Giant oscillations of the magnetothermoelectric power of a metal near an electronic topological transition

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An investigation is reported of the longitudinal magnetothermoelectric power of a metal which is in a state close to an electronic topological transition. Under certain conditions this power may exhibit giant oscillations as a function of the applied magnetic field. These oscillations arise because the electron relaxation time in a magnetic field depends on the electron energy and are typical of normal metals, irrespective of the shape of the Fermi surface. Nevertheless, both the form and the amplitude of these oscillations change greatly near a topological transition. The results of recent investigations of the magnetothermoelectric power of cadmium single crystals subjected under pressure in a magnetic field are compared with the predictions of the proposed theory.

1. INTRODUCTION

Detailed experimental and theoretical investigations have recently been made of the kinetics of electronic topological transitions.[1-12] The thermoelectric power of metals and alloys undergoing such transitions has attracted particular attention because they are accompanied by giant anomalies if the temperature of a sample is sufficiently low. These anomalies have been reported for $Li_{1-x}Mg_x$, $Cd_{1-x}Mg_x$, and $Bi_{1-x}Sb_x$ alloys, for Ta and In bulk samples, and for Bi and Al whiskers. The results of all the experiments are in good qualitative agreement with the current theoretical ideas.[8-12]

The next stage in the investigation of the kinetics of electron transitions in the vicinity of a topological transition was reached in the recent studies of the influence of a magnetic field on the anomalies of the thermoelectric power of Cd crystals near transitions induced by hydrostatic pressures.[13] Cadmium undergoes a complex change in the Fermi surface topology[14] at pressures close to $P_c = 16$ kbar in the absence of a magnetic field: the "arms" of a hole "monster" collapse in the second Brillouin zone and electron "needles" appear in the third zone. Varying the pressure near P_c also gives rise to a clear peak in the pressure dependence of the thermoelectric power; the amplitude of this peak depends strongly on the field but the field does not affect the position of the peak.[13] It is interesting to note that on one side of such a transition the thermoelectric power is practically independent of the magnetic field and on the other side the dependence is strong. The dependence of the magnetothermoelectric power (MTEP) on the applied magnetic field reported in Ref. 13 demonstrates a tendency to reach saturation far from an electronic topological transition, whereas near such a transition the MTEP peak rises monotonically with the field up to 6 T.

The MTEP of a metal undergoing an electronic topological transition in the absence of a magnetic field exhibits a giant peak of the thermoelectric power due to a change in the probability of the electron–impurity scattering near a transition point, as demonstrated in Refs. 9–11. On the other hand, it is well known that transport characteristics of a metal exhibit oscillations of the de Haas–van Alphen type as a function of the applied magnetic field H. However, whereas the de Haas–van Alphen oscillations are related to a periodic dependence of the thermodynamic potential $\Omega(\mu)$ on the applied field H, the transport characteristics oscillate because of changes in the processes of conduction electron scattering in a magnetic field. For example, an oscillatory correction to the electron–impurity relaxation time[15] is shown to be responsible for electrical conductivity of the Shubnikov–de Haas type. In other words, both in a magnetic field and near an electronic topological transition the expression for the relaxation time of conduction electrons includes corrections that depend in different ways on the energy and these corrections give rise to anomalous contributions to the thermoelectric power.

We shall use an electronic topological transition model of the "neck-breaking" type to calculate the longitudinal thermoelectric power of a metal near a transition in the presence of a magnetic field applied parallel to the neck axis. This geometry is selected in order to avoid the complicating influence of effects of the magnetic breakthrough type, which appear in this model of the Fermi surface when the magnetic-field orientation is transverse.

2. MODEL AND ELECTRON RELAXATION TIME NEAR AN ELECTRONIC TOPOLOGICAL TRANSITION IN THE PRESENCE OF A MAGNETIC FIELD

Following the usual treatments,[9-11] we simulate breaking of the neck of a Fermi surface of a metal by simulating this neck with a hyperboloid of revolution (Fig. 1). A one-sheet hyperboloid then represents an open Fermi surface and a two-sheet hyperboloid corresponds to a closed surface. The number of conduction electrons n_e determines the limiting value of the longitudinal momentum p_0. In this model the energy of a conduction electron in the presence of a magnetic field H can be described by

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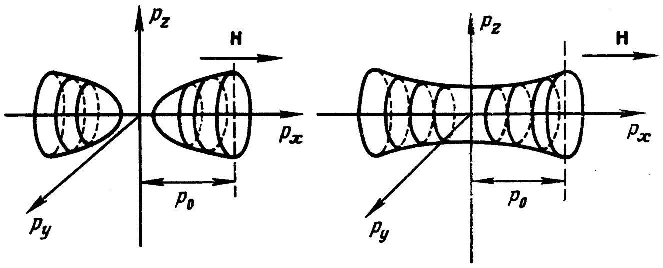

FIG. 1. Topological transition of the neck-breaking type in a magnetic field. If $\Delta = 0$ an open Fermi surface becomes closed, p_0 is the limiting value of the longitudinal momentum.

$$E_{n\pm}(p_z)=E_c-\frac{p_z^2}{2m_z}+\hbar\Omega\left(n+\frac{1}{2}\right)\mp\frac{eH\hbar}{2m_Sc},\qquad(1)$$

where E_c is the critical energy corresponding to the transition point at $T=0$ in the absence of a magnetic field and of impurity scattering; $\Omega = eH/m_\perp c$ is the cyclotron frequency; m_z, $m_\perp$, and m_S are the longitudinal, transverse, and spin masses of electrons. We assume $\hbar = 1$ and, for the sake of simplicity, we assume $m_S = m_\perp$. To each Landau level and "$\pm$" spin index corresponds a Fermi momentum $p_{0n\pm}$ defined by

$$p_{0n\pm}=\left\{2m_z\left[E_c-\mu+\Omega\left(n+\frac{1}{2}\right)\mp\frac{eH}{2m_Sc}\right]\right\}^{1/2},\qquad(2)$$

where μ is the chemical potential.

We introduce a parameter $\Delta_\pm$ representing the proximity of the electron system in a metal to a topological transition:

$$\Delta_\pm=\mu-E_c\pm\frac{eH}{2m_Sc}=\mu-E_c\pm\frac{eH}{2m_\perp c}=\Delta\pm\frac{\Omega}{2},\qquad(3)$$

where

$$p_{0n\pm}=\left\{2m_z\left[\Omega\left(n+\frac{1}{2}\right)-\Delta_\pm\right]\right\}^{1/2}.\qquad(4)$$

The maximum number of levels $N_{\max}$ is found from

$$n_e=\frac{eH}{2\pi^2c}\sum_{n=0}^{N_{max}}p_{0n\pm},\qquad(5)$$

which allows for the Landau level degeneracy. The summation indicated above is carried out over all real momenta $p_{0n\pm}$ [i.e., the summation in Eq. (5) is carried out also over the spin index]. For simplicity, we assume that the interaction of electrons with impurities is independent of the electron spin; the projection is then not affected by the scattering processes and we can consider independently the currents of electrons with different spin projections.

We now discuss the specific case of an electron with the spin projection $+\frac{1}{2}$. We are interested in the case of a pure metal ($T\tau\gg 1$) in a moderately strong magnetic field ($\Omega\ll\mu$). The states of conduction electrons in a magnetic field $H\parallel p_z$ are characterized by quantum numbers n and p_z, so that the expression for the probability of electron scattering by an impurity is then given by[16]

$$W_+(\varepsilon)=2\pi N_i|U|^2\int dp_z\sum_n\delta(E_{n+}(p_z)-\varepsilon)\frac{eH}{c(2\pi)^2},\qquad(6)$$

where N_i is the number of scattering centers per unit volume, U is the Born scattering amplitude (assumed to be isotropic in order to simplify the treatment), and the energy spectrum of an electron is described by Eq. (1). Integrating Eq. (6) with respect to the momenta p_z, we obtain

$$W_+(\omega)=W_0\frac{\Omega}{2\varepsilon_0^{1/2}}\sum_{m=0}^{N_{max}}\frac{\theta(-\varepsilon_m(\omega))}{|\varepsilon_m(\omega)|^{1/2}},\qquad(7)$$

where $\varepsilon_m(\omega)=\omega+\Delta_+-\Omega(m+1/2)$, $\omega=\varepsilon-\mu$, $\theta(x)$ is the Heaviside function, $\varepsilon_0=p_0^2/2m_z$, and $W_0=\pi^{-1}N_i|U|^2m_\perp p_0$ is the Born probability of electron scattering by an impurity far from a transition.

In moderately strong fields (when $\Omega\ll\mu$), we can apply the Poisson formula and go over from summation to integration in Eq. (7). Using the explicit shape of the Fermi surface of Eq. (1) in the selected model, we have

$$\sum_{m=0}^{N^+_{max}}\frac{\theta(-\varepsilon_m)}{|\varepsilon_m|^{1/2}}=\int_0^{N^+_{max}}\frac{\theta(S_N-\Omega t-\omega-\Delta-{}^1/_2\Omega)}{|S_N-\Omega t-\omega-\Delta-{}^1/_2\Omega|^{1/2}}dt$$
$$+2\,\mathrm{Re}\sum_{k=1}^{\infty}\int_0^{N^+_{max}}\frac{\theta(S_N-\Omega t-\omega-\Delta-{}^1/_2\Omega)}{|S_N-\Omega t-\omega-\Delta-{}^1/_2\Omega|^{1/2}}e^{2\pi ikt}dt,\qquad(8)$$

where $N^+_{\max}$ is the maximum number of the Landau levels governing the largest section S_0 of the Fermi surface:

$$S_0=2\pi m_\perp\Omega(N_{max}+{}^1/_2),$$
$$S_N=S_0/2\pi m_\perp=\varepsilon_0+\Delta.\qquad(9)$$

Using the integrals in Eq. (8), we find that the scattering probability $W_+(\omega)$ is described by

$$W_+(\omega)=W_0\Bigg\{\left(1-\frac{\omega}{\varepsilon_0}\right)^{1/2}-\frac{|\omega+\Delta|^{1/2}}{\varepsilon_0^{1/2}}\theta(-\omega-\Delta)$$
$$+\left(\frac{\Omega}{2\varepsilon_0}\right)^{1/2}\sum_{k=1}^{\infty}\frac{(-1)^k}{k^{1/2}}\cos\left[\varphi(\varepsilon_0)-\frac{\pi}{4}\right]$$
$$-\theta(-\omega-\Delta)\left(\frac{\Omega}{\varepsilon_0}\right)^{1/2}$$
$$\times\sum_{k=1}^{\infty}\frac{(-1)^k}{k^{1/2}}[\cos\varphi(\varepsilon_0)C(\varphi(-\Delta))$$
$$+\sin\varphi(\varepsilon_0)S(\varphi(-\Delta))\Bigg]\Bigg\},\qquad(10)$$

where $\varphi(\varepsilon_0)=(2\pi k/\Omega)(\varepsilon_0-\omega)$, $\varphi(-\Delta)=(2\pi k/\Omega)\times(-\Delta-\omega)$, and $C(a)$ and $S(a)$ are the Fresnel integrals:

$$\begin{bmatrix}C(a)\\S(a)\end{bmatrix}=(2\pi)^{-1/2}\int_0^a\begin{bmatrix}\cos t\\\sin t\end{bmatrix}t^{-1/2}dt.\qquad(11)$$

Using Eq. (10), we now find the relaxation time $\tau(\omega,\Delta_+,\Omega)=W_+^{-1}(\omega)$. Note that we obtained Eq. (6) in the Born approximation. We can go over from the Born scattering amplitude U to the true amplitude A in the presence of a magnetic field if we use the results of Ref. 15 and 17:

$$A=U\left[1+i\frac{(2m_z)^{1/2}eHU}{4\pi c}\sum_{m=0}^{N_{max}}\frac{\theta(-\varepsilon_m(\omega))}{|\varepsilon_m(\omega)|^{1/2}}\right], \qquad (12)$$

which then gives the following final expression for the relaxation time $\tau(\omega,\Delta_+,\Omega)$

$$\tau(\omega,\Delta_+,\Omega)=\tau_1(\omega,\Delta)+\tau_2(\omega,\Omega)+\tau_3(\omega,\Delta,\Omega), \qquad (13)$$

where

$$\tau_1(\omega,\Delta)=\tau_0\left[1+\frac{\omega}{2\varepsilon_0}+\left(\frac{|\omega+\Delta|^{1/2}}{\varepsilon_0^{1/2}}+\frac{\omega+\Delta}{\varepsilon_0}\right)\theta(-\omega-\Delta)\right],$$

$$\tau_2(\omega,\Omega)=\tau_0R\left(\frac{\Omega}{2\varepsilon_0}\right)^{1/2}\sum_{k=1}^{\infty}(-1)^k k^{-1/2}\cos\left(\varphi(\varepsilon_0)-\frac{\pi}{4}\right),$$

$$\tau_3(\omega,\Omega,\Delta)=-\tau_0R\left(\frac{\Omega}{\varepsilon_0}\right)^{1/2}\sum_{k=1}^{\infty}(-1)^k k^{-1/2}[\cos\varphi(\varepsilon_0)C(\varphi(-\Delta))$$
$$+\sin\varphi(\varepsilon_0)S(\varphi(-\Delta))]\theta(-\omega-\Delta),$$

and $\tau_0=\pi(N_i|A|^2m_\perp p_0)^{-1}$ is the relaxation time of an electron scattered by an impurity far from an electronic topological transition in the absence of a magnetic field, while the coefficient $R=2|A|^2(p_0m_\perp/2\pi)^2-1$ is due to the difference–in a magnetic field–between the true and Born scattering amplitudes. The above expression for the relaxation time is limited to terms on the order of $(\Delta/\varepsilon_0)^{1/2}$, and $(\Omega/\varepsilon_0)^{1/2}$, which govern the behavior of the MTEP near an electronic topological transition. A more detailed analysis of the expression for $\tau(\omega,\Delta,\Omega)$ is carried out in Ref. 17 using the temperature diagram technique and expanding a one-electron Green function in terms of the eigenfunctions of the electron in a magnetic field.[15]

Clearly, since Eq. (13) does not contain spin indices, the relaxation time of an electron with the spin projection $-\frac{1}{2}$ is described by the same equation. The quantity τ_1 in Eq. (13) has been encountered before.[9-11] It describes the likely scattering processes near an electronic topological transition in the absence of a magnetic field: τ_0 is the usual scattering in the peripheral part of the Fermi surface to the peripheral part again, whereas the term with the θ function represent the scattering from the a peripheral to a singular region. It should be pointed out that the probability of the scattering to a singular region described by the last term $\tau_0^{-1}(|\omega+\Delta|/\varepsilon_0)^{1/2}\theta(-\omega-\Delta)$ is effectively independent of the parameter ε_0 (since $\tau_0^{-1}\propto\varepsilon_0^{1/2}$ holds) and we can expect that the model nature of the description of the Fermi surface neck will not affect the final results.

The influence of a magnetic field on $\tau(\omega,\Delta,\Omega)$ is manifested by the presence of the term $\tau_2+\tau_3$. For example, at $T=0$ the term τ_2 represents the correction to the relaxation time in the case of a one-sheet hyperboloid (for $\Delta>0$). In the opposite case (for $\Delta<0$) when the neck is broken, the contribution of τ_3 also becomes nonzero and we shall show that this results in a strong cancellation of the oscillatory part of the thermoelectric power. However, the parameter ε_0 in the expressions for τ_2 and τ_3 not only occurs as the factor explained above, but also performs the function of a phase of the oscillatory trigonometric and Fresnel functions. This is because this model applies in reality only near a neck. If we use it literally, then all the integrals with respect to p_z in Eq. (6) and with respect to t in Eq. (8) can be found accurately, but then the result is governed by the limiting area of the cross section S_0 which is not generally an extremal cross section of the Fermi surface because $\partial S/\partial p_z|_{\varepsilon=\varepsilon_0}\neq0$. In fact, it is clear that far from a neck the real Fermi surface has certain maximum cross sections and these determine the magnitude of the correction to the relaxation time as well as the oscillation period when the magnetic field is altered.[18] If the latter is indeed the cross section $S_{\max}$, then the correction calculated by the steepest-descent method near the point $S_{\max}$ will depend on the derivatives $\partial^2S/\partial p_z^2|_{\mu,p_z^{\max}}$ and $\partial S/\partial\varepsilon|_{\mu,p_z^{\max}}=2\pi m^*$. Therefore, using the expressions in the system (13) for τ_2 and τ_3 we must bear in mind that the model of an electronic topological transition with the spectrum described by Eq. (1) predicts an oscillation amplitude governed by the derivative $\partial S/\partial p_z|_{p_z=p_0}\neq0$. This is a shortcoming of the model and it suggests that the results obtained cannot be used to provide a quantitative description of the effect, but the qualitative picture is correct because in reality near one extremal cross section far from the neck of the Fermi surface the derivative $\partial S/\partial p_z$ vanishes and the oscillation amplitude is then governed—in accordance with Ref. 18—by m^* and $\partial^2S/\partial p_z^2|_{\mu,p_z^{\max}}$. We can therefore say that the real behavior of the Fermi surface near an extremal cross section increases the amplitude [by a factor of $(\varepsilon_F/\omega_0)^{1/2}$] of the oscillations in the relaxation time, compared with the results given by Ref. 13.

3. GENERAL EXPRESSION FOR THE THERMOELECTRIC POWER

We now discuss our main task, which is calculation of the thermoelectric power of a metal closs to an electronic topological transition point in the presence of a magnetic field. We are interested in the a thermoelectric tensor component β_{zz} on the assumption that not only a magnetic field, but also a temperature gradient ∇T ($\nabla T\|E\|H\|p_z$) acts along the hyperboloid axis p_z (Fig. 1). It is known[16] that

$$\beta_{zz}=-\frac{e}{4T^2}\int v_z^2\tau(\varepsilon)(\varepsilon-\mu)\operatorname{ch}^{-2}\frac{\varepsilon-\mu}{2T}d^3p. \qquad (14)$$

Integrating with respect to angles, we obtain

$$\beta_{zz}=-\frac{em_\perp\varepsilon_0^{3/2}}{24\pi^2T^2m_z^{1/2}}\int_{-\infty}^{\infty}\omega\,d\omega\,\tau(\omega,\Delta,\Omega)\operatorname{ch}^{-2}\frac{\omega}{2T}. \qquad (15)$$

Note that Eq. (14) describes the contribution of electrons with the spin projection $+\frac{1}{2}$. Obviously, the contribution of electrons with the opposite spin is exactly the same. Summation over the spin projections gives rise to a factor 2 in the numerator of Eq. (15). Substituting the explicit expression for $\tau(\omega,\Delta,\Omega)$ from Eq. (13), we find that near an electronic topological transition in the presence of a magnetic field the electron component of the thermoelectric power can be represented as a sum of three contributions: β_S are the usual background and singular parts of the thermoelectric power near an electronic topological transition in the absence of a magnetic field; β_H is an oscillatory Shubnikov–de Haas correction due to the influence of a magnetic field on the kinetics of conduction electrons far from a transition, and β_{HS} is due to the influence of the applied magnetic field on the singular part of the thermoelectric power near the transition (as in Sec. 2, this division is quite arbitrary).

The contribution of β_S calculated from Eq. (15) is identical with that found earlier:[9]

$$\beta_S(\Delta,T)=\beta_0\begin{cases}2+\dfrac{1}{2}\left(\dfrac{\varepsilon_0}{|\Delta|}\right)^{1/2}, & \Delta\ll -T\\ \dfrac{3}{2}+0.28\left(\dfrac{\varepsilon_0}{T}\right)^{1/2}\left(1-0.29\dfrac{\Delta}{T}\right), & |\Delta|\ll T,\\ 1+0.36\left(\dfrac{\varepsilon_0}{T}\right)^{1/2}\dfrac{\Delta}{T}\exp\left(-\dfrac{\Delta}{T}\right), & T<\Delta\leqslant T\ln T\tau\end{cases} \quad (16)$$

where

$$\beta_0=\frac{em_\perp T\tau_0}{9}\left(\frac{2\varepsilon_0}{m_z}\right)^{1/2}$$

is the thermoelectric power far from the transition on the open Fermi surface side.[1)] We stress that the singular part of the thermoelectric power in the direct vicinity of an electronic topological transition is $(\varepsilon_0/T)^{1/2}$ times greater than the background value. Note that we have ignored here the range $\Delta \gtrsim T\ln(T\tau)$, where $\beta_s = \beta_0[1+0.12(\varepsilon_0\tau)^{1/2}/(\tau\Delta)^{3/2}]$, in order to avoid unnecessary complications.[9]

4. OSCILLATIONS OF THE THERMOELECTRIC POWER OF A NORMAL METAL IN A MAGNETIC FIELD

We now consider the contribution made to the thermoelectric power β_H by the magnetic-field dependence of the relaxation time. The results obtained in the present section are in no way related to the specific model of the Fermi surface and they describe oscillations of the Shubnikov–de Haas type exhibited by the thermoelectric effect of a normal metal.

Substituting in Eq. (15) the expression for $\tau_2(\omega,\Omega)$ from the system of equations (13), we obtain

$$\beta_H=-\beta_0\frac{3\varepsilon_0}{4\pi^2T^2}\left(\frac{\Omega}{2\varepsilon_0}\right)^{1/2}\int_{-\infty}^{\infty}R\omega\,d\omega\,\mathrm{ch}^{-2}\frac{\omega}{2T}\times\sum_{k=1}^{\infty}\frac{(-1)^k}{k^{1/2}}\cos\left[\frac{2\pi k}{\Omega}(\varepsilon_0-\omega)-\frac{\pi}{4}\right]$$
$$=-\beta_0\frac{3R}{\pi^2}\left(\frac{2\varepsilon_0}{T}\right)^{1/2}\left(\frac{\Omega}{T}\right)^{1/2}\sum_{k=1}^{\infty}\frac{(-1)^k}{k^{1/2}}\times\Psi_1\left(\frac{2\pi^2kT}{\Omega}\right)\sin\left(\frac{2\pi k\varepsilon_0}{\Omega}-\frac{\pi}{4}\right). \quad (17)$$

In contrast to the background component β_0, calculation of the integral in Eq. (15) shows that the main frequency dependence (expressed in terms of the parameter ε_0/Ω) is due to the strongly oscillating cosine and not due to the usual term proportional to ω. It is interesting that the function

$$\Psi_1(x)=\int_0^\infty t\sin\frac{2xt}{\pi}\,\mathrm{ch}^{-2}t\,dt=\frac{\pi}{2\,\mathrm{sh}\,x}[x\,\mathrm{cth}\,x-1]=\begin{cases}\dfrac{\pi}{6}x\left(1-\dfrac{7}{30}x^2\right), & x\ll 1\\ \pi xe^{-x}, & x\gg 1\end{cases} \quad (18)$$

occurring in the above expression is a total derivative of the function $\Psi(x)=x/\mathrm{sh}\,x$ $[\Psi_1(x)=-\frac{1}{2}\pi\Psi'(x)]$, which appears in a description of the Shubnikov–de Haas oscillations of the conduction of a normal metal in a magnetic field.[16]

Note that Eq. (17) is identical with an expression obtained earlier.[19] We now analyze these results. In weak fields $(\Omega \ll 2\pi^2 T)$ the exponential fall of the terms of the series in Eq. (17) means that we need to consider only the first term of the sum with $k=1$, whereas the thermoelectric power is

$$\beta_H=6\pi R\beta_0\left(\frac{2\varepsilon_0}{\Omega}\right)^{1/2}\exp\left(-\frac{2\pi^2T}{\Omega}\right)\sin\left(\frac{2\pi\varepsilon_0}{\Omega}-\frac{\pi}{4}\right). \quad (19)$$

In strong fields $(\Omega \gg 2\pi^2 T)$ an analysis of the sum in Eq. (17) is more difficult. The exponential fall of the terms in this series begins only from $k_0 \sim (\Omega/2\pi^2 T)$; moreover, in accordance with the asymptotic form $\Psi_1(x)$ described by Eq. (18), these terms increase in magnitude and alternate in sign. Bearing in mind this fact, we estimate the sum by calculating only the terms $k \lesssim (\Omega/2\pi^2 T)$ in the series (17) by replacing the function $\Psi_1(2\pi^2 T/\Omega)$ with its asymptotic value valid in the case of small arguments:

$$\sigma=\sum_{k=1}^{\infty}(-1)\,k^{-1/2}\Psi_1\left(\frac{2\pi^2kT}{\Omega}\right)\times\sin\left(\frac{2\pi\varepsilon_0}{\Omega}k-\frac{\pi}{4}\right)-\frac{2^{1/2}\pi^3}{6}\left(\frac{T}{\Omega}\right)\sum_{k=1}^{k_0}(-1)^kk^{1/2}\times\left[\sin\left(\frac{2\pi\varepsilon_0}{\Omega}k\right)-\cos\left(\frac{2\pi\varepsilon_0}{\Omega}k\right)\right]. \quad (20)$$

If $\Omega_{N+1/2}$ satisfying $\varepsilon_0/\Omega_{N+1/2}=N+1/2$ (N is an integer), we have $\sin(2\pi\varepsilon_0 k/\Omega_{N+1/2})\equiv 0$, and $\cos(2\pi\varepsilon_0 k/\Omega_{N+1/2})=(-1)^k$. We then obtain

$$\sigma_{N+1/2}\propto-\left(\frac{T}{\Omega}\right)\sum_{k=1}^{k_0}k^{1/2}\sim-\left(\frac{\Omega_{N+1/2}}{T}\right)^{1/2}.$$

Therefore, at the point $\Omega_{N+1/2}$ where all the terms of the sum (17) have the same sign and the cosine reaches its maximum value, we have

$$\beta(\Omega_{N+1/2})\propto\beta_0(\varepsilon_0/T)^{1/2}(\Omega_{N+1/2}/T)^{1/2}. \quad (21)$$

We can also calculate the thermoelectric power at a different singular point $\Omega=\Omega_{N+1/4}$. Now the expression in square brackets in Eq. (17) assumes the value ± 1 and the sum can be calculated using just the first terms [and not the upper limit as was done for $\sigma(\Omega_{N+1/2})$]; $\sigma(\Omega_{N+1/4})=0.22$. Therefore, we have

$$\beta(\Omega_{N+1/4})=-0{,}65R\beta_0(\varepsilon_0/\Omega_{N+1/4})^{1/2}. \quad (22)$$

A similar analysis of the sum in Eq. (17) at the points $\Omega_{N-1/4}$ and the sign of the latter function $\beta(\Omega_{N-1/4})\propto\beta_0 R(\varepsilon_0/T)^{1/2}$, varies rapidly with Ω. Therefore, the dependence of the oscillations of the thermoelectric power on the cylotron resonance frequency (i.e., on the magnetic field) has the form shown in Fig. 2. Figure 3 gives the results of a computer calculation of the values of $\beta_H(\Omega)$ valid in the range of moderate and strong fields. We can see that in weak fields there are indeed sinusoidal oscillations

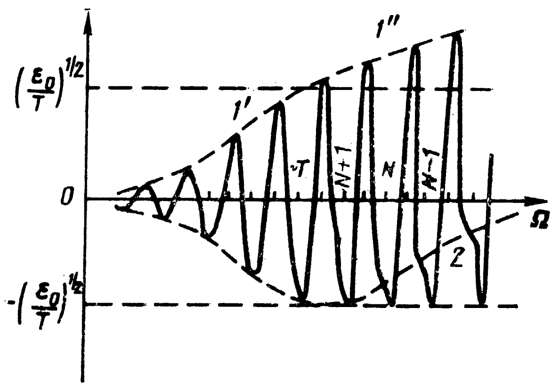

FIG. 2. Schematic dependence of the relative value of the thermoelectric power $\beta_H/\beta_0 R$ on the cyclotron frequency $\Omega = eH/m_\perp c$, The points $N + a$ identify the values of Ω_{N+a} at which we have $\varepsilon_0/\Omega = N + a$ (N is an integer). 1') $\sim(\varepsilon_0/T)^{1/2}\exp(-2\pi_2 T/\Omega)$; 1") $\sim(\varepsilon_0/T)^{1/2}(\Omega/T)$; 2) $\sim(\varepsilon_0/\Omega)^{1/2}$.

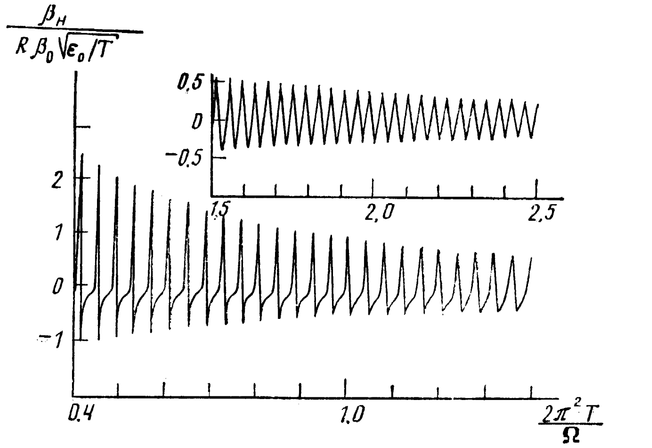

FIG. 3. Results of a numerical calculation of the dependence β_H on the external magnetic field (in the case when $\varepsilon_0/T = 500$). We can easily see that the strongly nonharmonic oscillations in the range $2\pi^2 T/\Omega \lesssim 1$ change rapidly to ordinary quasiharmonic oscillations in weaker fields.

with a gradually increasing amplitude, whereas in strong fields the curve becomes clearly asymmetric about the Ω^{-1} axis, there are large peaks in the range $\beta > 0$ corresponding to the points $\varepsilon_0/(N + 1/4)$, separated by $1/\varepsilon_0$ exactly, whereas at the points $(N + 1/4)/\Omega$ the value in question is $\beta < 0$ and the dependence exhibits inflections which become larger as the magnetic field is increased.

5. OSCILLATIONS OF THE SINGULAR PART OF THE THERMOELECTRIC POWER NEAR AN ELECTRONIC TOPOLOGICAL TRANSITION IN A MAGNETIC FIELD

We now turn to calculating the singular part of the thermoelectric power $\beta_{HS}(\Delta, T, S)$, allowing for the influence of the applied magnetic field on the singularity of the thermoelectric power near an electronic topological transition. This quantity is determined formally by the contribution τ_3 to the general expression given by Eq. (13). Ignoring small terms in the integrand of Eq. (15), which contains powers $(\omega/\varepsilon_0)^{1/2}$ and higher, and transforming the Fresnel integrals into an incomplete Euler gamma function, we find

$$\beta_{HS}(\Delta,\Omega,T) = \frac{3R}{\pi^{5/2}}\beta_0\left(\frac{\varepsilon_0}{2T}\right)^{1/2}\left(\frac{\Omega}{T}\right)^{1/2}\operatorname{Re}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^{1/2}} \times \exp\left[i\left(\frac{2\pi k\varepsilon_0}{\Omega}-\frac{\pi}{4}\right)\right] \times \int_{\Delta/2T}^{\infty}\frac{x\,dx}{\mathrm{ch}^2 x}\exp\left(\frac{4\pi ikT}{\Omega}x\right)\gamma\left(\frac{1}{2},\frac{4\pi ikT}{\Omega}\left(x-\frac{\Delta}{2T}\right)\right). \tag{23}$$

We now investigate the general expressions in different regions in the vicinity of an electronic topological transition.

A. $\Delta \ll -T$ (two-sheet hyperboloid). In this range of values of Δ, after expanding of the γ function in terms of the parameter $xT/|\Delta|$ (when $x \lesssim 1$ are important), we can transform the integral in Eq. (23) to the simpler expression

$$\beta_{HS} = \beta_0\frac{6R}{\pi^{5/2}}\left(\frac{\varepsilon_0}{2T}\right)^{1/2}\left(\frac{\Omega}{T}\right)^{1/2}\operatorname{Re}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^{1/2}} \times\left[\exp\left(i\frac{\pi}{4}\right)\gamma\left(\frac{1}{2},\frac{2\pi ik|\Delta|}{\Omega}\right)\right. \times\Psi_1\left(\frac{2\pi^2kT}{\Omega}\right)+2T\left(\frac{2\pi k}{|\Delta|\Omega}\right)^{1/2}\exp\left(-\frac{2\pi ik|\Delta|}{\Omega}\right) \times\left.\Psi_2\left(\frac{2\pi^2kT}{\Omega}\right)\right]\exp\left(\frac{2\pi i\varepsilon_0}{\Omega}k\right), \tag{24}$$

where the function

$$\Psi_2(x) = \int_0^{\infty}\frac{t^2\cos(2xt/\pi)}{\mathrm{ch}^2 t}dt = \left(\frac{\pi}{2}\right)^2\frac{2\,\mathrm{sh}\,x\,\mathrm{ch}\,x - x(1+\mathrm{ch}^2 x)}{\mathrm{sh}^3 x} = \begin{cases} -\dfrac{\pi^2}{2}xe^{-x}, & x\gg 1\\ \dfrac{\pi^2}{12}\left(1-\dfrac{7}{10}x^2\right), & x\ll 1\end{cases} \tag{25}$$

is in turn found to be the total derivative of $\Psi_1(x)$.

In moderately strong fields ($\Omega \lesssim |\Delta|$) the incomplete gamma function remaining in Eq. (24) can also be replaced by the asymptotic form at large values of the argument:

$$\gamma(1/2, x) = \pi^{1/2} - x^{-1/2}e^{-x}, \tag{26}$$

where it is found that the main part of the contribution of the first term in square brackets of Eq. (24) is exactly equal in magnitude and opposite in sign to the contribution $\beta_H(\Omega, T)$ found in the preceding section. Therefore, in the range $\Omega \lesssim |\Delta|$ ($|\Delta| \gg T$) we have

$$\beta_{HS}+\beta_H = \frac{3R}{\pi^3}\beta_0\left(\frac{\varepsilon_0}{|\Delta|}\right)^{1/2} \times\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^{1/2}}\left[4\pi k\Psi_2\left(\frac{2\pi^2kT}{\Omega}\right)\right. \left.-\frac{\Omega}{T}\Psi_1\left(\frac{2\pi^2kT}{\Omega}\right)\right]\cos\left(\frac{S_0k}{\Omega m_\perp}\right). \tag{27}$$

In weak fields, i.e., for $\Omega \ll 2\pi^2 T \ll |\Delta|$, in view of the exponential nature of the terms in this series, we can sum it retaining only just the first term:

$$\beta_{HS}+\beta_H=-6\beta_0 R\left(\frac{\varepsilon_0}{|\Delta|}\right)^{1/2}\left(\frac{2\pi^2T}{\Omega}\right)\exp\left(-\frac{2\pi^2T}{\Omega}\right)\cos\frac{S_0}{\Omega m_\perp}. \tag{28}$$

In strong fields for $2\pi^2T\ll\Omega<|\Delta|$ the terms of this series rise again to $k_0\sim\Omega/2\pi^2T$ and then begin to fall exponentially (with increasing k). Terminating the summation at $\sim k_0$, we find

$$\beta_{HS}+\beta_H=\frac{49}{60}\left(\frac{\varepsilon_0}{|\Delta|}\right)^{1/2}\beta_0\left(\frac{2\pi^2T}{\Omega}\right)^2\sum_{k=1}^{k_0}(-1)^k k^2\cos\frac{S_0 k}{\Omega m_\perp}. \tag{29}$$

This sum can be calculated explicitly for $k_0\gg1$:

$$\sum_{k=1}^{k_0}(-1)^k k^2\cos(\alpha k)=(-1)^{k_0}\frac{k_0^2}{2}\frac{\cos[\alpha(k_0+1/2)]}{\cos(\alpha/2)}. \tag{30}$$

The remarkable properties of the above sum are strong oscillations of its sign depending on the value of k_0 within a band of width $\sim k_0^2$ and the presence of strong ($\sim k_0^3$) positive peaks at certain values of α which gives $\Omega_{N+1/2}=S_0/2\pi m_\perp(N+1/2)$.

Therefore, the quantity $\beta_{HS}+\beta_H$ found for $T\ll\Omega<|\Delta|$ (but $\Omega\neq\Omega_{N+1/2}$) oscillates rapidly [no longer harmonically, but in accordance with a more complicated law, because the sum of Eq. (29) contains many harmonics] within a band

$$|\beta_{HS}(\Omega)+\beta_H(\Omega)|\lesssim\beta_0(\varepsilon_0/|\Delta|)^{1/2}. \tag{31}$$

However, near singularities $\Omega_{N+1/2}$ this dependence has sharp peaks of amplitude which increases linearly with Ω:

$$\beta_{HS}(\Omega_{N+1/2})+\beta_H(\Omega_{N+1/2})\sim\beta_0\left(\frac{\varepsilon_0}{|\Delta|}\right)^{1/2}\left(\frac{\Omega}{2\pi^2T}\right). \tag{32}$$

Finally we consider the range of strong fields $\Omega>|\Delta|$. As in the preceding case, the exponential fall of the terms of the series begins from $k_0\sim\Omega/2\pi^2T\gg1$, but for $1\lesssim k\lesssim\Omega/|\Delta|$ (although $\Omega/|\Delta|\ll\Omega/2\pi^2T$), we cannot use the asymptotic form (26) for the gamma function. An analysis of Eq. (24) shows that the corrections to the first terms are of order $(T/|\Delta|)^{1/2}$ and the quantity $\beta_{HS}(\Omega)+\beta_H(\Omega)$ is still given by Eqs. (31) and (32) in this range of fields.

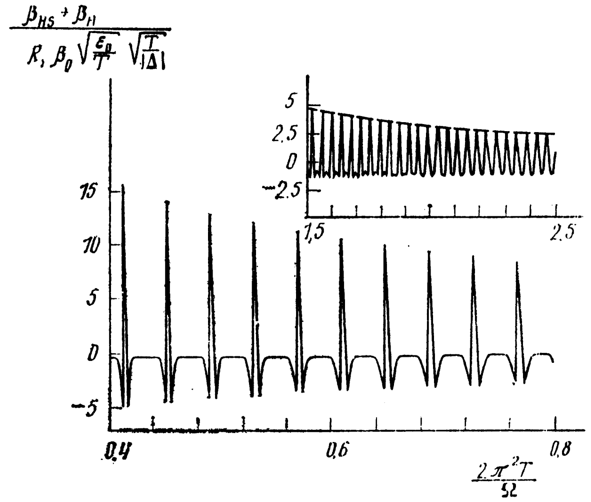

FIG. 4. Results of a numerical calculation of the total magnetothermoelectric power $\beta_H+\beta_{HS}$ as a function of the external magnetic field [in the case when $(\varepsilon_0+\Delta)/T=500$, $\Delta\ll-T$).]

It therefore follows that for $\Delta\ll-T$ (two-sheet hyperboloid) the amplitude of the magnetic-field-dependent contribution $\beta_H+\beta_{HS}$ is considerably smaller [by a factor $(T/|\Delta|)^{1/2}$] than the amplitude of the normal magnetoelectric effect $\beta_H(\Omega)$. The corresponding dependence of $\beta_{HS}(\Omega)+\beta_H(\Omega)$ is shown in Fig. 4. In strong magnetic fields this dependence is far from harmonic. In accordance with analytic results, there are strong positive peaks at points $2\pi^2T/\Omega_{N+1/2}$, but the magnitude of this effect in the range $\Delta\ll-T$ is less than that of the oscillations of β_H in terms of the parameter $(T/|\Delta|)^{1/2}$.

B. $\Delta>0$ (one-sheet hyperboloid). Going back to the integral representation of the gamma function in Eq. (23), altering the order of integration, and bearing in mind that

$$\int_t^\infty\begin{pmatrix}\cos[a(x-t)]\\ \sin[a(x-t)]\end{pmatrix}\frac{x\,dx}{\operatorname{ch}^2x}=4\sum_{n=1}^\infty(-1)^{n+1}e^{-2nt}\left[\frac{tn}{4n^2+a^2}\begin{pmatrix}2n\\ a\end{pmatrix}+\frac{n}{(4n^2+a^2)^2}\begin{pmatrix}4n^2-a^2\\ 4an\end{pmatrix}\right], \tag{33}$$

we obtain

$$\beta_{HS}=\frac{3R\beta_0}{2\pi^{1/2}}\left(\frac{\varepsilon_0}{T}\right)^{1/2}\left(\frac{\Omega}{2\pi T}\right)^2\sum_{n=1}^\infty(-1)^{n+1}n^{1/2}\times\exp\left(-\frac{n\Delta}{T}\right)D_n(\Delta,\Omega,T), \tag{34}$$

$$D_n(\Delta,\Omega,T)=\sum_{k=1}^\infty(-1)^{k+1}\left\{\cos\frac{S_0k}{m_\perp\Omega}\left[\frac{2n\Delta/T-1}{k^2+(a_n/\pi)^2}+\frac{4a_n^2}{\pi^2(k^2+(a_n/\pi)^2)^2}\right]-\sin\frac{S_0k}{m_\perp\Omega}\left[\frac{\pi k(2n\Delta/T+1)}{a_n(k^2+(a_n/\pi)^2)}+\frac{4a_n}{\pi(k^2+(a_n/\pi)^2)^2}\right]\right\}, \tag{35}$$

with $a_n=n\Omega/2T$.

The sums occurring in Eq. (35) can be calculated exactly. For example, we find that

$$\frac{2}{\pi}\sum_{k=1}^\infty\frac{(-1)^{k+1}}{k^2+a^2}\begin{pmatrix}\cos kx\\ k\sin kx\end{pmatrix}=\begin{pmatrix}-\dfrac{1}{a}\dfrac{\operatorname{ch}(ax)}{\operatorname{sh}(\pi a)}+\dfrac{1}{2a^2}\\ \dfrac{\operatorname{sh}(ax)}{\operatorname{sh}(\pi a)}\end{pmatrix},\quad |x|<\pi. \tag{36}$$

We can use an expression of the form (36) if we express the arguments of the trigonometric function in Eq. (35) in the form kx, where $|x|<\pi$. This can be done by representing $S_0/2\pi m_\perp\Omega$ in the form $S_0/2\pi m_\perp\Omega=N+r$, where N is an integer and

$$r=\left\{\frac{S_0}{2\pi m_\perp\Omega}\right\}-\frac{1}{2}\in\left[-\frac{1}{2},\frac{1}{2}\right)$$

represents the fractional part; then, $D_n(\Delta,\Omega,T)$ assumes the following form in the range under investigation ($\Delta>0$):

$$D_n(\Delta,\Omega,T)=\frac{1}{2}\left\{\frac{\pi^2}{a_n^2}\left(4n\frac{\Delta}{T}+3\right)\right.$$
$$-\frac{2\pi^2}{a_n}\,\mathrm{sh}^{-1}a_n\left[\exp(2a_nr)\left[n\frac{\Delta}{T}+\frac{1}{2}+\frac{a_n\exp a_n}{\mathrm{sh}\,a_n}\right]\right.$$
$$\left.\left.+\exp(-2a_nr)\,2a_n\left(r+\frac{1}{2}\right)\right]\right\}, \qquad (37)$$

where β_{HS} is still given by Eq (34).

Far from a transition (where $\Delta\gg T$), the sum of Eq. (34) can be limited to just the first term because of the exponential dependence of the terms on $n\Delta/T$:

$$\beta_{HS}=-\beta_0\frac{3R}{2\pi^{3/2}}\left(\frac{\varepsilon_0}{T}\right)^{1/2}\frac{\Delta}{T}\exp\left(-\frac{\Delta}{T}\right)\left\{\frac{\Omega}{T}\frac{\exp(\Omega r/T)}{2\,\mathrm{sh}(\Omega/2T)}-1\right\}. \qquad (38)$$

In weak fields characterized by $\Omega\ll T$, this expression simplifies to

$$\beta_{HS}(\Omega)=\frac{3}{2\pi^{3/2}}\beta_0R\left(\frac{\varepsilon_0}{T}\right)^{1/2}\frac{\Delta}{T}$$
$$\times\exp\left(-\frac{\Delta}{T}\right)\frac{\Omega}{T}\left[\frac{1}{2}-\left\{\frac{S_{max}}{2\pi m_\perp\Omega}\right\}\right]. \qquad (39)$$

In the opposite case when $\Omega\gg T$ but $\Delta\gg\Omega$, we find that

$$\beta_{HS}(\Omega)=\frac{3}{2\pi^{3/2}}\beta_0R\left(\frac{\varepsilon_0}{T}\right)^{1/2}\frac{\Delta}{T}\exp\left(-\frac{\Delta}{T}\right)\frac{\Omega}{T}$$
$$\times\begin{cases}1, & |r-{}^1\!/_2|\gg T/\Omega,\ \Omega\neq\Omega_{N+1/2}\\ -\Omega/T, & |r-{}^1\!/_2|<T/\Omega,\ \Omega=\Omega_{N+1/2}\end{cases}. \qquad (40)$$

For the sake of completeness we also give the expression for β_{HS} very strong magnetic fields ($\Delta\gg T$, $\Omega\gg\Delta$), which are not attainable experimentally:

$$\beta_{HS}=\frac{3R\beta_0}{2\pi^{3/2}}\left(\frac{\varepsilon_0}{T}\right)^{1/2}\exp\left(-\frac{\Delta}{T}\right)$$
$$\times\begin{cases}\Delta/T, & |r-{}^1\!/_2|\gg T/\Omega,\ \Omega\neq\Omega_{N+1/2}\\ {}^1\!/_2(\Omega/T)^2, & |r-{}^1\!/_2|<T/\Omega,\ \Omega=\Omega_{N+1/2}\end{cases}. \qquad (41)$$

Figure 5 shows the dependence of β_{HS} on the magnetic field in the range defined by $\Delta\gg T$ and $\Delta\gg\Omega$. It should be noted that β_{HS} now becomes exponentially small compared with β_H and the latter determines the magnetoelectric effect in the investigated range. Then, the MTEP behaves like the MTEP of an isotropic metal.

Figure 6 gives the results of a numerical calculation β_{HS} for the intermediate range $\Omega\sim T\sim\Delta$. We can see how an increase in Ω alters the oscillation profile from sawtooth to dip-like.

C. $\Delta\ll T$ (direct vicinity of an electronic topological transition). The thermoelectric power now exhibits a giant anomaly in the absence of a magnetic field, which depends weakly on Δ (Refs. 9–11). Therefore, for the sake of simplicity we shall be interested in the dependence $\beta_{HS}(\Omega,\Delta=0)$. Substituting $\Delta=0$ into Eqs. (34) and (37), we obtain

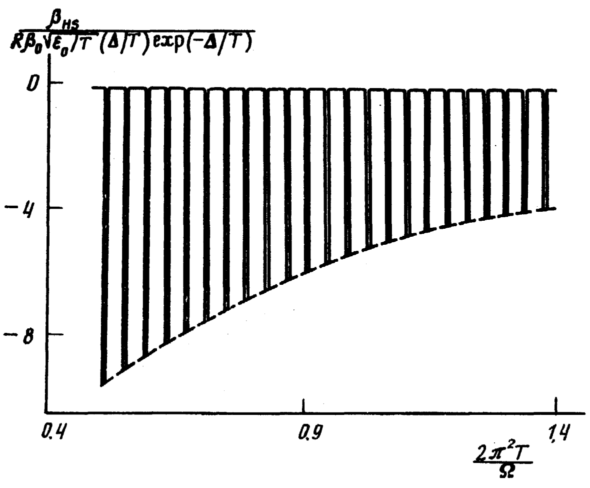

FIG. 5. Graph of the function $\beta_{HS}(2\pi^2T/\Omega)$ plotted for $\varepsilon_0/T=500$ and $\Delta\gg T$. The results of numerical calculations are in good agreement with analytic expressions given by Eqs. (38) and (39).

$$\beta_{HS}=\frac{9R\beta_0}{4\pi^{3/2}}\left(\frac{\varepsilon_0}{T}\right)^{1/2}\left\{\frac{2^{1/2}-1}{2^{1/2}}\zeta\left(\frac{3}{2}\right)\right.$$
$$-\frac{\Omega}{6T}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^{1/2}\,\mathrm{sh}\,a_n}\left[\exp(2a_nr)\right.$$
$$\left.\left.\times\left(\frac{1}{2}+\frac{a_n\exp a_n}{\mathrm{sh}\,a_n}\right)+\exp(-2a_nr)\,2a_n\left(r+\frac{1}{2}\right)\right]\right\}. \qquad (42)$$

As before, we have $r=\{S_0/2\pi m_\perp\Omega\}-1/2$ and Eq. (23) is transformed to Eq. (42) using the properties of the sums in Eq. (36).

In weak fields ($\Omega\ll T$) the sum in the expression (42) generally contains only terms with $n\sim T/\Omega\gg1$. However, the sum obtained in the limit $\Omega\to0$ is dominated by the terms with $n\ll T/\Omega$ and it is found that this term exactly cancels the first term independent of Ω which occurs in the braces. The coefficient in front of the linear term of the expansion in powers of β_{HS} can be determined in exactly the same way and the higher contributions found in this range are small $[\sim(\Omega/T)^{3/2}]$. We therefore have

$$\beta_{HS}(\Omega\ll T,\Delta=0)$$
$$=\frac{3(2^{1/2}-1)\zeta({}^1\!/_2)}{2\pi^{3/2}}\beta_0R\left(\frac{\varepsilon_0}{T}\right)^{1/2}\frac{\Omega}{T}\left(1+\frac{5}{2}r(\Omega)\right)$$
$$=0.16R\beta_0\left(\frac{\varepsilon_0}{T}\right)^{1/2}\frac{\Omega}{T}\left(1+\frac{5}{2}r\right). \qquad (43)$$

In the opposite case when $\Omega\gg T$, for any value $n=1,2,3,\ldots$, we can write down

$$\mathrm{cth}\frac{\Omega n}{2T}=1+2\exp\left(-\frac{\Omega n}{T}\right).$$

An analysis of the sum in Eq. (42) obtained after the simplification shows that $\beta_{HS}(\Omega)$ is strongly dependent on the value of the parameter $r(\Omega)$. If $|r(\Omega)-1/2|\gtrsim T/\Omega$

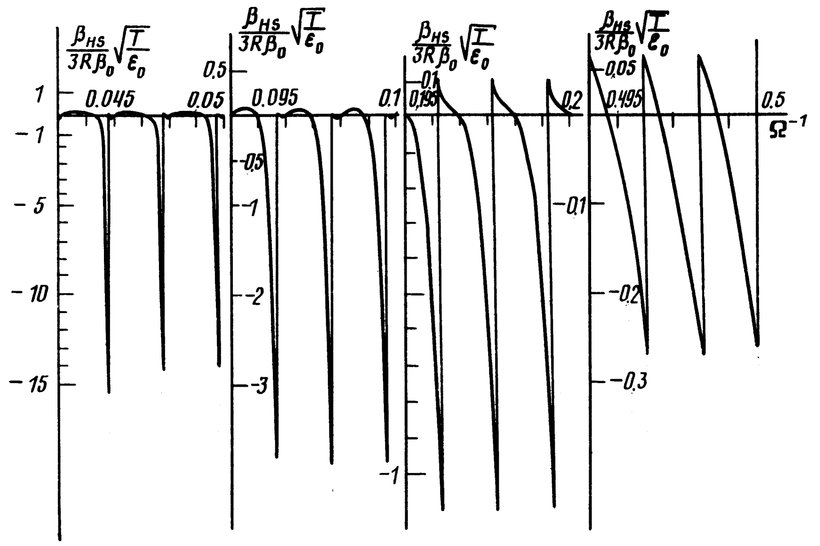

FIG. 6. Graph of the function $\beta_{HS}(1/\Omega)$ in the intermediate range $\Delta>0$ ($\varepsilon_0/T=500$, $\Delta/T=4$). For clarity, the oscillatory peaks are shown on different scales for different values of Δ.

($\Omega\neq\Omega_{N+1/2}$), the sum in Eq. (42) converges rapidly and we have

$$\beta_{HS}(\Omega\neq\Omega_{N+1/2},\Delta=0)=\frac{9R}{4\pi^{3/2}}\beta_0\left(\frac{\varepsilon_0}{T}\right)^{1/2}\left\{\frac{2^{1/2}-1}{2^{1/2}}\zeta\left(\frac{3}{2}\right)-\frac{2}{3}\left(\frac{\Omega}{T}\right)^2\exp\left(-\frac{\Omega}{T}\left|\frac{1}{2}-r\right|\right)\right\}\sim\beta_0R\left(\frac{\varepsilon_0}{T}\right)^{1/2}.$$

Therefore, if Ω differs from the resonant values of the magnetic field ($\Omega_{N+1/2}$), which we have already identified, the contribution β_{HS} remains practically constant and of the order of $\beta_S(\Delta=0)$. Therefore, very close to the values of $\Omega_{N+1/2}$ (where $|r(\Omega)-1/2|\lesssim T/\Omega$) the terms of the series (42) no longer fall exponentially, but the series can be summed and β_{HS} is described by

$$\beta_{HS}(\Omega=\Omega_{N+1/2},\Delta=0)=\frac{3(2^{1/2}-1)}{8\pi^{3/2}}\zeta\left(\frac{1}{2}\right)R\beta_0\left(\frac{\varepsilon_0}{T}\right)^{1/2}\frac{\Omega}{T},$$

i.e., against the background of the constant value of β_{HS}, if $\Omega\neq\Omega_{N+1/2}$ then at the points $\Omega=\Omega_{N+1/2}$ there are deep and extremely narrow ($\Delta\Omega\sim\Omega T/\varepsilon_0$) dips. The general nature of the dependence $\beta_{HS}(\Omega,\Delta=0)$ is demonstrated in Fig. 7.

Following exactly the same procedure, and expanding Eq. (23) in terms of small values of the ratio Δ/T, we can find the Δ-dependent correction to the nonlinear dependence $\beta_{HS}(\Omega,\Delta=0)$, but it is of no special interest.

It is important to stress that the position of the absolute maximum of $\beta_{HS}(\Delta)$ is completely independent of the applied magnetic field, which can be demonstrated directly by differentiating β_{HS} of Eq. (23) with respect to Δ. The equation for an extremal point

$$\int_{\Delta/2T}^{\infty}\frac{x\,dx}{(x-\Delta^*/2T)^{1/2}}\frac{1}{\mathrm{ch}^2\,x}=0$$

obtained in this case is then independent of the magnetic field and it is identical with the corresponding equation found in Ref. 9: $\Delta^*=-1.28T$.

6. DISCUSSION OF RESULTS

We analyze these results by considering first the behavior of a longitudinal thermoelectric power of a normal metal (in the absence of an electronic topological transition) in a magnetic field. We can see from these expressions that, in accordance with the general ideas from the theory of galvanomagnetic phenomena, the longitudinal component of the

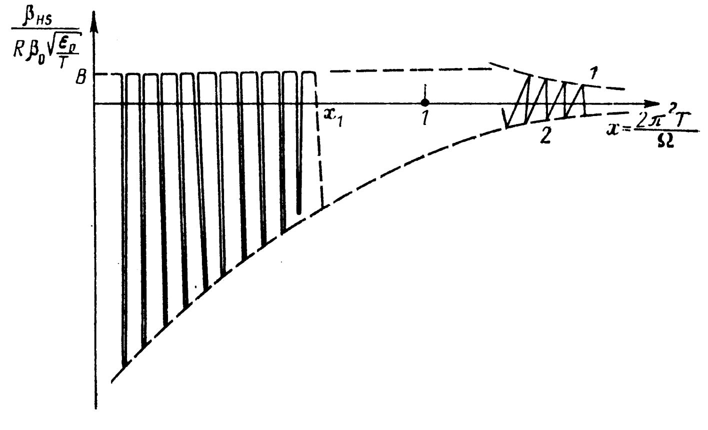

FIG. 7. Schematic dependence $\beta_{HS}(2\pi^2T/\Omega,\ \Delta=0)$ near an electronic topological transition. Here, $B=(9/4\pi^{3/2})(2^{1/2}-1)\cdot2^{-1/2}\zeta(3/2)$ and $x_1=2\pi^2T/\Omega_{N+1/2}$. The envelopes 1 and 2 correspond to $\pm\,{}^3/_4\pi^{1/2}(2^{1/2}-1)|\zeta(1/2)|/x$.

thermoelectric power of a normal metal considered as a function of the applied magnetic field exhibits oscillations of the Shubnikov–de Haas type. The origin of these oscillations is obvious: the magnetic field is increased, the Landau levels cross the Fermi level and this results in a periodic change in the density of states and a consequent change in the relaxation time. It is important to note that, beginning with fields

$$H \geqslant \frac{\pi^2 mc}{e} \frac{T}{\ln(\varepsilon_F/T)}$$

the amplitude of these oscillations exceeds the background value β_0 ($\sim T/\varepsilon_F$) for a normal metal in the absence of a magnetic field and in this sense the oscillatory contribution can be regarded as giant [by a factor $(\varepsilon_0/T)^{1/2}$].

It should be pointed out that the problem of oscillations of the transverse component of the thermoelectric power in a magnetic field was considered a long time ago (in the sixties).[20-22] However, the amplitude of the oscillations found in these early studies, like the oscillations of the conductivity, does not exceed the background value.

This is because the thermodynamic approach used in these early studies ignored the energy dependence of the electron relaxation time this dependence is responsible—in accordance with Eq. (15)—for the anomalously large amplitude of oscillations of the longitudinal thermoelectric power we have found.

Since the function $\omega \cosh^{-2}(\omega/2T)$ which occurs in the expression for β is odd, any noneven (in ω) contribution to $\tau(\omega)$ gives rise to a correction to the thermoelectric power. In view of the smallness of the background value, such contributions are usually considerable. An anomalously large thermoelectric power is found in measurements on Kondo alloys, near electronic topological transitions, and in other cases. We can see from the above analysis that a magnetic field has a similar effect on the thermoelectric power, because it bends the electron trajectories and gives rise to an energy dependence of the relaxation time of these electrons.

We now analyze oscillations of the longitudinal MTEP of a metal near an electronic topological transition. Far from the transition on the one-sheet hyperboloid side ($\Delta \gg T$) the associated corrections β_{HS} are exponentially small and the whole field-dependent part of β is governed by β_H (Figs. 2 and 3), as discussed above. Nevertheless, we must point out that the nature of the small-amplitude oscillations of β_{HS} is now very different from the oscillations of β_H: in weak fields ($\Omega \ll 2\pi^2 T$) these oscillations are sawtooth-shaped [see Eq. (39)] whereas in stronger fields ($\Omega \gtrsim 2\pi^2 T$) the dependence of β_{HS} is in the form of narrow and deep dips in fields in which the next Landau level crosses the Fermi level. This characteristic behavior of $\beta_{HS}(\Omega)$ is also retained near the transition ($\Delta \ll T$), but the amplitude of such oscillations becomes of the same order of magnitude as those of $\beta_H(\Omega)$ and the field-dependent part of the thermoelectric power $\beta_H(\Omega) + \beta_{HS}(\Omega)$ is a sum of oscillations of these two types. It is interesting to note that in the range $\Delta \ll -T$ the contribution of β_{HS} largely cancels the value of β_H. This is due to our artificial division of the magnetic-field-dependent part of the relaxation time in Eq. (13) into two parts (τ_2 and τ_3). Consequently, in the case of a two-sheet hyperboloid the magnetic-field dependent part of the thermoelectric power $\beta_H + \beta_{HS}$ also oscillates [see Eq. (27)]. However, the amplitude of these oscillations expressed in terms of the parameter $(T/|\Delta|)^{1/2}$ is less than on the other side of the transition ($\Delta \gg T$). Nevertheless, in terms of the parameter $(\varepsilon_F/|\Delta|)^{1/2}$ these oscillations exceed the background value β_0 and are of the same order of magnitude as the anomaly β_{HS} which appears as we approach an electronic topological transition.

The oscillations found by the above analysis can, under certain conditions, be observed experimentally. This is easily done for a normal metal in the absence of an electronic topological transition. The conditions for experimental observation are that the magnetic field in the sample be uniform, $\Delta H/H_N \ll H_N/\varepsilon_F$, the temperature gradient be small, and the electron mean free path be long ($T\tau \gg 1$).

In the experiments described in Ref. 13 these requirements were satisfied (with the possible exception of the last), so that no oscillations were observed. However, the above results indicate that some features of the dependence of β on the proximity to the transition point and on the magnetic field nevertheless can be explained qualitatively by such oscillations.

Firstly, the position of the thermoelectric power peak is independent of the applied longitudinal magnetic field, which is in good agreement with the experimental results. Secondly, one wing of the dependence is affected strongly by the applied magnetic field. The profile of the other wing is almost unaffected by an increase in the field. If we average the dependences (24) and (34) over the magnetic field, we find that in sufficiently strongly magnetic fields the asymmetry of these functions has the effect that the thermoelectric power behaves like the envelopes of the peaks, i.e., to the left of the transition the dependence $\beta(\Omega,\Delta \ll -T)$ should be much weaker [by a factor $(T/|\Delta|)^{1/2}$], than on the right ($\Delta \gg T$).

Finally, when the magnetic field is increased, the value of the thermoelectric power at such a peak increases and this is again in qualitative agreement with the experimental results.

We now consider the limiting case of weak fields. We can see from the above results that the oscillatory corrections then fall exponentially in magnitude. We must remember that we postulated that electrons are rarely scattered by impurities ($\Omega\tau \gg 1$) and, we assumed that $r_L \ll l = v_F\tau$. For $H \lesssim mc/e\tau$, this condition is no longer obeyed and our theory is invalid in such very weak fields.

We shall conclude by analyzing the influence of spin splitting on the characteristics of the behavior of the MTEP near an electronic topological transition in the case when the spin mass differs considerably from the cyclotron mass ($m_s \neq m_\perp$). We shall allow for this influence by modifying the expression for the relaxation time (13), so as to replace Δ with $\Delta(\uparrow\downarrow) = \Delta \pm {}^1/_2\Omega(m_\perp/m_S - 1)$, which describes electrons with the spin projections $\pm 1/2$. Moreover, we can show that in the system of equations (13) the expressions describing $\tau_2(\omega,\Omega)$ and $\tau_3(\omega,\Delta,\Omega)$ now have not the factor $(-1)^h$ but $\exp(i\pi k m_\perp/m_S)$, which in the calculation of the thermoelectric power of a normal metal β_H of Eq. (15) transforms to $\cos(\pi k m_\perp/m_S)$, in agreement with the theoretical treatments given in Refs. 16 and 18. The situation is more complex in the case of the contribution β_{HS} which is due to the effect of the applied magnetic field and due to the proximity of the system to an electronic topological transition. If the effects associated with the thermal spreading of the Fermi surface predominate over the spin splitting effects $[T\gg\Omega(m_\perp/m_S-1)]$ or if the system is far from a transition point $[\Delta\gg\Omega(m_\perp/m_S-1)]$, then (as in the case of β_H) the summation over the spin projections in the relationship (23) gives rise to a factor $\cos(\pi k m_\perp/m_S)$. In the opposite case the currents of electrons must be calculated separately for $s=\pm 1/2$ and then added. This alters not only the profile of the oscillation peaks, but also shifts the electronic topological transition point by an amount $\Omega(m_\perp/m_s-1)$.

The authors are deeply grateful to A. A. Abrikosov, A. G. Aronov, V. S. Egorov, and M. I. Kaganov for discussing these results and valuable comments.

[1)] The above value differs by a factor of 2 from that given in Refs. 9–11. This is due to the fact that the background value is selected on the side of the open Fermi surface ($\Delta\gg T$) and not a closed one, as was done in Refs. 9–11. It should also be mentioned that the quantity ε_0 is a parameter of our model and therefore we cannot literally compare the experimental results with the expressions in Eq. (16). Nevertheless, the factor $(\varepsilon_0/T)^{1/2}$ provides a correct indication of the order of magnitude by which the thermoelectric power increases [the factor is $(\varepsilon_F/T)^{1/2}$,] in the vicinity of a transition. This applies also to other quantities (for example, τ_0) described by expressions that contain the parameter ε_0. We are grateful to M. I. Kaganov for drawing our attention to this point.

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