Lasers based on photorefractive gratings written by the synchronous detection mechanism

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We have carried out a theoretical analysis of the conditions for the onset of lasing in a ring resonator and in a linear resonator with one photorefractive crystal and in a ring resonator with two crystals where the grating has been written using the mechanism of synchronous detection of a running interference pattern by an external variable field.

1. INTRODUCTION

Ring oscillators and linear oscillators based on photorefractive crystals (PRC) with different mechanisms for recording the diffraction grating have been studied in great detail,[1,2] especially those with a diffusion nonlinearity,[3–7] and with a drift nonlinearity in constant[8,9] and in variable[10,11] external fields. Such oscillators are self-tuning, i.e., if the eigenfrequencies of the cavity do not coincide with the frequencies of the longitudinal modes of the pumping laser, a wave builds up in the cavity with a mixed frequency relative to the pumping frequency. Here two lasing conditions are fulfilled: an amplitude condition and a phase condition. Energy losses of the cavity wave occur as a result of diffraction of the pump wave by the grating. Phase detuning of the cavity is compensated by the nonlinear phase shift in two-wave mixing of the cavity wave with the pump wave in photorefractive crystals. The grating on which the mixing takes place is recorded by beams with a frequency difference $\delta\omega$ smaller than or of order the inverse relaxation time of the grating, $\delta\omega \approx \tau_{sc}^{-1}$. In other words, the mixing is quasi-degenerate in frequency.

As has already been shown both theoretically and experimentally,[12,13] a quasistatic grating can be written by a fast-moving interference pattern formed by two beams having a frequency difference $\Omega + \Delta\Omega \gg \tau_{sc}^{-1}$. Coupling of the waves on such a grating is highly nondegenerate. This mechanism has come to be called synchronous detection (SDM). In a resonator with fixed mirrors the frequency difference between the beams incident on the crystal is insufficient for efficient recording of the grating by SDM. To construct an oscillator based on SDM, the required frequency shift Ω can be produced periodic motion of one of the mirrors, thereby providing feedback. In the present paper we examine some schemes for such oscillators based on one and two photorefractive crystals. We study the stationary states of these schemes, their stability and lasing characteristics. To start with, in the case of a unidirectional ring oscillator with one crystal, we consider in detail the base model including a description of the nondegenerate mixing on an SDM grating. Then on the basis of this model we consider various modifications of linear and ring resonators.

2. RING OSCILLATOR WITH ONE CRYSTAL

Let us consider the standard ring oscillator consisting of a piezoelectric mirror and two plane mirrors (Fig. 1). A sawtooth voltage with frequency Ω equal to the frequency of the variable voltage applied to the crystal, and an amplitude which shifts the phase of the reflected signal by 2π, is applied to the piezoelectric mirror. This kind of phase modulation shifts the frequency of the reflected signal by $\pm\Omega$ with respect to that of incident signal. The sign of the frequency shift is determined by the direction of motion of the piezoelectric mirror.

When a pump beam with amplitude P is incident on the crystal, a cavity wave is formed from the noise scattered by the inhomogeneities of the crystal. After the first reflection from the piezoelectric mirror, this wave acquires a frequency shift Ω, and, interacting with the pump wave, writes the quasistatic SDM grating. The frequency shift grows with each successive pass. The gratings written by waves with frequencies $\omega + n\Omega$ ($n > 1$) are small[14] and the effect of these waves reduces to a simple decrease in the contrast of the main interference pattern moving with velocity Ω/q. Here q is the spatial frequency of the gratings. In the cavity beam incident on the crystal there is no wave with unshifted frequency ω, for which reason quasi-degenerate mixing is excluded in this scheme.

Let us consider the mixing of waves on a dynamic grating in the plane wave approximation. Two waves incident on the crystal (see Fig. 1) take part in the process of writing the grating:

$$P(\mathbf{r}, t) = P\exp\{i(\mathbf{k}_1\mathbf{r}-\omega t)\},\qquad S(\mathbf{r}, t) = S\exp\{i[\mathbf{k}_2\mathbf{r}-(\omega+\Omega)t]\}. \tag{1}$$

Here P is the amplitude of the pump wave and S is the amplitude of the cavity wave having frequency shift Ω. Diffraction by the recorded static grating leads to the formation of two new waves:

$$S_1(\mathbf{r}, t) = S_1\exp\{i(\mathbf{k}_2\mathbf{r}-\omega t)\},\qquad P_1(\mathbf{r}, t) = P_1\exp\{i[\mathbf{k}_1\mathbf{r}-(\omega+\Omega)t]\}. \tag{2}$$

By interfering, the wave pairs (1) and (2) produce travelling interference patterns and write the grating, thanks to

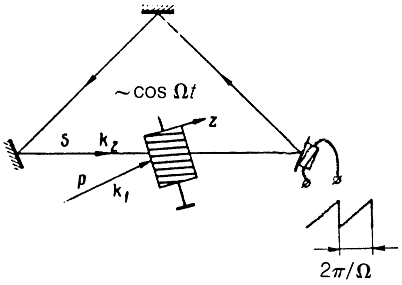

FIG. 1. Ring oscillator with periodic sawtooth modulation of the resonator length.

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SDM. The amplitude of the grating is described by the relaxation equation

$$\dot{Q}+Q=-\frac{g}{I_0}(P^*S+P_1^*S_1). \tag{3}$$

Here I_0 is the total intensity of the waves illuminating the crystal, including all the frequency components of the cavity wave and g is the nondegenerate coupling constant, determined by the amplitude of the space charge field E_{sc} (Ref. 12), normalized to the contrast of the interference pattern: $g=-(\omega/c)(n^3r/2)E_{sc}$, where n is the unperturbed index of refraction, r is the electrooptic coefficient, and c is the speed of light. The quantity τ_{sc} is chosen in Eq. (3) as the unit of time. The quantity Q characterizes the amplitude of the permittivity grating:

$$\delta\varepsilon(\mathbf{r})=\frac{2cn}{\omega}[Q\exp\{i(\mathbf{k}_1-\mathbf{k}_2)\mathbf{r}\}+\text{h.c.}]. \tag{4}$$

Dynamic diffraction of waves (1) and (2) on grating (4) is described by the system of simplified equations

$$\frac{\partial S}{\partial z}=iQP_1,\quad \frac{\partial P}{\partial z}=iQ^*S_1,$$
$$\frac{\partial S_1}{\partial z}=iQP,\quad \frac{\partial P_1}{\partial z}=iQ^*S. \tag{5}$$

The system of equations (3), (5) is closed. In the derivation of Eq. (3) we have neglected the recording of the gratings by the static interference patterns P^*S_1 and P_1^*S, assuming the efficiency of degnerate mixing to be small. Such a situation can always be achieved in a crystal which does not have a photovoltaic effect at low spatial frequency. The efficiency of recording of the grating by a static interference pattern in an external variable field or as a result of diffusion in this case is small since its dependence on the spatial frequency is linear.[2,15] On the contrary, the efficiency of SDM in this case is maximal.

One can easily convince oneself by direct substitution that the solution of Eqs. (3), (5) has the form

$$S=S(0,t)\cos|Q|z,\qquad P=P_0\cos|Q|z,$$
$$S_1=i\frac{Q}{|Q|}P_0\sin|Q|z, \tag{6}$$

$$P_1=i\frac{Q}{|Q^*|}S(0,t)\sin|Q|z,\qquad \dot{Q}+Q=-\frac{g}{I_0}S(0,t)P^*(0,t), \tag{7}$$

where P_0 is the input amplitude of the pump wave and $S(0,t)$ is the amplitude of the component of the cavity wave as it enters the crystal. Because the mixing on an SDM grating is steady, the spatial and temporal dependences of all the quantities separate.

The resulting relations (6) are completely equivalent to the case of diffraction of a wave pair S and P on a grating whose hologram amplitude is uniform with depth[16,17] and which is determined by the slowly varying contrast of the interference pattern upon entrance to the crystal, in accordance with Eqs. (7).

Let R be the complex amplitude transmission coefficient of the resonator for one pass. The amplitude-phase condition for lasing, taking Eqs. (6) into account, gives

$$S(0,t)=RS_1(l,t)=R\frac{iQP_0}{|Q|}\sin|Q|l. \tag{8}$$

Physically, this condition means that the wave $S(0,t)$ is built up at the entrance to the crystal as a result of diffraction of the pump wave by the grating with a subsequent round-trip through the resonator and reflection by the piezoelectric mirror. The value of the phase of the complex coefficient R deserves individual attention. In a static resonator this phase is determined by the detuning from a whole number of wavelengths. In the case of the sawtooth motion of one of the mirrors considered here this detuning varies linearly in time within the limits 0–2π. To determine the phase of R, it is necessary to take the value of the detuning (the length of the resonator) when the applied field is at its maximum. It is precisely this phase that the grating is in antiphase with the instantaneous position of the running interference pattern,[12,13] as was assumed in the choice of the phase on the right-hand side of Eq. (3).

Substituting Eq. (8) in Eq. (7), we obtain an equation describing the grating dynamics.

$$\dot{Q}+Q=-\frac{ig|P|^2QR}{I_0|Q|}\sin|Q|l. \tag{9}$$

The amplitude of the frequency components of the cavity wave at the entrance to the crystal are related by the recurrence relation

$$E_{n+1}=R(1-\eta)^{1/2}E_n. \tag{10}$$

Here E_n is the amplitude of the component with frequency shift $n\Omega$ ($n>0$) relative to the pump. The amplitude of the first principal component is related to the pump amplitude.

$$S=E_1=R\eta^{1/2}P_0. \tag{11}$$

Here $\eta=\sin^2|Q|l$ is the instantaneous diffraction efficiency of the grating. Taking these relationships into account and carrying out an elementary summation, we obtain for the total intensity of the waves irradiating the crystal

$$I_0=|P_0|^2+\sum_{n=1}^{\infty}|E_n|^2=\frac{1-|R|^2+2|R|^2\sin^2|Q|l}{1-|R|^2+|R|^2\sin^2|Q|l}|P_0|^2. \tag{12}$$

Near the threshold for the onset of lasing, the amplitudes of the cavity waves and of the grating can be taken to be small for $|R|<1$, whence Eq. (12) into account we have

$$I_0\approx|P_0|^2,\qquad \frac{\sin|Q|l}{|Q|}\approx l. \tag{13}$$

In this approximation we obtain a linearized equation describing the behavior of the grating:

$$\dot{Q}=-Q-iglRQ. \tag{14}$$

From Eq. (14) it follows that the grating and the cavity wave due to it result when the coupling constant gl exceeds a threshold value:

$$gl>1/\text{Im}R>0. \tag{15}$$

Similar calculations for the case of an ideal resonator $|R|=1$ give the threshold $gl>2$.

As the value of gl passes through the threshold, the

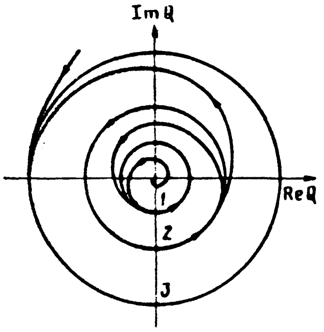

FIG. 2. Phase portrait of the grating amplitude.

phase portrait of the system in the (ReQ, ImQ) plane near the null equilibrium position transforms from a stable focus into an unstable one. The velocity with which the grating moves when lasing begins and the detuning $\Delta\Omega$ of the cavity waves from exact synchronism with the external field are given in the first approximation by the imaginary part of the root of the characteristic equation (14)

$$\Delta\Omega = gl\,\mathrm{Re}R. \tag{16}$$

In the steady lasing regime the grating moves with constant velocity and its amplitude is bounded. The corresponding limiting phase trajectory in the (ReQ, ImQ) plane is a limit cycle (Fig. 2). Finding the steady-state amplitude of the grating (the radius of the limit cycle) and the velocity with which it moves requires the solution of the nonlinear equation (9) with intensity I_0 determined by relation (12). Searching for the steady-state solution in the form $Q = Q_0 \exp\{-i\Delta\Omega t\}$ gives for the amplitude Q_0 and the frequency $\Delta\Omega$

$$\Delta\Omega = \mathrm{Re}R/\mathrm{Im}R = \mathrm{ctg}\varphi, \tag{17}$$

$$\sin Q_0 l = \frac{Q_0 l}{gl\,\mathrm{Im}\,R}\left[\frac{1-|R|^2+2|R|^2\sin^2 Q_0 l}{1-|R|^2+|R|^2\sin^2 Q_0 l}\right]. \tag{18}$$

Here φ is the phase detuning of the resonator length from a whole number multiple of 2π when the field in the crystal attains its maximum. Recall that as the unit of time we have chosen the relaxation time of the grating τ_{sc} determined by the total intensity I_0. The roots of the transcendental equation (18) determine the steady-state values of the grating amplitude. The rational function of $\sin^2 Q_0 l$ on the right-hand side of Eq. (18) increases monotonically in the interval (0,1) from 1 to $1 + |R|^2$. For small values of $|R|$, which, as a rule, are encountered in practice, we will replace this function in the analysis of the solutions (18) everywhere by unity. As a result, for the steady-state values of the amplitude Q_0 we obtain

$$\sin Q_0 l = Q_0 l/(gl\,\mathrm{Im}R). \tag{19}$$

The first nonzero root of Eq. (19) appears for $gl > 1/\mathrm{Im}\,R > 0$, which, as could be expected, coincides with the instability threshold (15). As the coupling constant gl increases further, additional roots of Eqs. (19) appear in pairs. In each pair one root corresponds to an unstable limit cycle, and the other to a stable one.

In an oscillator with degenerate mixing and a medium capable of sufficient gain γl simultaneous generation of several modes is possible,[1,10,11] differing in the velocity with which the grating moves and, consequently, in the frequency shift relative to the pump. In contrast to this, in the oscillator under consideration one grating is always excited. Its velocity does not depend on the coupling constant, but is determined, as in the degenerate case, by the phase detuning of the resonator (17). For when the coupling constant is the amplitude of the grating can take on several values, determined by Eq. (18). Such an oscillator can be considered as a multistable system in which the number of stable states (limit cycles) is greater by one than the number of unstable ones. However, the transition of the oscillator to a state with number greater than unity is possible only in the case of drastic loss of stability, i.e., as a result of a large initial perturbation.

One of the important characteristics of any laser is the power of the generated radiation. In the given case this power I is determined by the diffraction efficiency of the grating. Its limiting value for $\eta = 1$ is given by Eq. (12): $I = I_0 - |P|^2 = |R|^2|P|^2$. In an ideal resonator ($|R| = 1$) the lasing intensity reaches the level of the pumping power. Note that the power of the lasing wave consists of many components which have different frequency shifts $n\Omega + \Delta\Omega$. As the coupling constant increases from the threshold value, the diffraction efficiency, and with it the power, vary nonmonotonically, reaching their maxima at $gl = \pi/(2\mathrm{Im}R)$ in agreement with the pendulum solution of Eqs. (6).

Let us consider the limit cycles which are determined by the magnitude of R. For an imaginary transmission coefficient (Re $R = 0$) the generated grating has a steady character [see Eq. (16)], but the conditions of lasing for a given coupling constant gl are optimal, in agreement with inequality (15). These facts have a simple physical interpretation. Indeed, let a noisy grating arise in the crystal. When the wave is diffracted by it, its phase changes by $\pi/2$ [the factor of i in the simplified equations (5)]. If after traversing the resonator the diffracted wave acquires an additional phase shift $\pi/2$, the interference entering the crystal is in antiphase, but the grating recorded by it is in phase with the starting perturbation of the refractive index. When the threshold condition (15) is exceeded, the noisy grating grows without spatial displacement, but the steady-state, static position of the grating is determined by the position of the initial perturbation. In oscillators with degenerate mixing on a shifted grating caused by a diffusion nonlinearity,[2] the condition Im $R = 0$ is optimal. Such a difference is due to a shift of the grating by $\pi/2$ relative to the interference pattern in the case of diffusion mixing. A resonator with a piezoelectric mirror can be tuned to the optimal lasing condition Im $R = 0$ by regulating the phase difference between the variable field applied to the crystal and the sawtooth modulating signal.

In the other limiting case, Im $R = 0$, lasing is impossible. The picture of the grating dynamics in this case is the following. Upon diffraction from a random starting grating the wave acquires a phase shift of $\pi/2$, which is not compensated by the round-trip through the resonator. As a result, this wave, by interacting with the pump, writes the grating shifted by $\pm\pi/2$ with respect to the starting grating. Thus, the noisy grating does not receive the in-phase input arising from diffraction of the pump. This situation is completely analogous to degenerate mixing on an unshifted grating, which does not allow the pump to amplify the cavity beam and does not bring about the onset of lasing.

3. A LINEAR OSCILLATOR WITH ONE CRYSTAL

Linear oscillators, which can be used for wavefront reversal (WFR),[6] can be built on the basis of an analogous principle (Fig. 3). The principle of their operation can be easily understood from inspection of Fig. 3. In the steady-state regime the pump is diffracted by the grating with subsequent reflection from the static mirror. Part of the reflected wave gives the reversed component after diffracting on the grating. The remaining part is reflected from the piezoelectric mirror after passing through the grating, thereby acquiring a frequency shift, and is reflected into the crystal, where, interacting with the pump, it writes the SDM grating. As with a ring oscillator, here components with higher frequency shifts also occur here.

In the given scheme, in the absence of degenerate mixing the grating is written by the component of the cavity wave with frequency $\omega + \Omega + \Delta\Omega$, propagating in the same direction as the pump while interacting with the latter. The counterpropagating cavity wave does not write a grating with the pump, since the electric field in the direction of the wave vector of this grating is small and its spatial frequency is large.[12] Waves with frequency shifts $\omega + n\Omega + \Delta\Omega$ $(n > 1)$ only decrease the contrast of the interference pattern being written, since the amplitudes of the gratings excited by them are small.[14] Thus, the grating dynamics, as before, are described by Eq. (7). The amplitude of the component of the cavity wave taking part in the writing is related to the pump in the following way:

$$S = PR[\eta(1-\eta)]^{1/2}, \tag{19'}$$

where R is the complex transmission coefficient of the resonator after one pass and $\eta = \sin^2|Q|l$, as before, is the diffraction efficiency of the grating.

In contrast to a ring oscillator, in the calculation of the total intensity I_0 the intensity of the counterpropagating cavity wave must be taken into account. With this, we have for the linear oscillator

$$I_0 = \frac{1-|R|^2+3|R|^2\eta-2|R|^2\eta^2}{1-|R|^2+2|R|^2\eta-|R|^2\eta^2}|P|^2. \tag{20}$$

Combining this equation with Eqs. (7), (19') and (20), we obtain a nonlinear equation which describes the grating dynamics:

$$\dot{Q}+Q = -gR[\eta(1-\eta)]^{1/2}\left[\frac{1-|R|^2+2|R|^2\eta-|R|^2\eta^2}{1-|R|^2+3|R|^2\eta-2|R|^2\eta^2}\right]. \tag{21}$$

Linearizing Eq. (21) near the zero equilibrium position, we obtain its stability threshold, which coincides with the threshold of the ring oscillator: $gl > 1/\text{Im } R$. For an ideal resonator ($|R| = 1$) this threshold becomes $gl > 3/2$. The phase portrait of the system in the (Re Q, Im Q) plane is also analogous to that for a ring oscillator. To find the circulation frequency around the limit cycles and their amplitudes, we replace the slowly varying factor in brackets in Eq. (21) everywhere by unity. We then seek the solution of the so-obtained nonlinear equation in the form $Q = Q_0\exp(-i\Delta\Omega t)$. We easily find

$$\Delta\Omega = \text{Re}R/\text{Im}R = \text{ctg }\varphi, \tag{22}$$

$$\sin 2Q_0 l = 2Q_0 l/gl\text{Im}R. \tag{23}$$

Equation (22) relates the detuning $\Delta\Omega$ of the frequency of the generator wave from exact synchronism with the detuning of the resonator length from a whole number of wavelengths φ. The roots of Eq. (23) give the steady-state values of the grating amplitude Q_0. Let us turn our attention now to the coefficient 2 in Eq. (23) in comparison with Eq. (19). The first nonzero solution of Eq. (23) appears, as could be expected, at $gl > 1/\text{Im } R$. Roots then appear in pairs as the coupling constant gl increases continuously. In the limit $gl \to \infty$ the roots of Eq. (23) asymptotically approach $(Q_0 l)_k = k\pi/2$, where k is an integer. The diffraction efficiency and the reversal coefficient of the WFR mirror in this case asymptotically approach unity, never reaching it. This result has a simple physical interpretation. In the case of unit diffraction efficiency the pump wave, by diffracting twice on the grating, would be completely reflected into the reversed wave. In this case there is no mixed component entering into the crystal and the grating is erased.

The reversed wave in the present scheme contains all the frequency components, including the unmixed one. The latter can be eliminated if the mirrors in this scheme change places.

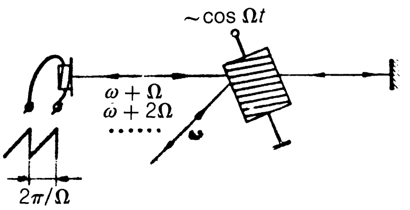

FIG. 3. Optical diagram of the linear oscillator based on the mechanism of synchronous detection.

4. RING OSCILLATOR WITH TWO CRYSTALS

It is also of interest to consider the scheme of a ring oscillator with two crystals, each of which is pumped by its own beam, and whose frequencies are shifted by the amount Ω, coinciding with the frequency of the applied field (Fig. 4a). With increase of the threshold, which is given below, a wave arises in the resonator which has both a mixed and an unmixed component. Here the unmixed component writes the grating in the first crystal, and the mixed, in the second. Crystal 1 is illuminated by three waves:

$$\begin{aligned} P(\mathbf{r}, t) &= P\exp\{i[\mathbf{k}_2\mathbf{r}-(\omega+\Omega)t]\}, \\ A(\mathbf{r}, t) &= A\exp\{i[\mathbf{k}_1\mathbf{r}-\omega t]\}, \\ B(\mathbf{r}, t) &= B\exp\{i[\mathbf{k}_1\mathbf{r}-(\omega+\Omega)t]\}. \end{aligned} \tag{24}$$

For crystal 2 these waves have the form

$$\begin{aligned} P(\mathbf{r}, t) &= P\exp\{i(\mathbf{k}_2\mathbf{r}-\omega t)\}, \\ C(\mathbf{r}, t) &= C\exp\{i(\mathbf{k}_1\mathbf{r}-\omega t)\}, \\ D(\mathbf{r}, t) &= D\exp\{i[\mathbf{k}_1\mathbf{r}-(\omega+\Omega)t]\}. \end{aligned} \tag{25}$$

For the dynamic variables of the grating Q_1 and Q_2 in crystals 1 and 2, respectively, we have the equations

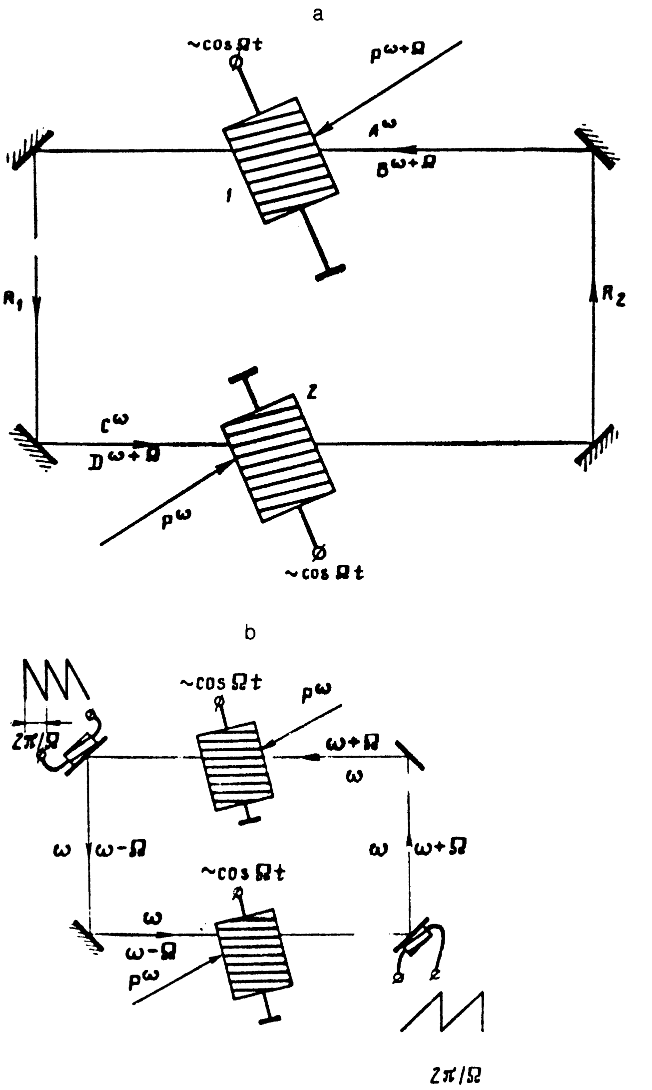

FIG. 4. Diagram of the unidirectional ring oscillator with two crystals: a) fixed mirrors and frequency-shifted pumps, b) movable mirrors and single-frequency pumps.

$$\dot{Q}_1+Q_1=-\frac{g_1P^*A}{|P|^2+|A|^2+|B|^2},$$
$$\dot{Q}_2+Q_2=-\frac{g_1P^*D}{|P|^2+|C|^2+|D|^2}. \qquad (26)$$

In Eqs. (24) and (25) the amplitudes of the pumps have been taken to be equal. The amplitudes of the waves P, A, B, C, and D are connected by the laws of diffraction on the two gratings and the complex transmission coefficients:

$$A=(1-\eta_1)^{1/2}(1-\eta_2)^{1/2}R_1R_2A+R_2\eta_2^{1/2}P,$$
$$B=(1-\eta_1)^{1/2}(1-\eta_2)^{1/2}R_1R_2B+R_1R_2\eta_1^{1/2}P,$$
$$C=(1-\eta_1)^{1/2}(1-\eta_2)^{1/2}R_1R_2C+R_1R_2\eta_2^{1/2}P,$$
$$D=(1-\eta_1)^{1/2}(1-\eta_2)^{1/2}R_1R_2D+R_1\eta_1^{1/2}P. \qquad (27)$$

Here $\eta_1=\sin^2|Q_1|l_1$ and $\eta_2=\sin^2|Q_2|l_2$ are the grating diffraction efficiencies in the first and second crystals, respectively, R_1 and R_2 are the complex transmission coefficients of the left and right arms of the resonator (see Fig. 4a). Taking both crystals to be identical and assuming that $\eta_1=\eta_2$ and $R_1=R_2=R$, and taking into account relations (27), Eqs. (26) transforms to one nonlinear equation for the dynamic variable $Q=Q_1=Q_2$:

$$\dot{Q}=-Q-i\,\frac{QRg\sin|Q|l}{|Q|}\times\left[\frac{1-(1-\eta)R^{*2}}{|1-(1-\eta)R^2|^2+\eta|R|^2+\eta|R|^4(1-\eta)}\right], \qquad (28)$$

where $\eta=\sin^2|Q|l$ is the diffraction efficiency of the gratings. Near the lasing threshold we set $\eta\to0$, $\sin|Q|l\approx|Q|l$, and obtain the linearized equation

$$\dot{Q}=-Q-\frac{iglR}{1-R^2}Q, \qquad (29)$$

from which follows the threshold condition for the onset of lasing

$$gl>\operatorname{Im}\frac{R}{1-R^2}. \qquad (30)$$

Recall that here $R^2=R_1R_2$ is the complex transmission coefficient for a complete round-trip of the resonator. As can be seen, the threshold condition (30) is different from that obtained for single-crystal generators. The behavior of the phase portrait of the system at large values of gl is similar to that described above. The rotation frequency around the limit cycles in this case is

$$\Delta\Omega=\frac{1-R^2}{1+R^2}\operatorname{ctg}\varphi. \qquad (31)$$

The amplitudes of the limit cycles is determined by the equation

$$\sin|Q_0|l=\frac{|Q_0|L}{gl\operatorname{Im}[R/(1-R^2)]}. \qquad (32)$$

The minimum threshold of onset of lasing $(gl)_{th}$ is attained for the following values of the phase φ of the coefficient R:

$$(gl)_{th}=\begin{cases}\dfrac{1+|R|^2}{|R|} & \text{for } \sin\varphi=1, \quad 0<|R|<2^{1/2}-1\\[2ex] \dfrac{4(1-|R|^2)}{1+|R|^2} & \text{for } \sin\varphi=\dfrac{1-|R|^2}{2|R|}, \quad 2^{1/2}-1<|R|<1\end{cases}. \qquad (33)$$

Completely analogous results obtain from considering a generator containing two crystals pumped by beams of the same frequency (Fig. 4b). The frequency shift in such a generator is produced by two piezoelectric mirrors moving in opposite directions. Pairs of waves with frequencies $(\omega,\omega+\Omega)$ or $(\omega-\Omega,\omega)$ propagate in the cavity, depending on the cross section. The lengths of resonators with two crystals (Figs. 4a and b) do not vary in time and the phase tuning of R can be realized only by continuous, smooth motion of one of the mirrors.

5. CONCLUSION

The main difference between the generators considered here and those based on degenerate mixing, in our view, is that lasing occurs in only one mode, whose frequency detuning $\Delta\Omega$ depends on the resonator length. In the schemes considered, the magnitude of $\Delta\Omega$ can be easily regulated by the

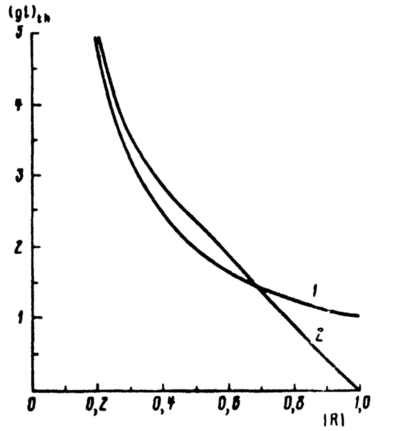

FIG. 5. Dependence of the threshold coupling constant $(gl)_{th}$ on the magnitude of the transmission coefficient $|R|$ for optimal value of its phase: 1) single-crystal scheme, 2) two-crystal scheme.

phase difference between the applied field and the sawtooth signal controlling the piezoelectric mirror. The single-mode regime means, apparently, more stable generation in the presence of fluctuations of the parameters of the pump laser in comparison with degenerate-mixing generators. The lasing threshold of these schemes is relatively high and finite even for ideal resonators with $|R| = 1$ in contrast with quasidegenerate lasers based on PRC gratings. Figure 5 shows the values of the threshold coupling constants $(gl)_{th}$ in the schemes considered for optimal phase of the transmission coefficient R. As can be seen, for values of R realizable in practice ($R < 0.7$) the single-crystal scheme is more advantageous. The value of the constant gl needed for the realization of the generators described here ($gl \approx 2$) means a diffraction efficiency of $\sin(mgl/2) \approx \sin 1 \approx 84\%$ for recording of SDM gratings by an interference pattern with unit contrast.[17] In experiments being carried out at present on the recording of SDM gratings in crystals of $Bi_{12}Ti(Si)O_{20}$ a diffraction of only 25% has been reached.[14] In this connection, it is interesting to consider the recording of gratings in a variable field in segnetoelectric crystals with a higher electrooptical coefficient.

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