Resonance optics of the low-temperature quantum gases H and D

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The optical properties of dense low-temperature atomic spin-polarized hydrogen and deuterium gases are examined in a near-resonance frequency region corresponding to strong interaction of the gas with electromagnetic radiation. It is shown that new nontrivial quantum effects can appear here even in the case of nondegenerate gases. In contrast to a low-temperature gas whose particles have a de Broglie wavelength shorter than the effective resonance-interaction region, the elastic scattering and the excitation transfer in pair interaction between excited and unexcited atoms leads to formation of a collective mode, a sort of exciton (analogs of such excitons are the known spin waves in atomic hydrogen gas). Hybridization of the excitons with photons leads in turn to new elementary excitations—polaritons—which are characterized, just as in the case of crystal optics, by two energy-spectrum branches separated by a gap. It is the polariton character of the elementary excitations which governs the main features of the interaction between resonance radiation and a gas medium, including total reflection of light from a volume of gas.

1. INTRODUCTION

Low-temperature spin-polarized atomic hydrogen ($H_\downarrow$) and deuterium ($D_\downarrow$) gases, which are intensively investigated of late (Refs. 1 and 2), satisfy the condition

$$k_T R_0 \ll 1, \qquad (1.1)$$

where $k_T = (2mT)^{1/2}$ (m is the atom mass and $\hbar = 1$) is the thermal momentum of the particles, and $R_0 \approx 3.5$ Å is the effective interatomic-interaction radius. This condition leads to the onset of quantum phenomena even in Boltzmann $H_\downarrow$ and $D_\downarrow$ gases (specific spin waves,[3] direct manifestation of quantum correlations in the optical characteristics[4]). In this sense these can be referred to as quantum gases, although the temperature can also be higher than the degeneracy temperature.

The optical properties of gases in the temperature region (1.1) differ in principle from the known analogous properties of these gases at higher temperature. We shall show here that unique excitons and polaritons, which are collective excitations that are not trivial for a gas medium, can appear in resonance optics of sufficiently dense, low-temperature $H_\downarrow$ and $D_\downarrow$ gases satisfying the condition

$$n\lambda^3 \gg 1 \qquad (1.2)$$

(n is the gas density and λ is the photon wavelength), and can alter the dielectric constant radically.

Under condition (1.2) the resonance interaction of an excited atom with an unexcited one predominates over natural broadening, and is therefore decisive for the optical properties. We shall be interested in the region $|\Delta\omega| \lesssim nd^2$ near the dipole-transition frequency ($\Delta\omega$ is the frequency detuning, d is the reduced dipole moment of the atomic transition), which corresponds under this condition to strong interaction of the medium with electromagnetic radiation.[1)] We consider a gas without a condensate. Resonance optics of a Bose gas with a condensate has a number of additional peculiarities and will be considered separately.

The resonance-optics problem was treated in the binary approximation by Vdovin and Galitskiĭ[5] for a high-temperature gas, when

$$k_T r_0 \ll 1 \qquad (1.3)$$

where $r_0 \propto md^2 > R_0$ is the effective radius of the resonance-interaction potential $V(\mathbf{r}) \propto d^2/r^3$. Condition (1.3) yields for the collision broadening of the excited atom a cross section $\sigma \propto d^2/v$ (v is the relative collision velocity). The scales of the coherent interaction of the excited and unexcited atoms, and the scale $n\langle\sigma v\rangle$ of its collision width, turn both out to be proportional to nd^2, so that both the real and the imaginary part of the dielectric constant are of the order of unity. This means that the reflection and transmission coefficients of the incident radiation become of the order of unity, and the transmitted photons are absorbed over a distance on the order of the wavelength λ.

In the considered low-temperature quantum gases $H_\downarrow$ and $D_\downarrow$ meeting the criterion (1.1), a condition inverse to (1.3) can be realized for transitions to highly enough excited states of the atom, with the binary approximation preserved. In this situation, which we shall call the "quantum collision" regime, we have an entirely different picture. The characteristic collision-broadening cross section is now $\sigma \propto r_0^2$, and the collision width $n\langle\sigma v\rangle \propto T^{1/2}$ is much smaller than the coherent interaction, for which the scale nd^2 is preserved. As a result, it is just this interaction which can lead to formation of unique weakly damped excitons whose coherent interaction with the electromagnetic radiation leads in turn to formation of polaritons, which are combined photon–exciton excitations. The optics of a low-temperature quantum gas should thus be determined essentially by the polariton character of the spectrum of the elementary excitations, including in particular the possibility of complete reflection of radiation from a gaseous medium.

2. QUALITATIVE ANALYSIS

As already noted, in a high-temperature gas satisfying condition (1.3) the cross section for scattering and transfer of excitation in pair interaction of an excited atom with an unexcited one is $\sigma \propto d^2/v$ (Ref. 5). When the gas temperature is lowered, the characteristic value of σ increases, and two different situations are possible. The first corresponds to

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violation of the binary approximation and to inclusion of multiparticle effects at a definite value of T, when the characteristic scattering radius $\sigma^{1/2}$ becomes of the order of the average distance $\bar{r}$ between the particles. The second situation (quantum-collision regime) corresponds to preservation of the binary approximation even when the cross section reaches its limiting low-temperature value $\sigma \propto r_0^2$ at

$$k_T r_0 \gg 1, \tag{2.1}$$

and requires satisfaction of the criterion

$$r_0 \propto md^2 \ll \bar{r} \propto n^{-1/3}. \tag{2.2}$$

We shall consider only the quantum-collisions regime.

The need for meeting simultaneously conditions (2.2) and (1.2) imposes an upper bound on the quantity d^2 indicative of the quantum-collision regime:

$$md^2 \ll \lambda. \tag{2.3}$$

This bound corresponds to radiative transitions from the ground state of the hydrogen atom to a state with principal quantum number $\nu \gg 1$. In turn, at the temperatures of several times ten millidegrees Kelvin that can actually be attained in $H_\downarrow$ gas,[2)] the condition (2.1) can be met only for $\nu \gtrsim 5$.

In the quantum collision regime the characteristic collision width of the excited atom is of the order of

$$\Gamma \propto n\langle\sigma v\rangle \propto (nd^2)(k_T r_0) \ll nd^2. \tag{2.4}$$

The quantity nd^2 is the characteristic size of the frequency region in which resonance radiation interacts strongly with the medium. One can therefore expect the optics of low-temperature gases to be coherent under condition (2.4). On the other hand, the quantity nd^2 determines (just as in the high-temperature case) the scale of the coherent interaction of an excited atom with a gas. The condition (2.4) means then the possibility of formation of a weakly damped collective mode—sort of a unique exciton [this mode is the analog of the aforementioned spin waves in $H_\downarrow$ gas (Ref. 3)[3)]]. As a result of the strong coherent interaction of the resonance radiation with the excitons, the true elementary excitation in the system is a polariton—a quasiparticle that is a superposition of a photon and an exciton, which will determine in fact the optics of the gas.

The situation, however, is made complicated by the presence of additional excited-atom dispersion due to the nonsphericity of the resonance interaction. The additional dispersion results in a dielectric-constant part of order unity in the frequency interval $\Delta\varepsilon = 4\pi nd^2$ near the resonance frequency ω_0. In the frequency region $\Delta\varepsilon$ the polaritons are strongly damped and the picture of the reflection, transmission, and absorption of the radiation turns out to be qualitatively the same as in a high-temperature gas. At the same time, a weakly damped exciton can exist even for additional dispersion. In this case its frequency ω_* (to which a dielectric-constant pole corresponds) is located outside the interval $\Delta\omega$. At frequencies outside $\Delta\omega$ the optical properties of the gas are then determined by the existence of weakly damped excitonic polaritons. The presence of an energy gap $\Delta E \propto 4\pi nd^2$ between two branches of the polariton spectrum should lead to total reflection of the radiation from the gas volume. This effect, naturally, occurs only in that part of the frequency interval ΔE which does not overlap the strong-absorption region $\Delta\omega$ (the degree of overlap of the intervals ΔE and $\Delta\omega$ is determined by the specific features of the short-range interaction between an atom and an excited atom).

If there is no real exciton, the optics of the gas has outside the interval $\Delta\omega$ qualitatively the same character, except that now the dielectric constant has no pole corresponding to an exciton. In this sense one can speak of a virtual exciton that determines the optics outside the interval $\Delta\omega$.

The entire foregoing analysis pertains to the case of small Doppler width

$$\omega_0 v_T / c \ll nd^2. \tag{2.5}$$

For d^2 values corresponding to the quantum-collision regime ($\nu \gtrsim 5$), at realistic gas temperatures, this condition can be met for densities exceeding[4)] 10^{19} cm^{-3}.

If the inverse of condition (2.5) holds, we have the usual Doppler broadening of the spectral line. The photon passes through the gas medium with near-unity probability and is adsorbed over a distance substantially longer than its wavelength. It is of interest to note that this results in a collective mode (exciton) that is weakly damped at momenta $k \ll nd^2/v_T$ (cf. Ref. 3). Such an exciton hybridizes with a photon only to a low degree.

The magnetic field strength H needed to produce the gases $H_\downarrow$ and $D_\downarrow$ themselves is of no principal significance for the considered picture of the interaction of resonance radiation with a gas. We shall consider, however, a more lucid situation, in which $\mu_B H \gg nd^2$, ε_f (ε_f is the fine splitting for the excited atom). In this case the component of the electron orbital momentum M of the atom ($M = 0, \pm 1$) along the magnetic-field axis is a good quantum number, and the frequency regions corresponding to transitions into states with different M are distinctly separated.

3. FUNDAMENTAL RELATIONS

We present in this section a number of general relations on which the exposition that follows will be based.

In our case of a strong magnetic field, for the frequency region corresponding to transitions into states with orbital-momentum component M, we can express the polarization operator $\Pi_{\alpha\beta}$ in the form

$$\Pi_{\alpha\beta}(\mathbf{k}, \omega) = e_M^{\alpha} e_M^{\beta *} \Pi_M(\mathbf{k}, \omega), \tag{3.1}$$

where the unit vectors $\mathbf{e}_M$ are given in terms of the excited-atom polarization vectors by the equations

$$\mathbf{e}_0 = \mathbf{e}_z, \quad \mathbf{e}_\pm = (\mathbf{e}_x \pm i\mathbf{e}_y)/2^{1/2}$$

(the z axis is directed along the magnetic field).

For a polarization operator in the form (3.1) the frequency ω, the wave vector $\mathbf{k}$, and the polarization vector $\mathbf{g}$ of an elementary excitation propagating in a medium are related by

$$[k^2 - (\omega/c)^2]\mathbf{g} - \mathbf{k}(\mathbf{kg}) + \Pi_M \mathbf{e}_M (\mathbf{e}_M^* \mathbf{g}) = 0. \tag{3.2}$$

It follows from (3.2) that for arbitrary direction of $\mathbf{k}$ one of the modes of the elementary excitations is a photon that does not interact with the medium. Corresponding to this mode is a polarization vector $\mathbf{g}_1$ uniquely defined by the conditions $\mathbf{k} \cdot \mathbf{g}_1$ and $\mathbf{e}_M^* \cdot \mathbf{g}_1 = 0$. (An exception is the case $M = 0$, $\mathbf{k} \| \mathbf{e}_0$, when the two transverse modes do not interact with the medium.)

The dispersion law that follows from (3.2) for a mode interacting with a medium (the polarization vector is orthogonal to $\mathbf{g}_1$) can be written in the form

$$\left(\frac{ck}{\omega}\right)^2 = 1 - \frac{1-a}{(\omega/c)^2 \Pi_M^{-1}(\mathbf{k},\omega) - a}, \tag{3.3}$$

where $a = \cos^2\theta$ for $M = 0$, $a = \frac{1}{2}\sin^2\theta$ for $M = \pm 1$, and θ is the angle between the vector $\mathbf{k}$ and the magnetic-field direction.

To find the polarization operator $\Pi_M(\mathbf{k},\omega)$ we use the Keldysh diagram technique (see, e.g., Ref. 7) which, in contrast to the Matsubara technique used in Ref. 5, obviates the need for an analytic continuation in frequency. It is then necessary to sum over the "+" and "−" indices at the vertices, for it can be easily verified that diagrams containing "+" and "−" vertices simultaneously are negligibly small. It is possible then to consider vertices and Green's functions with only one fixed index, which we choose to be "−".

For transitions into states with given M, we represent the Hamiltonian of the interaction of a gas medium with resonant electromagnetic radiation in the form

$$H_{int} = (\omega_0 d/c)\int d\mathbf{r}\,(\hat{\varphi}_M^+ \hat{\psi} \mathbf{e}_M^* + \hat{\varphi}_M \hat{\psi}^+ \mathbf{e}_M)\,\hat{\mathbf{A}}. \tag{3.4}$$

Here $\hat{\mathbf{A}}$ is the electromagnetic-field operator, while $\hat{\varphi}_M$ and $\hat{\psi}$ are the respective field operators for the excitation of atoms with orbital-momentum component M and for unexcited atoms.

Taking the particle elastic interaction into account in the mean-field approximation, we can express the Green's function of a gas of unexcited atoms in the form

$$G(\mathbf{k},\omega) = (\omega - E(k) + i\delta)^{-1} \pm 2\pi i n_k \delta(\omega - E(k)),$$
$$E(k) = k^2/2m + (1\pm 1) n U, \tag{3.5}$$

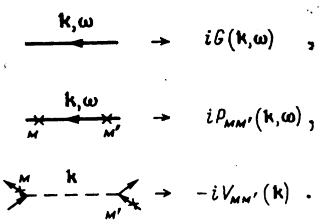
where n_k are the occupation numbers for the gas particles and U is the effective elastic-interaction vertex. Here and below, the upper and lower signs refer to Fermi and Bose gas, respectively. Relation (3.5), which corresponds to complete absence of photons and excited atoms in the system, will be used for the case when the photon and excited-atom densities are low. We use the angular-momentum representation for the Green's functions of excited atoms and photons. The zeroth Green's function of excited atoms is

$$P_{MM'}^{(0)}(\mathbf{k},\omega) = \delta_{MM'}(\omega - \omega_0 - \varepsilon_M - k^2/2m + i\delta)^{-1}, \quad \varepsilon_M = \mu_B M H. \tag{3.6}$$

In our problem, only the longitudinal part of the photon propagator $D_{MM'}(\mathbf{k},\omega)$ plays an important role in the diagrams. This part is equivalent to the potential of resonance interaction between an atom and an excited atom

$$V_{MM'}(\mathbf{k}) = (\omega_0 d/c)^2 D''_{MM'}(\mathbf{k},\omega_0) = 4\pi d^2 (\mathbf{e}_M \mathbf{k})(\mathbf{e}_{M'} \mathbf{k})/k^2, \tag{3.7}$$

since only frequencies close to ω_0 are effective for the photon propagators in the diagrams. After neglecting the transverse photon propagator and replacing the longitudinal photon propagator by the potential (3.7), the diagram technique becomes graphically identical with that of Ref. 5. The analytic perusal of the diagram corresponds in this case to a Keldysh technique with all the vertices having an index "−". The rules for correspondence of the graphic and analytic elements take then the form



(3.8)

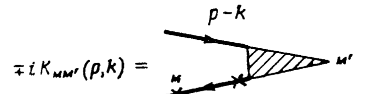
(3.9)

(3.10)

From the form of the Hamiltonian (3.4) follows a general expression for the polarization operator. In a strong magnetic field, accurate to the small parameter $nd^2/(\mu_B H)$, only the diagonal element is significant (k, p, and p' denote below 4-momenta)

$$\Pi_{MM}(k) \equiv \Pi_M(k) = 4\pi(\omega_0 d/c)^2 \int K_{MM}(p,k)\, d^4p/(2\pi)^4, \tag{3.11}$$

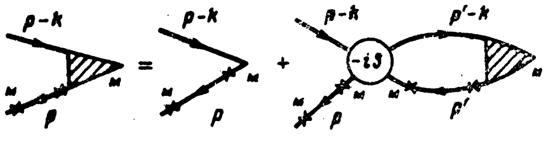
where



(3.12)

The sign in (3.12) was chosen such as to compensate, in the case of a Fermi gas, for the additional "−" sign that appears in the polarization operator because the diagram (3.12) enters in this operator with closed left-hand ends. The diagrams entering in K should be irreducible in terms of the line (3.10), in the sense that it is impossible to separate the right-hand end of the diagram from the two left-hand ones by cutting one such line. This property of the function K is obvious if it is recognized that it defines a polarization operator that must be irreducible in the photon propagator, and the dashed line is the redesignated longitudinal part of this propagator.

K_{MM} satisfies the relation

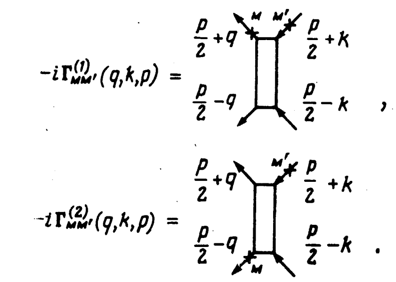


(3.13)

in which $S_{MM'}(p,k,p')$ denotes the aggregate of four-pole diagrams that are irreducible both with respect to cutting the line pairs (3.8)–(3.9) and with respect to cutting the line (3.10).

Lines with oppositely directed arrows are not taken into account in (3.12) and (3.13), owing to their nonresonant character, so that in the frequency region of interest to us they make a contribution that is small in terms of the parameter nd^2/ω_0. Also disregarded are the off-diagonal terms of $P_{MM'}$ and $K_{MM'}$, which are small in the parameter $nd^2/(\mu_B H)$.

In the gas approximation it is possible to change in the diagrams from the interaction potential to the ladder four-pole diagrams



(3.14)

Together with the resonance interaction we shall take into account also the pure elastic short-range interaction of the excited and unexcited atoms, an interaction essential for transitions with small enough values of d.

Ladder series for the four-pole diagrams (3.14) are summed in standard fashion by transforming to the functions

$$\Gamma^{(\pm)}=\Gamma^{(1)}\pm\Gamma^{(2)}. \qquad (3.15)$$

As a result (see Ref. 5),

$$\Gamma^{(\pm)}(q,\ k,\ p)=\Gamma^{(\pm)}(\mathbf{q},\ \mathbf{k},\ \Omega),\ \Omega=\omega_p-\omega_0-\mathbf{p}^2/4m, \qquad (3.16)$$

where $\Gamma^{(\pm)}(\mathbf{q},\mathbf{k},\Omega)$ is a solution of a known integral equation for a four-pole diagram corresponding to an elastic potential

$$U^{(\pm)}=U\pm V, \qquad (3.17)$$

where U is the potential of the short-range interaction of an excited and an unexcited atom.

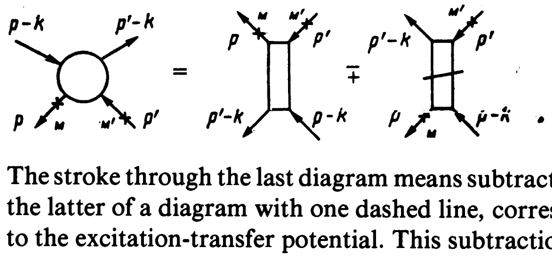
It will be shown below that the optical properties of a gas at the frequencies $|\omega-\omega_0|\propto 4\pi nd^2$ of interest to us are determined by values of $\Gamma^{(\pm)}(\mathbf{q},\mathbf{k},\Omega)$ that differ little from their values on the mass shell $\Omega=\mathbf{k}^2/m$, on which the equation for $\Gamma^{(\pm)}(\mathbf{q},\mathbf{k},\Omega)$ goes over into the scattering-problem equation. It is convenient in this case to express $\Gamma^{(\pm)}$ in terms of the interaction potential and the wave function of a particle pair in the coordinate representation

$$\Gamma^{(\pm)}_{MM'}(\mathbf{q},\mathbf{k})\equiv\Gamma^{(\pm)}_{MM'}(\mathbf{q},\mathbf{k},\Omega=\mathbf{k}^2/m)$$
$$=\int d\mathbf{r}\, e^{-i\mathbf{q}\mathbf{r}}U^{(\pm)}_{MM''}(\mathbf{r})\chi_{\mathbf{k}M'}(\mathbf{r},M''). \qquad (3.18)$$

Here $\chi_{kM'}(\mathbf{r},M'')$ is the wave function of the relative motion of a particle pair in a potential $U^{\pm}_{MM''}$, a function corresponding to a momentum $\mathbf{k}$ and an angular-momentum component M' prior to scattering. As $|\mathbf{r}|\to\infty$ this function has an asymptote

$$\chi_{\mathbf{k}M'}(\mathbf{r},\ M'')\to e^{i\mathbf{k}\mathbf{r}}\delta_{M'M''}. \qquad (3.19)$$

The expression for the function S in terms of the amplitudes $\Gamma^{(1)}$ and $\Gamma^{(2)}$ is



(3.20)

The stroke through the last diagram means subtraction from the latter of a diagram with one dashed line, corresponding to the excitation-transfer potential. This subtraction is necessary to satisfy the aforementioned irreducibility requirement for S. We designate the function corresponding to the crossed-out diagram by $\tilde{\Gamma}^{(2)}$. The "$-$" sign in front of this diagram in the case of a Fermi gas is due to the fact that its substitution for s in (3.13) leads to the appearance of a closed loop of Fermi lines. With allowance for Eqs. (3.13), (3.14), (3.16), and (3.20) we get

$$S(\mathbf{p},\ k,\ \mathbf{p}')=\Gamma^{(1)}(\mathbf{k}/2+\mathbf{q},\ \mathbf{k}/2-\mathbf{q},\ \Omega)\mp\tilde{\Gamma}^{(2)}(-\mathbf{q}-\mathbf{k}/2,$$
$$-\mathbf{q}+\mathbf{k}/2,\ \Omega), \qquad (3.21)$$

where

$$\mathbf{q}=(\mathbf{p}-\mathbf{p}')/2,\ \Omega=\omega_k-\omega_0+\mathbf{q}^2/m-\mathbf{k}(2\mathbf{p}+2\mathbf{p}'-3\mathbf{k})/4m.$$

The Green's function of the excited atom is obtained from the Dyson equation which yields ($P_M\equiv P_{MM}$) apart from off-diagonal terms small in the parameter $nd^2/(\mu_B H)$

$$P_M(p)=[\omega_p-\omega_0-\varepsilon_M-\mathbf{p}^2/2m-\Sigma_M(p)+i\delta]^{-1}, \qquad (3.22)$$

where the mass operator Σ takes in the gas approximation the form[5]

$$\Sigma_M(p)=i\int G(p')[\Gamma^{(2)}_{MM}(-\mathbf{q},\mathbf{q},\tilde{\Omega})\mp\Gamma^{(1)}_{MM}(\mathbf{q},\mathbf{q},\tilde{\Omega})]d^4p'/(2\pi)^4, \qquad (3.23)$$

where $\Omega=\omega_p+\omega_{p'}-\omega_0-(\mathbf{p}+\mathbf{p}')^2/4m$. The reason for the "$-$" sign in the case of a Fermi gas is that the diagram of a mass operator with four-pole $\Gamma^{(1)}$ contains a closed fermion loop. Substituting in (3.23) the explicit expression for G makes possible integration over $d\omega_{p'}$, which leads to replacement of $iG(p')d^4p'/(2\pi)^4$ and $\pm n_{p'}d\mathbf{p}'/(2\pi)^3$. The parameter $\tilde{\Omega}$ becomes equal then to $\omega_p-\omega_0-\mathbf{p}^2/2m+\mathbf{q}^2/m$.

4. THE FOUR-POLE $\Gamma^{(\pm)}$ AND THE POLARIZATION OPERATOR IN THE LOW-ENERGY LIMIT. EXCITONS IN A LOW-TEMPERATURE GAS

Let us find the general form of the functions $\Gamma^{(\pm)}$ in the limit of small momenta $\mathbf{q}$ and $\mathbf{k}$ corresponding to the quantum-collision regime. The condition (1.3) permits a clear-cut division of the integration region in (3.18) into two. In the strong-interaction region $r\lesssim r_0$ the function $\chi_{\mathbf{k}M'}(\mathbf{r},M'')$ does not depend on $\mathbf{k}$, the exponential is $e^{-i\mathbf{k}\mathbf{r}}\approx 1$, and we have a contribution independent of the momenta, which we designate by $4\pi d^2\zeta^{(\pm)}_{MM'}$. In the weak interaction region $r\gg r_0$ the function $\chi_{\mathbf{k}M'}(\mathbf{r},\mathbf{M}'')$ takes its asymptotic value (3.19), while $U^{(\pm)}_{MM''}(\mathbf{r})=\pm V_{MM'}(\mathbf{r})$. The resultant integral builds up over distances $\propto|\mathbf{q}-\mathbf{k}|^{-1}$. In view of the condition (1.3), we can therefore set formally the lower integration limit at $r=0$, and the resultant contribution to $\Gamma^{(\pm)}$ is equal to $\pm V_{MM'}(\mathbf{q}-\mathbf{k})$ (3.7). We have thus as a result

$$\Gamma^{(\pm)}_{MM'}(\mathbf{q},\mathbf{k})=4\pi d^2\{\zeta^{(\pm)}_{MM'}\pm[\mathbf{e}_M^*(\mathbf{q}-\mathbf{k})][\mathbf{e}_{M'}(\mathbf{q}-\mathbf{k})]/(\mathbf{q}-\mathbf{k})^2\}. \qquad (4.1)$$

Note that even if the elastic-interaction effective radius is $R_0\ll r_0$, the quantities $\zeta^{(\pm)}_{MM'}$ are determined essentially by the behavior of the potential $U^{(\pm)}_{MM'}(\mathbf{r})$ at short distances $r\lesssim R_0$. The reason is that it is precisely this range of distances which governs the form of the function χ for all $r\lesssim r_0$. The characteristic values $\zeta^{(\pm)}_{MM'}$ should be of the scale of unity (cross section $\sigma\propto r_0^2$). In principle, however, a situation is possible in which $|\zeta^{\pm}_{MM'}|$ becomes anomalously large because of the resonance on a weakly bound level (real or virtual).

In accordance with (3.15) we obtain for the amplitudes $\Gamma^{(1)}$ and $\Gamma^{(2)}$

$$\Gamma^{(1)}_{MM'}(\mathbf{q},\mathbf{k})=2\pi d^2(\zeta^{(+)}_{MM'}+\zeta^{(-)}_{MM'}), \qquad (4.2)$$

$$\Gamma^{(2)}_{MM'}(\mathbf{q},\mathbf{k})=2\pi d^2(\zeta^{(+)}_{MM'}-\zeta^{(-)}_{MM'})+4\pi d^2\frac{[\mathbf{e}_M^*(\mathbf{q}-\mathbf{k})][\mathbf{e}_{M'}(\mathbf{q}-\mathbf{k})]}{(\mathbf{q}-\mathbf{k})^2}. \qquad (4.3)$$

We consider now the expression (3.21) for the function S near the mass shell. The quantities $|\mathbf{p}|$ and $|\mathbf{p}'|$ in (3.21) are of the order of the characteristic momentum k_T of the particles, and the momentum $|\mathbf{k}|\lesssim\omega_0/c\ll k_T$. We have hence on the basis of (4.2) and (4.3) [the expression for $\Gamma^{(2)}$ contains only the first term of (4.3)]

$$S_{MM'}=4\pi d^2\zeta^{(\mp)}_{MM'}. \qquad (4.4)$$

We call attention to the fact that a contribution to S is made only by the amplitude $\Gamma^{(-)}$ in the case of a Fermi gas and only by the amplitude $\Gamma^{(+)}$ in the case of a Bose gas. This distinguishes the low-temperature case from the case of fast particles,[5] where the result is independent of the statistics, since the contribution made to s by the first term of (3.21) is small (owing to the almost backward scattering).

Taking into account the explicit form of the function S and introducing the functions ($\omega\equiv\omega_k$)

$$K_M(k)=4\pi d^2\int K_M(p,k)\,d^4p/(2\pi)^4, \qquad (4.5)$$

$$Q_M(k)=4\pi d^2\int n_{|\mathbf{p}-\mathbf{k}|}P_M(\mathbf{p},\omega_p=\omega+E(|\mathbf{p}-\mathbf{k}|))\,d\mathbf{p}/(2\pi)^3, \qquad (4.6)$$

we obtain from relations (3.11)–(3.13) ($\zeta^{(\mp)}_M\equiv\zeta^{(\mp)}_{MM}$)

$$\Pi_M(k)=(\omega_0/c)^2K_M(k), \qquad (4.7)$$

$$K_M(k)=(Q_M^{-1}(k)\pm\zeta^{(\mp)}_M)^{-1}. \qquad (4.8)$$

To determine the mass operator Σ_M that enters in the expression (3.23) for the Green's function P_M in the mass-shell vicinity of interest to us, we can use in (3.23) the values (4.2) and (4.3) of Γ. As a result we get

$$\Sigma_M(p)=4\pi nd^2\left[\zeta^{(\mp)}_M\mp\int\frac{n_{p'}}{n}\frac{|(\mathbf{p}-\mathbf{p}')\mathbf{e}_M|^2}{(\mathbf{p}-\mathbf{p}')^2}\,d\mathbf{p}'/(2\pi)^3\right]. \qquad (4.9)$$

The first term in the right-hand side of (4.9) is connected with the particle interaction at short distances $r\lesssim r_0$. The momentum-dependent second term is due to the interaction over large distances $r\gg r_0$. Depending on $\mathbf{p}$, it ranges from 0 to $4\pi nd^2$ for $M=0$ and to $2\pi nd^2$ for $M=\pm1$.

The function $\mathbf{K}_M$ to which the polarization operator Π_M is proportional is in fact a two-particle Green's function constructed on two ψ and two φ operators. This function contains information on the excitation transport in the system. Strictly speaking, we are dealing with an imaginary system in which the interaction with transverse degrees of freedom of the electromagnetic field (with the photons) is "turned off." The latter will hybridize in our case with the natural mode of the system, and the true excitation-transport picture will be described not by the function K_M but by Eq. (3.2). It is advantageous, however, to first mentally turn off the interaction of the gas with the photons and to analyze the natural mode of the excitation transport. The more so, since it will be seen below that in some cases its distortion by hybridization with photons is of no fundamental importance.

If $K_M(\mathbf{k},\omega)$ has a pole near the real ω axis, this means that the system contains a mode of weakly damped elementary excitations—excitons. It follows from (4.8) that the dispersion equation is then

$$Q_M^{-1}(\mathbf{k},\omega)\pm\zeta^{(\mp)}_M=0. \qquad (4.10)$$

In view of the presence of additional dispersion of the excited atom the second term in the right-hand side of (4.9), a real solution of (4.10) contributing a pole to K_M does not exist for all values of $\zeta^{\mp}_M$. In the case of Boltzmann gases a numerical analysis of Eq. (4.10) using (3.23), (4.6), and (4.9) shows that there is no real solution as $k\to0$ if $-0.29\leqslant\zeta^{(\pm)}_M\leqslant0.64$.

A dispersion law for the excitons can be obtained analytically in the limit as $|\zeta^{(\mp)}_M|\to\infty$. One can neglect in this case the momentum-dependent second term, which does not exceed $4\pi nd^2$. Accordingly, the expressions that follows for Q_M from (4.9), (3.23), and (4.6) takes the form

$$Q_M=4\pi d^2\int n_p[\omega-\omega_0-\varepsilon_M-4\pi nd^2\zeta^{(\mp)}_M-\mathbf{pk}/m+i\delta]^{-1}\,d\mathbf{p}/(2\pi)^3$$
$$\approx(\xi-\zeta^{(\mp)}_M)^{-1}\{1+{}^1/_3\langle p^2\rangle k^2/[4\pi nd^2m(\xi-\zeta^{(\mp)}_M)]^2\},$$
$$\xi=(\omega-\omega_0-\varepsilon_M)/(4\pi nd^2),\qquad \langle p^2\rangle=\int\frac{n_p}{n}p^2\,d\mathbf{p}/(2\pi)^3. \qquad (4.11)$$

Assuming the momentum k to be small enough, we have expanded here in powers of k up to the quadratic term. Hence, substituting (4.11) in (4.10), we obtain for the natural frequency ω of the collective mode, corresponding to a momentum $k=0$,

$$\omega_*=\omega_0+\varepsilon_M+4\pi nd^2\xi_*,\qquad \xi_*=(1\mp1)\zeta^{(\mp)}_M. \qquad (4.12)$$

The expansion procedure in (4.11) is legitimate if the term proportional to k^2 is small compared with unity. Replacing in this term ζ by ζ_* we arrive at the dispersion law

$$\xi=\xi_*\mp{}^1/_3\frac{\langle p^2\rangle k^2}{[4\pi nd^2m]^2\zeta^{(\mp)}_M} \qquad (4.13)$$

and we can represent the small expansion parameter in the form

$$\frac{v_Tk}{4\pi nd^2|\zeta^{(\mp)}_M|}\ll1. \qquad (4.14)$$

The characteristic width of the energy band for the excitons, as seen from (4.13) and (4.14), turns out to be much smaller than $4\pi nd^2$.

The dispersion law (4.13) is very similar to that which obtains for spin waves in a low-temperature gas, and is based on an identical small parameter (see Ref. 3). This reflects the fact that if $|\zeta_M^{(\mp)}| \gg 1$, when the additional dispersion can be neglected, the excitation-transport picture in the gas under condition (2.1) becomes qualitatively the same as the spin-transport pattern under condition (1.1). If the parameter (4.14) becomes of the order of unity we can verify, by determining the imaginary part of Q_M, that strong collisionless Landau damping takes place. If the parameter (4.14) is small the Landau damping is exponentially small relative to this parameter (cf. Ref. 3).

In addition to collisionless damping, collisional damping of excitons is also present, but we have neglected it by retaining in the four-pole Γ only the real part. The characteristic width of the collisional term is $\tau^{-1} \propto n r_0^2 v_T$. The condition for weak collisional damping (short de Broglie wavelength compared with the mean free path), namely $(d\omega/dk)\tau \gg k^{-1}$, where $\omega(k)$ is the exciton frequency defined by relation (4.13), imposes a lower bound on the exciton momentum

$$k^2 \gg (n r_0^3)(n/k_T)\,|\zeta_M^{(\mp)}|. \qquad (4.15)$$

By virtue of the criterion (2.1) it is possible to satisfy simultaneously in the quantum-collision regime the condition (4.15) as well as (4.14), imposing an upper bound on k.

The criteria (4.14) and (4.15) remain in force for real solutions of (4.10) at $|\zeta_M^{(\mp)}| \propto 1$, while the dispersion law for the exciton can be described qualitatively by Eq. (4.13) as before. A noteworthy nontrivial fact is that a collective mode exists in the presence of strong additional dispersion—the excitation energy of the particles depends on their momentum, but coherent interaction leads to formation of collective mode nonetheless.

5. EXCITONIC POLARITONS. DIELECTRIC CONSTANT

Interaction between a medium and transverse degrees of freedom of an electromagnetic field leads, generally speaking, to hybridization of the excitons with the photons, i.e., to formation of excitonic polaritons. The polariton dispersion law is determined by relation (3.3) which we write, with allowance for (4.7) and (4.8), in the form

$$\left(\frac{ck}{\omega}\right)^2 = \varepsilon_M(\mathbf{k},\omega) = 1 - \frac{1-a}{Q_M^{-1}(\mathbf{k},\omega) \pm \zeta_M^{(\mp)} - a}. \qquad (5.1)$$

Assuming the criterion of weak collisionless damping of the excitons (4.14) to be met, with k of the order of the photon momentum k_0, which is equivalent in fact to the condition that the Doppler broadening be small, we can put $\mathbf{k} = 0$ in $Q_M(\mathbf{k},\omega)$ by virtue of the small width of the energy band for excitons compared with $4\pi n d^2$. As a result, the scalar dielectric constant ε_M becomes independent of $\mathbf{k}$.

In the limit $|\zeta_M^{(\mp)}| \gg 1$, putting $Q_M^{-1} = \xi - \zeta_M^{(\mp)}$, we have on the basis of (5.1)

$$\left(\frac{ck}{\omega}\right)^2 = \varepsilon_M(\omega) = 1 - \frac{1-a}{\xi - (1\mp 1)\zeta_M^{(\mp)} - a}. \qquad (5.2)$$

Two modes of the polariton energy spectrum correspond to Eq. (5.2). Just as for the well-known case of polaritons in a crystal (see Ref. 8), they are separated by an energy gap, whose value in our case is

$$\Delta = 4\pi n d^2 (1-a). \qquad (5.3)$$

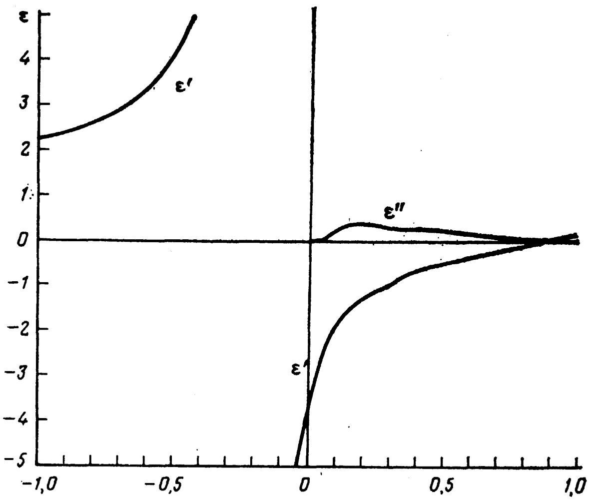


FIG. 1. Dielectric constant as a function of $\xi - \zeta_0^{(+)}$ for a Bose gas at $T \gg T_c$, $M = 0$, $\theta = \pi/2$, $\zeta_0^{(+)} = -0.5$.

This gap occurs in the frequency region

$$\xi_1 \leqslant \xi \leqslant \xi_2,$$
$$\xi_1 = (1\mp 1)\zeta_M^{(\mp)} + a, \quad \xi_2 = (1\mp 1)\zeta_M^{(\mp)} + 1. \qquad (5.4)$$

The real part of the dielectric constant $\varepsilon'_M(\omega)$, defined by the right-hand side of (5.2) becomes negative in the region of the gap (5.4). The imaginary part is then $\varepsilon''_M(\omega) = 0$, and total reflection of the radiation should take place from the principal volume in the region of the polariton-gap spectrum. Allowance for the additional dispersion when determining Q_M yields a nonzero ε'_M, but only in the frequency region $\zeta_M^{(\mp)} \leqslant \xi \leqslant \zeta_M^{(\mp)} \mp 1$, which does not overlap in the considered limit $|\zeta_M^{(\mp)}| \gg 1$ with the region of the total reflection (5.4).

The pole of the dielectric constant is close to the exciton natural frequency ω (corresponding to $\mathbf{k} = 0$). At $a = 0$ it coincides with ω_*.

Under conditions when an exciton exists at $|\zeta_M^{(\mp)}| \propto 1$ the polariton spectrum and the dependence of ε''_M on ε''_M remain qualitatively the same as for $|\zeta_M^{(\mp)}| \gg 1$. The additional-dispersion region, however, in which $\varepsilon''_M \neq 0$, can "eat up" part of the total-reflection region. The values of ε'_M and ε''_M calculated in this situation for a Bose gas at $T \gg T_c$ (T_c is the Bose-condensation temperature) are shown in Fig. 1 for the case $M = 0$, $\theta = \pi/2$, $\zeta_0^{(+)} = -0.5$.

At these values of $\zeta_M^{(\mp)}$, when there are now weakly damped excitons in the gas, the $\varepsilon'_M(\omega)$ dependence retains nonetheless the features inherent in the exciton case. Although there is indeed no pole on the $\varepsilon'_M(\omega)$ curve, a maximum and a minimum are clearly pronounced. This is seen from Fig. 2, which shows the computer-calculated dielectric constant of a Bose gas at $T \gg T_c$, $M = 0$, $\theta = \pi/2$, $\zeta_0^{(+)} = 0.25$. Outside the region of additional dispersion corresponding to $\varepsilon''_M \neq 0$, there exists again a total-reflection region. This picture can be treated as a result of hybridization of a photon with a "virtual" exciton.

Allowance for the imaginary part in the four-pole Γ yields the collisional damping of the polaritons. The width of

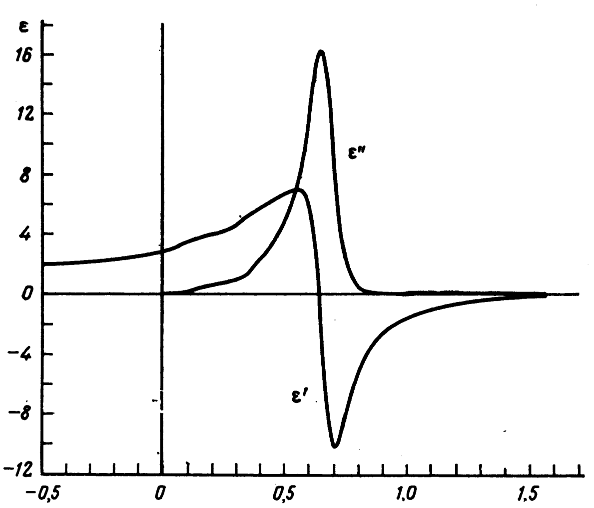


FIG. 2. Dielectric constant as a function of $\xi - \zeta_0^{(+)}$ for a Bose gas at $T \gg T_c$, $M = 0$, $\theta = \pi/2$, $\zeta_0^{(+)} = 0.25$.

the collisional damping turns out to be small compared with characteristic polariton energy $4\pi nd^2$ to the extent that the parameter $k_T r_0 \ll 1$ is small. As to the radiative damping of the polariton at $k \propto k_0$ the radiative width is $\tau_0^{-1} \propto d^2/\lambda^3 \ll 4\pi nd^2$ by virtue of the condition (1.2).

The case $a \to 1$, which is possible for $M = 0$ and $\theta \to 0$, is special. In this case one of the solutions of (5.1) is $\omega = ck$, i.e., it corresponds to a photon that hardly interacts with the gas medium. The second solution corresponds to an exciton with a dispersion law defined by the equation

$$Q_M^{-1}(\mathbf{k}, \omega) \pm \zeta_M^{(\mp)} - 1 \approx 0. \tag{5.5}$$

The situation is similar for strong Doppler broadening, when

$$v_T k_0 / 4\pi nd^2 \gg 1. \tag{5.6}$$

In this case, for $k \propto k_0$, we have $|Q_M(\mathbf{k},\omega)| \ll 1$ and from (5.1) we get $\omega \approx kc$, i.e., a photon that hardly polarizes the medium. For $k \ll k_0$ we can set equal to zero the left-hand side of (5.1), obtaining thus Eq. (5.5) corresponding to an exciton. By virtue of the condition (5.5), there exists a wide momentum range $k \ll k_0$ in which (4.5) is satisfied and its collisional damping is weak.

The last two cases correspond to the presence in the gas of "pure excitons" not hybridized with photons. It should be noted that the characteristic mean free path $l(k) = (d\omega/dk)\tau_0$ of these excitons with respect to spontaneous radiative decay exceeds their de Broglie wavelength under the condition

$$k \gg k_* \propto (nd^2/T)^{1/2} (r_0/\lambda)^{1/2} k_0. \tag{5.7}$$

The criterion (5.7) may turn out to be more stringent than (4.15). It can, however also be readily met in the quantum-collision regime even for momenta $k \ll k_0$, by virtue of the inequality (2.3) and the condition (2.2) that leads at $T > T_c$ to $nd^2 \ll T$.

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[1)] The region $|\Delta\omega| \gg nd^2$, which is of interest from the standpoint of the appearance of spatial quantum correlations in optical characteristics of a gas, was considered earlier in Ref. 4.

[2)] We do not consider open traps, where the temperature may be significantly lower, but $n\lambda^2 \ll 1$.

[3)] Strictly speaking, the collective mode connected with excitation transport in a gas, exists also in the high-temperature region, as can be seen directly from the solution of the problem in Ref. 5. In the high-temperature region, however, the collective mode is always strongly damped (diffuse).

[4)] Such densities are attainable by compressing atomic hydrogen in a strongly non-uniform electric or magnetic field (electric or magnetic needle).[6]

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