Pseudophoton diffusion and radiation of a charged particle in a randomly inhomogeneous medium

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The radiation emitted by a charged particle moving in a randomly inhomogeneous medium is considered. In determining the intensity of the radiation the diffusion of the pseudophoton excited by the moving charge is taken into account. It is shown that in the shortwave range $\lambda \ll l$ (l is the photon mean free path in the medium) the corresponding contribution to the intensity of the radiation is the predominant one. Intensity fluctuations and polarization of the radiation are also investigated.

1. INTRODUCTION

As is well known,[1] a charge moving in a medium with an inhomogeneous dielectric constant radiates. This is by its very nature transition radiation. The nature of this radiation can be described in the following way (see, e.g., Ref. 2). A charge moving in a medium creates an electromagnetic field (a pseudophoton), which is scattered from the fluctuations of the dielectric constant and converted into radiation.

In earlier articles which have addressed this problem (see Refs. 2 and 3) multiple scattering of the electromagnetic field was not taken into account. In the long-wave region where the wavelength of the radiated wave is much larger than the characteristic dimensions of the inhomogeneity, it is valid to neglect multiple scattering and the radiation has a dipole character.[1] In the short-wave region $\lambda \ll l$ (l is the photon mean free path in the medium), although the scattering is weak as a result of the presence of the small parameter λ / l, as we will show below, it is still necessary to take multiple scattering into account. It turns out that multiple scattering of the pseudophoton leads to its diffusion in the medium and the diffusion contribution to the radiation intensity is the dominant one.

Note that although the pseudophoton is not a real photon (a plane electromagnetic wave), nevertheless its weak scattering obeys the ordinary laws of Rayleigh light scattering.

Note that here we are considering a situation in which the main interaction of the medium with the electromagnetic field is its own elastic scattering. For this reason, along with the condition $\lambda \ll l$, the condition $l \ll l_{\rm in}$ must also be fulfilled, where $l_{\rm in}$ is the inelastic mean free path of the photon in the medium. Such media can be, in particular, those systems in which peaks are observed in the backscattering diagram.[4]

2. INITIAL RELATIONS

The electromagnetic field created by a charge moving in a medium is described by the Maxwell equations

$$\operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \quad \operatorname{rot}\mathbf{B} = \frac{4\pi}{c} e\mathbf{v}\delta(\mathbf{r}-\mathbf{v}t) + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t},$$
$$\operatorname{div}\mathbf{D} = 4\pi e\delta(\mathbf{r}-\mathbf{v}t), \qquad \mathbf{D} = \varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r}, t). \tag{1}$$

Here $\mathbf{v}$ is the constant velocity of the particle, which we will take to be directed along the z axis, $\mathbf{v}\|\hat{z}$. Transforming to its Fourier components, we obtain from Eqs. (1) the following equation for $\mathbf{E}(\mathbf{r},\omega)$:

$$\nabla^2\mathbf{E}(\mathbf{r}, \omega) - \operatorname{grad}\operatorname{div}\mathbf{E}(\mathbf{r}, \omega) + \frac{\omega^2}{c^2}\varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r}, \omega) = \mathbf{j}(\mathbf{r}, \omega). \tag{2}$$

We assume that the dielectric constant has the form $\varepsilon(\mathbf{r}) = \varepsilon_0 + \varepsilon_1(\mathbf{r})$, where ε_1 is its fluctuating part, which we choose to be in the form of a random field with Gaussian distribution and a δ-like correlation function:

$$\frac{\omega^4}{c^4}\langle\varepsilon_1(\mathbf{r})\varepsilon_1(\mathbf{r}')\rangle = g\delta(\mathbf{r}-\mathbf{r}'), \tag{3}$$

and $\mathbf{j}$ is the source, which for a moving charge with velocity $\mathbf{v}$ in the direction z has the form

$$\mathbf{j}(\mathbf{r}, \omega) = \frac{4\pi i e\omega\mathbf{v}}{vc^2}\delta(x)\delta(y)\exp\left(-\frac{i\omega z}{v}\right). \tag{4}$$

To separate the radiation due only to the fluctuations from the Cherenkov radiation, we represent the field $\mathbf{E}$ in the form of a sum of two components: $\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_1$, where the field $\mathbf{E}_0$ is the background field created by the moving charge in the homogeneous medium with dielectric constant ε_0, and $\mathbf{E}_1$ is the scattering field associated with the fluctuating dielectric constant $\varepsilon_1(\mathbf{r})$. In light of the foregoing, the fields $\mathbf{E}_0$ and $\mathbf{E}_1$ satisfy the equations

$$\nabla^2\mathbf{E}_0 - \operatorname{grad}\operatorname{div}\mathbf{E}_0 + \frac{\omega^2}{c^2}\varepsilon_0\mathbf{E}_0 = \mathbf{j}(\mathbf{r}),$$
$$\nabla^2\mathbf{E}_1 - \operatorname{grad}\operatorname{div}\mathbf{E}_1 + \frac{\omega^2}{c^2}\varepsilon_0\mathbf{E}_1 + \frac{\omega^2}{c^2}\varepsilon_1\mathbf{E}_1 = -\frac{\omega^2}{c^2}\varepsilon_1\mathbf{E}_0. \tag{5}$$

Note that usually the small term $\sim\varepsilon_1\mathbf{E}_1$ is discarded. We however will keep it since it is precisely this term which gives rise to the multiple scattering which leads to the diffusion of the pseudophoton, which in turn drastically alters the radiation intensity.

At large distances from the system the electromagnetic field can be considered to be a plane wave in which the moduli of the electric and magnetic field strengths are equal. Therefore the radiation intensity at the frequencies ω and $\omega + d\omega$ and in the direction of the solid angles Ω and $\Omega + d\Omega$ can be written as follows:

$$dI(\omega, \mathbf{n}) = {}^1\!/_2 c\varepsilon_0^{1/2}|\mathbf{E}_1(\mathbf{R})|^2R^2d\Omega d\omega. \tag{6}$$

Here $\mathbf{n}$ is a unit vector in the direction of the observation

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point $\mathbf{R}$, and Ω is the corresponding solid angle. As usual, at large distances from the system $R \gg L$, where L is the characteristic dimension of the system, $|\mathbf{E}_1(\mathbf{R})|^2$ behaves like $1/R^2$, so the intensity does not depend on R.

The expression for the intensity (6) must be averaged over all realizations of the random field $\varepsilon_1(\mathbf{r})$. For this purpose we introduce the Green's function of the second of Eqs. (5):

$$\left[(\nabla^2+k^2)\delta_{im}-\frac{\partial^2}{\partial x_i\,\partial x_m}+\frac{\omega^2}{c^2}\varepsilon_1(\mathbf{r})\delta_{im}\right]G_{mj}(\mathbf{r},\mathbf{r}')$$

$$=\delta_{ij}\delta(\mathbf{r}-\mathbf{r}'). \quad (7)$$

Here we have introduced the notation $k=(\omega/c)\varepsilon_0^{1/2}$ and summation over the repeated indices is understood. Using Eqs. (7) and (5), we have

$$I_{ij}(\mathbf{R})=\langle E_{1i}(\mathbf{R})E_{1j}^*(\mathbf{R})\rangle$$

$$=\frac{\omega^4}{c^4}\int d\mathbf{r}'\,d\mathbf{r}''\langle G_{ij}(\mathbf{R},\mathbf{r}')G_{jk}^*(\mathbf{R},\mathbf{r}'')\varepsilon_1(\mathbf{r}')\varepsilon_1(\mathbf{r}'')\rangle$$

$$\times E_{0j}(\mathbf{r}')E_{0k}^*(\mathbf{r}''). \quad (8)$$

It is clear that the radiation intensity is expressed in a simple way in terms of the tensor I_{ij}:

$$I=\sum_i I_{ii}.$$

3. AVERAGE PSEUDOPHOTON GREEN'S FUNCTION

In this section we will find in the ladder approximation a pseudophoton averaged Green's function which satisfies Eq. (8). In order to do this, we first find the free ($\varepsilon_1\equiv 0$) Green's function, which in the momentum representation satisfies the equation

$$[(k^2-q^2-i\delta)\delta_{im}+q_iq_m]G_{mj}{}^0(\mathbf{q})=\delta_{ij}. \quad (9)$$

We seek the solution of Eq. (9) in the form of a combination of transverse and longitudinal parts:

$$G_{mj}{}^0(\mathbf{q})=(\delta_{mj}-\hat{q}_m\hat{q}_j)G_\perp{}^0(q)+\hat{q}_m\hat{q}_jG_\parallel{}^0(q), \quad (10)$$

where $\hat{q}_i$ are the components of the unit vector in the $\mathbf{q}$ direction. Substituting Eq. (10) in Eq. (9) and solving, we find

$$G_\perp{}^0(q)=\frac{1}{k^2-q^2-i\delta},\qquad G_\parallel{}^0(q)=\frac{1}{k^2}. \quad (11)$$

Using Eqs. (10) and (11), we obtain the following expression for the free Green's function of the pseudophoton:

$$G_{ij}{}^0(\mathbf{q})=\frac{\delta_{ij}-q_iq_j/k^2}{k^2-q^2-i\delta}. \quad (12)$$

Note that the photon Green's function contains only a transverse part. Therefore the difference between the free Green's functions of the photon and the pseudophoton consists in the fact that for the photon the numerator in Eq. (12) contains q^2 in place of k^2. However, for weak scattering, values of q close to k play a major role, i.e., in this case the Green's functions of the photon and the pseudophoton coincide. It is precisely for this reason that we say that the scattering of the photon and the pseudophoton take place according to the same laws. For example, we will show below that the mean free paths of the photon and the pseudophoton coincide.

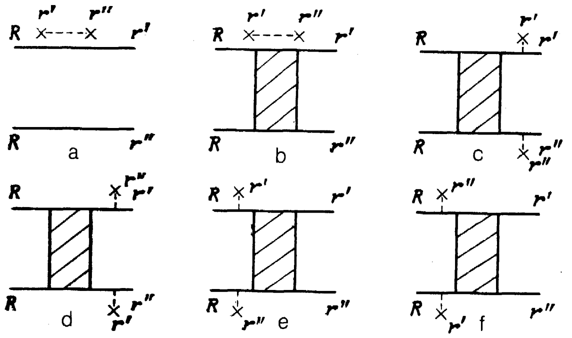

FIG. 1. Diagram for the average radiation intensity.

In the coordinate representation it is possible to obtain from Eq. (10) the following expression for the Green's function:

$$G_{ij}{}^0(\mathbf{R})=\frac{\delta_{ij}}{4\pi R}\exp(ikR)+\frac{1}{4\pi k^2}\frac{\partial^2}{\partial R_i\,\partial R_j}\frac{\exp(ikR)}{R}. \quad (13)$$

Using the extrinsic diagram technique,[5] we obtain an equation which the average Green's function satisfies in the coherent potential approximation:

$$G_{ij}(\mathbf{q})=G_{ij}{}^0(\mathbf{q})+gG_{im}{}^0(\mathbf{q})\int\frac{d\mathbf{p}}{(2\pi)^d}G_{ml}{}^0(\mathbf{p})G_{lj}(\mathbf{q}). \quad (14)$$

The solution of Eq. (14) can be represented in the form

$$G_{ij}(\mathbf{q})=\frac{\delta_{ij}-q_iq_j/k^2}{k^2-q^2-i\,\mathrm{Im}\,\Sigma}, \quad (15)$$

where $\mathrm{Im}\,\Sigma=kg/6\pi$. In this approximation the mean free path is given by $l=k/\mathrm{Im}\,\Sigma=6\pi/g$, i.e., it coincides with the photon mean free path (cf. Ref. 6). Note that in Eq. (15) we have discarded the term $\mathrm{Re}\,\Sigma$, which, as usual, gives an insignificant renormalization of the parameters.

4. AVERAGE RADIATION INTENSITY

As was shown above, the average radiation intensity is given by expressions (6) and (8). Since we intend to calculate the diffusion contribution to the radiation intensity, we present the simplest diagrams (Fig. 1) containing the diffusion mode and contributing to expression (8). In Fig. 1 a rectangle denotes the so-called diffusion (see, e.g., Ref. 7), defined in Fig. 2.

In the diagrams the solid lines denote the average Green's function, and the dashed lines—the random field correlator. We note only that the terminal Green's functions containing the observation point outside the system are free. The vector indices are written out for brevity only for the

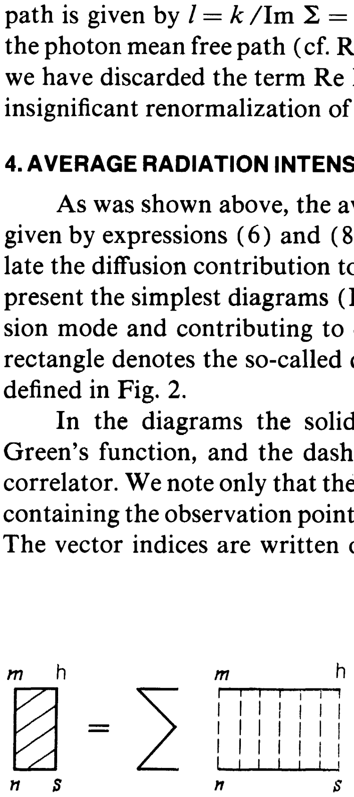

FIG. 2. Definition of the diffusion for the pseudophoton.

diagram that defines the diffusion. To start with, we calculate the contributions of the irreducible diagrams a and b. Using Eqs. (3) and (8), we have

$$I_{ij}{}^{0}=\frac{g}{\varepsilon_0{}^2}\int d\mathbf{r}G_{if}{}^{0}(\mathbf{R},\mathbf{r})G_{jk}{}^{0*}(\mathbf{R},\mathbf{r})E_{0f}(\mathbf{r})E_{0k}{}^{*}(\mathbf{r}),$$

$$I_{ij}{}^{D}=\frac{g}{\varepsilon_0{}^2}\int d\mathbf{r}\,d\mathbf{r}_1\,d\mathbf{r}_2 G_{im}{}^{0}(\mathbf{R},\mathbf{r}_1)G_{jn}{}^{0*}(\mathbf{R},\mathbf{r}_1)P_{mnhs}(\mathbf{r}_1,\mathbf{r}_2)$$
$$\times G_{hf}(\mathbf{r}_2,\mathbf{r})G_{sk}{}^{*}(\mathbf{r}_2,\mathbf{r})E_{0f}(\mathbf{r})E_{0k}{}^{*}(\mathbf{r}). \qquad (16)$$

Here P_{mnhs} is the diffusion propagator of the pseudophoton (Fig. 2). The background field $\mathbf{E}_0$ in Eqs. (16) can be found from Eq. (5):

$$E_{0j}(\mathbf{q})=\frac{8\pi^2 e\omega}{ic^2}\left[\frac{v_j}{k^2-q^2}-\frac{vq_zq_j}{k^2(k^2-q^2)}\right]\delta(\omega-q_zv). \qquad (17)$$

At large distances from the system ($R\gg r$) the function $G_{ij}^{0}(\mathbf{R},\mathbf{r})$ can be represented in the form

$$G_{ij}{}^{0}(\mathbf{R},\mathbf{r})=\frac{\delta_{ij}-n_in_j}{4\pi R}\exp\{ik(R-\mathbf{nr})\}. \qquad (18)$$

Substituting Eq. (18) in Eqs. (16), we obtain (θ is the observation angle)

$$I_{ij}{}^{0}(\theta,\omega)=\frac{g}{16\varepsilon_0{}^2\pi^2R^2}(\delta_{if}-n_in_f)(\delta_{jk}-n_jn_k)\int d\mathbf{r}E_{0f}(\mathbf{r})E_{0k}{}^{*}(\mathbf{r}). \qquad (19)$$

Using Eq. (17) for the part of the intensity I^0 due to single scattering, we find from Eqs. (19) and (6)

$$I^0(\theta,\omega)=\frac{e^2\omega^2}{\varepsilon_0{}^{3/2}c^3}\frac{gd}{16k^2}\left[2\sin^2\theta\left(\frac{k_0{}^2}{k^2}-1\right)+(1+\cos^2\theta)\frac{k_0{}^2}{k^2}\left(\ln\frac{1}{a_{min}{}^2(k_0{}^2-k^2)}-1\right)\right]. \qquad (20)$$

Here d is the path taken by the particle in the medium, $k_0=\omega/v$, and it was assumed in obtaining relation (20) that $k<k_0$ (the condition of the absence of Cherenkov radiation). The logarithmic divergence in Eq. (20) at small distances is connected with the δ-like character of the correlation function of the random field. Taking the finite radius of the correlation function into account, the divergent integral is truncated at this radius[8] and $a_{\min}$ is simply this radius.

Now let us calculate the diffusion contribution to the radiation intensity. Substituting Eq. (18) into Eq. (16), we obtain

$$I_{ij}{}^{D}=\frac{g}{16\pi^2R^2\varepsilon_0{}^2}(\delta_{im}-n_in_m)(\delta_{jn}-n_jn_n)P_{mnhs}(\mathbf{q}=0)$$
$$\times\left[\int d\mathbf{r}G_{hf}(\mathbf{r})G_{sk}{}^{*}(\mathbf{r})\right]\left[\int d\mathbf{r}E_{0f}(\mathbf{r})E_{0k}{}^{*}(\mathbf{r})\right]. \qquad (21)$$

As was stated above, the scattering of the photon and the pseudophoton take place in the same way. This means in particular that the corresponding diffusion propagators coincide. Therefore we can use here the diffusion propagator for the photon which at small momenta and for an unbounded medium is equal to (see, e.g., Ref. 6)

$$P_{mnhs}(\mathbf{q})\approx\delta_{mn}\delta_{hs}P(q)\approx\delta_{mn}\delta_{hs}g/q^2l^2. \qquad (22)$$

Hence it follows that for an unbounded medium the radiation intensity diverges. However, it is clear that if we allow for the dimensions of the system the smallest possible momentum in the system will be $\sim 1/L$, where L is the characteristic dimension of the system. Therefore for a finite system the quantity $P(q=0)$ is replaced by $\sim gL^2/l^2$. Substituting relations (22) into Eq. (21) and calculating the integral over the Green's functions with the help of Eq. (12), we obtain

$$I_{ij}{}^{D}=\frac{1}{16\pi^2\varepsilon_0{}^2R^2}(\delta_{ij}-n_in_j)P(q=0)\int d\mathbf{r}|\mathbf{E}_0(\mathbf{r})|^2. \qquad (23)$$

After calculating the integral of $|\mathbf{E}_0(\mathbf{r})|^2$ for the diffusion component of the intensity we find from Eq. (6)

$$I^D=\frac{e^2\omega^2}{\varepsilon_0{}^{3/2}c^3}\frac{gd}{4\pi k^2}\frac{L^2}{l^2}\left[\frac{k_0{}^2}{k^2}\ln\frac{1}{a_{min}{}^2(k_0{}^2-k^2)}-1\right]. \qquad (24)$$

As follows from Eqs. (24) and (20), $I^D/I^0\sim L^2/l^2\gg 1$. This means that the main radiation mechanism is pseudophoton diffusion. We note that the radiation associated with this mechanism is isotropic.

In the derivation of Eq. (24) it was assumed for definiteness that $l_{\rm in}\gg L$. If the inverse inequality $L\gg l_{\rm in}$ obtains then L in Eq. (24) is replaced by $l_{\rm in}$.

Note that for expression (22) to be finite if suffices to have the system bounded in one direction. For example, if the system is bounded in the z direction (even if $L_z\gg l$), then using the method of images, which permits us to express the diffusion propagator for a semi-infinite medium in terms of the corresponding propagator of the infinite medium, it can be shown that in this case $P(q=0)$ is replaced by $\sim L_z{}^2/l^2$.

Let us now estimate the contribution of the remaining diagrams. Calculations show that even though diagrams c and d are also proportional to $P(q=0)$, they are still small in relation to I^D. The first is of the order of $(1/k^2l^2)(1/k^2a_{\min}^2)$, and the second is of the order of $1/k^2l^2$. As to diagrams e and f, they are in general nonsingular. Thus, the main contribution to the radiation intensity comes from diagram b. We emphasize that this conclusion bears a direct relation to the contribution to the radiation intensity. If $\mathbf{r}'$ and $\mathbf{r}''$ coincide, the contributions of the reducible (b) and irreducible (c and d) diagrams are of the same order and if we calculate a quantity containing the tensor I_{ij} (8) for coincident $\mathbf{r}'$ and $\mathbf{r}''$, then the contributions of the irreducible diagrams must also be taken into account.

5. POLARIZATION OF THE RADIATION

Let us now determine the polarization of the radiation due to pseudophoton diffusion. At large distances from the system $R\gg r$, the radiation field can be taken to be a plane wave whose direction of propagation coincides with $\mathbf{n}$, but whose electric field vector lies in the plane perpendicular to $\mathbf{n}$ (see Fig. 3).

The polarization tensor is defined as follows:[9]

$$\rho_{\alpha\beta}=\langle E_\alpha E_\beta{}^{*}\rangle/I^D. \qquad (25)$$

Here $\alpha,\beta=1$, 2, and E_1 and E_2 are the projections of the vector $\mathbf{E}$ on the mutually perpendicular directions 1 and 2

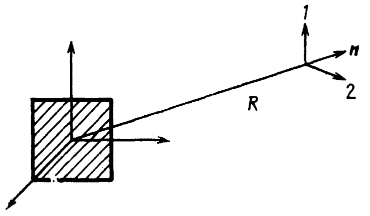

FIG. 3. Calculating the polarization tensor.

(see Fig. 3). To find the polarization tensor $\rho_{\alpha\beta}$ we express the components E_α in terms of the components E_i:

$$E_\alpha = e_{\alpha i} E_i. \qquad (26)$$

Since the directions 1, 2, and **n** are mutually perpendicular, the following relations hold between the components $e_{\alpha i}$:

$$e_{\alpha i} e_{\beta i} = \delta_{\alpha\beta}, \qquad e_{\alpha i} n_i = 0. \qquad (27)$$

Substituting Eq. (26) into Eq. (25) and using Eqs. (27) and (23), we find

$$\rho_{\alpha\beta} = {}^1/_2 \delta_{\alpha\beta}. \qquad (28)$$

This form of the tensor corresponds completely to unpolarized radiation. Note that the polarization of the radiation due only to single scattering of the pseudophoton[1] differs sharply from expression (28).

6. RADIATION-INTENSITY FLUCTUATIONS

Our analysis shows that the main contribution to the variance of the radiation intensity is given by the term

$$\langle \delta I^2(\mathbf{R}) \rangle = \frac{\sigma^2}{c^4} \int d\mathbf{r}_1 \, d\mathbf{r}_2 \langle G_{ij}(\mathbf{R}, \mathbf{r}_1) G_{mh}^*(\mathbf{R}, \mathbf{r}_1) \rangle \times \langle G_{mg}(\mathbf{R}, \mathbf{r}_2) G_{ik}^*(\mathbf{R}, \mathbf{r}_2) \rangle \times E_{0j}(\mathbf{r}_1) E_{0k}^*(\mathbf{r}_2) E_{0g}(\mathbf{r}_2) E_{0h}^*(\mathbf{r}_1). \qquad (29)$$

The main contribution to the averages in expression (29) is given by the diagram containing one diffusion. Using Eqs. (12) and (18), it can be shown that

$$\langle G_{ij}(\mathbf{R}, \mathbf{r}_1) G_{mh}^*(\mathbf{R}, \mathbf{r}_1) \rangle = \frac{1}{16\pi^2 R^2} (\delta_{im} - n_i n_m) P(q=0) \frac{l}{6\pi} \delta_{jh}. \qquad (30)$$

Substituting Eq. (30) into Eq. (29), we finally find for the variance

$$\langle \delta I^2(\mathbf{R}) \rangle = {}^1/_2 \langle I \rangle^2. \qquad (31)$$

Thus we see that the radiation intensity fluctuations are anomalously large, in fact of the order of the intensity itself. Such a behavior can be explained in the following way. We know[10] (see also Ref. 11) that during the diffusive propagation of light in a randomly inhomogeneous medium the intensity undergoes large fluctuations. Clearly the same thing happens during the propagation of a pseudophoton in a randomly inhomogeneous medium. It is precisely these fluctuations that are manifested in the radiation intensity.

7. CONCLUSION

The aim of the present article has been to take account of the effects of multiple scattering of an electromagnetic field during the radiation of a charge moving in a randomly inhomogeneous medium. The pseudophoton concept, with the help of which many well-known results in the propagation of electromagnetic waves in randomly inhomogeneous media can be applied to the problem of radiation, turns out to be quite convenient.

We have shown that multiple scattering leads to pseudophoton diffusion and that the contribution of this diffusion to the radiation intensity is the main one in the shortwave region $\lambda \ll l$. The main features of this radiation are its isotropy and its completely unpolarized character. This behavior is explained by the fact that in diffusion there is no preferred direction. The radiation intensity fluctuations, similar to many other disordered systems, are anomalously large.

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