Plasmon mechanism of high-temperature superconductivity in cuprate metal-oxide compounds

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The plasmon mechanism of Cooper pairing of almost-free "light" carriers in a wide 2D band is considered in the framework of standard superconductivity theory in the intermediate-coupling approximation. The model used is that of a layered metal with quasi-two-dimensional electron spectrum and a narrow band near the Fermi level, with exchange of virtual quanta of low-frequency collective excitations of the charge density of almost localized "heavy" carriers in a narrow 2D band (acoustic plasmons), which are hybridized with dipole-active oxygen vibrational modes (optical phonons) in the entire volume of the Brillouin band. It is shown that when the account is taken of the multilayer structure of cuprate metal-oxide compounds and of multiparticle Coulomb correlations (of the type of "local-field" effects), such a mechanism can ensure quite high values of the critical temperature of the superconducting transition temperature $T_c$, and describes correctly all the main properties of high-temperature superconductors, such as the nonmonotonic dependence of $T_c$ on the density of the dopant impurity or on the oxygen content, the rise of $T_c$ with increase of the number $n$ of the cuprate CuO$_2$ layers in the primitive cell of the crystal with a tendency to saturation at $n>3$, the anomaly of the oxygen isotopic effect, and others.

1. INTRODUCTION

1. It is known that high-temperature superconductors based on cuprate metal-oxide compounds (MOC) have many unusual physical properties, including: a) anomalously high critical superconducting (SC) transition temperatures $T_c$ ($30-125$ K at optimal composition, b) an anomalously weak isotope effect (IE) which tends to decrease with increase of $T_c$ (Refs. 7–10), c) a nonmonotonic dependence of $T_c$ on the density of the dopant and on the oxygen content, i.e., on the density of the carriers (holes, electrons) in the conducting CuO$_2$ layers, d) rise of $T_c$ with increase of the number $n$ of the cuprate layers in the primitive cell of the crystal, e) correlation between the maximum values of $T_c$ and the frequencies of the "oxygen" vibrational modes for various classes of cuprate MOC, and others.

It will be shown below that all the above as well as other features of high-temperature superconductivity (HTSC) in cuprate MOC can be explained (at least qualitatively) on the basis of a simple model, proposed in Refs. 21 and 22, of a two-band quasi-two-dimensional metal with light ($d$) and (h) carriers (electrons, holes) there exists a low-frequency (1f) oscillation mode of the $h$-carrier charge density, with a quasi-acoustic dispersion law, called acoustic plasmons (AP), the possible existence of which had been discussed earlier as applied to transition metals and their compounds, semimetals, and semiconductors.

In contrast to metals (or semiconductors), however, which have relatively wide bands (valleys) and a quadratic spectrum of degenerate $l$-or $h$-carriers, when the AP mode is bounded on the high-momentum side by a region of strong Landau quantum damping, in layered metals with very narrow 2D bands, for which the spectrum takes the form

$$ F_{h}(k_x, k_y) = \frac{W_h}{2} \left( \cos k_x a + \cos k_y b \right), $$

(1)

where $W_h$ is the band width, and $a$ and $b$ are the lattice constants in the plane of the layers $(a=b=0.34)$, the AP branch lies higher than the upper boundary of the region of strong Landau damping by $h$-carriers in the entire volume of the Brillouin zone (BZ) under certain conditions (particularly when

$$ W_h < \epsilon_{2d} / \omega_{ap} $$

where $\epsilon_{2d}$ is the high-frequency (HF) dielectric constant of the crystal.

In other words, in the strong coupling approximation, which corresponds to $h$-carriers almost localized on the sites of the crystal lattice and consequently to a periodically inhomogeneous distribution of the electron density, the $h$-plasmon spectrum $\omega_{h}(q)$ is a periodic function of the quasimomentum having the period of the reciprocal lattice (similar to the phonon spectrum). Therefore the real part of the effective permittivity $\epsilon(q, \omega)$ of the metal is negative in the energy region

$$ \omega_{ap}(q, a/2) < |\omega| < \omega_{h}(q) $$

in the entire range $q_1 < 2k_{a1}$ of importance for $l$-carrier Cooper pairing ($k_{a1}$ is the Fermi momentum of the degenerate $l$-carriers). That is to say, attraction appears in the screened Coulomb interaction (SCI) and is due to exchange of virtual AP:

$$ \text{Re } V_C(q, \omega) = V_C(q) \text{Re } \epsilon^{-1}(q, \omega) = 0, $$

(2)

where $V_C$ is a matrix element of unscreened Coulomb repulsion.

This attraction, due to the "dynamic rescreening" effect on account of the retarded electron-plasmon interaction
(EPI), contributes to the superconductivity and is characterized, in view of the Kramers–Kronig relation for the response function $e^{-i\omega}(q,\omega)$ by the dimensionless coupling constant

$$\lambda_{H} = 2 \frac{\omega_0}{\omega_0^2 - \omega^2} \int_0^\infty d\omega \left( \frac{G_0(q,\omega)}{\omega} \right) e^{-i\omega},$$

(3)

where $\omega_0$ is the density of states (DS) on the Fermi level in a wide 2D band (of width $W_\phi W_\phi$), and the angle brackets $\langle \rangle$ denote averaging over a weakly rippled cylindrical Fermi surface (FS). The increase of $\lambda_{H}$ compared with the constant $\lambda_{H}$ of the electron-phonon interaction (EPI) leads on the one hand to an increase of $T_c$ and on the other to a suppression of the IE (see Ref. 21), in qualitative agreement with the experimental data. \textsuperscript{8,10}

3. It must be emphasized that the feasibility of a "plasmon" mechanism of superconductivity was considered theoretically long before the discovery of HTSC for transition metals and their alloys and compounds, \textsuperscript{26-31,33} and also for degenerate multivalley semiconductors, semimetals, and layered semiconductor structures (Refs. 34–36). \textsuperscript{26}

This mechanism is being discussed of late as one of the possible causes of HTSC in cuprate MOC (see, e.g., Refs. 36–43). It must be borne in mind here, however, that in single-band layered metal, notwithstanding the quasielectronic dispersion law for long-wave plasma oscillations propagating across the layers (see Refs. 39 and 41), the energies of virtual plasmons in the region of large momentum transfers (of the order of the Fermi momentum $k_F$) exceed the Fermi energy $E_F$, so that their contribution to the attraction near the FS, and hence in the Cooper-pairing mechanism, is suppressed by quasiparticle damping. \textsuperscript{28}

As shown in Refs. 34 and 44 (see also Refs. 21 and 42), in multiband or in multivalley degenerate semiconductors (semimetals) with large carrier number the ECI effectiveness can increase substantially in the case of strong ion coupling, when the static permittivity of the crystal is $\epsilon_0 = \epsilon_{\phi}$. In this case, owing to hybridization of the AP with the optical (longitudinal (LO) and transverse (TO) phonons), the attraction region broadens in energy all the way to

$$\omega = \sqrt{\omega_0^2 + \Omega_0^2},$$

where $\omega_0$ is the LO-phonon frequency and $\Omega_0$ is the plasma frequency of the h-carriers. In this case, however, the attraction-region upper-bound momentum remains the same

$$q_{max} = \sqrt{q_0^2 + \Omega_0^2/\beta_0},$$

(see Fig. 1a) if the spectrum of the degenerate h-carriers with Fermi velocity $v_\parallel$ is quadratic. The attraction region can broaden substantially if the h-carriers form a Wigner crystal and the spectrum of their collective excitations (phonons) becomes a periodic function of the quasimomentum.$^4$

In a layered metal with a narrow 2D band in an essentially nonparabolic h-carrier spectrum the AP spectrum remains by definition periodic in $q_\parallel$ (with a period $2\pi/a$), and the hybridization of the AP with the LO and TO phonons proceeds over the entire BZ volume (Fig. 1b), so that an attraction between the l-carriers in region (1) exists over the entire momentum-transfer interval $0 < q < 2k_F$ (where $k_F$ = $\sqrt{4nN_1}$ is the Fermi momentum of the l-carriers and $N_1$ is their surface 2D density in the layer. We assume hereafter that in cuprate MOC the most effectively hybridized with the AP are those optical phonon modes corresponding to oscillations of the oxygen ions $O^{2-}$ in the CuO$_2$ layers. This explains, in particular, the experimental observed the shift, and the broadening into the $h_f$ region, of the corresponding peaks of the "phonon" DS determined from tunnel experiments$^{40}$ or from inelastic neutron scattering$^{41,42}$ in transitions from non-superconducting or low-temperature superconducting phase of cuprate MOC to HTSC phases.

The assumption of a predominant role of such a hybrid phonon-plasma mode in the mechanism of Cooper pairing of l-carriers explains also the linear connection, revealed$^{29}$ by MOC Raman optical spectra, between the maximum values of $T_c$ and the frequencies $f_1 \equiv 2nT_c^{*\alpha}$, of the corre-
spending vibrational modes, which is typical of the single-mode spectrum.

The monotonic increase of the h-carrier density in the course of doping of cuprate MOC by a non-isovalent impurity or by changing the composition and the oxygen content\(^{18}\) leads, in the framework of the described model, to a non-monotonic change of \(T_c\), since the increase of the plasma frequency \(\Omega\) and the broadening of the attraction region take place simultaneously with an enhancement of the Coulomb repulsion on account of the decrease of the Bogolyubov-Tolmachev logarithm\(^{22}\) in the Morel–Anderson pseudopotential.\(^{23}\) Such a \(T_c\) dependence with a maximum at a certain optimal carrier density agrees qualitatively\(^{11-15}\) with experiment.\(^{16}\)

Finally, by taking into account the specific features of the Coulomb interaction and of the "local-field" effects, in multilayer structures with a spatially inhomogeneous electron-density distribution in a direction perpendicular to the plane of the layers (along the \(c\) axis) we can obtain, within the framework of the proposed ECI model, high \(T_c\) \(\geq 100\) K and the dependence of \(1/T_c\) on the number \(n\) of the cuprate CuO\(_2\) layers in the unit cell of the crystal, in good agreement with the experimental data for MOC of the type, Bi\(_2\)Sr\(_2\)Ca\(_n\)Cu\(_{2n+1}\)O\(_{8-\delta}\), and Tl\(_n\)Bi\(_2\)Sr\(_{2n-1}\)Cu\(_{2n}\)O\(_{8-\delta}\) (\(n = 1, 2\)) (Refs. 16–18), and also for \((\text{Ca}_{1-x}\text{Sr}_x)\text{CuO}_2\) (Ref. 54).

2. ELECTRON-PHONON INTERACTION AND ELECTRON-PHONON INTERACTION MODEL IN LAYERED METAL-OXIDE COMPOUNDS

1. We examine the "plasmon" mechanism of HTSC in cuprate MOC by starting with a simple model of the band spectrum of a layered metal with two partially filled overlapping 2D-bands of substantially different width \((W_p > W_e)\) near the Fermi level. Such a model was used\(^{18}\) to describe qualitatively correctly the various anomalous properties of the optical conductivity of MOC in the infrared (IR) part of the spectrum\(^{20}\) and of the reciprocal Hall constant\(^{19}\) with allowance for the inelastic scattering of the majority degenerate l-carriers in a wide 2D band damping acoustic plasmons and the finite contribution made to the conductivity by nondegenerate h-carriers in a narrow 2D band; b) the presence of a dip (minimum) in the frequency dependence of the optical conductivity of MOC in the infrared (IR) band\(^{20}\) an analog of the Holstein effect on phonons due to renormalization of the quasiparticle spectrum as a result of the electron-plasma interaction; c) the deviation from the Köring law for the relaxation rate of the nuclear spins of copper ions\(^{21}\) due to their interaction with collective excitations of the spin density of h-carriers (paramagnons), and others.

The assumed presence, in the electron spectrum of cuprate MOC, of a narrow band with a high state density is indirectly confirmed by the "pinning" of the Fermi level in the photoelectron emission and x-ray absorption spectra\(^{-1,22}\) in the course of doping. Detailed numerical calculations\(^{-23,24}\) of the band structure of cuprate MOC, with allowance for hybridization of a large number of atomic orbitals and for correlation (exchange) effects, likewise point to the possible appearance of sufficiently narrow 2D bands near the FS. Furthermore, these bands can become consider-
FIG. 2. Arrangement of CuO	extsubscript{2} layers of multilayer cuprate MOC of type TlBa	extsubscript{2}Ca	extsubscript{2}Cu	extsubscript{3}O	extsubscript{7-2x} with \(d = 9.6 \text{Å}\) and \(d_0 = 3.2 \text{Å}\) for \(n = 3\) and 5. The curves show schematically the distribution of the electron density, which decreases exponentially with increasing distance from the plane of a layer with characteristic length \(\varphi_0 d\).

3. If the probability of electron tunneling between neighboring layers is low enough, so that the variables of the "fast" longitudinal motion of the \(\text{I}-\) and \(\text{h}-\) carriers in the layer plane and of the "slow" adiabatic transverse motion separate,\(^a\) the corresponding PO take the form

\[
\hat{\Pi}_A(q, \omega, n) = \Pi_j(q, \omega) \hat{\beta}(n)/\hat{d}(n),
\]

where \(\hat{\beta}(n)\) is a structure factor that take into account the spatially inhomogeneous distribution of the electron density along the \(c\) axis (see the Appendix and Fig. 3).

In the energy \(\omega\) and momentum \(q\) transfer region defined by the conditions

\[
W_0 \sin(qd/2) < |\omega| < \min(\omega \Omega_j), \quad \varphi^{-1}(n) < q_j < n/a,
\]

where \(v_f\) is the Fermi velocity of the \(\text{I}-\) carriers, and \(\Omega_j\) is their plasma frequency, we have in the random-phase approximation (RPA)

\[
\mathop{Re}\Pi_A = -2\pi \rho_d, \quad \mathop{Re}\Pi_A = \frac{\alpha^2 q^2}{4\pi^2 \rho_0^2}.
\]

Here \(v_f = m^* \omega_0^2 / 2\pi\) is the density of states of the degenerate \(\text{I}-\) carriers near the edge of the relatively wide 2D band, \(m^* = 4\alpha^2 W_0\) is their effective mass, and \(\Omega_0\) is the plasma frequency of the \(\text{h}-\) carriers (see Ref. 22).

The real part of the permittivity \(\tilde{\varepsilon}\) (5) can then be approximately represented, in the frequency region \(W_0 \leq |\omega| < \Omega_j\), where there is no strong Landau damping by the \(\text{h}-\) carriers, and the Landau damping by the \(\text{I}-\) carriers is small, in the form

\[
\mathop{Re} \tilde{\varepsilon}(q, \omega) = \frac{1 + \frac{2 \alpha^2 \omega_0^2}{\alpha^2 q^2}}{1 + \frac{2 \alpha^2 \omega_0^2}{\alpha^2 q^2}} \left[ \frac{\omega^2}{\omega_j^2(q)} - \frac{\omega^2}{\omega_j^2(q)} \right] - \frac{\omega^2}{\omega_j^2(q)}.
\]

Here \(\alpha^2 = \xi/\pi \varphi_0^2\) is the effective Bohr radius of the \(\text{I}-\) carriers, and \(\xi = \varphi_0 \varphi_0\) at frequencies higher than those of the polar optical phonons (but low compared with the frequencies of the interband transitions).
As follows from (10), in the regions of \( \omega \) and \( q \) defined by the inequalities

\[
[\text{max} |\omega(q)|, W_{\Sigma}(\omega(p/2)) |\omega| < Q_{\Sigma}(q),
\]

we have \( \text{Re} \, f(q, \omega) < 0 \), i.e., a retarded interelectron attraction takes place:

\[
\text{Re} \, f(q, \omega) = \frac{V_{\Sigma}(q) e^{-i \omega q}}{\omega q} < 0.
\]

The main contribution to the Cooper pairing of the \( I \)-carriers is made by the attraction region due to the exchange of virtual quanta of hybrid phonon-plasma oscillations of frequency

\[
\Omega(q) = |\omega(q)| + \frac{1}{2} \omega^2(p/2) |\omega(q)|^{1/2}
\]

at \( \omega(q) \to \omega(q)_0 \) [see Eq. (11)].

The retarded EPFI with the remaining phonon modes having frequencies \( \omega(q) \) and matrix elements \( g_q(p) \) can be described with the aid of the usual Green's function: \( \Delta(p, \omega_n) = \sum_{\omega} \frac{\Delta(p, \omega_n) \Psi^2(p)}{\omega_n - \omega(q)} \).

3. EQUATION FOR THE GAP PARAMETER AND THE EPI CONSTANT

The present opinion is \( \Sigma_0(a) \) that a change of the oxygen content in a layered cuprate MOC leads to a Hubbard or Mott insulator-metal phase transition that coincides practically with the transition that takes into the HTSC phase when the hole density in the volume of the unit cell is 0.05 \( a \). \( a \) is 0.05 per CuO

We assume henceforth that at the point of transition into the metallic phase \( (p_0 = 0.05) \) the Fermi level is in the immediate vicinity of the edge of a narrow 2D band located at a distance \( E_0 \) from the nearest edge of a broad 2D band (see Refs. 21 and 22), i.e., the Fermi energy of degenerate I-carriers is \( E_F = E_0 < \frac{W}{3} \) but \( E_0 < W_0 \). In the doping process, for like type of I and \( a \)-carriers (electrons or holes), as the narrow band is filled at high density of states \( (\nu > \nu_c) \) the position of the Fermi level and the density of the \( a \)-carriers remain practically unchanged, whereas the density \( N_a \) of the \( a \)-carriers and their plasma frequency \( \Omega_a \) increase.

It is assumed also that to describe practically free degenerate I-carriers one can use the standard Fermi-liquid approach (with allowance for the quasi-two-dimensional character of the electron spectrum) and for the usual Cooper pairing mechanism in the superconducting state at \( T \leq T_c \). At the same time, almost-localized \( a \)-carriers can remain nondegenerate all the way to \( T \leq T_c \) (see Refs. 21 and 22).

In this two-component model, the integral equation for the SC order parameter of I-carriers, linearized for \( T < T_c \) and with allowance for the "local-field" and "anomalous" vertex-particle effects takes the form

\[
\Delta(p, \omega_n) = \frac{1}{2} \sum_{p'} \int \frac{d^3p}{(2\pi)^3} \, G(p', \omega_n) \times \left[ \sum_{\omega} \frac{\Delta(p', \omega_n) \Psi^2(p')}{\omega_n - \omega(q)} \right] \times \left[ \sum_{\omega} \frac{\Delta(p, \omega_n) \Psi^2(p)}{\omega_n - \omega(q)} \right],
\]

where \( \omega_n = (2n + 1)\pi T \), are discrete frequencies \( (n = 0, \pm 1, \pm 2, \ldots) \), \( T \) is the normal Coulomb (three-pole) vertex, \( \sum^\prime \) and \( \sum^\prime \) are the Gor'kov anomalous temperature function, \( T^\prime \) which takes in the case of a weak ripple of the cylindrical Fermi surface in a layered medium and in the case of separable (in the adiabatic approximation) variables of the longitudinal and transverse motion of the I-carriers in 2D layers, the form

\[
G(p, p', \omega_n) = \frac{\Delta(p, \omega_n) \Psi^2(p)}{\omega_n - \frac{i}{\hbar} [\sum_{\omega} \frac{\Delta(p, \omega_n) \Psi^2(p)}{\omega_n - \omega(q)}] + \frac{\Delta(p', \omega_n) \Psi^2(p')}{\omega_n - \omega(q)} + \frac{\Delta(p, \omega_n) \Psi^2(p)}{\omega_n - \omega(q)}},
\]

Here \( \omega_n = (2n + 1)\pi T \) is the Fermi level renormalized by the interelectron interaction

\[
E_F = E_{P} - \Sigma_{EP}(p_0, 0).
\]

and \( \Sigma_{EP} \) is the self-energy part, defined by the integral equation

\[
\Sigma_{EP}(p, \omega_n) = \frac{1}{2} \sum_{p'} \int \frac{d^3p}{(2\pi)^3} \, G(p', \omega_n) \times \left[ \sum_{\omega} \frac{\Delta(p', \omega_n) \Psi^2(p')}{\omega_n - \omega(q)} \right] \times \left[ \sum_{\omega} \frac{\Delta(p, \omega_n) \Psi^2(p)}{\omega_n - \omega(q)} \right].
\]

Here \( G_{EP} \) is the normal Green's function of the I-carriers:

\[
G_{EP}(p, p'; \omega_n) = \frac{\Psi^2(p)}{\omega_n - \frac{i}{\hbar} [\sum_{\omega} \frac{\Delta(p, \omega_n) \Psi^2(p)}{\omega_n - \omega(q)}] + \frac{\Delta(p', \omega_n) \Psi^2(p')}{\omega_n - \omega(q)}},
\]

and \( f_{EP}(\omega_n) \) is the part of \( \Sigma_{EP} \) which is odd with respect to the sign of \( \omega_n \).

The Coulomb vertex \( \Gamma_{C} \) is known to satisfy on the FS of a charged Fermi liquid the Ward-Pinesiskii identity (see Ref. 70) which takes, in the presence of 1P collective (plasma) excitations of the charge density the form (see Ref. 21)

\[
\Gamma_{C} = 1 - \frac{1}{\hbar} \frac{d\Sigma_{EP}(\omega)}{d\omega} |_{\omega = 0} = 1 - \frac{d\theta_{EP}(\omega)}{d\omega} |_{\omega = 0},
\]

where \( \theta_{EP}(\omega) \) is that part of the function \( f_{EP}(\omega) \) which is due to the EPI (see below).

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Changing in (17) from integration over $\rho_1^i$ to an integral over
\[ \eta' = \bar{\eta}(\rho_2^i) \]
from \(-\infty\) to \(+\infty\) accurate to terms of order \((T_c/\Phi_0)^2\) and recognizing that the residue at the pole with respect to \(\eta'\) in the anomalous Green's function (18) has a singularity \((-1/\omega_{\text{an}})\) at small \(\omega_{\text{an}} = \sigma T_c\), we see that near the FS \((\omega = \sigma T_c, \rho_1^i \approx \rho_1^i \equiv \bar{\eta}_1\)) under the condition that \(\Gamma_c\) depends little on the energy transfers \(|\omega_{\text{an}} - \omega_{\text{ac}}|\) and momentum transfers \(q = |\rho - \rho'\|\), the square of the vertex part \(\Gamma_c^2\) contained in (17) can be approximately replaced by \((\Gamma_c^2)^2\). We shall henceforth use \(\Gamma_c^2\) to replace only one ("excess") vertex \(\Gamma_c\) in (17), and neglect for simplicity the contribution of the remaining vertices \(\Gamma_c\) in (17) and (19), since the vertex \(\Gamma_c\) tends to unity with increase of the distance from the Fermi surface. This lowers somewhat the estimates of the EPI constant (see Ref. 36), whereas replacement of all the \(\Gamma_c\) vertices in (17) and (19) by \(\Gamma_c^2\) and the use of relation (21) as the definition of \(\Gamma_c\) (see Ref. 21) overestimates the values of the coupling constant.

3. Using the spectral representation for the phonon Green's function \(D_{\omega\omega}(q,\omega)\) and the Kramers-Kronig relation for the response function \(\tilde{\tau}(q,\omega)\), we reduce Eqs. (17) and (19), for weak damping in the region \(0 < \omega - \omega_{\text{max}}\), under the condition
\[ \tilde{\Omega}_c = \Omega_\omega (3k_F) \ll \omega_{\text{max}} \]
to the standard Eliashberg equations for the renormalization gap parameter
\[ \lambda_\omega(\omega) = \delta_\omega(\omega)/2f_\omega(\omega) \]
and the renormalization factor
\[ Z_\omega(\omega) = (1 - f_\omega(\omega)/\omega) \]
as \(T \rightarrow T_c:\)
\[ Z_\omega(\eta_0)\delta_\omega = \int_0^{\infty} \frac{d\omega'}{2\omega'} \Delta_\omega(\omega')\lambda(\omega', \omega, T_c)Z_\omega^0 \]
\[ + \zeta_\omega(\omega', \omega, T_c) - \mu_\omega \frac{d\omega'}{2\omega'} \Re \delta_\omega(\omega')\Theta\Theta(\omega'/2T_c)Z_\omega^0 \]
(22)
\[ Z_\omega(\omega) = 1 - \int_0^{\infty} \frac{d\omega'}{2\omega'} \left( \zeta_\omega(\omega', \omega, T_c) + \zeta_\omega(\omega', \omega, T_c) \right) \]
(23)
where
\[ \lambda_\omega(\omega', \omega, T_c) = \int \frac{d\omega''}{2\omega''} \left[ \Theta\Theta(\omega''/2T_c) \right] \]
(24)
is the square of the EPI matrix element averaged over the FS, \(F_{\omega\omega}(\omega)\) is the phonon density of states for those phonon modes which are not hybridized with acoustic plasmons and are described by the Green's function (16),
\[ \tilde{\eta}(\eta) = \eta(\eta_0)\Theta(\eta) \]
is the renormalized state density of the l-carriers on a cylindrical FS in a layered metal with \(n\) conducting layers per unit cell, and \(\beta(\eta)\) is the corresponding structure factor of the normal and anomalous self-energy parts (see the Appendix and Fig. 3).

Note that the averaging over a weakly rippled cylindrical FS reduces approximately to integration over the transverse momentum \(p_\perp\) and over the azimuthal angle \(\phi\) between \(p_\perp\) and \(\rho_\parallel\). The latter can be replaced in the case of an isotropic (in the plane of the layers) electron spectrum by integration over the longitudinal momentum transfer \(q_{\|}\) and the Jacobian of the transition has a root singularity at the point \(q_{\|} = 2k_F\) (see Refs. 21 and 22).

In a real cuprate MOC crystal with initially isotropic spectrum and a wide 2D band (e.g., with a flattened congruent sections of the FS and the band almost half-filled, see Ref. 72), elastic scattering of the l-carriers by the lattice defects produce during the Cooper pairing almost effective isotropization (averaging over the angle \(\phi\) of both the quasiparticle spectrum and of the interaction between the quasiparticles, in accordance with the Anderson theorems for "dirty" superconductors. Consequently the main contribution to the electron-phonon and electron-plasma interactions is made, just as in the isotropic case, by the region \(q_{\|} \approx 2k_F\).
As a result, with account taken of Eqs. (6) and (27), the dimensionless Coulomb repulsion constant can be estimated from the equation\(^1\)

\[
\mu_C(n) = \frac{\alpha_j}{2} \beta(n), \quad \alpha_j = \frac{\mu_j^2}{e_{\text{eff}}^2} = \frac{1}{b_{\text{eff}}^2},
\]

(28)

where the quantity

\[
e_{\text{eff}}(q) = \langle \text{Re} \sigma^{-1}(q, \omega_{\text{opt}}) \rangle^{-1}
\]

at \(q_1 = 2k_F\) can differ somewhat from the optical permittivity \(e_{\text{opt}}\) of the crystal as \(q \to 0\).

4. From Eq. (23) under the condition \(T_C < \min\{\omega_\alpha, \omega_{\text{opt}}\}\), where \(\omega_\alpha\) is the average frequency of the phonons that hybridize with the acoustic plasmons, we obtain in the limit as \(\omega \to 0\)

\[
Z_0(0) = 1 + \lambda_\alpha + \lambda_{\text{pl}}
\]

(29)

where

\[
\lambda_\alpha = \frac{1}{2} \int_0^\infty \frac{d\omega}{\omega} \sigma_\alpha(\omega), \quad \lambda_{\text{pl}} = \frac{1}{2} \int_0^\infty \frac{d\omega}{\omega} \sigma_{\text{pl}}(\omega).
\]

(30)

On the other hand, from Eq. (19) taking into account (4) with \(T_C \approx 1\), which corresponds to the random phase approximation, we obtain in the limit as \(\omega \to 0\) and \(T \to 0\)

\[
f_\alpha(\omega) = -\gamma \omega \int_0^\infty \frac{d\omega'}{\omega'} (D_\alpha(\omega, \omega')) = -\omega (\mu_\alpha - \mu_\epsilon),
\]

(31)

where

\[
\mu_\alpha = \frac{1}{2} (V_C(q_0))_{\text{opt}}^{-1}(q_0), \quad \mu_\epsilon = \frac{1}{2} (V_C(q_0))_{\text{opt}}^{-1}(q_0, 0).
\]

(32)

Comparing (29) with (31) we see that the role of the electron-plasma interaction constant (by analogy with the electron-phonon interaction) is played by the quantity

\[
\lambda_\alpha = \lambda_\alpha + \lambda_{\text{pl}}
\]

(see Refs. 21, 22), and from Eq. (21) with taking into account Eq. (21), it follows that

\[
\Gamma_C^* = 1 + \lambda_{\text{pl}}
\]

(31)

so that the effective coupling constant in (22) is equal to

\[
\lambda^* = \lambda_\epsilon (1 + \lambda_{\text{pl}}).
\]

Thus, in the case of predominant electron-plasma interaction (\(\lambda_{\text{ele}} \gg \lambda_\epsilon\)), according to (21), (22), and (29), the non-adiabatic renormalization \(Z_\alpha(0) = 1 + \lambda_{\text{pl}}\) of the interaction responsible for the Cooper pairing of the \(l\)-carriers is almost completely cancelled on the Fermi surface by the local-field corrections to the Coulomb vertex \(\Gamma_C\), as noted earlier in Ref. 36 (see also Ref. 21).

Note that in crystal with high ionization, where\(^{11}\)

\[
e_{\text{eff}}^2 \to e_0^2 \epsilon, \quad \mu_{\text{eff}} \to \mu, \quad \mu_{\text{eff}} \approx \mu_C,
\]

In the case of nondegenerate almost localized \(h\)-carriers, \(\gamma_{\text{pl}}\) is an almost linear function of temperature in a wide \(T\) range. This explains, in particular, the nearly linear \(T\) dependence of the resistance of cuprate MOC.\(^{3,5}\)

4. CRITICAL TEMPERATURE OF SC TRANSITION AND ISOTOPIC SHIFT INDEX \(T_C\)

Various approximate methods of solving the Eliashberg integral equations\(^7\) for superconductors with strong electron-phonon interactions have been previously considered in numerous studies (see the reviews, Refs. 29 and 68) with an aim at obtaining relatively simple analytic equations for the calculation of \(T_C\). An empirical analysis and model calculations have shown (see, e.g., Refs. 74–79) that satisfactory results in the intermediate-coupling region (\(\lambda_{\text{ele}} \lesssim 1\)) are obtained with an exponential equation for \(T_C\) in the form

\[
T_C = K \mu_{\text{opt}} \exp\left(\frac{1 + \lambda_{\text{pl}}}{\lambda_{\text{pl}} - \mu_{\text{eff}}^2 (1 + D_{\text{pl}})}\right)
\]

(33)

where \(\mu_{\text{opt}}\) is the Coulomb pseudopotential\(^7\) and the parameters \(K\) and \(L\) depend on the form of the phonon spectrum and on the actual choice of the approximation, with \(K < 1\) and \(L < 1\) in all cases.

On the other hand, according to Ref. 80, in the case of strong coupling (\(\lambda_{\text{pl}} > 1\)) the equation for \(T_C\) should be

\[
T_C = 0.18 \mu_{\text{opt}} \mu_{\text{eff}}
\]

(34)

which is valid, strictly, if \(\omega \sigma_{\text{pl}} \approx \omega_{\text{opt}}\). Account must be taken here of the strong quasiparticle damping which was shown in Ref. 79 to lead to a "gapless" state with a complex gap parameter \(\Delta(\omega)\) whose real and imaginary parts vanish on the FS and take as \(\omega \to 0\) and \(T \to T_c\), the form

\[
\text{Re} \Delta(\omega) = \frac{\omega}{\gamma_{\text{pl}} - \Delta}, \quad \text{Im} \Delta(\omega) = \frac{\omega}{\gamma_{\text{pl}} - \Delta}
\]

(35)

where \(\Delta_0\) and \(\Delta_1\) are certain constants, and \(\gamma_{\text{pl}} \approx \lambda_{\text{pl}} T_c / \nu_{\text{pl}}\) is the rate of the quasiparticle damping via inelastic relaxation on acoustic phonons.

2. In the framework of the considered model of layered two-band metal with predominant EPI and with a "plasmon" mechanism of Cooper pairing of degenerate \(l\)-carriers, the quasiparticle relaxation is mainly on virtual acoustic plasmons and is determined by their Landau damping on \(h\)-carriers (see Refs. 21 and 22). The damping decrement of the quasiparticles near the FS (\(\omega \to 0\)) in the normal state (\(T > T_c\)) is given by

\[
\gamma_{\text{pl}}(T) = \frac{1}{\omega_{\text{pl}}(T)} \sum_{\omega_{\text{pl}}(T) > \omega} \frac{S_{\text{pl}}(\omega)}{\omega (2\pi)^2 \rho_{\text{pl}}(T) \tau_{\text{pl}}(T)}
\]

(36)

In the case of nondegenerate almost localized \(h\)-carriers, \(\gamma_{\text{pl}}\) is an almost linear function of temperature in a wide \(T\) range. This explains, in particular, the nearly linear \(T\)-dependence of the resistance of cuprate MOC.\(^{3,5}\)

In the region \(\omega > E_{\text{pl}} - \epsilon_{\text{eff}}\), it follows from (22) and (23) that

\[
\text{Re} \{Z_\alpha(\omega)\} \Delta_0(\omega) = C_{\text{pl}}, \quad \text{Im} \{Z_\alpha(\omega)\} \Delta_0(\omega) = \omega C_{\text{pl}}
\]

(37)

\[
\text{Re} \{Z_\alpha(\omega)\} = 1 + \lambda, \quad \text{Im} \{Z_\alpha(\omega)\} = \gamma / \omega
\]

(38)

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where \( R = \lambda_A + \lambda_\omega \) and \( y_i = y_{A_i} + y_{\omega_i} \), while \( C_{i,j} \) are quantities that depend little on \( \omega \)(see Ref. 79). Hence

\[
\text{Re} \Delta_i(\omega) = \frac{\omega^2 \Delta_{0i}}{\gamma_i^2 + \omega^2(1 + \lambda_i)^2}, \quad \Delta_{0i} = (1 + \lambda_i)C_{i} + y_i C_{2}.
\]

\[
\text{Im} \Delta_i(\omega) = \frac{\omega \gamma_i \Delta_{0i}}{\gamma_i^2 + \omega^2(1 + \lambda_i)^2}, \quad \Delta_{0i} = (1 + \lambda_i)(C_{2} - (1 + \lambda_i)\Delta_{2i} - (1 + \lambda_i)\Delta_{1i}).
\]

(39)

\[
\Delta_{1i} = C_{i},
\]

(40)

From the dispersion equation for \( \Delta_i(\omega) \) (Ref. 81), as \( \omega \to 0 \) and \( T \to T_c \),

\[
\frac{[\text{Im} \Delta_i(\omega)]}{\omega} \equiv -\frac{1}{2} \int_0^\pi \text{Re} \Delta_i(\theta)
\]

we obtain, taking (39) and (40) into account, the connection between the parameters \( \Delta_{0i} \) and \( \Delta_{1i} \):

\[
\Delta_{0i} = \Delta_{1i}(1 + \lambda_i).
\]

(42)

It follows from (39) and (22) that the presence of a "gapless" region near the FS at

\[
\omega \ll \gamma_i(1 + \lambda_i)
\]

must be taken into account in the calculation of \( \tau_i \), if the condition \( \gamma_i(1 + \lambda_i) \ll T \) is satisfied. In the case of a strong electron-phonon interaction \( (\lambda_{\phi} \gg 1) \) this condition reduces to the inequality \( T \ll D_{\phi} \), i.e., corresponds in fact to the condition that Eq. (34) be valid.

3. For an approximate solution of Eq. (22) we approximate \( \text{Re} \Delta_i(\omega) \) in the integrands, with allowance for (39),

\[
\text{Re} \Delta_i(\omega) = \frac{\omega^2 \Delta_{0i}}{\gamma_i^2 + \omega^2(1 + \lambda_i)^2},
\]

(43)

where \( \Delta_{0i} = \Delta_i(1 + \lambda_i)^2 \), and the frequency \( \Delta_{0i} \) is determined from the condition

\[
\text{Re} \Delta_i(\omega) = 0
\]

(see below).

Substituting (43) in (22) we easily verify that if \( T_c < \gamma_i \), the Cooper logarithm has a lower cutoff limit \( \gamma_i \), so that \( T_c \) depends on the character of the temperature dependence of the decrement \( \gamma_i(T) \) of the damping of quasiparticles by virtual acoustic phonons and acoustic plasmons as \( T \to T_c \) [see (36)].

On the other hand, if \( T_c > \gamma_i \) (but \( T_c \ll D_{\phi}, \Delta_{0i} \)), the "zero gap" region (\( \omega \ll \gamma_i \)) can be neglected [17] and \( T_c \) can be calculated using a previously obtained [16] approximate exponential equation having, with allowance for (42) and the local-field corrections (21) the form

\[
T_c = \tilde{\mu}_c \exp \left[ -\frac{1}{y_i} \frac{1 + \lambda_i}{y_i - \mu_c^{-2}} \right],
\]

(44)

where

\[
\lambda_i = \lambda_{B_i} + \lambda_{\omega_i}, \quad \lambda_i - \tilde{\lambda}_i = \lambda_{B_i} + \tilde{\lambda}_i, \quad \tilde{\lambda}_i = \lambda_{B_i}(1 + \lambda_i).
\]

(45)

\[
\tilde{\lambda}_i(\Delta_0) = 2 \int_0^\pi \frac{d\theta}{\omega} \frac{1}{\omega} \left[ S_{\phi}(\omega) + (1 + \lambda_i)S_{\phi}(\omega) \right] \ln \left( \frac{1 + \epsilon_i}{\mu_i} \right),
\]

(46)

\[
\tilde{\lambda}_i(\Delta_0) = 2 \int_0^\pi \frac{d\theta}{\omega} \frac{1}{\omega} \left[ S_{\phi}(\omega) + (1 + \lambda_i)S_{\phi}(\omega) \right] \ln \left( \frac{1 + \epsilon_i}{\mu_i} \right).
\]

(47)

\[
\tilde{\mu}_c(\Delta_0) = \tilde{\mu}_c(1 + \mu_c \ln (E_2/E_1))^{-1}. \quad \tilde{\mu}_c = \mu_c(1 + \lambda_i).
\]

(48)

To estimate \( T_c \) and to analyze the dependences of \( T_c \) on various parameters, we begin with the Einstein model of a spectrum with two \( \delta \)-function peaks:

\[
\tau_i = \frac{1}{2} \Omega_{\phi} \tau_i(\omega_\phi - \Omega_{\phi}) + \frac{1}{4} \chi_{\phi} \tau_i(\omega - \Omega_{\phi}),
\]

(49)

where \( \Omega_{\phi} \) is the frequency of the hf dipole-active (polar) optical mode corresponding to oxygen-ion oscillations in dielectric oxide layers and appears in the Raman spectra of cuprate MOC in \( \omega_{\phi} \) polarization transverse to the plane of the layers, \( \omega_{\phi} \) while \( \chi_{\phi} \) is the frequency of the hybrid phonon-plasma oscillations (11) at \( \omega_i = 2\pi\tau_i \) and \( \mu_{\omega_i} \).

\[
\tilde{\Omega}_\phi(n) = \left[ \frac{2\sigma_0^2 \tau_i(n) \sigma_i^n + n_{12}^2}{1 + \sigma_0^2 \tau_i(n)^2} \right]^{1/2},
\]

(50)

Note that the frequency of the hf optical mode \( \omega_{\phi} \), which corresponds to oxygen-ion oscillations in CuO$_2$ layers, should be observed in a longitudinal polarization \( \mu_{\phi} = \omega_{\phi} = \omega_{\phi} \) in the Raman spectra of nonsuperconducting (dielectric) phases of the cuprate MOC, \( \omega_{\phi} \), whereas a higher hybrid mode \( \Omega_{\phi} \) is apparently observed in metallic superconducting phases.

The parameters \( \lambda_{\omega_i} \) and \( \lambda_{\omega_i} \) for the spectrum (49) take according to (45) and (46) the form

\[
\tilde{\lambda}_i(\Delta_0) = \tilde{\lambda}_i \ln \left( 1 + \frac{\mu_i}{\mu_i} \right) + \tilde{\lambda}_i \ln \left( 1 + \frac{\mu_i}{\mu_i} \right).
\]

(51)

\[
\tilde{\lambda}_i(\Delta_0) = \tilde{\lambda}_i \ln \left( 1 + \frac{\mu_i}{\mu_i} \right) + \tilde{\lambda}_i \ln \left( 1 + \frac{\mu_i}{\mu_i} \right).
\]

(52)
while the frequency $\tilde{\omega}_0$ is determined, according to (22) and (24), with allowance for (37)-(43), by the equation (cf. Ref. 76):

$$\bar{\omega}_0 \equiv \bar{\omega}_0 \left[ 1 + \alpha \tilde{\omega}(\bar{\omega}) \right]^{-1},$$

where, $\bar{\omega}_0 = \frac{\tilde{\omega}_0}{1 + \bar{\omega}_0}$.

Under the condition

$$\bar{\gamma}_L < \bar{\omega}_0 \ll \bar{\omega}_0$$

we can put in (53), with good accuracy,

$$\bar{\omega}_0 = \frac{\tilde{\omega}_0}{1 + \bar{\omega}_0} = \frac{\tilde{\omega}_0}{1 + \bar{\omega}_0}$$

This yields at $\bar{\gamma}_L \ll \tilde{\omega}_0$ and $\mu \pi \ll \frac{\tilde{\omega}_0}{\bar{\omega}_0}$

$$\bar{\omega}_0 = \frac{\tilde{\omega}_0}{1 + \bar{\omega}_0}$$

Note that expressions (55) and (56) yield satisfactory results and comparable values of the electron-phonon and electron-polaron interaction constants $\Lambda_{e-p}$ and $\Lambda_{e-p}$, whereas $\Lambda_{e-p} = 0$ or $\Lambda_{e-p} = 0$ Eqns. (55) and (56) lead to the incorrect limiting values $\tilde{\omega}_0 = \tilde{\omega}_0 = \tilde{\omega}_0 = \tilde{\omega}_0$, thus pointing to the need for a more logical self-consistent definition of the point $\bar{\omega}_0$. We therefore choose $\tilde{\omega}_0$ to be the rms spectrum frequency, which is defined by (56) and which leads as $\bar{\omega}_0 = \frac{\tilde{\omega}_0}{\tilde{\omega}_0}$ and $\bar{\omega}_0 = \frac{\tilde{\omega}_0}{\tilde{\omega}_0}$ to the correct limiting values $\omega^{1/2} = \tilde{\omega}_0 = \tilde{\omega}_0 = \tilde{\omega}_0$.

4. From (44), taking (50) and (56) into account, we obtain for the oxygen isotopic-effect index:

$$\tilde{\omega}_0 = \frac{\omega}{\omega} \frac{\tilde{\omega}_0}{\tilde{\omega}_0} \frac{\tilde{\omega}_0}{\tilde{\omega}_0}$$

Thus, the electron-plasmon and polar electron-phonon interactions are considerably enhanced by the multiparticle correlation effects in the Coulomb interaction (see Refs. 21, 36, 43).

5. COMPARISON OF THEORY WITH EXPERIMENT

1. To analyze the results and to compare them with the experimental data on HTSC in cuprate MOC, we shall estimate some realistic values of the model parameters. We estimate first the longitudinal effective mass of the $l$-carriers in a wide 2D band, starting with calculated and empirical data on the width of the cuprate-MOC hybrid $p_d$ band located in the interval $W_l = \pm 4$ eV, corresponding at a $= 4 \AA$ to values

$$m^* = 4a^2 W_l = (6,5 - 2)m_0$$

where $m_0$ is the free-electron mass.

The 2D $l$-carrier density $N_l$ per unit cuprate-layer area in layered MOC with $n$ conducting CuO$_2$ layers per unit cell, gathered into packets of thickness...
\[ \text{L}(n) = (n - 1)\sigma \]

(see Fig. 2), with the electrons (holes) uniformly distributed among the layers in the packet, is connected with the average \( l \)-carrier bulk density \( \bar{n}_l \) by the relation
\[ \bar{n}_l(n) = \frac{n}{a} \text{c}(n) \quad \text{and} \quad \text{c}(n) = \frac{a + L(n)}{n}. \]  

(60)

When the number of \( \text{CuO}_2 \) layers in the packet is increased, the number of \( x, l \)-carriers in the primitive-cell volume, \( \bar{v}_l(n) = \bar{n}_l(n)/a \), connected with the constant valency. In this case the density \( \bar{n}_l \), the Fermi momentum \( k_F = \sqrt{2\pi \bar{n}_l} \) and the Fermi energy \( E_F = k_F^2/2m_l \) of the degenerate \( l \)-carriers in the \( \text{CuO}_2 \) layers all decrease, and their dimensionless density parameter
\[ \alpha_l = \frac{1}{k_F^2} \]

increases with increase of \( n \) in accordance with the relation (for \( a = b \)):

\[ \bar{n}_l(n) = \frac{x}{a^2} \quad \text{and} \quad \bar{v}_l(n) = \frac{1}{a} \sqrt{\frac{2\pi \bar{n}_l}{a^2 n}}, \]

(61)

Experiment,\textsuperscript{37,82-84} however, shows that in most cuprate MOC the plasma frequency of the majority carriers depends little on the number of \( \text{CuO}_2 \) layers in the unit cell. Thus, for example in the compounds \( \text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8 \) with \( n = 3 \) and \( \text{Tl}_2\text{Ba}_2\text{CaCu}_2\text{O}_8 \) with \( n = 2 \) the longitudinal (in the layer plane) plasma frequency is equal to \( \omega_{pl} \approx 1.1-1.2 \) eV (Ref. 83), and in \( \text{YBa}_2\text{Cu}_3\text{O}_7 \) it is somewhat lower (\( \omega_{pl} \approx 1.4 \) eV). Here, however, there are in addition to the \( \text{CuO}_2 \) layers in the packet an increase of the bulk density of the \( l \)-carriers with increase of \( n \), for example on account of the variable valencies of the Bi and Ti ions or on account of the excess oxygen, we obtain according to (60)

\[ \bar{n}_l(n) = \frac{x}{2a^2} \quad \text{and} \quad \bar{v}_l(n) = \frac{\sqrt{\frac{2\pi \bar{n}_l}{a^2 n}}}{a}, \]

(62)

\[ \bar{v}_l(n) = \frac{\sqrt{2\pi \bar{n}_l}}{\frac{2\pi \bar{n}_l}{a^2 n}} \quad \text{and} \quad \alpha_l(n) = \frac{a}{\bar{n}_l(n)^{1/2}} \sqrt{\delta_n + n^{1/2}}. \]

(63)

On the other hand, the effective \( l \)-carrier mass can be estimated from the experimental values \( \omega_{pl} \approx 1.4-1.5 \) eV and \( \varepsilon_n \approx 3.8-5.2 \) for \( \text{YBa}_2\text{Cu}_3\text{O}_7 \) (Refs. 82, 83) at a hole density \( \bar{n}_l \approx 5.8 \times 10^{21} \text{ cm}^{-3} \), which corresponds to one extra hole per unit-cell volume (according to the chemical valencies of the components). Assuming that the longitudinal effective masses and densities of the holes in the 2D-layers of \( \text{CuO}_2 \) and 1D-chains of \( \text{CuO}_2 \) are approximately equal (see Ref. 57), we obtain for the \( l \)-carrier band effective mass and for the band width the respective estimates \( m_l^\ast = (0.7-1.1)m_0 \) and \( W_l \approx (1.8-2.9) \) eV.

It must be taken into account at the same time that the definitions of \( E_p \) and \( \alpha_l \) (61) and (62) contain the quasi-particle effective mass renormalized to multiparticle correlations

\[ m_l^\ast = m_l^0 \left[ 1 - 2m_l^0 \bar{\nu}_l(k_F)^2 \right]^{-1}, \]

(64)

which can be considerably larger than the optical (band) mass \( m_l^0 \).

2. As noted above, the effective \( h \)-carrier mass \( m_h^0 = 4/\varepsilon_k^2 W_k \) of the \( h \)-carriers in a narrow 2D band of width \( W_k \ll W_h \) can be increased by polaron effects, which are manifested also under conditions of screening by \( l \)-carriers (see Ref. 65). Since \( m_h \) is unknown, we use hereafter, as the parameter that depends on the \( h \)-carrier density, the square of the ratio of their plasma frequency and the LO-phonon frequency, contained in the expression for the frequency of hybrid phonon-plasma oscillations [see (50)]:

\[ \bar{\Omega}_h(n) = \frac{a_{lo}}{\bar{n}_h(n)^{1/2}} \bar{v}_h(n)^{1/2}, \]

(65)

where

\[ \bar{\Omega}_h(n) = \frac{a_{lo}}{\bar{n}_h(n)^{1/2}} \bar{v}_h(n)^{1/2}, \]

Here \( \bar{\Omega}_h \) is the effective 2D density of the \( h \)-carriers in a narrow band.

It is assumed in the considered band-spectrum model that at the point of dielectric-metal phase transition the FS of the degenerate \( l \)-carriers is "pinned" on the edge of the narrow 2D band \( (E_p = E_0) \) with high density of states \( \bar{n}_l \), and the subsequent doping increases mainly the density \( \bar{n}_l \) of the \( h \)-carriers, whereas the \( l \)-carrier density remains almost constant \( (\bar{n}_l \approx \text{const}) \), for example on account of the variable valencies of the Bi and Ti ions or on account of the excess oxygen, we obtain according to (60)

\[ \bar{n}_l(n) = \frac{x}{2a^2} \quad \text{and} \quad \bar{v}_l(n) = \frac{\sqrt{2\pi \bar{n}_l}}{\frac{2\pi \bar{n}_l}{a^2 n}} \]

(62)

\[ \bar{v}_l = \frac{\sqrt{2\pi \bar{n}_l}}{\frac{2\pi \bar{n}_l}{a^2 n}}, \]

(63)
FIG. 4. Dependences of $T_c$ on $x$, for various $n$ at $d = 6$, with $d = 7.5$, $x_e = 4$, $x = 0.2$, $\omega_0 = 300$, $\delta = 0.6$, $m^* = 1.8$, $\gamma_1 = 1$, corresponding to $\text{Ti}_2\text{Ba}_2\text{Ca}_n\ldots\text{Cu}_n\text{O}_{2n+1}$ ($T_c = 125\text{ K}$ at $n = 3$) for $n = 4$. The curves with $n = 4$ and 5 pertain to hypothetical compounds of this class.

4. Figure 7 shows those dependences of $T_c$ on $x$, which have been obtained for the same parameters but at $d = 9.6\text{ b}$; ($d = 3$), corresponding to a layered MOC of the type $\text{Ti}_2\text{Ba}_2\text{Ca}_n\ldots\text{Cu}_n\text{O}_{2n+1}$, with a TiO monolayer, for different values of $n$. These agree well with experiment. The most characteristic feature of this compound, observed in experiment and obtained theoretically on the basis of the considered model, is inversion of the lowering of the $T_c$ curve at $n > 3$ with $T_c (x) \approx 110\text{ K}$ at $n = 3$.

Figure 8 shows the dependence of $T_c$ on $n$ for the parameters of Fig. 7. It is seen that $T_c$ is a nonmonotonic function of the number of the CuO$_2$ cuprate layers in the unit cell of the multilayer compound $\text{Ti}_2\text{Ba}_2\text{Ca}_n\ldots\text{Cu}_n\text{O}_{2n+1}$ (Ref. 18) shows also plots of $T_c (n)$ corresponding to the parameters of Fig. 4 for $\text{Ti}_2\text{Ba}_2\text{Ca}_n\ldots\text{Cu}_n\text{O}_{2n+1}$, and also for $\text{Bi}_2\text{Sr}_2\text{Ca}_n\ldots\text{Cu}_n\text{O}_{2n+1}$, and demonstrating the deviation from the “chew rule” (tendency of $T_c$ to saturate with increase of $n$) in multilayer cuprate MOC.

Note that the relation

$$T_c = \frac{\pi}{2\alpha},$$

which is indicative of the single-mode models is satisfied with good accuracy for the maximum value of $T_c$ at $n = 3$ in Fig. 4 and for the corresponding hybrid frequency $\bar{\Omega}$ at $n = 3$. This attests to predominating role of the hybrid mode of the phonon-plasma oscillations $\Omega_1 (q)$ in Cooper pairing of $F$-carriers.

Figure 5 shows the dependences of the effective coupling constant $\Lambda$ (59) and of the effective Coulomb pseudopotential $\mu \xi$ (48) on $x$, while Fig. 6 shows the functions $\bar{\Omega}_1$ (65) and $\bar{\Omega}_2 = (\omega^2)^{1/2}$ (56), corresponding to the parameters of Fig. 4. It follows hence that the nonmonotonic dependence of $T_c$ on $x$, (i.e., on $p_0$ or $x$), within the framework of the considered model of the “plasmon” HTSC mechanism, is due to competition between two effects: expansion of the region of the effective interelectron attraction, due to the increase of the frequency of the hybrid phonon-plasma oscillations $\bar{\Omega}_1$ (Fig. 6a) and consequently the average frequency $\bar{\omega}$ of the spectrum (Fig. 6b), on the one hand, and the enhancement of the Coulomb repulsion on account of the decrease of the Bogolyubov-Tolmachev logarithm $\ln (\bar{\Omega}/R)$ in $E_F (\bar{\Omega})$ in the Morel-Anderson pseudopotential $\mu \xi$ (Fig. 5b), which makes $\Lambda$ nonmonotonic (Fig. 5a).

Note that the relation

$$T_c = \frac{\pi}{2\alpha},$$

is satisfied with good accuracy for the maximum value of $T_c$ at $n = 3$ in Fig. 4 and for the corresponding hybrid frequency $\bar{\Omega}$ at $n = 3$. This attests to predominating role of the hybrid mode of the phonon-plasma oscillations $\Omega_1 (q)$ in Cooper pairing of $F$-carriers.

Figure 7 shows those dependences of $T_c$ on $x$, which have been obtained for the same parameters but at $d = 9.6\text{ b}$; ($d = 3$), corresponding to a layered MOC of the type $\text{Ti}_2\text{Ba}_2\text{Ca}_n\ldots\text{Cu}_n\text{O}_{2n+1}$, with a TiO monolayer, for different values of $n$. These agree well with experiment. The most characteristic feature of this compound, observed in experiment and obtained theoretically on the basis of the considered model, is inversion of the lowering of the $T_c (x)$ curve at $n > 3$ with $T_c (x) \approx 110\text{ K}$ at $n = 3$.

Figure 8 shows the dependence of $T_c$ on $n$ for the parameters of Fig. 7. It is seen that $T_c$ is a nonmonotonic function of the number of the CuO$_2$ cuprate layers in the unit cell of the multilayer compound $\text{Ti}_2\text{Ba}_2\text{Ca}_n\ldots\text{Cu}_n\text{O}_{2n+1}$ (Ref. 18) shows also plots of $T_c (n)$ corresponding to the parameters of Fig. 4 for $\text{Ti}_2\text{Ba}_2\text{Ca}_n\ldots\text{Cu}_n\text{O}_{2n+1}$ and demonstrating the deviation from the “chew rule” (tendency of $T_c$ to saturate with increase of $n$) in multilayer cuprate MOC.

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which is indicative of the single-mode models is satisfied with good accuracy for the maximum value of $T_c$ at $n = 3$ in Fig. 4 and for the corresponding hybrid frequency $\bar{\Omega}$ at $n = 3$. This attests to predominating role of the hybrid mode of the phonon-plasma oscillations $\Omega_1 (q)$ in Cooper pairing of $F$-carriers.

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Figure 8 shows the dependence of $T_c$ on $n$ for the parameters of Fig. 7. It is seen that $T_c$ is a nonmonotonic function of the number of the CuO$_2$ cuprate layers in the unit cell of the multilayer compound $\text{Ti}_2\text{Ba}_2\text{Ca}_n\ldots\text{Cu}_n\text{O}_{2n+1}$ (Ref. 18) shows also plots of $T_c (n)$ corresponding to the parameters of Fig. 4 for $\text{Ti}_2\text{Ba}_2\text{Ca}_n\ldots\text{Cu}_n\text{O}_{2n+1}$ and also for $\text{Bi}_2\text{Sr}_2\text{Ca}_n\ldots\text{Cu}_n\text{O}_{2n+1}$, and demonstrating the deviation from the “chew rule” (tendency of $T_c$ to saturate with increase of $n$) in multilayer cuprate MOC.
Such an $n$-dependence of $T_{EPI}^m$, which agrees with experiment, is due in this model to the decrease of the Fermi momentum $k_F(n)$ with increase of $n$ (see Eq. (62)), where by the parameter $\alpha_i(n) - k_F^{-1}(n)$ increases while the quantities $\bar{E}_i(n) - k_F^{-1}(n)$, $\Omega_i(n)$ and $\bar{a}_i(n)$ decreases (see Figs. 6a,b), with $\bar{E}_i(n)$ decreasing much faster than $\bar{a}_i(n)$, so that the Coulomb pseudopotential $\bar{\mu}_C(n)$ increases with $n$ (Fig. 5b). As a result, the increase of the EPI constant $\lambda_{EPI}(n) = \alpha_i(n)\beta_i(n)/2$ with increase of $n$ in the region of the saturation of the exponential in (44) is offset by the decrease of the pre-exponential factor $\bar{a}_i(n)$ and by the increase of $\bar{\mu}_C(n)$ due to the decrease of the ratio $\bar{E}_i(n)/\bar{a}_i(n)$.

On the other hand, the increase of the distance $d$ between the packets of the alternating CuO$_2$ and Ca layers (i.e., of the dimensionless parameter $\delta(d)/d$) leads, according to (62), to an increase of $k_F$, i.e., to a decrease of the parameter $\alpha_i - k_F^{-1}$, but on the other hand to a faster increase of $E_{EPI} - k_F^{-1}$, which is accompanied by a weakening of the Coulomb repulsion (a decrease of $\bar{\mu}_C$) and an increase of...
weaker (of the Josephson type).

The dashed lines of Fig. 8 show the dependences of $T_c$ on $n$ in the case of a predominant nonpolar electron-phonon interaction, when the coupling constant is

$$\Lambda_\mu(n) = \Lambda_\mu(1) \beta(n) \alpha(n).$$

We see that these dependences differ greatly from the corresponding ones in the case of EPI with a coupling constant

$$\Lambda_n = \frac{(1 + \kappa)[\ln(1 + \alpha_0^2)] + \ln(1 + \alpha_0^2)}{\ln(1 + \alpha_0^2)}.$$

(67)

where

$$\alpha_0^2 = \frac{1}{\nu}(\omega_0^2 + (2\pi)^2) \Omega^2, \quad \Omega^2 = \Omega^2 \sqrt{\frac{\nu}{n}}, \quad \Omega_0^2 = \frac{1}{\nu \omega_0^2}, \quad \mu^2 = (2n)^2 \Omega^2,$$

(69)

$T_c$ reaches then its limiting value (for $\Lambda_n > 0$)

$$T_{c0} = \alpha_0^2 \exp{-1/\Lambda_n},$$

(70)

which depends on the parameters and can be high enough at $E_F \gg \Delta_0^*$. In particular, for the parameters of Fig. 4, when the effective mass decreases from the value $m^* = 1.8m_0$ to $m^* = m_0$, the value of $T_c$ increases, according to (67)-(70), from 100 to 180 K.

Since the compound $(\text{Ca}, \ldots, \text{Sr})_n \ldots \text{CuO}_2$ with an "infinite" number of cuprate layers corresponds to stoichiometric composition $(x = y = 0)$ to a simple layered crystal with one CuO$_2$ layer per unit cell, it might seem that, by analogy with LA$_{2-x}$(Ba,Sr)$_{2-x}$CuO$_4$ or Bi$_{2-x}$Sr$_x$CuO$_4$, it should have a low $T_c \lesssim 20-40$ K. However, the proximity of the CuO$_2$ superconducting layers, with distances $d \approx 3$ Å between them comparable to the transverse coherence length $\xi_T$, reduces the effective electron-phonon coupling and contributes to a rise of $T_c$ in contrast to lanthanum or bismuth MOC with $n = 1$, in which single superconducting CuO$_2$ layers are much farther spaced, $d \approx 6-12$ Å apart, and the coupling between is weaker (of the Josephson type).

6. Assuming that near the Fermi level there is no narrow 2D band partly filled with $\sigma$-carriers ($N_\sigma = 0$ and $N_\pi = 0$), thereby excluding the EPI and retaining only the polar electron-phonon interaction with oxygen optical modes $\alpha_{\sigma 0}$ and $\alpha_{\pi 0}$ (with the same coupling constant $\Lambda_\mu$), the maximum values of $T_c$ are drastically decreased and the dependence of $T_c$ on the carrier density and on the number of cuprate layers in the packets is radically altered. In fact, in this case, in the course of doping, as the broad 2D band is filled and the $\sigma$-carrier density $n$ is increased, the values of $k_{\pi}$ and $E_{\pi}$ increase but the density parameter $\alpha_0 \sim k_{\pi}^n$ increases, and with it the coupling constant $\Lambda_\mu \sim \alpha(x)$, which turns out to be more substantial than the decrease of the Coulomb pseudopotential $\mu$ owing to the increase of the logarithm $\ln(\rho_{\pi}/\rho_0)$.

Figure 9 shows the dependence of $T_c$ on the dimensionless parameter $N_{\pi} = (2n^2\pi)^{1/2}$ for various $n$ at $N_\sigma = 0$ and at the same value of the remaining parameters as in Fig. 4. Evidently the $T_c(N_{\pi})$ dependences do not correspond to the experimental data of Ref. 14 for multilayer cuprate MOC (such as Bi$_2$Sr$_2$Ca$_{n-1}$Cu$_2$O$_{y-1}$ or Tl$_{n-1}$Ba$_2$Ca$_{n-1}$Cu$_2$O$_{y}$), and also the maximum values of $T_c$ do

\[ T_c (n) - B(n), \] and have nothing in common with experiment.\textsuperscript{16-18}

5. Of particular interest is the recently synthesized layered compound\textsuperscript{9} $(\text{Ca}, \ldots, \text{Sr})_n \ldots \text{CuO}_2$ with alternating layers of CuO$_2$ and Ca(Sr). This compound has a rather high $T_c \approx 110$ K. Within the framework of the considered model of close-packed packets (Fig. 2) this corresponds to going to the limit as $n \rightarrow \infty$ and to infinite constants $\Lambda_\mu(n)$ and $\mu_\pi(n)$, since $\beta(n) \rightarrow \infty$. However, owing to the mutual cancellation of the strong-coupling and local-field effects, the effective coupling constants (59) tends as $n \rightarrow \infty$ to a finite asymptotic value.

\[ T_c (n) - \infty. \]
not exceed 40 K. Thus, the contribution of the electron-plasmon interactions turns out in the present model to be decisive for HTSC. At the same time, the rapid (exponential) decrease of $T_c$ with increase of $E_p - E_p^c$ at $n = 1$ in Fig. 9 correlates with the abrupt decrease of $T_c$ in La$_{2-y}$ (Ba,Sr)$_y$CuO$_4$ when the dopant content is increased in the region $x > 0.15$, which may indicate a predominant role of electron-phonon interactions in this cuprate MOC.

We proceed to analyze the oxygen IE in the SC phases of cuprate MOC, which tend to suppress the IE with rise of $T_c$, in contrast to ordinary superconductors in which the opposite tendency is observed as a rule, with the IE weakening as $T_c$ is lowered until the IE exponent vanishes or reverses sign at $T_c < 1$ K because of the strong Coulomb repulsion at a weak electron-phonon interaction.

An exception among cuprate MOC is La$_{2.5}$Sr$_{0.5}$CuO$_4$ with an anomalous behavior of the IE exponent for oxygen as a function of the composition, (viz., $a_0$, first increase with increase of the Sr content up to a maximum $a_0 = 0.6$, and at the point $x = 0.15$ near the maximum of $T_c$ it decreases jumpwise to $a_0 = 0.1$ (Ref. 87), apparently because of the lattice instability and the anharmonicity of the phonons in the region of the structural transition when the electron-phonon interaction is strong.

The isotopic effect was measured in Ref. 10 with $^{16}$O replaced by $^{18}$O in the compound Y$_{1-x}$Pr$_x$Ba$_2$Cu$_3$O$_{6.5+}$, as a function of the content of the Pr dopant that suppresses the superconductivity (lowers $T_c$). This can be due both to the large magnetic moment of the Pr atoms and to the lowering of the hole density on account of the higher valency of Pr compared with Y, and in this sense a decrease of Pr is equivalent to a decrease of the oxygen deficit $\delta$. It was observed in Ref. 10 that as $x$ is decreased and $T_c$ is increased, with $\delta$ constant ($\delta < 1$), the IE index decreases from $a_0 = 0.5$ at $x = 0.5$ and $T_c = 30$ K to $a_0 = 0.02$ at $x = 0$ and $T_c = 90$ K (see Fig. 10a).

FIG. 10. Experimental plots of the oxygen IE exponent $a_0$ on $T_c$ for the mixed compounds Y$_{1-x}$Pr$_x$Ba$_2$Cu$_3$O$_{6.5+}$ (a, Ref. 10) and YBa$_{1.6}$ (La,Sr)$_{0.4}$Cu$_3$O$_{6.5+}$ (b, Ref. 88).

FIG. 11. Dependence of $\alpha_0$ on $\delta$ (a) and on $T_c$ (b) for the parameters of Fig. 4.
A similar result was obtained in Ref. 88 with the Ba in YBa$_2$Cu$_3$O$_{y}$ partially replaced by La, the atoms of which have a higher valency (than Ba) and have no magnetic moment. The oxygen isotopic-effect exponent in YBa$_{1-x}$La$_x$Cu$_3$O$_y$ compound (with $x \geq 7$) decreases from $\alpha_{O}=0.4$ at $x=0.5$ and $T_c \approx 40$ K to $\alpha_{O}<0.02$ at $x=0$ and $T_c \approx 92$ K, and a zero or even negative isotopic effect was observed when Ba was replaced by Sr (see Fig. 10b).

Calculations of the isotopic-effect index in the above "plasmon" mechanism of superconductivity in a layered metal with a narrow 2D band near the Fermi surface, using Eqs. (57) and taking Eqs. (48), (51), (52), (56), and (58) into account, show that Coulomb repulsion, owing to the anomaly large (compared with ordinary metals) values of the pseudopotential $\mu_r^+ \approx 0.3-0.8$ (see Fig. 5b), leads to strong suppression of the isotopic effect and causes $\alpha_{O}$ to vanish and reverse sign in the region of maximum $T_c$, where $\mu_r^+=\lambda$.

Figure 11 shows the dependences of $\alpha_{O}$ on $\delta_{k}$ (a) and on $T_c$ (b), plotted for the parameters of Fig. 4 at various $n$. As seen from the figure, the oxygen isotopic effect index vanishes and then becomes negative in the region where $T_c$ is lower. It must be emphasized that this is precisely the isotopic-effect anomaly observed in Ref. 88 for Y(Ba$_{1-x}$Sr$_x$)$_2$Cu$_3$O$_y$ (Fig. 10b). The reason may be that the smaller radius of the Sr$^{2+}$ ions (compared with Ba$^{2+}$) makes possible supersaturation of the samples with oxygen ($x \geq 7$) and a shift towards higher hole densities, where $T_c$ decreases and $\alpha_{O}<0$. A similar reversal of the sign of $\alpha_{O}$ on passing through the maximum of $T_c$ should be observed in the compounds BiSrCaCu$_2$O and TlBaCaCuO.

It is thus possible to explain, on the basis of the "plasmon" mechanism of HTSC, the anomalies of the isotopic effect in cuprate MOC, in agreement with the experimental data.

On the other hand, if the EPI interaction is excluded ($\Omega_{E}=0$, $\delta_{k}=0$) and only the polar EPHI with oxygen-bridational modes $\omega_{E}$ and $\Omega_{E}$, the behavior of the IE changes radically: increases rapidly as $\delta_{k}$ increases (Fig. 12a) and as $T_c$ increases (Fig. 12b) from negative values of $\alpha_{O}=-0.05-0.75$ at low carrier densities ($\delta_{k}=0.5$) and $T_c \approx 5-20$ K to positive values $\alpha_{O}=0.25-0.45$ at higher densities ($\delta_{k}=2$) and $T_c \approx 25-40$ K. These dependences agree qualitatively with the character of the IE for ordinary EPHI in low-temperature superconductors, but does not agree with experiment for the oxygen IE in cuprate MOC.

6. CONCLUSIONS

The following conclusion can be drawn from the foregoing. The proposed model of a layered metal, with quasi-two-dimensional electron spectrum and with two 2D bands having substantially different widths ($W_{f}>W_{t}$) and overlapping near the Fermi level, can account, on the basis of the standard superconductivity theory with Cooper pair-formation for the main peculiarities and regularities of HTSC in cuprate MOC in the intermediate coupling approximation ($\lambda \approx 1$, $T_c \approx T_{c0}$). This is due to allowance for factors such as:

a) interaction of degenerate $f$-carriers in a wide band with $l$ collective excitations of the charge density of almost localized $f$-carriers in a narrow band (acoustic plasmons) with a spectrum periodic in momentum, b) hybridization of the AP with dipole-active (polar) optical oscillations of oxygen ions, c) multilayer "packet" crystal structure of cuprate MOC with close-packed conducting CuO$_2$ layers, d) multi-particle Coulomb correlations in a charged quasi-two-dimensional Fermi liquid of $l$-carriers (of the type of "local-field" effects).

In particular, the experimentally observed nonmonotonic dependence of $T_c$ on the dopant density or on the oxygen content in this model (Figs. 4 and 7) is the result of competition between the increase of the width of the inter-electron attraction region, on the one hand, and the enhance-
ment of the Coulomb repulsion, on the other, due to the increase of the h-carrier plasma frequency $\Omega_1$ and the frequency $\Omega_2$ of the hybrid phonon-plasma oscillations in the course of filling the narrow band with h-carriers at almost constant l-carrier density (due to "pinning" of the Fermi level near the edge of the narrow band with high density of states).

The rise of $T_c$ with increase of the number of cuprate CuO$_2$ layers in the unit cell of the crystal is due to the almost additive contribution of each layer to the EPI constant, owing to the proximity effect, while the tendency to saturation and even to lowering of $T_c$ at sufficiently large $n$ (Fig. 8) is due to the almost full mutual cancellation of the effects of the local field and of the strong coupling.

The abrupt decrease (down to zero and below) of the oxygen isotopic effect with rise of $T_c$ (Fig. 11), which is typical of cuprate MOC$^{25-36}$ (Fig. 10) is due to the anomalously strong (compared with ordinary superconductors) Coulomb repulsion on account of the relatively small charged oxygen vacancies and by non-isovalent impurities states).

Hybridization of hf oxygen vibrational modes with AP should lead, on the one hand, to an increase of the frequencies of the corresponding peaks of the phonon density of states, which agrees with the results of tunnel and neutron experiments$^{49-50}$ for HTSC phases of cuprate MOC, and on the other to different anomalies in the phonon dispersion revealed by neutron scattering, and to breaks in the spectra and with frequency jumps in the region of "polariton" splitting of the branches (Fig. 1), i.e., not scattered by "plasmalike" electron-density perturbations with a quasiacoustic dispersion law.$^{17}$ However, strong AP damping on account of elastic scattering of h-carriers by charged oxygen vacancies and by non-isovalent impurities (Drude damping) can make the hybridization of AP with optical phonon unobservable on account of smearing of the phonon peaks. This damping, nonetheless, should not influence the value of $T_c$ so long as the AP lifetime exceeds the l-carrier Cooper-pairing time.

We note in conclusion that the model considered in the present paper makes it possible to predict certain possibilities of further raising $T_c$, say by increasing the distance $d$ between CuO$_2$ cuprate layers separated by Ca$^{2+}$ layers and by increasing the number of intermediate dielectric BiO and TIO layers. It seems that this is the very reason why, in multilayer cuprate MOC with BiO or TIO layers in the unit cell, the maximum value $T_c^{max}=125$ K (at $n=3$) is higher than in compound with TIO monolayers ($T_c^{max}=110$ K).

Figure 13a shows plots of $T_c$ vs $n$, for the same parameters as in Fig. 4, but with $\delta_0=5.25$ ($d=16.8$ Å), corresponding to introduction into the unit cell of two additional BiO or TIO layers (for example, when single-crystal films are grown by molecular epitaxy). It is seen from the figure that in this case the maximum of $T_c$ at $n=5$ reaches $T_c^{max}=155$ K.

Even higher maxima of $T_c$ can be attained by increasing the l-carrier density $\bar{\rho}_l$ (i.e., the parameter $\bar{\rho}_l$) provided the positions of the Fermi level and of the narrow band coincide (peak of the density of states). Thus, for example, if $\bar{\rho}_l$ in thallium MOC is increased to $\bar{\rho}_l=8\times10^{11}$ cm$^{-2}$ (i.e., $\bar{\rho}_l=1.5$), we obtain $T_c^{max}=220$ K at $n=5$ for Tl$_2$Ba$_2$Ca$_2$Cu$_2$O$_{y+4}$. This occurs prior to saturation and inversion of $T_c$ with respect to the number $n$ of the cuprate 2D CuO$_2$ layers in the unit cell (cf. Fig. 7).

Thus, according to the considered "plasmon" mechanism of HTSC, simultaneous increase of the parameters $\bar{\rho}_l$, $d$, and $\alpha$ in multilayer cuprate MOC should lead to a substantial rise of $T_c$ of high-temperature superconductors, notwithstanding F. Anderson's known statement that high $T_c$ cannot be obtained with the aid of "electronic" (in par-

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig13.png}
\caption{Dependences of $T_c$ on $n$, for hypothetical cuprate MOC with $d=16.8$ Å (a) when $T_c^{max}=155$ K at $n=5$ and with $\bar{\rho}_l=1.5$ (b), when $T_c^{max}=220$ K at $n=5$ (the remaining parameters are the same as in Fig. 4).}
\end{figure}
ticular, an "excitonic," Refs. 91 and 92) superconductivity mechanisms, a statement which is valid only in the random-phase approximation, with allowance for multiparticle Coulomb correlations (of the local-field-effects type) and other factors that enhance the interelectron attraction (ionicity of the lattice, multilayer character of the structure, multivalley character of the band spectrum, and others).

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APPENDIX

We have then for the Coulomb matrix element in the momentum approximation the expression:

$$V_C(q_z, n) = \frac{2\pi e^2}{q_z} \left[ \sum_{n=0}^\infty \cos m_0 q_z \exp(-nq_zd) + 2 \sum_{n=1}^\infty \cos m_0 q_z \chi(n) \right]$$

$$\times \left[ \frac{\sin q_z(n) - \cos q_z(n)}{\sin q_z(n)} \right]$$

(A2)

For sufficiently large \( q_z \), when \( q_z c(n) \gg 1 \) and \( m_0 q_z \gg 1 \), Eq. (A2) leads with exponential accuracy to expression (6). Note that as \( n \to \infty \) the matrix element (A2) tends to infinity, but the Coulomb potential in real 3D space, which is defined (disregarding Umklapp processes) by the expression

$$V_C(R, z, n) = \frac{n}{2\pi} \int_{-\pi/n}^{\pi/n} d\varphi \int_{-\pi/n}^{\pi/n} \frac{d\varphi'}{2\pi} \sum_{\vartheta} V_C(q_z, \vartheta, n) \exp(i\varphi z),$$

(A3)

where \( J_n(x) \) is a Bessel function, can be readily seen to remain finite as \( n \to \infty \) and \( c(n) \to \infty \) everywhere except at the singular point \( R = 0 \). The relation \( V_C(q_z, q_x, n) / c(n) \) is finite as \( n \to \infty \) and goes over, according to (A1), into the known expression for the Coulomb matrix element in a simple layered crystal with layer spacing \( d \) (in 2D space):

$$V_C(q_z, q_x) = \frac{2\pi e^2}{q_z} \frac{\sin q_z d}{\sin q_z(n) - \cos q_z(1)}$$

(A4)

Finally, at \( n = 1 \) the ratio \( V_C(q_z, q_x, 1) / d \) becomes equal to expression (A4) for a crystal with distances \( d \) between layers.

We now calculate the structure factors \( \beta(n) \) which are determined by the Fourier component \( \Psi_i(p_5) \) of the transverse part \( \Psi_i(z) \) of the wave function of the carriers in the layers. The presence of the factor \( \Psi_i^{-1}(p_5) \), which is contained in the Green's functions (19) and (21), is due to separation of the variables of the "fast" longitudinal (in the plane of the layers) and "slow" transverse (across the layers) motion of the electrons. This separation is possible in the framework of the "adiabatic" approximation in the case of a sufficiently strong two-dimensional anisotropy of the electron spectrum, when the probability of tunneling between layers is exponentially small, and the cylindrical Fermi surface is weakly rippled along \( p_z \), corresponding to strong anisotropy of the effective masses \( (m^*/m^* \ll 1) \). Representing in this case the electron (hole) wave function in the 2D layer in the multiplicative form

$$\Psi_i = \Psi_i(x, y)\Psi_i(z)$$

and introducing normal and anomalous Green's functions in longitudinal variables at a fixed coordinate \( z \) (cf. Ref. 70)

$$G(x' - x, y' - y; z) = \langle \Psi_i(x, y)\Psi_i(x', y')\rangle_{\Psi_i(z)}$$

(A5)

$$P(x' - x, y' - y; z) = \langle \Psi_i(x, y)\Psi_i(x', y')\rangle_{\Psi_i(z)}$$

(A6)

we obtain, after changing to the momentum representation in terms of the independent variables, the expressions (19) and (21) in which the function \( \Psi_i(p_5) \) is assumed for simplicity to be real.

Separating the variables in the normal and anomalous self-energy parts responsible for the EPI and the polar EPI, and taking (6) or (A2) into account, we can separate the dimensionless structure factor that depends on the number \( n \) of the layers:

$$\beta(n) = c(n) \int_{-\pi/n}^{\pi/n} \frac{d\varphi}{2\pi} \sum_{\vartheta} V_C(q_z, \vartheta, n)$$

and

(A7)

$$P = \frac{2\pi e^2}{q_z} \frac{\sin q_z d}{\sin q_z(n) - \cos q_z(1)}$$

The integration over \( p_z \) is carried out here within the limits of the first BZ, and the sum over \( K \) describes the contribution of the Umklapp processes.

Similarly, to calculate the polarization operator of the \( I \)-and \( K \)-carriers localized in the 2D layers:
with allowance for its normalization to unity at the point \( z = z_0 \), we obtain

\[
\frac{\Psi_1^2(p, n = 2k - 1)}{\frac{1}{1 + \langle \Phi_p \rangle^2}} [1 + 2 \sum_{\nu = 1}^{n_l} \cos(n \Phi_p \rho_d)],
\]

(A11)

\[
\frac{\Psi_1^2(p, n = 2k)}{\frac{1}{1 + \langle \Phi_p \rangle^2}} \sum_{\nu = 1}^{n_l} \cos \left( \frac{2m - 1}{2} \rho_p \rho_d \right).
\]

(A12)

For a simple layered crystal with \( n = 1 \), substituting (A11) in (A7) and (A9), we get

\[
\rho(1) = \frac{d}{d_0} \arctan \left( \frac{a d_0}{d} \right), \quad \tilde{\rho}(1) = \frac{1}{2} \left[ \rho(1) + \frac{1}{1 + (a d_0/d)^2} \right].
\]

(A13)

In the general case \( n \neq 1 \) and \( l_{c0} \neq 0 \), the integrals over \( \rho_p \) in (A7) and (A9), with allowance for (A11) and (A12), cannot be calculated in explicit form. In the limiting case \( l_{c0} = 0 \), when \( \rho(1) = \tilde{\rho}(1) = 1 \), one can obtain for \( \rho(\alpha) \) and \( \tilde{\rho}(\alpha) \) at \( n > 1 \):

\[
\rho(\alpha) = \frac{1 + 2(\alpha a d_0 \rho_d)^2}{1 + (\alpha a d_0 \rho_d)^2} \frac{\sin(\frac{\pi}{n_l} \rho_d)}{\cosh(n_l \rho_d)}, \quad n = 2k - 1,
\]

(A14)

\[
\rho(\alpha) = \frac{4(\alpha a d_0 \rho_d)^2}{1 + (\alpha a d_0 \rho_d)^2} \frac{\sin(\frac{2m - 1}{2} \rho_d)}{2(\alpha a d_0 \rho_d)^2}, \quad n = 2k,
\]

(A15)

It is easily seen that when \( n \gg 1 \) and \( c(\alpha) \gg \sigma d_0 \), expressions (A14) and (A15) take with good accuracy the form

\[
\rho(\alpha) = \alpha, \quad \tilde{\rho}(\alpha) = \alpha + \frac{n - 1}{n_l} \alpha^2.
\]

(A16)

Figure 3 shows plots of \( \rho(\alpha) \) and \( \tilde{\rho}(\alpha) \) for \( l_{c0} = 0 \),\( d = 12 \, \text{Å} \) and \( d_0 = 3.2 \, \text{Å} \), and also the dependences of \( \rho(1) \) and \( \tilde{\rho}(1) \) on \( l_{c0}(a) \).

This model was used in Refs. 21 and 22 to describe the anomalies of the kinetic, thermodynamic, optical, and magnetoresistance properties of the normal metallic state of cuprate MOC.

This was demonstrated earlier in Ref. 27 for quasi-one-dimensional (chain) metals with narrow one-dimensional (1D) band, and in Ref. 26 for intermetallic compounds of transitions (such as Nb,Ge) with a narrow three-dimensional (3D) band of cubic symmetry, and in Ref. 22 for layered metals with narrow 2D bands.

"Earlier papers" discusses an "electronic" mechanism of superconductivity in transition metals on account of interband static screening of the Coulomb interaction by s- and d-electrons, without allowance for retardation, i.e., for exchange of virtual If plasmons.

Such a possibility was considered in connection with the problem of raising \( T_c \) in semiconductors (semimetals) with substantially different...
According to Ref. 14, the maximum of \( n_{\text{In}} \) in photocatalytic reactions is observed when the number of dopant holes in \( n_{\text{In}} = 0.1-0.3 \) per CuO2 square layer is in a unit cell, which corresponds to a lattice constant of \( a = 3.8 \text{ Å} \) and \( \rho = 2 \times 10^{-2} \text{ cm}^2 \). 

It is assumed that \( \alpha_{\text{MOC}} < \alpha_{\text{Ba}} \), i.e., in the region of relatively weak Landau damping \( \alpha_{\text{MOC}} < \alpha_{\text{Ba}} \). 

Static screening by \( I \)-carriers in the region of \( \alpha_{\text{MOC}} < \alpha_{\text{Ba}} \) is caused by polarization effects in the region of \( \alpha_{\text{MOC}} < \alpha_{\text{Ba}} \). 

It is assumed that the oxygen ion mass (see Refs. 69 and 70). 

Both M and oxygen peaks with \( \omega_{\text{M}} = 300-400 \text{ K} \) and \( \omega_{\text{O}} = 600-900 \text{ K} \) are distinctly observed in the excitations of oxygen, i.e., to orbital excitations. 

It is recognized that \( \omega_{\text{M}} < \omega_{\text{Ba}} \), i.e., in the region of relatively weak Landau damping \( \omega_{\text{M}} < \omega_{\text{Ba}} \). 

We have in mind here the exponential decrease of the carrier density with \( \omega_{\text{M}} < \omega_{\text{Ba}} \) due to the increase of the distance from the plane of the conducting CO layers (see Fig. 2). 

The number of “phononlike” modes in the phonon spectrum is preserved here. 

In the present model we take into account only conducting CO layers in an excitonic matrix having a certain pertinence. The influence of nonequivalent layers on Coulomb interaction in layered crystal is taken into account in Ref. 93.


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