same time, even a weak interaction between the particles of the gas is found to be sufficient for the quadratic dispersion law $\omega\simeq k^2/2m$ to be replaced by a linear dispersion law $\omega\propto k$ (see below, in Sec. 5). Because of this, a nonideal Bose gas with a fixed number of particles at $T<T_c$ possesses a number of specific hydrodynamical properties.[15,16]

Thus, when the velocity of the medium is low [$u<u_{c1}\sim\ln(R/R_c)R^{-1}$, where R is the size of the system and R_c is the correlation length], the Bose-condensate component and normal component do not interact. When the velocity of the medium is greater than u_{c1} vortex filaments and rings are formed in the Bose-condensate component, as a result of which friction is established between the normal and condensate components. There also exists an upper critical velocity (u_{c2}), at which the condensate disappears completely. The vortices are solitons possessing a nonzero topological quantum number. The discovery of such pion solitons in relativistic collisions of nuclei would be of undoubted interest. There are also other specific properties of the hydrodynamics of a π-condensate system. Here, however, we shall not go into details, but confine ourselves to the above list. The detailed description of the hydrodynamics of a π^+,π^-,π^0 liquid at $T<T_c$ is of interest in its own right.

To conclude this section, we note that we have deliberately not used the terms "superfluidity" and "superconductivity," since in the relativistic region at temperatures $T\gtrsim m$ there are always dissipative processes associated with the creation and annihilation of particles. Over the time interval that we are considering ($t\ll\tau_{\rm abs}$) we can neglect these processes and assume that the number of particles is approximately fixed. Only to this accuracy can we speak of the absence of interaction between the normal (n) and condensate (s) subsystems.

4. PION CONDENSATION IN THE $\lambda\varphi^4$ MODEL AT $T=0$

Lagrangian and equations of motion

The Lagrangian density of the pions in the model with interaction $H'=\lambda\varphi^4/4$, with $\varphi=(\varphi_1,\varphi_2,\varphi_3)$, has the form

$$L=\frac{(\partial_\mu\varphi)^2-m^2\varphi^2}{2}-\frac{\lambda\varphi^4}{4}, \tag{4.1}$$

where λ is the coupling constant and the fields of the π^+, π^-, and π^0 mesons are given in terms of the isotopic vector $\varphi=(\varphi_1,\varphi_2,\varphi_3)$:

$$\varphi_{\pi^\pm}=\frac{\varphi_1\pm i\varphi_2}{\sqrt{2}},\quad \varphi_{\pi^0}=\varphi_3. \tag{4.2}$$

In the general case the field operator of the pions is represented in the form

$$\begin{aligned}
&\hat\varphi_{\pi^\mp}=\sum_{\mathbf k}[\hat a_{\mathbf k}c_{\mathbf k}^+(t)e^{i\mathbf{kr}}+\hat b_{\mathbf k}^+c_{\mathbf k}^{-*}(t)e^{-i\mathbf{kr}}],\\
&\hat\varphi_{\pi^0}=\sum_{\mathbf k}[\hat{\tilde a}_{\mathbf k}\tilde c_{\mathbf k}(t)e^{i\mathbf{kr}}+\hat{\tilde a}_{\mathbf k}^+\tilde c_{\mathbf k}^*(t)e^{-i\mathbf{kr}}],
\end{aligned} \tag{4.3}$$

where $\hat a_{\mathbf k}$ is the π^--annihilation operator, $\hat b_k$ is the π^+-annihilation operator, $\hat{\tilde a}_{\mathbf k}$ is the π^0-annihilation operator, and c_k and $\tilde c_k$ are certain time-dependent coefficients. Here, the processes of creation and annihilation of pions are included in the analysis. We note that the π^0 meson has a one-component wave function, since the π^0 coincides with the anti-π^0 meson. This certainly excludes the possibility of superfluidity of a π^0 subsystem with a nonfixed number of π^0 mesons, since the velocity of the superfluid motion is determined by the gradient of the phase of the complex wave function.

As already stated above, when the pion system is considered over times $t \ll \tau_{\text{abs}}$ the processes of particle creation and annihilation can be assumed to be suppressed. This implies that cross terms of the type $\hat{a}_{\mathbf{k}}\hat{b}^+_{\mathbf{k}}$ appear in the Hamiltonian with small coefficients. Taking this into account, as a simplification for $t \ll \tau_{\text{abs}}$ we may set, approximately,

$$\varphi_{\pi^-} = \sum_{\mathbf{k}} \hat{a}_{\mathbf{k}} c^+_{\mathbf{k}} e^{i\mathbf{kr}}, \quad \varphi_{\pi^+} = \sum_{\mathbf{k}} \hat{b}_{\mathbf{k}} c^-_{\mathbf{k}} e^{i\mathbf{kr}},$$
$$\varphi_{\pi^0} = \sum_{\mathbf{k}} \hat{\tilde{a}}_{\mathbf{k}} \tilde{c}_{\mathbf{k}} e^{i\mathbf{kr}}. \tag{4.4}$$

Then all three fields are represented by complex wave functions, and, in the semiclassical approximation, are described by the equations of two-fluid hydrodynamics.

In the more general case we shall start from a certain Lagrangian describing an interacting pion gas with a fixed number of particles of each kind. In the model with $\lambda\varphi^4$ interaction we have

$$\begin{aligned}\mathscr{L} = \int d\mathbf{r} \Big\{ & |(i\partial_t + \mu_-)\varphi_-|^2 - |\nabla\varphi_-|^2 - m^2|\varphi_-|^2 \\ & + |(i\partial_t + \mu_+)\varphi_+|^2 - |\nabla\varphi_+|^2 - m^2|\varphi_+|^2 \\ & + |(i\partial_t + \mu_0)\varphi_0|^2 - |\nabla\varphi_0|^2 - m^2|\varphi_0|^2 \\ & - \frac{\lambda}{2}(|\varphi_+|^2 + |\varphi_-|^2 + |\varphi_0|^2)^2 \Big\},\end{aligned} \tag{4.5}$$

where we have introduced the notation $\varphi_{\pi^-} = \varphi_- e^{-i\mu_- t}$, $\varphi_{\pi^0} = \varphi_0 e^{-i\mu_0 t}$, $\varphi_{\pi^+} = \varphi_+ e^{-i\mu_+ t}$, and μ_-, μ_0, and μ_+ are the chemical potentials of the π^-, π^0, and π^+ mesons, determined from the conditions for fixing the numbers of the corresponding particles:

$$\frac{\partial\mathscr{L}}{\partial\mu_-} = N_-, \quad \frac{\partial\mathscr{L}}{\partial\mu_+} = N_+, \quad \frac{\partial\mathscr{L}}{\partial\mu_0} = N_0. \tag{4.6}$$

The fields φ_-, φ_+, and φ_0 are found from the equations of motion obtained by variation of the Lagrangian:

$$\begin{aligned}(i\partial_t + \mu_i)^2\varphi_i + \Delta\varphi_i - m^2\varphi_i - \lambda(|\varphi_+|^2 + |\varphi_-|^2 \\ + |\varphi_0|^2)\varphi_i = 0, \quad i = +, -, 0.\end{aligned} \tag{4.7}$$

In the mean-field approximation, from (4.7) we obtain

$$|\varphi_+|^2 + |\varphi_-|^2 + |\varphi_0|^2 = \varphi_c^2 = \frac{\mu_i^2 - m^2}{\lambda}, \tag{4.8}$$

whence it follows that $\mu_- = \mu_+ = \mu_0 = \mu$.

From (4.6) and (4.8), for the density of particles of a given kind we have

$$\rho_i = 2\mu|\varphi_i|^2, \quad \rho = \rho_+ + \rho_- + \rho_0 = 2\mu\frac{\mu^2 - m^2}{\lambda}. \tag{4.9}$$

Hence, for $\rho \ll 4m^3/\lambda$,

$$\mu \simeq m + \frac{\lambda\rho}{4m^2}, \tag{4.10}$$

while for $\rho \gg 2m^3/\lambda$,

$$\mu \simeq (\rho\lambda/2)^{1/3}. \tag{4.11}$$

As well as the mean field we must also take into account the contribution of quantum and thermal (for $T \neq 0$) fluctuations. The former introduce unimportant changes. Therefore, for $T = 0$ we can confine ourselves to the mean-field approximation. In this case the free energy coincides with the energy and is given by the expression

$$F = \sum_i \mu_i \frac{\partial\mathscr{L}}{\partial\mu_i} - \mathscr{L} = \left[\mu\rho - \frac{(\mu^2 - m^2)^2}{2\lambda}\right]V. \tag{4.12}$$

In the limiting case of a low pion density [Eq. (4.10)] we have

$$F \simeq \left[\rho m + \frac{\lambda\rho^2}{8m^2}\right]V, \tag{4.13}$$

while in the high-density limit [Eq. (4.11)] we obtain

$$F \simeq \frac{3}{2^{7/3}}\lambda^{1/3}\rho^{4/3}V. \tag{4.14}$$

5. EXCITATION SPECTRUM IN THE PRESENCE OF A BOSE CONDENSATE

The Goldstone boson

For simplicity we shall consider the case when there are only pions of one kind. Let these be π^0 mesons. We shall describe the excitations superposed on the Bose condensate by introducing real, small-amplitude fields $\rho'(t,\mathbf{r})$ and $\chi'(t,\mathbf{r})$ such that

$$\varphi_0 = \varphi_c(1 + \rho)e^{i\chi'}, \quad \mu_0 > m. \tag{5.1}$$

Substituting φ_0 into the equation of motion (4.7) and separating the real and imaginary parts, we find in the linear approximation in the fields ρ' and χ'

$$\begin{aligned}& -\partial_t^2\rho' - 2\mu_0\partial_t\chi' + \Delta\rho' - [3\lambda\varphi_c^2 - \mu_0^2 + m^2]\rho' = 0, \\ & -\partial_t^2\chi' + 2\mu_0\partial_t\rho' + \Delta\chi' = 0.\end{aligned} \tag{5.2}$$

Hence, setting

$$\rho' = A_k\sin(\omega t - \mathbf{kr}), \quad \chi' = B_k\cos(\omega t - \mathbf{kr}), \tag{5.2a}$$

we have

$$\begin{aligned}& 2\mu_0\omega A_k + (\omega^2 - k^2)B_k = 0, \\ & (\omega^2 - k^2 - 3\lambda\varphi_c^2 + \mu_0^2 - m^2)A_k + 2\mu_0\omega B_k = 0.\end{aligned} \tag{5.2b}$$

From (5.2a) we obtain the excitation spectrum

$$\omega^2 - k^2 = \frac{3\lambda\varphi_c^2 + 3\mu_0^2 + m^2}{2} \pm \sqrt{\frac{(3\lambda\varphi_c^2 + 3\mu_0^2 + m^2)^2}{4} + 4\mu_0^2k^2}. \tag{5.3}$$

Substituting into (5.3) the value φ_c (4.8) obtained in the mean-field approximation, we obtain

$$\omega^2 \simeq 3\mu_0^2 - m^2 + k^2 \pm \sqrt{(3\mu_0^2 - m^2)^2 + 4\mu_0^2 k^2}. \qquad (5.3a)$$

The upper branch of the spectrum corresponds to excitations with mass $[2(3\mu_0^2 - m^2)]^{1/2}$, and the lower branch corresponds to gapless oscillations. For $k \ll \mu_0$ the latter are given by the expression

$$\omega^2 \simeq \frac{\mu_0^2 - m^2}{3\mu_0^2 - m^2} k^2 + \frac{2\mu_0^4}{(3\mu_0^2 - m^2)^3} k^4. \qquad (5.4)$$

Hence, for an ideal gas we have a quadratic dispersion law ($\omega \simeq k^2/2m$). For a nonideal gas, for small values of the coupling constant ($\lambda\rho_0 \ll 4m^3$), according to (4.10) we have

$$\omega \simeq \sqrt{\frac{\lambda\rho_0(k=0)}{4m^3}} k, \qquad (5.5)$$

while for large values of λ ($\lambda\rho_0 \gg 4m^3$), taking (4.9) into account, we have

$$\omega \simeq \frac{1}{\sqrt{3}} k. \qquad (5.6)$$

Thus, in the case of a nonideal gas there are Goldstone excitations ($\omega \propto k$) in the spectrum. Therefore, according to the Landau criterion, a nonideal π^0 gas with a rigorously fixed number of particles ought to possess superfluidity. But in the case of the ideal gas ($\omega \propto k^2$) the property of superfluidity is absent.

The massive photon

Since the π^+ and π^- meson are charged particles, to determine the spectrum of the $\pi^\pm$ excitations we must take into account the interaction with the electromagnetic field. For this it is necessary to perform the gauge replacement

$$\partial_\mu \varphi_- \to (\partial_\mu - ieA_\mu)\varphi_-, \quad \partial_\mu \varphi_+ \to (\partial_\mu + ieA_\mu)\varphi_+. \qquad (5.7)$$

Suppose that there are only π^- mesons. Their Lagrangian, with allowance for the interaction with the electromagnetic field, has the form

$$\mathscr{L} = \int d\mathbf{r} \left\{ |(\partial_\mu - ie\tilde{A}_\mu)\varphi_-|^2 - m^2|\varphi_-|^2 - \frac{\lambda}{2}|\varphi_-|^4 - \frac{F_{\mu\nu}^2}{16\pi} \right\},$$

$$\tilde{A}_\mu = (A_0 - i\mu_-/e, A), \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \qquad (5.8)$$

Since the charge of the π^- system is not neutralized in any way, the distribution of the π^- mesons over the volume is, generally speaking, nonuniform. However, for $\mu_- \gg |eA_0| \sim N_- e^2/R$ this nonuniformity can be neglected. Thus, for a not too large number of pions ($N_- \ll R\mu_-/e^2$) their distribution can be assumed to be approximately uniform.

Representing the field φ_- in the form

$$\varphi_- = \tilde{\varphi}(1+\rho')e^{i\chi'}, \quad \tilde{\varphi} \simeq \sqrt{\frac{\mu_-^2 - m^2}{\lambda}}, \quad \mu_- > m, \qquad (5.9)$$

where ρ' and χ' are small-amplitude real fields, we obtain

$$|(\partial_\mu - ie\tilde{A}_\mu)\varphi_-|^2 \simeq \tilde{\varphi}^2(\partial_\mu\rho')^2 + \tilde{\varphi}^2(\mu_- - eA_0')^2 \times (1+\rho')^2 - e^2\tilde{\varphi}^2\mathbf{A}'^2. \qquad (5.10)$$

Here we have introduced the new field $A_\mu' = A_\mu - \partial_\mu\chi'/e$. Since the quantity $F_{\mu\nu}$ is gauge-invariant it can also be expressed in terms of the field A_μ', i.e., $F_{\mu\nu} = F_{\mu\nu}' = \partial_\mu A_\nu' - \partial_\nu A_\mu'$. As a result, the Goldstone field χ' is absorbed by the gauge transformation. Taking this into account and varying the Lagrangian with respect to the remaining fields A_μ' and ρ', in the linear approximation we obtain the equations

$$\Box A_\mu' + 8\pi e^2\tilde{\varphi}^2 A_\mu' + 16\pi e\mu_-\tilde{\varphi}^2\rho'\delta_{\mu 0} = 0, \quad \partial^\mu A_\mu' = 0,$$
$$\Box\rho' + 2(\mu_-^2 - m^2)\rho' - 2\mu_- eA_0' = 0. \qquad (5.11)$$

We note that in the equation for the field A_μ' we have discarded the term $-16\pi e\mu_-\tilde{\varphi}^2\delta_{\mu 0}$, which is responsible for the spatial nonuniformity of the distribution of π^- mesons. As already stated, this nonuniformity can be neglected for $N_- \ll R\mu_-/e^2$, $\mu_- \gg m$.

The equations (5.11) determine the spectrum of the excitations about the "new" Bose-condensate vacuum. There are three branches of excitations:

$$\omega^2 \simeq 8\pi e^2\tilde{\varphi}^2 + k^2,$$
$$\omega^2 \simeq \mu_-^2 - m^2 + 4\pi e^2\tilde{\varphi}^2 \pm \sqrt{(\mu_-^2 - m^2 + 4\pi e^2\tilde{\varphi}^2)^2 - 16\pi e^2\tilde{\varphi}^2(3\mu_-^2 - m^2)}, \qquad (5.12)$$

The lowest mass

$$m_A^2 \simeq 8\pi e^2\tilde{\varphi}^2 \simeq 8\pi(\mu_-^2 - m^2)e^2/\lambda \qquad (5.12a)$$

is possessed by the "magnetic" excitations, a larger mass

$$m_V^2 \simeq \frac{8\pi e^2(3\mu_-^2 - m^2)}{\lambda}, \quad \mu_-^2 - m^2 \gg \frac{16\pi e^2(3\mu_-^2 - m^2)}{\lambda} \qquad (5.12b)$$

is possessed by the "electric" excitations, and a still larger mass

$$m_\varphi^2 \simeq 2(\mu_-^2 - m^2) \qquad (5.12c)$$

is possessed by the meson excitations. Here, only the "magnetic" oscillations are independent. Therefore, our separation of the branches is rather arbitrary.

Thus, the Goldstone field χ' has been absorbed by the gauge transformation. As a consequence of this, the photon has acquired an additional polarization and a nonzero mass.

We draw attention to the fact that in the presence of charged pions of only one kind the characteristic absorption time $\tau_{\rm abs}$ is considerably longer than in the case when both π^+ and π^- mesons are present. Because of the conservation of electric charge, the absorption in the former case is determined entirely by weak processes.

We shall consider now the case when there are both π^+ and π^- mesons, and, for simplicity, $N_{\pi^+} = N_{\pi^-}$ and $N_{\pi^0} = 0$. In this case the Lagrangian has the form

$$\mathscr{L} = \int d\mathbf{r} \Big\{ |(i\partial_t + \mu - V)\varphi_-|^2 - |(\nabla - ieA)\varphi_-|^2$$

$$+|(i\partial_t+\mu+V)\varphi_+|^2-|(\nabla+ieA)\varphi_+|^2$$
$$-m^2(|\varphi_-|^2+|\varphi_+|^2)-\frac{\lambda(|\varphi_-|^2+|\varphi_+|^2)^2}{2}$$
$$\left.-\frac{F_{\mu\nu}^2}{16\pi}\right\}. \quad (5.13)$$

The pion distributions are uniform, since the total charge is equal to zero. Analogously to (5.9), we represent the fields φ_- and φ_+ in the form

$$\varphi_-=\tilde{\varphi}(1+\rho'_-)e^{i\chi'_-},\quad \varphi_+=\tilde{\varphi}(1+\rho'_+)e^{-i\chi'_+},$$
$$\tilde{\varphi}=\sqrt{\frac{\mu^2-m^2}{2\lambda}}. \quad (5.14)$$

Introducing the new field $A'_\mu = A_\mu - (\partial_\mu\chi'_- + \partial_\mu\chi'_+)/2e$, we easily see that only a dependence on the fields ρ'_-, ρ'_+, A'_μ, and $\tilde{\chi}=\frac{1}{2}(\chi'_- - \chi'_+)$ remains. Varying the Lagrangian with respect to these fields we can determine the excitation spectrum.

As a result, for $k\to 0$ there are gap excitations with masses

$$m_A^2\simeq m_V^2\simeq\frac{8\pi e^2(\mu^2-m^2)}{\lambda},\quad m_{\rho_-}^2=m_{\rho_+}^2\simeq 3\mu^2-m^2, \quad (5.15)$$

and also one Goldstone oscillation

$$\omega\simeq\sqrt{\frac{\mu^2-m^2}{3\mu^2-m^2}}\,k. \quad (5.15a)$$

An entirely analogous treatment can also be carried out for $T\neq 0$, and also in the general case, when there are π^+, π^-, and π^0 mesons. As a result, for $\rho_+=\rho_-=\rho_0=\rho/3$, for the mass of the "magnetic" excitations we obtain

$$m_A=\sqrt{8\pi}e[\rho(k=0)/3\mu]^{1/2}, \quad (5.16)$$

where $\rho(k=0)$ is given by the expression (2.10) in the case of a nonrelativistic gas and by the expression (2.19) for an ultrarelativistic gas. Setting $\rho(k=0)\simeq\rho_b \simeq(0.5\text{–}3)m_\pi^3$ (see Sec. 2), and assuming $\lambda\ll 1$ and $T\ll T_c$, we obtain $m_\gamma\simeq(25\text{–}60)$ MeV. The nonzero value of the photon mass could, in principle, be manifested experimentally in a certain suppression of the yield of soft photons $(\omega<m_\gamma)$ by a factor $\exp(-m_\gamma/T)$. In ultrarelativistic collision of nuclei, however, it is likely that $T\gg m_\gamma$ holds, and this factor is approximately equal to unity. An excess yield of soft photons could be explained by nonequilibrium effects.

The extra contribution to the mass of the ρ^0 meson

The ρ meson is a vector particle, described by the field $\rho_\mu=(\rho_\mu^0,\rho_\mu^+,\rho_\mu^-)$, $\mu=0, 1, 2, 3$. Therefore, the interaction of the ρ^0 meson with the π^+- and π^--meson fields is analogous to the $\gamma\pi^+\pi^-$ interaction. Thus, the $\rho\pi^+\pi^-$ interaction is implemented by the replacement[18]

$$\partial_\mu\varphi_-\to(\partial_\mu-ig_{\rho\pi\pi}\varphi_\mu^0)\varphi_-,\ \partial_\mu\varphi_+$$
$$\to(\partial_\mu+ig_{\rho\pi\pi}\varphi_\mu^0)\varphi_+,g_{\rho\pi\pi}$$
$$\simeq 6.1. \quad (5.17)$$

To determine the excitations in the presence of condensates of π^+, π^-, and π^0, we must represent the corresponding fields in a form analogous to (5.9). The difference from photons lies in the fact that in the ρ-meson case the Goldstone field χ' cannot be absorbed into the gauge transformation of the ρ^0 field, since the latter (because of the nonzero ρ^0-meson mass $m_\rho^0\neq 0$) is not gauge-invariant. However, the field χ' can be absorbed into the gauge transformation of the electromagnetic field, since the interaction of ρ^0 and the photon field A_μ with the $\pi^\pm$ fields is implemented by the general replacement

$$\partial_\mu\varphi_-\to(\partial_\mu-ieA_\mu-ig_{\rho\pi\pi}\rho_\mu^0)\varphi_-,$$
$$\partial_\mu\varphi_+\to(\partial_\mu+ieA_\mu+ig_{\rho\pi\pi}\rho_\mu^0)\varphi_+. \quad (5.18)$$

As a result, as above, we find that the photon acquires a nonzero mass (5.16), and the initially massive ρ^0 meson acquires extra mass. For $\rho_+=\rho_-=\rho_0=\rho/3$ we have

$$m_\rho^2=(m_\rho^0)^2+\tilde{m}_\rho^2,\quad \tilde{m}_\rho^2=\frac{2}{3}\frac{g_{\rho\pi\pi}^2}{\mu}\rho(k=0). \quad (5.19)$$

Setting $\rho(k=0)\simeq\rho_{\text{nucl}}$, $T=0$, and $\mu\simeq m$ for the estimate, we obtain $m_\rho\simeq 6.5m_\pi$, while for $\rho(k=0)\simeq 6\rho_{\text{nucl}}$ we find $m_\rho\simeq 10m_\pi$, which is considerably greater than the bare ρ^0-meson mass ($\simeq 5.5m_\pi$).

As is well known, by virtue of vector dominance the yield of dileptons is principally determined by the ρ^0 channel of the reaction:

$$\pi^+\pi^-\to\rho^0\to l^+l^-. \quad (5.20)$$

Because of the excess mass of the ρ^0 meson the probability of this reaction in the presence of Bose condensation of pions should be reduced (by a factor $\gamma\sim\exp[-(m_\rho - m_\rho^0)/T]$). Thus, we may expect substantial suppression ($\gamma\sim 0.1$–0.3) of the yield of dileptons from the region of the pion fireball in the presence of a developed pion condensate within it.

6. INTERACTING PION GAS AT NONZERO TEMPERATURE

For simplicity we shall consider the case when there are only π^0 mesons. Then $\varphi_+=\varphi_-=0$ holds, and φ_0 has the form (5.1) with quantities ρ' and χ' given by (5.2a). Taking this into account and expressing the amplitude B_k in terms of A_k by means of (5.2c), from the Lagrangian (4.5) we obtain

$$\mathscr{L}=\mathscr{L}_c+\mathscr{L}_{\text{fl}},\quad \mathscr{L}_c=V\left\{(\mu^2-m^2)\varphi_c^2-\frac{\lambda\varphi_c^4}{2}\right\},$$
$$\mathscr{L}_{\text{fl}}=\int\frac{d^4kV}{(2\pi)^4}\text{Re}(G^R(\omega,k))^{-1}\varphi_k'^2, \quad (6.1)$$

where

$$[G^R(\omega,k)]^{-1}=3\lambda\varphi_c^2+m^2-\mu^2-(\omega^2-k^2)+\frac{(\omega^2-k^2)^3}{4\mu^2\omega^2}+\frac{(\omega^2-k^2)^2(3\lambda\varphi_c^2-\mu^2+m^2)}{2\mu^2\omega^2}-\frac{(\omega^2-k^2)(3\lambda\varphi_c^2-\mu^2+m^2)}{2\mu^2\omega^2}+i0. \quad (6.1a)$$

The quantity $G^R(\omega,k)$ plays the role of the retarded Green's function of the excitations. Its pole determines the excitation spectrum (5.3). Near its poles the Green's function G^R has the simpler form

$$G^R=Z_i/(\omega-\omega_i(k)+i0), \quad i=1,2, \quad (6.2)$$

where $\omega_i(k)$ is the corresponding branch of the excitation spectrum (5.3), and

$$Z_i=\left|\left(\frac{\partial \operatorname{Re}(G^{R-1})}{\partial\omega}\right)_{\omega_i(k)}\right|^{-1}. \quad (6.2a)$$

The quantity

$$\Gamma_i=2\omega_i(k)\left(\frac{\partial \operatorname{Re}(G^R)^{-1}}{\partial\omega}\right)^{-1}_{\omega_i(k)} \quad (6.3)$$

shows the weight with which the given branch of the excitation spectrum is populated by real mesons. According to (5.3)–(5.6), we have $\Gamma_1=\Gamma_2=1/2$ for $k\to\infty$, i.e., as they merge at high momenta the two branches of the excitation spectrum are represented with the same weight. As $k\to 0$, for the lower branch the weight $\Gamma_1\to 0$, and for the upper branch the weight $\Gamma_2\to 1$.

The thermodynamic potential $\Omega(\mu,T)$ is obtained by averaging the Lagrangian over the Gibbs distribution. According to (6.1) we have

$$\Omega(\mu,T)=-\langle\mathcal{L}\rangle_T=V\left\{-(\mu^2-m^2)\varphi_c^2+\frac{\lambda\varphi_c^4}{2}\right\}-\int\frac{d^\varphi kV}{(2\pi)^4}\operatorname{Re}(G^R(\omega,k))\langle\varphi_k'^2\rangle_T, \quad (6.4)$$

where the quantity $\langle\varphi_k'^2\rangle_T$ is[19]

$$\langle\varphi_k'^2\rangle_T=-\frac{2\operatorname{Im}G^R(\omega+\mu,k)}{\exp(\omega/T)-1}. \quad (6.5)$$

The chemical potential μ and the density of free pions are related by

$$\rho=2\mu\varphi_c^2+2\omega(k)\langle\varphi'^2\rangle_T, \quad (6.6)$$

where the quantity $\langle\varphi'^2\rangle_T$ is determined in terms of $\langle\varphi_k'^2\rangle_T$ by means of the expression (6.5)

$$\langle\varphi'^2\rangle_T=\int\frac{d^4k}{(2\pi)^4}\langle\varphi_k'^2\rangle_T=-\int\frac{d^4k}{(2\pi)^4}\frac{2\operatorname{Im}G^R}{\exp(\omega/T)-1}. \quad (6.7)$$

Taking (6.2) and (6.2a) into account, we have

$$\rho=2\mu\varphi_c^2+\sum_{i=1}^{2}\int\frac{d^3k}{(2\pi)^3}\frac{\Gamma_i}{\exp(\omega_i(k)/T)-1}. \quad (6.8)$$

The quasiparticle density ρ^q is determined by the expression

$$\rho^q=-\frac{\partial\Omega}{\partial\mu}=2\mu\varphi_c^2-\int\frac{d^4k}{(2\pi)^4}\frac{(\partial G^{R,-1}/\partial\omega)\,2\operatorname{Im}G^R}{\exp(\omega/T)-1}\simeq 2\mu\varphi_c^2+\sum_{i=1}^{2}\int\frac{d^3k}{(2\pi)^3}[\exp(\omega_i(k)/T)-1]^{-1}. \quad (6.9)$$

For $T\gg m'\simeq\sqrt{2(3\mu^2-m^2)}$ the characteristic values of k in the integrals (6.7)–(6.9) are of order T. Therefore, taking (6.1a) and (5.3) into account we have

$$\langle\varphi'^2\rangle_T\simeq T^2/12, \quad (6.10)$$

$$\rho\simeq 2\mu\varphi_c^2+\rho(T/T_c)^3, \quad (6.10a)$$

$$\rho^q\simeq 2\mu\varphi_c^2+2\rho(T/T_c)^3, \quad (6.10b)$$

where the quantity T_c is given by the expression (2.16). The quantity $\rho(k=0)=2\mu\varphi_c^2$ is the density of condensate particles. We note that the expression (6.10a) coincides with the expression (2.19) obtained for an ultrarelativistic ideal gas. The quasiparticle density ρ^q differs from the particle density ρ because of the presence of the two branches in the quasiparticle spectrum, populated by real particles in proportion to the weight factors Γ_i.

The classical field φ_c is obtained from the equation of motion obtained by variation of the thermodynamic potential $\Omega(\mu,T)$ with respect to φ_c. Taking (6.4), (6.1a), and (6.10) into account, we can show that for $T\gg m'$ and $\lambda\ll m\pi^2/T$ the dependence $\varphi_c(\mu)$ has the same form (4.9) as for $T=0$.

The momentum distribution of free pions in the model of sudden breakup of the fireball[8,19,20] has the form

$$\frac{d^3N}{[d^3k/(2\pi)^3]V_b}=\sum_{i=1}^{2}\frac{\Gamma_i(\omega_k/\omega_i(k))}{\exp(\omega_i(k)/T_b)-1},$$

$$k>0, \quad \omega_k=\sqrt{m^2+k^2}-m, \quad (6.11)$$

$$\frac{d^3N}{[d^3k/(2\pi)^3]V_b}=\frac{m}{\mu}\rho(k=0)\delta(k),$$

$$k\simeq 0, \quad \rho(k=0)\simeq\rho\left[1-\left(\frac{T}{T_c}\right)^3\right]. \quad (6.12)$$

It is this distribution, and not the distribution of quasiparticles, that must be compared with the experimental distribution. Here we must keep in mind that in the dynamical system we are considering the Bose-condensate pions are spread over small momenta. Therefore, instead of a δ-function distribution of them there is in fact a narrow but finite peak at $k=0$. In this way it would be possible to explain the excess yield of soft pions that is observed experimentally.[12]

Only quadratic fluctuations were taken into account above. It is possible to do this only if $\varphi_c^2 \gg \langle \varphi'^2 \rangle_T$, whence we obtain the estimate

$$|T-T_c| \gg m/6. \tag{6.13}$$

Thus, there is a narrow fluctuation region $|T-T_c| \lesssim m/6$ in which the fluctuating field is so large that it cannot be treated using perturbation theory.

Using the expressions (6.4) and (6.5) one can establish other thermodynamic characteristics of the system. Thus, for the energy we have

$$E = V\mu\rho(k=0) + \int \frac{d^4kV}{(2\pi)^4} \frac{\partial \operatorname{Re} G^{R,-1}}{\partial \omega} \langle \varphi_k'^2 \rangle_T - \langle \mathscr{L} \rangle_T. \tag{6.14}$$

Using (6.1a), we obtain

$$E \simeq V\mu\rho(k=0) - \frac{(\mu^2-m^2)^2}{2\lambda} V + V \sum_{i=1}^{2} \int \frac{d^3k}{(2\pi)^3} \frac{\omega_i(k)}{\exp(\omega_i(k)/T)-1}. \tag{6.15}$$

Here the first term is the contribution of the condensate, and the second is the contribution of the quasiparticles superposed on the condensate.

Above, we considered the case when there are only π^0 mesons. In an analogous way we can also consider the general case, when there are pions of all three kinds. Here, however, we must bear in mind that, as a consequence of the interaction with the electromagnetic vacuum, the spectra of the π^+ and π^- mesons differ from the spectra of the π^0 mesons. It is obvious that some changes of the spectra arise also as a consequence of the interaction of the π^+, π^-, and π^0 excitations with each other.

7. PION CONDENSATION IN THE σ-MODEL

As is well known, in strong interactions chiral $SU(2) \times SU(2)$ symmetry is well fulfilled. Its simplest realization is the linear σ-model, the Lagrangian of which is usually represented in the form (see Ref. 21)

$$\mathscr{L} = \int d\mathbf{r} \left\{ \frac{\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \pi \partial^\mu \pi}{2} - \frac{\Delta}{4} (\sigma^2 + \pi^2 - f_\pi^2)^2 + \bar{N} i \gamma^\mu \partial_\mu N - g \bar{N} (\sigma + \tau \pi \gamma_5) N \right\} + \mathscr{L}_{SB}, \tag{7.1}$$

where $f_\pi \simeq 93$ MeV is the pion-decay constant. For generality we have introduced an interaction with initially massless nucleons, and Λ and g are constants. The vacuum is realized in a spontaneously broken mode corresponding to one of the roots $\sigma = \pm f_\pi$ and $\pi^2 = 0$. As a result, the nucleon acquires the mass

$$m_N = g\sigma. \tag{7.2}$$

The constants Λ and g are chosen in order to obtain the value of the mass of the nucleon in the vacuum, and, for the σ meson, to obtain a mass in the region of 700 MeV, in agreement with the phase-analysis data. This can be achieved with $g \simeq 10$ and $\Lambda \simeq 20$. The term $\mathscr{L}_{SB}$ corresponds to weak breaking of the chiral symmetry, as a consequence of which the pion acquires mass. It is usually chosen either in the form

$$\mathscr{L}_{SB} = -\int d\mathbf{r} m_\pi^2 \pi^2/2, \tag{7.3}$$

or in the form

$$\mathscr{L}_{SB} = -\int d\mathbf{r} f_\pi \sigma m_\pi^2. \tag{7.4}$$

In the case we are considering, for characteristic times $t \ll \tau_{\text{abs}}$, the number of π^+, π^-, and π^0 mesons is approximately conserved. To take this circumstance into account, we shall start from a modified Lagrangian, just as was done Sec. 4. From (7.1) we obtain

$$\begin{aligned} \mathscr{L} \simeq \int d\mathbf{r} \Bigg\{ & \frac{(\partial_t \sigma)^2 - (\nabla \sigma)^2}{2} + \frac{|(i\partial_t + \mu_-)\pi_-|^2 - |\nabla \pi_-|^2}{2} \\ & + \frac{|(i\partial_t + \mu_+)\pi_+|^2 - |\nabla \pi_+|^2}{2} \\ & + \frac{|(i\partial_t + \mu_0)\pi_0|^2 - |\nabla \pi_0|^2}{2} \\ & - \frac{\Delta}{4} (\sigma^2 + |\pi_+|^2 + |\pi_-|^2 + |\pi_0|^2 - f_\pi^2)^2 \Bigg\} \\ & + \mathscr{L}_{SB} + \mathscr{L}_N + \mathscr{L}_{\pi N} + \mathscr{L}_{\sigma N}, \end{aligned} \tag{7.5}$$

where $\mathscr{L}_N$, $\mathscr{L}_{\pi N}$, and $\mathscr{L}_{\sigma N}$ are terms corresponding to the free nucleon field and its interaction with the π and σ mesons. Unlike the pions, the σ mesons are heavy particles ($m_\sigma \simeq 700$ MeV). Therefore the probability of their production as a result of decay of resonances in the initial stages of the collision of nuclei can be much smaller than the probability of production of pions. Thus, the initial number of σ mesons is much smaller than the number of pions, and the σ mesons can be neglected. To this accuracy we can set $\mu_\sigma \simeq 0$.

Next, for simplicity, we shall consider the case when there are only pions of one kind, say π^-. Varying the Lagrangian (7.4) with respect to the σ and π^- fields, and taking the term $\mathscr{L}_{SB}$ in the form (7.3), we obtain the equations of motion

$$\begin{aligned} & \Lambda(\sigma^2 + |\pi_-|^2 - f_\pi^2)\pi_- - (i\partial_t + \mu_-)^2 \pi_- - \Delta \pi_- \\ & + m^2 \pi_- = 0, \\ & \Lambda(\sigma^2 + |\pi_-|^2 - f_\pi^2)\sigma + \partial_t^2 \sigma - \Delta \sigma = 0. \end{aligned} \tag{7.6}$$

From this, in the mean-field approximation, we obtain two possible solutions:

$$1) \quad \sigma = 0, \quad |\pi_-|^2 = f_\pi^2 + \frac{\mu_-^2 - m^2}{\Lambda},$$

$$\rho_- = \mu_- |\pi_-|^2 = \mu_- \left(f_\pi^2 + \frac{\mu_-^2 - m^2}{\Lambda} \right), \quad (7.7)$$

2) $\sigma^2 = f_\pi^2 - |\pi_-|^2, \quad \mu_- = m,$

$$\rho_- = m |\pi_-|^2 = m(f_\pi^2 - \sigma^2), \quad \rho_- < m^2 f_\pi^2. \quad (7.8)$$

We note that the second solution holds only for $\rho_- < m^2 f_\pi^2$.

To make a choice between these two solutions we must compare the energies corresponding to them. According to (7.5), the energy is represented in the form

$$E = V \left\{ \frac{\mu^2 + m^2}{2} |\pi_-|^2 + \frac{\Lambda}{4} (\sigma^2 + |\pi_-|^2 - f_\pi^2)^2 \right\}, \quad (7.9)$$

where we have taken into account that π_- and σ are constant fields. Substituting the solution (7.7) into (7.8), we obtain

$$E = V \left\{ \frac{\mu_-^2 + m^2}{2} \left(f_\pi^2 + \frac{\mu_-^2 - m^2}{\Delta} \right) + \frac{\mu_-^2 - m^2}{4\Lambda} \right\}. \quad (7.10)$$

Taking into account that, for large values of Λ, according to (7.7),

$$\mu_- \simeq \rho_- / f_\pi^2, \quad (7.11)$$

we have

$$E \simeq V \left\{ \frac{m^2 f_\pi^2}{2} + \frac{\rho_-^2}{2 f_\pi^2} + O\left(\frac{1}{\Lambda}\right) \right\}. \quad (7.12)$$

For the second solution (7.8) we obtain

$$E \simeq V m_\pi \rho_- . \quad (7.13)$$

Comparing the expressions (7.12) and (7.13), we easily see that for $\rho < \rho_c \simeq f_\pi^2 m \simeq \rho_{\text{nucl}}$ the second solution holds, while for $\rho > \rho_c$ the first solution holds. Thus, for $\rho > \rho_c$ a chiral phase with a vacuum average $\sigma = 0$ is realized. For this reason, the nucleon mass, determined by (7.2), vanishes for $\rho \geqslant \rho_c$.

The vanishing of the nucleon mass should be manifested experimentally in a significant increase in the yield of $N\bar{N}$ pairs from the region of the pion fireball. In fact, in the model of sudden breakup of the fireball[8,19,20] we have

$$\delta\rho_N = \delta\rho_{\bar{N}} = 4 \int \frac{d^3k}{(2\pi)^3} \frac{\Gamma}{\exp(k/T_b) + 1}, \quad (7.14)$$

where $\delta\rho_N$ ($\delta\rho_{\bar{N}}$) is the density of nucleons (antinucleons) created in the region of the pion fireball,

$$\Gamma = \sqrt{m_N^2 + k^2} / k \quad (7.14a)$$

is the factor by which the quasiparticle branch of the spectrum in the pion fireball ($\omega \simeq k$) is populated by free nucleons, and the factor 4 takes into account the spin-isospin degeneracy. It is obvious that the contribution (7.14) must be added to the corresponding expression for the density of nucleons and antinucleons emerging from the region of the nuclear fireball. In addition, the excess yield of resonances, which subsequently decay into nucleons, antinucleons, and pions, must be taken into account in an analogous way. The estimate (7.14) for $T_b \ll m_N$ gives for the number of nucleons and antinucleons

$$\delta N_N \simeq \delta N_{\bar{N}} \simeq \frac{m_N}{b} T_b^2 V_b \gtrsim m_\pi^3 V_b. \quad (7.15)$$

For $R_b \simeq 4$ fm we have $\delta N_N \simeq \delta N_{\bar{N}} \sim 10^2$. An excess yield of nucleons and antinucleons from the region of the pion fireball would be evidence in favor of the substantial nucleon-mass decrease predicted by the chirally symmetric models.

CONCLUDING REMARKS

Thus, Bose condensation of pions leads to a number of specific distinctive features that, in principle, can be manifested in experiment. For example, in the presence of a condensate (or in the case of proximity to the critical condensation point) the yield of soft pions is substantially enhanced. Such an enhancement has indeed been detected experimentally. In the presence of a condensate the hydrodynamic description of the system changes because of the presence of two weakly interacting subsystems—the normal and the condensate subsystem. The excitation spectrum is greatly softened, and this has an effect on the equation of state of the system. These features should be manifested in the description of the dynamics of the pion liquid. In the presence of a condensate, pion vortices (solitons) are formed, the detection of which would be of undoubted interest. In the presence of a pion condensate the photon acquires mass and an extra polarization, while the ρ meson acquires a large additional mass, and this can lead to substantial suppression of the yield of dileptons. The nucleon-mass decrease that has been demonstrated in the framework of the σ model can be manifested in an increase of the yield of $N\bar{N}$ pairs. Finally, the presence of long-range order in the condensate may be responsible for the large pion-source radius measured in experiments on $\pi\pi$ correlations.

Many questions related to Bose condensation in the dynamical problem require further analysis. For example, it is necessary to take explicit account of effects associated with the possible absorption of pions, starting from the fundamental Lagrangian (4.1). It is of interest to describe the kinetics of the transition of a relativistic system to a quasiequilibrium state with a Bose condensate. In analyzing the hydrodynamic stage one must take into account the anisotropy of the expansion of the pion fireball.

This work has been carried out with the financial support of the Russian Fund for Fundamental Research (project code 93-02-3816). The author is also grateful to Yu. Ivanov, B. Friman, J. Knoll, W. Nörenberg, H. Schulz, and A. Nefed'ev for discussions.

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Translated by P. J. Shepherd