Alignment of the angular momentum of the "odd nucleons" in connection with low-frequency anomalies in the rotational spectra of nuclei

A. M. Kamchatnov

I. V. Kurchatov Institute of Atomic Energy, Moscow

(Submitted 12 March 1990; resubmitted 15 May 1990)
Zh. Eksp. Teor. Fiz. **98**, 754–763 (September 1990)

Equations of motion are constructed for a "rotator + particle" model describing the alignment of the angular momentum of the "odd nucleons" during the rotation of nuclei. A general solution of the equations of motion is found. The steady-state rotations pertinent to nuclear physics are studied in detail. The results are compared with experimental data.

1. INTRODUCTION

The experimental discovery[1] of "backbending" in the rotational bands of nuclei has attracted much interest to the interaction of nuclear rotation with the internal degrees of freedom of the nucleus, i.e., essentially to the interaction with the angular momenta of the individual nucleons.[2,3] Since the individual angular momenta also interact with each other, the rotation of the nucleus leads to a complex restructuring of their coupling: Whereas at low angular velocities the angular momenta are oriented toward the symmetry axis of the nucleus, **n**, so the projection of the total angular momentum onto its axis is conserved, at high angular velocities these angular momenta are oriented toward the (vector) angular rotation velocity $\mathbf{\Omega}$. There is accordingly a transition from a (**jn**) coupling of the angular momenta **j** of the individual nucleons to a ($\mathbf{j\Omega}$) coupling.[4] Since the angular momenta **j** interact with each other, and since none of them plays any particularly special role, this restructuring of the coupling is of a collective nature. Thermodynamic methods can accordingly be used to analyze it; they lead to a satisfactory phenomenological description[5–7] of backbending.

There is, however, another limiting case, which may occur (for example) for certain rotational bands of odd nuclei when the coupling of the angular momentum of the odd nucleon with the angular momenta of the other particles is weak in comparison with the coupling of these other angular momenta with each other. If the interaction of this odd nucleon with the other angular momenta is slight, this fact will be manifested experimentally as so-called untied rotational bands (Ref. 8, for example), in which the distances between energy levels are nearly the same as in the corresponding neighboring even-even nucleus. However, one also runs into cases in which the coupling of the odd angular momentum is not particularly weak, and this angular momentum becomes aligned along the angular rotation velocity of the nucleus, $\mathbf{\Omega}$, only if the rotation is sufficiently fast, i.e., only if $\Omega = |\mathbf{\Omega}|$ is larger than a certain critical value Ω_c. On the other hand, Ω_c cannot be so large that the other particles of the core begin to align. The "rotator + particle" model,[9–11] in which all the core particles are collected in a single rotating rotator, is widely used to describe this situation. The angular momentum **j** of the odd nucleon interacts with both the angular rotation velocity of the rotator, $\mathbf{\Omega}$, and the rotator axis **n**. The angular momenta of the particles constituting the rotator, in contrast, are regarded as being coupled so closely that the rotation does not restructure them.

The rotational bands of even-even nuclei constructed from excited states with a nonzero angular momentum may have some similar properties. In speaking of the built-in angular momentum of the "odd nucleons" below, we will mean any situation in which an additional angular momentum is aligned along the angular velocity vector. The built-in angular momentum is usually quite large ($j \sim 5$–10, in units of $\hbar$), so the classical approach to the model can be taken as a first approximation.[10]

In the present paper we examine the version of the rotator + particle model in which the equations of motion are completely integrable.[1)] In this case, the motion of the system can be studied in some detail over the entire range of angular velocities Ω, including the alignment point Ω_c. From the experimental standpoint, the most important conclusion reached here is that a plot of the angular rotation velocity of the nucleus, Ω, versus the angular momentum of the nucleus, J, has a change of slope at the alignment point, and the corresponding effective moment of inertia decreases abruptly at this point. In the final section of this paper we discuss experimental data which have revealed "low-frequency anomalies" of this sort in the rotational spectra of atomic nuclei.

2. EQUATIONS OF MOTION

We write the Lagrangian of a system consisting of a rotator with a moment of inertia I, rotating at an angular velocity $\mathbf{\Omega}$, and an angular momentum $\mathbf{j} + K\mathbf{n}$, which is "built into" the rotator, in the following form:

$$L = I(\Omega_1^2 + \Omega_2^2)/2 + (\mathbf{j} + K\mathbf{n})\mathbf{\Omega} - j\omega_0 U(\chi). \qquad (1)$$

Here $(\Omega_1, \Omega_2, \Omega_3)$ are the projections of the angular velocity vector onto the axes of the coordinate system moving with the rotator. As usual, axis 3 is directed along the rotator, and the corresponding moment of inertia is zero. This situation corresponds to an inability of the nucleus to rotate around its symmetry axis.[14] We assume that the built-in angular momentum consists of two parts. The angular momentum **j** of the odd nucleons interacts with the rotator axis **n** through the potential $U(\chi)$, where χ is the angle between the vectors **j** and **n**. The coefficient $j\omega_0$ of U is chosen from dimensionality considerations. The angular momenta of the particles constituting the rotator are combined in the angular momentum $K\mathbf{n}$, which is directed strictly along the rotator. As was pointed out in the Introduction, the rotation is assumed to be adiabatically slow for these particles. The second term on the right side of (1) describes the interaction of the built-in angular momentum $\mathbf{j} + K\mathbf{n}$ with the angular velocity $\mathbf{\Omega}$. If the angular momentum **j** is fixed with respect to

 0038-5646/90/090418-06$03.00

the coordinate system moving with the rotator, then Lagrangian (1) (without the last term, which is constant in this case) leads to equations of motion which are equivalent to the Volterra equations which are used to describe this system.[15] This circumstance shows that choosing the interaction term in the Lagrangian in this particular form is valid at large as well as small angular velocities. When we go over to the Hamilton's variables, on the other hand, this simple form of the interaction is valid only at small angular velocities $\mathbf{\Omega}$.

We express the angular velocity vector in terms of Euler angles:[16]

$$\Omega_1=\dot\varphi \sin\theta \sin\psi + \dot\theta \cos\psi,\quad \Omega_2=\dot\varphi \sin\theta\cos\psi-\dot\theta\sin\psi,\quad \Omega_3=\dot\varphi\cos\theta+\dot\psi. \quad (2)$$

Making use of the inability of the nucleus to rotate around its symmetry axis, mentioned above, we fix the coordinate system moving with the rotator in such a way that its second axis runs perpendicular to the plane defined by the vectors $\mathbf{j}$ and $\mathbf{n}$. We then have

$$\mathbf{j}=(j\sin\chi,\ 0,\ j\cos\chi),\quad n=(0,\ 0,\ 1), \quad (3)$$

and the Lagrange equations for the variables θ, φ, ψ, χ take the following form, after some simple manipulations:

$$I\dot\Omega_1\cos\psi-I\dot\Omega_2\sin\psi-I\Omega_1\Omega_3\sin\psi-I\Omega_2\Omega_3\cos\psi -j\Omega_3\sin\chi\sin\psi+j\dot\chi\cos\chi\cos\psi+(K+j\cos\chi)\dot\varphi\sin\theta=0, \quad (4)$$

$$I\dot\Omega_1\sin\psi+I\dot\Omega_2\cos\psi+I\Omega_1\Omega_3\cos\psi-I\Omega_2\Omega_3\sin\psi +j\Omega_3\sin\chi\cos\psi+j\dot\chi\cos\chi\sin\psi-(K+j\cos\chi)\dot\theta=0, \quad (5)$$

$$\dot\chi=-\Omega_2 \quad (6)$$

and

$$\Omega_1\cos\chi-\Omega_3\sin\chi-\omega_0U'=0,\quad U'=dU/d\chi. \quad (7)$$

Taking the obvious linear combinations of Eqs. (4) and (5), we can put system (4)–(6) in the form

$$\dot\Omega_1=\Omega_2\Omega_3-K\Omega_2/I,$$
$$\dot\Omega_2=-\Omega_1\Omega_3+K\Omega_1/I+j\omega_0U'/I, \quad (8)$$
$$\dot\chi=-\Omega_2.$$

By virtue of relation (7), the number of independent variables is three.

Equations (7) and (8) are the equations of motion for our mechanical model. It is not difficult to see that they have two integrals of motion: the square of the total angular momentum,

$$J^2=(I\Omega_1+j\sin\chi)^2+(I\Omega_2)^2+(K+j\sin\chi)^2, \quad (9)$$

and the total energy,

$$E=I(\Omega_1^2+\Omega_2^2)/2+j\omega_0U(\chi). \quad (10)$$

These two integrals are sufficient to reduce the integration of Eqs. (7) and (8) to quadrature form. However, to carry out a detailed study we would naturally like to specify the potential $U(\chi)$ in some way; we do this in the following section of this paper.

3. ALIGNMENT OF THE ANGULAR MOMENTUM OF THE ODD NUCLEONS

The discussion below is restricted to the case $K=0$, which embodies all the characteristic features of the phenomenon of interest here. The potential U can be chosen on the basis of the following considerations. We consider the particular solution $\dot\chi=\Omega_2=0$, which corresponds to a steady-state rotation. In this case we have $\dot\Omega_1=0$, since we have $\Omega_1=\text{const}$ and $\Omega_3=j\omega_0U'/I\Omega_1$. Relation (7) then gives us the following expression for Ω_1:

$$\Omega_1=[\omega_0U'+(\omega_0^2U'^2+4j\omega_0U'\sin\chi\cos\chi/I)^{1/2}]/2\cos\chi. \quad (11)$$

It is not difficult to see that the problem simplifies dramatically if we choose

$$U(\chi)=-\cos^2\chi, \quad (12)$$

where the minus sign corresponds to an attraction of $\mathbf{j}$ to $\mathbf{n}$. Physical considerations also lead us to make this choice, since it satisfies the requirement of invariance under the substitution $\mathbf{n}\to-\mathbf{n}$. Furthermore, potential (12) is quadratic in χ at $|\chi|\ll1$ and also in $\chi-\pi/2$ at $|\chi-\pi/2|\ll1$, so the choice in (12) actually does not restrict the generality of the analysis in the regions of most importance: at the beginning of the rotational band and near the point at which $\mathbf{j}$ becomes aligned along $\mathbf{\Omega}$.

For potential (12), expression (11) becomes

$$\Omega_1=[\omega_0+(\omega_0^2+2j\omega_0/I)^{1/2}]\sin\chi. \quad (13)$$

The angle χ takes on the value $\pi/2$; i.e., the angular momentum $\mathbf{j}$ becomes completely aligned along $\mathbf{\Omega}$ at

$$\Omega_1=\Omega_c\equiv\omega_0+(\omega_0^2+2j\omega_0/I)^{1/2}. \quad (14)$$

This angular velocity corresponds [according to (9) with $K=0$, $\Omega_2=0$, and $\chi=\pi/2$] to a critical value of the angular momentum,

$$J_c=j+I\omega_0+(I^2\omega_0^2+2jI\omega_0)^{1/2}. \quad (15)$$

We thus have the useful relationship

$$I\omega_0=(J_c-j)^2/2J_c. \quad (16)$$

Specifying $U(\chi)$ as in (12), and introducing J_c, we can proceed to the integration of equations of motion (7) and (8) (with $K=0$). The integral in (9) makes it possible to express Ω_1 in terms of Ω_2 and χ. When the resulting expression is substituted into (10), we obtain an equation from which we can find Ω_2 as a function of χ. As a result, the last equation of system (8) gives us a differential equation for χ, which can be written in the form

$$\left(\frac{d\cos\chi}{dt}\right)^2=\frac{(J_c^2-j^2)^2}{4I^2J_c^2}(\mu_1^2-\cos^2\chi)(\cos^2\chi-\mu_2^2), \quad (17)$$

where

$$\mu_{1,2}^2=\frac{4IJ_c^2}{(J_c^2-j^2)^2}\left\{-E-\frac{(J_c-j)^2}{2jJ_c}\left(E-\frac{J^2+j^2}{2I}\right)\pm[(E-E_1)(E-E_2)]^{1/2}\right\}, \quad (18)$$

$$E_1=\frac{J_c-j}{2IJ_c}(J^2-jJ_c),\qquad E_2=-\frac{J_c-j}{2IjJ_c}(J_cJ^2-j^3). \quad (19)$$

We know that solutions of Eq. (17) can be expressed in terms of elliptic functions.[17] If the parameter values are such that the relations $0\leqslant\mu_2^2\leqslant\mu_1^2\leqslant1$ hold, then $\cos\chi$ oscillates in the interval $\mu_2\leqslant\cos\chi\leqslant\mu_1$ in accordance with

$$\cos\chi=\mu_1\,\mathrm{dn}\left[\frac{\mu_1(J_c^2-j^2)}{2IJ_c}(t-t_0),k_a\right],\qquad(20)$$

where $k_a=(1-\mu_2^2/\mu_1^2)^{1/2}$ is the modulus of the elliptic function. With $\mu_1=\mu_2$, so that we have $k_a=0$, we find $\cos\chi=\mu_1$; i.e., we return to the case of a steady-state rotation. According to (18), the energy E takes on its minimum value E_1 in this case, so the dependence of the energy on the angular momentum for a steady-state rotation is described by

$$E_a=(J^2-jJ_c)/2I_a,\qquad(21)$$

where the effective moment of inertia is

$$I_a=\frac{J_cI}{J_c-j}>I.\qquad(22)$$

These expressions hold in the region $j<J<J_c$, which we call the "a phase," and in which we have

$$\cos\chi=\mu_1=\left(\frac{J_c^2-J^2}{J_c^2-j^2}\right)^{1/2}.\qquad(23)$$

The equation $\Omega_2=0$ leads to $\psi=\pi/2$ here (since we have $\dot\theta=0$, $\varphi=\text{const}=\Omega_a$), and conservation laws (9) and (10) take the following form in this case:

$$J^2=(I\Omega_a\sin\theta+j\sin\chi)^2+j^2\cos^2\chi,\qquad(24)$$

$$E=I\Omega_a^2\sin^2\theta/2-j\omega_0\cos^2\chi$$
$$=[(J^2-j^2\cos^2\chi)^{1/2}-j\sin\chi]^2/2I-j\omega_0\cos^2\chi.\qquad(25)$$

At a fixed value of the angle χ, expression (25) without its last term is the same as the equation which Kramers and Pauli[18] derived by a different method in the theory of band spectra describing the interaction of the rotation of a molecule with the fixed angular momentum of electrons.

Since with $\psi=\pi/2$ we have $\Omega_1=\dot\varphi\sin\theta$, $\Omega_2=0$, $\Omega_3=\dot\varphi\cos\theta$, we find the following expression for the absolute value of the angular velocity:

$$\Omega_a=(\Omega_1^2+\Omega_3^2)^{1/2}=\dot\varphi=\frac{dE_a}{dJ}=\frac{J_c-j}{IJ_c}J.\qquad(26)$$

Using this expression and (24), we can find the angle θ:

$$J\cos\theta=j\cos\chi.\qquad(27)$$

This equation has a clear physical meaning: The projection of the total angular momentum onto the axis of the rotator is equal to the projection of the "built-in angular momentum" onto this axis.

It follows from (23) and (27) that at small values of $J-j$, i.e., at the beginning of the rotational band, the angles χ and θ increase with increasing $J-j$ in a square-root fashion:

$$\chi\approx\left[\frac{2j(J-j)}{J_c^2-j^2}\right]^{1/2},\quad\theta\approx\frac{J_c}{(J_c^2-j^2)^{1/2}}\left[\frac{2(J-j)}{j}\right]^{1/2},\quad J-j\ll j.\qquad(28)$$

The limiting expression for θ which we have found might be compared with the corresponding expression for the well-known case of the theory of an adiabatically slow motion, in which the built-in angular momentum j is directly strictly along $\mathbf{n}$: $j=j\mathbf{n}$. We then have $\cos\theta=j/J$, so at small values of θ and $J-j$ we have

$$\theta_{\rm ad}\approx\left[\frac{2(J-j)}{j}\right]^{1/2}.\qquad(29)$$

A comparison with the second expression in (28) shows that in the limit in which the angular momentum $\mathbf{j}$ is strongly coupled with the axis $\mathbf{n}$, in which we have $I\omega_0\gg j$ and $J_c\gg j$, this expression does indeed become expression (29) for adiabatically slow rotation.

As the point of total alignment is approached, i.e., as $J\to J_c$, the angle χ tends toward $\pi/2$ in accordance with

$$\pi/2-\chi\cong(2/J_c)^{1/2}(J_c-J)^{1/2},\quad J_c-J\ll J_c.\qquad(30)$$

It follows from solution (20) that the frequency of the small oscillations around this steady-state rotation is

$$\omega_a=\left.\frac{\pi\mu_1(J_c^2-j^2)}{2K(k_a)IJ_c}\right|_{k_a\to0}=[(J_c^2-j^2)(J_c^2-J^2)]^{1/2}/IJ_c,\quad(31)$$

where $K(k)$ is a complete elliptic integral of the first kind. The frequency in (31) vanishes in a square-root fashion as $J\to J_c-0$, and it becomes imaginary at $J>J_c$. The latter result is evidence that this steady-state rotation becomes unstable in the upper g phase, with $J>J_c$.

At $J>J_c$, where $\mathbf{j}$ is completely aligned along $\mathbf{\Omega}$, and the rotator axis is perpendicular to the rotation axis, along which both $\mathbf{J}$ and $\mathbf{\Omega}$ are directed, we obviously have

$$\varphi=\Omega_gt,\quad\theta=\psi=\chi=\pi/2,\quad J>J_c.\qquad(32)$$

Using (25), we find the energy and the angular momentum in the g phase:

$$E_g=(J-j)^2/2I,\qquad(33)$$

$$\Omega_g=\dot\varphi=\frac{dE_g}{dJ}=\frac{J-j}{I}.\qquad(34)$$

The frequency of the small oscillations around the steady-state rotation in the g phase can be found from the corresponding solution of Eq. (17). With $E=E_g$ we have

$$\mu_1^2=0,\ \mu_2^2=-4J_c(J-J_c)(J_cJ-j^2)/(J_c^2-j^2)^{1/2}<0.$$

The solution of the equations of motion, which degenerates in the case $\mu_1\to0$, $E\to E_g$ into a steady-state rotation with $J>J_c$, corresponds to parameter values such that we have $\mu_1^2>0$ and $\mu_2^2<0$. In this case, the solution of Eq. (17) is

$$\cos\chi=\mu_1\,\mathrm{cn}\left[\frac{(J_c^2-j^2)(\mu_1^2+|\mu_2^2|)^{1/2}}{2IJ_c}(t-t_0),k_g\right],\qquad(35)$$

where $k_g=(1+|\mu_2^2|/\mu_1^2)^{-1/2}$. This solution describes oscillations of $\cos\chi$ in the interval $-\mu_1\leqslant\cos\chi\leqslant\mu_1$, and the frequency of the small oscillations near the steady-state rotation in the g phase is

$$\omega_g=\left.\frac{\pi(J_c^2-j^2)(\mu_1^2+|\mu_2^2|)^{1/2}}{2K(k_g)IJ_c}\right|_{k_g\to0}=[(J-J_c)(J_cJ-j^2)]^{1/2}/IJ_c^{1/2}.\qquad(36)$$

This frequency becomes imaginary at $J<J_c$, telling us that this steady-state rotation, with a vector $\mathbf{j}$ completely aligned along $\mathbf{\Omega}$, is unstable at values of J corresponding to the lower (a) phase. Figure 1 shows a plot of the angular velocity of the steady-state rotation versus J [see (26) and (34)]. In the transformation from ($\mathbf{jn}$) coupling to ($\mathbf{j\Omega}$) coupling, the slope of the angular velocity changes at the alignment point

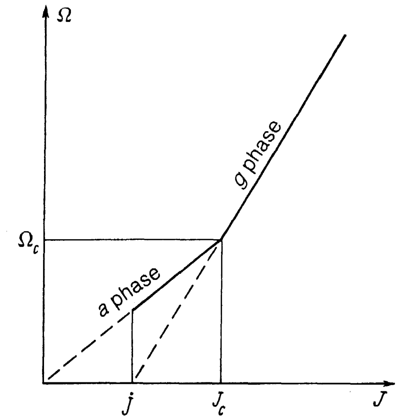

FIG. 1. Schematic plot of the angular velocity versus the angular momentum in the "rotator + particle" model.

J_c, while the effective moment of inertia decreases abruptly.

We can get an idea of the nature of the quantum-mechanical corrections by adding zero-point vibrations $\hbar\omega/2$, with frequencies (31) and (36), respectively, to the energies in (21) and (33):

$$E_a=\frac{\hbar^2}{2I}\frac{J_c-j}{J_c}(J^2-jJ_c)+\frac{\hbar^2}{2I}\times[(J_c^2-j^2)(J_c^2-J^2)]^{1/2}/J_c,\quad J<J_c, \tag{37}$$

$$E_g=\frac{\hbar^2}{2I}(J-j)^2+\frac{\hbar^2}{2I}\times[(J-J_c)(J_cJ-j^2)/J_c]^{1/2},\quad J>J_c. \tag{38}$$

All the angular velocities are expressed in units of $\hbar$. We should mention that near the alignment point J_c the oscillations of χ cannot be regarded as small, and expressions (37) and (38) are not applicable there. Evaluating the amplitude of the oscillations with the energy $\sim\hbar\omega_{a,g}$ and requiring that this amplitude be much smaller than unity, we find an applicability condition:

$$\frac{j^2}{J_c^2-j^2}\ll\frac{|J-J_c|}{J_c}. \tag{39}$$

It follows that a necessary condition for the applicability of expressions (37) and (38) near J_c is $j\ll J_c$; i.e., the coupling of the built-in angular momentum **j** with the **n** axis must be strong.

In addition to these corrections of a dynamic nature, we should also consider the corrections which arise because the angular momentum operators do not commute. At a formal level, these operators correspond to "zero-point vibration modes" of a system with frequencies equal to the rotation frequency, so that there is the typical quantum-mechanical replacement of the type $J^2\to J(J+1)$ (Ref. 12). Marshalek[19] has carried out a more rigorous analysis of these corrections with the help of Holstein-Primakoff transformations for the case of a built-in rotation in the model of Ref. 10. The results of Refs. 19 and 12 show that these corrections are relatively small at large values of J and are of a systematic nature at small values of J. They thus could not influence such important qualitative aspects of the motion as the nature of the singularity in the physical quantities at the alignment point J_c. If, on the other hand, we are interested in not only the vicinity of the point J_c but the entire *a* phase, then we should consider the deviations of potential $U(\chi)$ from the model (12) which we have been discussing. We should point out that when the *a* phase fills a large number of levels in the rotational band it becomes possible in principle to determine the actual potential $U(\chi)$ from experimental data.

4. STEADY-STATE ROTATION FOR AN ARBITRARY POTENTIAL $U(\chi)$

The basic characteristic measured in actual experiments is the dependence of the angular rotation velocity $\Omega_a=dE_a/dJ$ on the angular momentum J. In this section we show that the potential U can be expressed in terms of these two quantities Ω_a and J. In the steady state, there is no need to solve the equations of motion; it is sufficient to use simply the conservation laws. With $\Omega_1=\Omega_a\sin\theta$, $\Omega_2=\dot\chi=0$ these laws are

$$J^2=I^2\Omega_a^2\sin^2\theta+2j\sin\chi I\Omega_a\sin\theta+j^2, \tag{40}$$

$$E_a=I\Omega_a^2\sin^2\theta/2+j\omega_0U(\chi). \tag{41}$$

This becomes a closed system of equations when we make use of relationship (27), which is obvious for steady-state rotation. For convenience we replace $U(\chi)$ by

$$V(\chi)=2IE_a-2Ij\omega_0U(\chi). \tag{42}$$

Equations (40) and (41) then become

$$J^2-j^2=V+2j\sin\chi V^{1/2}, \tag{43}$$

$$V=I^2\Omega_a^2(1-j^2\cos^2\chi/J^2), \tag{44}$$

where $\sin\theta$ has been eliminated with the help of (27). Eliminating the angle χ from these two equations, we find the expression which we need for V:

$$V(J)=\frac{J^2-j^2}{(2J/I\Omega_a)-1}. \tag{45}$$

If the J dependence Ω_a is known from experiments, we can find the dependence $V(J)$ from (45), and we can then find $\cos^2\chi$ as a function of J from (44):

$$\cos^2\chi=(J^2/j^2)(1-V/(I\Omega_a)^2). \tag{46}$$

Combining $V(J)$ and $\cos^2\chi(J)$, we find the functional dependence $V(\chi)$; using (42), we then find the potential U as a function of the angle χ.

5. DISCUSSION

Examination of the experimental data presently available reveals that rotational bands of the type which we have been discussing here are rather rare. In the overwhelming majority of cases, the coupling of the built-in angular momentum **j** with the **n** axis is strong, on the same order of magnitude as the coupling of the angular momenta of the particles which constitute the rotator. Consequently, the restructuring of the coupling of the angular momenta is of a collective nature and is manifested as ordinary backbending. It follows that experimentally one should see primarily cases in which the coupling of the built-in angular momentum with the nuclear axis is weak, and the *a* phase consists of only a very small number of levels. It is important to note the following qualitative circumstance: In ordinary rotational bands, a plot of the angular velocity $\hbar\Omega = dE/dJ$ versus J in the g phase runs above the so-called solid-state line (I_0 is the solid-state value of the moment of inertia of the nucleus):

$$\hbar\Omega = \frac{\hbar^2}{I_0} J. \tag{47}$$

This is true with the possible exception of the very beginning of the rotational band. At the backbending point $J_c^{(b)}$, the plot of $\hbar\Omega$ descends below the solid-state line in (47) (Refs. 6 and 7). In contrast with backbending, the point at which the angular momentum of the odd nucleons becomes aligned is totally uncorrelated with the solid-state line. This qualitative circumstance makes it a simple matter to distinguish the alignment of the angular momentum **j** from a collective backbending. Furthermore, the moment of inertia of the ordinary rotational bands at the very beginning is usually smaller than the solid-state value. For a rotational band of the type which we have been discussing here, in contrast, the effective moment of inertia at the beginning of the band, where we find the a phase, may be above the solid-state value, as follows from (22), particularly in the case of weak coupling, with $J_c - j \ll J_c$. Beyond the alignment point J_c the effective moment of inertia drops abruptly to a value $I \sim I_0$, and as J is increased further we arrive at a comparatively well-studied ordinary backbending.

Let us look at some examples of rotational bands in which all these qualitative features can be seen clearly. The first example is the band constructed on the 7^- (1964 keV) excited state of the nucleus ^{154}Dy (Ref. 20). Figure 2 shows a plot of the angular velocity $\hbar\Omega = dE/dJ$ versus J. At angular momenta $J < 18$ the angular velocities lie above solid-state line (47), which corresponds to the solid-state value of the moment of inertia $\hbar^2/I_0 = 19.4$ keV. The levels with $J = 7, 9$, and 11 belong to the a phase, with a very high effective moment of inertia, $\hbar^2/I_a \approx 1$. At $J = J_c = 11$ there is a complete alignment of the angular momentum **j**, and the moment of inertia drops abruptly to a value close to the solid-state value. At $J = J_c^{(b)} = 18$, there is a typical backbending, and the curve of the angular velocity goes under the solid-state line. Shown for comparison in Fig. 3 is a plot of the angular velocity of the rotational band constructed on the ground state of the same nucleus. This band undergoes a backbending at $J_c^{(b)} = 15$; the angular velocity in the g phase lies below the solid-state line at all times (the solid-state line here corresponds to the same value of I_0 as in Fig. 2), and it has no anomalies of any sort at the beginning of the band.

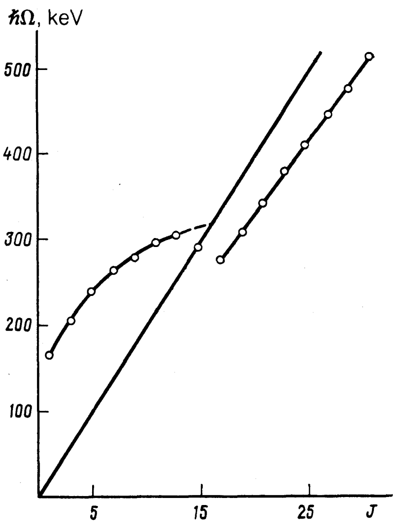

FIG. 3. Angular velocity of the yrast line of the nucleus ^{154}Dy.

Our second example is the $5/2^+$ band (99 keV) of the odd nucleus ^{163}Yb (Ref. 21). Figure 4 shows a plot of the angular velocity of this band versus J. Here again, the first three levels correspond to the a phase, which now lie entirely under the solid-state line. At $J_c = 13/2$, the odd angular momentum becomes aligned; this event is accompanied by a sharp decrease in the effective moment of inertia. This band has a fairly long g phase, and the backbending occurs at $I_c^{(b)} \approx 20$. In each case (Figs. 2 and 4), there is an obvious qualitative similarity between the initial part of the curve of the rotational bands and Fig. 1.

One might hope that further experimental research would result in the discovery of other rotational bands of this type, including some with a longer a phase, in which case the actual potential of the interaction of the odd angular momentum with the nuclear core could be determined, according to the results of the preceding section of this paper. At the same time, other versions of an early alignment of a small number of odd angular momenta may be encountered. In the

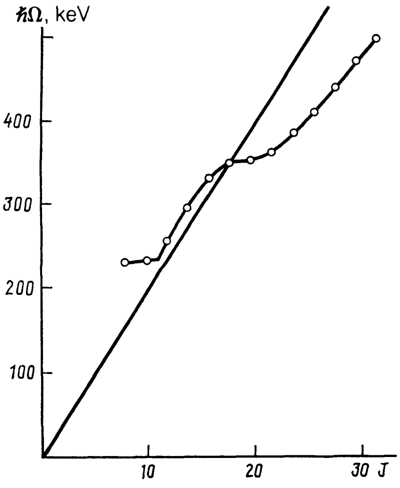

FIG. 2. Angular velocity of the 7^- band of the nucleus ^{154}Dy.

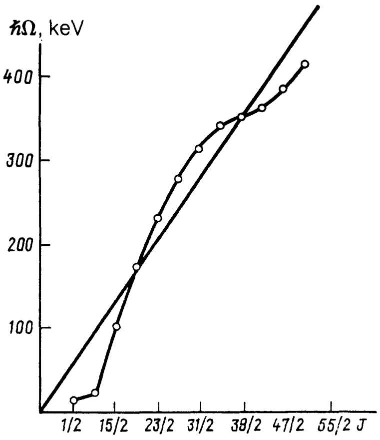

FIG. 4. Angular velocity of the $5/2^+$ band of the nucleus ^{163}Yb.

bands of even-even nuclei, for example, there may be cases in which the angular momenta of one pair of nucleons are easily aligned by the angular velocity, while the other nucleons are still combined in a common rotator. The yrast line of the nucleus ^{172}Os appears to be such a band; a low-frequency anomaly has been found there in the behavior of the effective moment of inertia.[22,23] The rotational band of the nucleus ^{171}Re exhibits a similar behavior.[24]

I am sincerely indebted to V. G. Nosov for numerous discussions of problems of nuclear rotation and also A. A. Vedenov, E. B. Levchenko, A. N. Timashev, A. L. Chernyakov, and V. R. Chechetkin for interesting discussions and useful comments.

[1)] A preliminary study of this model has been reported in some preprints.[12,13]

[1] A. Johnson, H. Ryde, and J. Starkier, Phys. Lett. B **34**, 605 (1971).
[2] F. S. Stephens, Rev. Mod. Phys. **47**, 43 (1975).
[3] R. M. Lieder and H. Ryde, Adv. Nucl. Phys. **10**, 1 (1978).
[4] F. S. Stephens and R. S. Simon, Nucl. Phys. A **183**, 257 (1972).
[5] V. G. Nosov and A. M. Kamchatnov, Zh. Eksp. Teor. Fiz. **73**, 785 (1977) [Sov. Phys. JETP **46**, 411 (1977)].
[6] V. G. Nosov and A. M. Kamchatnov, Zh. Eksp. Teor. Fiz. **76**, 1506 (1979) [Sov. Phys. JETP **49**, 765 (1979)].
[7] V. G. Nosov and A. M. Kamchatnov, Zh. Eksp. Teor. Fiz. **80**, 433 (1981) [Sov. Phys. JETP **53**, 221 (1981)].
[8] F. S. Stephens, P. Klenheinz, R. K. Sheline, and R. S. Simon, Nucl. Phys. A **222**, 235 (1974).
[9] A. Bohr and B. R. Mottelson, *Nuclear Structure*, Benjamin, New York, 1969 (Russ. Transl. Vol. 2, Mir, Moscow, 1977).
[10] A. Bohr and B. R. Mottelson, Phys. Scr. **22**, 461 (1980).
[11] I. M. Pavlichenkov, Zh. Eksp. Teor. Fiz. **96**, 404 (1989) [Sov. Phys. JETP **69**, 227 (1989)].
[12] A. M. Kamchatnov, Preprint IAÉ-4728/1, TsNIIatominform, Moscow, 1988.
[13] A. M. Kamchatnov, Preprint IAE-4854/1, Moscow, 1989.
[14] L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika (*Quantum Mechanics: Non-Relativistic Theory*), Nauka, Moscow, 1989.
[15] V. Volterra, Acta Mathematica 22, 201 (1898); Opere Matematiche 2 (1956).
[16] L. D. Landau and E. M. Lifshitz, *Mekhanika* (Mechanics), Nauka, Moscow, 1989.
[17] A. Erdélyi (editor), *Higher Transcendental Functions*, Vol. 3, McGraw-Hill, New York, 1953 (Russ. Transl. Vol. 3, Fizmatgiz, Moscow, 1967).
[18] H. A. Kramers and W. Pauli, Z. Phys. **13**, 351 (1923).
[19] E. R. Marshalek, Phys. Rev. C **26**, 1678 (1982).
[20] W. C. Ma, M. A. Quader, H. Emling *et al.*, Phys. Rev. Lett. **61**, 46 (1988).
[21] J. Kownacki, J. D. Garrett, J. J. Gaardhoje *et al.*, Nucl. Phys. A **394**, 269 (1983).
[22] J. L. Durell, G. D. Dracoulis, C. Fahlander, and A. P. Byrne, Phys. Lett. B **115**, 367 (1982).
[23] J. C. Wells, N. R. Johnson, C. Baktash *et al.*, Phys. Rev. C **36**, 431 (1987).
[24] R. A. Bark, G. D. Dracoulis, A. E. Stuchbery *et al.*, Nucl. Phys. A **501**, 157 (1989).

Translated by D. Parsons