# Characteristics of galvanomagnetic properties of compensated metals under static skin effect conditions in strong magnetic fields (tungsten)

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The anisotropy and the field dependences of the transverse magnetoresistance and of the Hall coefficient of tungsten single crystals with the resistivity ratio $\rho_{293.2\,K}/\rho_{4.2\,K}$ from 780 to 80 800 were determined at 4.2 K in 8–150 kOe magnetic fields. An investigation was made of the role of the static skin effect by measuring the galvanomagnetic properties of samples of different dimensions and shapes: rectangular plates and slabs, including some with a rectangular internal cavity, as well as Corbino disks. The experimental results demonstrated that a strong static skin effect was induced by the application of high magnetic fields to tungsten, so that the density of a direct electric current flowing near the surface of a crystal was between two and three orders of magnitude higher than the current density in the bulk. A new property of compensated metals was observed under the static skin effect conditions: the Hall coefficient exhibited an anomalously strong (by a factor of several tens) linear rise on increase in the magnetic field. The characteristic features of the behavior of the magnetoresistance and of the Hall coefficient in strong magnetic fields were analyzed using the mechanisms of surface scattering of conduction electrons.

## 1. INTRODUCTION

In the majority of compensated metals the Hall effect has been investigated in strong magnetic fields ($\omega_c\tau \gg 1$) and it has been found[1] that, in spite of the equality of the densities of electrons and holes ($n_e = n_h$), there is a finite Hall emf which is usually a linear function of the magnetic field $H$. The Hall emf may depend nonlinearly on the field when the field modifies the electron energy spectrum or alters the conditions for anisotropic scattering of conduction electrons. In such cases the Hall coefficient $R_H$ is a function of the magnetic field. For example, it has been found[2,3] that modification of the electron paths due to magnetic breakdown influences the high-field Hall effect and it is demonstrated in Ref. 4 that $R_H$ depends on $H$ in the case of anisotropic intersheet scattering of conduction electrons by phonons. In the absence of such effects the Hall coefficient should be independent of the magnetic field for metals with any topology of the Fermi surface.[5]

In an earlier brief paper[6] we reported an unusual near-quadratic rise of the Hall emf of tungsten with increasing magnetic field. It should be stressed that the investigation reported in Ref. 6 was not concerned with the effect of a transverse even field, which is known[5] to be proportional to $H^2$, but with the odd Hall emf including a contribution which is a quadratic function of the magnetic field. This is illustrated in Fig. 2 of Ref. 6, which gives the field dependences of the Hall emf of tungsten for two opposite directions of the magnetic field. The curves are asymmetric because of the contribution of the even effects, but this figure demonstrates that the Hall emf $U_H$ (representing the half-difference between these two curves) rises nonlinearly with the field.

Numerous investigations of the galvanomagnetic properties of tungsten single crystals in strong magnetic fields have shown that the magnetic breakthrough effect does not occur and that the intersheet electron–phonon scattering is "frozen out" below 25 K (Ref. 4). Therefore, attention is drawn in Ref. 6 to a strong linear rise of the Hall coefficient $R_H$ (by a factor amounting to tens of times) observed at $T < 20$ K when the mean free path $l$ of conduction electrons is greater than or of the order of the size $d$ of the investigated samples and much greater than the Larmor radius $r_H$ of electrons in a magnetic field ($l \gtrsim d \gg r_H$). A static skin effect can occur under these conditions.[7–9] This effect has been observed and investigated in high-purity tungsten single crystals in the course of studies of the magnetoresistance in fields up to 15 kOe (Refs. 10–12) using samples characterized by $d \lesssim l$. The hypothesis of Ref. 6 that the nonlinear field dependence of the Hall emf observed in high magnetic fields is related to the static skin effect must be confirmed experimentally.

The aim of the present investigation was to answer the following questions.

1. Is the static skin effect observed in tungsten in magnetic fields up to 150 kOe and how important is the influence of surface skin effect carriers on the magnetoresistance in the case of an inhomogeneous distribution of a static electric field in the bulk of the crystal?

2. Is the increase in the Hall coefficient with magnetic field the result of an interaction of conduction electrons with the surfaces of the samples or is this behavior due to some "bulk" properties of tungsten?

3. Which of the mechanisms for scattering of conduction electrons can be responsible for the contribution to the Hall emf which is a nonlinear function of the magnetic field?

The present paper is organized in such a way that Sec. 2 describes the main features of the experimental method used and the characteristics of the investigated samples. The results of our investigation of the magnetoresistance are presented in Sec. 3. From them we conclude that the distribution of the constant electric current in the samples is very nonuniform and is concentrated near the surface. The results of our investigation of the Hall effect in tungsten samples of different shape and with different crystallographic faceting are given in Sec. 4. The magnetoresistance and Hall effect data are analyzed in Sec. 5 on the basis of the static skin

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effect and intersheet surface scattering of conduction electrons.

## 2. SAMPLES AND EXPERIMENTAL METHOD

The magnetoresistance and the Hall effect were measured using tungsten single crystals with different dimensions, shapes, crystallographic orientations of the faces, and purity grades. The data on the samples are listed in Table I. The purest samples were cut from a single-crystal tungsten ingot for which the resistivity ratio was $\rho_{293.2K}/\rho_{4.2K} = 80\,800$. This ingot was cut by spark machining and then a defective surface layer 0.15–0.20 mm thick was removed by etching in a solution of the 20% $HNO_3$–40% $H_2SO_4$–40% HF composition, which was followed by electropolishing in a 2% solution of NaOH. The samples were then exposed to the atmosphere and no attempt was made to remove gases adsorbed on the surface. Therefore, in reality the surfaces of the investigated samples were not atomically clean. The magnetoresistance and the Hall emf were determined for samples which were in liquid or gaseous helium.

The magnetoresistance measurements were carried out employing the conventional four-point method under dc conditions. In high magnetic fields the resistivity was about 500 $\mu\Omega\cdot$cm and the current density in the samples was less than 10 A/cm$^2$. Ohm's law was satisfied well in this range of current densities. The Hall emf was measured by five- or six-point methods[14] under dc conditions. These methods made it possible to compensate the contributions even in the magnetic field and due to the magnetoresistance and the transverse even field. Moreover, the Hall emf was measured for two opposite directions of the magnetic field in order to eliminate completely the even contributions. The odd Hall emf $U_H$ was defined as the half-difference between these two measurements. The error in the magnetoresistance measurements was less than 0.1%, whereas in the Hall emf measurements it was 4%. The value of the Hall coefficient $R_H$ was determined with an error of at most 8–10%. Some other methodological features of the magnetoresistance and Hall coefficient measurements will be mentioned when reporting the results.

Superconducting UIS-1 magnets capable of creating magnetic fields up to 90 kOe and IGC-150 for fields up to 150 kOe were used as the magnetic field sources.[2)]

## 3. MAGNETORESISTANCE

The magnetoresistance measurements were carried out in fields up to 150 kOe in which the Larmor radius $r_H$ of a free electron reached $5\times10^{-4}$ mm. The average transport mean free path $l$, the dimensions of the samples $d$, and the specular reflection coefficients $q$ of conduction electrons incident on (110) and (100) surfaces are all listed in Table I. Clearly, the conditions for the static skin effect were satisfied.

Earlier experimental investigations of the static skin effect in compensated metals have involved an analysis of the field dependences of the specific magnetoresistance $\rho_{xx}(H)$ and of its anisotropy $\rho_{xx}(\varphi)$, due to the influence of the shape of the sample. This has been done at various temperatures using crystals of different purities and dimensions (see, for example, Refs. 10 and 11). In strong magnetic fields ($r_H \ll l$) when the dimension of a sample $d$ normal to the direction of **H** exceeds the Larmor radius ($d \gg r_H$) the power exponent n in the field dependences of the magnetoresistance $\rho_{xx}(H) \propto H^n$ is independent of the dimension $d$ and is close to $n = 2$ (Refs. 7 and 8), i.e., it is exactly the same as for bulk samples. In Ref. 6 we reported a deviation from the quadratic field dependence $\rho_{xx}(H)$, but—as explained in Ref. 15—this deviation was due to the presence of defects in the surface layer. No deviation from the quadratic dependence was found for crystals with a defect-free surface. Therefore, in strong magnetic fields an analysis of the asymptotes of the field dependences of the magnetoresis-

TABLE I. Properties of investigated samples.

| Sample No. | Dimensions, mm | Direction of axis of sample | Faces of sample | $\rho_{293.2K}/\rho_{4.2K}$ | $l$*, mm | $q$ ** |
|---|---|---|---|---|---|---|
| 1V | 1.39×1.43×12 | ⟨100⟩ | (110) | 52 340 | 3.0 | 0.6–0.7 |
| 2V | 0.34×1.38×12 | ⟨100⟩ | (110) | 39 790 | 3.0 | 0.6–0.7 |
| 3V | 0.35×0.39×12 | ⟨100⟩ | (110) | 19 320 | 3.0 | 0.6–0.7 |
| 4V | 1.28×1.27×12 | ⟨100⟩ | (100) | 47 140 | 3.0 | 0.1–0.2 |
| 5V | 0.32×1.59×12 | ⟨100⟩ | (100) | 36 120 | 3.0 | 0.1–0.2 |
| 6V | 0.33×0.34×12 | ⟨100⟩ | (100) | 17 310 | 3.0 | 0.1–0.2 |
| 7V | 1.61×1.58×12 | ⟨100⟩ | (100) | 780 | 0.03 | 0.1–0.2 |
| 8V | 0.36×1.60×12 | ⟨100⟩ | (100) | 780 | 0.03 | 0.1–0.2 |
| 9V | 0.56×0.57×12 | ⟨100⟩ | (100) | 780 | 0.03 | 0.1–0.2 |
| 10V | 2.49×2.45×12 | ⟨100⟩ | (110) | 61 030 | 3.0 | 0.6–0.7 |
| 10VP | Sample 10V with cavity of 0.86×1.48×12 dimensions | ⟨100⟩ | (110) | – | 3.0 | 0.6–0.7 |
| 11V | 2.88×2.82×9.0 | ⟨100⟩ | (100) | 780 | 0.03 | 0.1–0.2 |
| 11VP | Sample 11V with cavity of 1.74×2.00×9.0 dimensions | ⟨100⟩ | (100) | 780 | 0.03 | 0.1–0.2 |
| 12V | 0.63×0.36×12 | ⟨100⟩ | (110) | 22 140 | 3.0 | 0.6–0.7 |
| 12VK | Corbino disk (diameter 5.8 mm, thickness $d = 0.36$ mm) cut from the same plate as sample 12V | – | (110) | – | 3.0 | – |

*The mean free path of conduction electrons was estimated from measurements of the residual resistance allowing for the size effect in accordance with Ref. 13.
**Specular reflection coefficient of the faces.[10,11]

tance failed to give information on the static skin effect. In the present study of the magnetoresistance attention was focused on the influence of the dimensions and shape of the samples on this property.

Figure 1 shows the field dependences of the transverse magnetoresistance $R_{xx}(H)$ of tungsten samples 1V–3V with different transverse dimensions, with (110) faces, and with the current oriented along ⟨100⟩. For all these dependences the power exponent was $n = 1.96 \pm 0.05$. Clearly, the $R_{xx}(H)$ curves could be divided into two groups. The two upper curves with the higher value of $R_{xx}$ were obtained for samples with smaller transverse dimensions along **H**, whereas the two lower curves were obtained for samples with larger dimensions along **H**. Reduction of the dimensions at right-angles to **H** had very little effect on the value of $R_{xx}$. Similar field dependences of the magnetoresistance were obtained also for samples 4V–6V with (100) faces.

We shall now explain in greater detail the method used to determine $R_{xx}(H)$ for samples of different sizes and shapes. The method is illustrated schematically in Fig. 2. The four field dependences in Fig. 1 were obtained for three samples 1V–3V cut from the same part of a single-crystal rod with the resistivity ratio $\rho_{293.2\text{K}}/\rho_{4.2\text{K}} \approx 80\,800$ and with polished faces. The distance between the potential contacts was 10 mm for all the samples. The cross-sectional area of the plate 2V was approximately one-quarter of the cross-sectional area of the slab 1V, whereas the cross-sectional area of sample 3V was approximately one-quarter of the area of sample 2V (Fig. 2). The faces of all three samples were oriented to within $\pm 1°$ along the (110) planes and the long axis was directed along ⟨100⟩. The dependence $R_{xx}(H)$ was determined for the plate 2V in fields **H** parallel and perpendicular to the plane of the plate. Figure 2 shows the values of the resistances of samples 1V–3V in a field $H = 130$ kOe. Clearly, a reduction of the cross-sectional area of the samples by a factor of about 4 because of a reduction in the size $c$ by a factor of 4 (Fig. 2), increased only slightly the value of

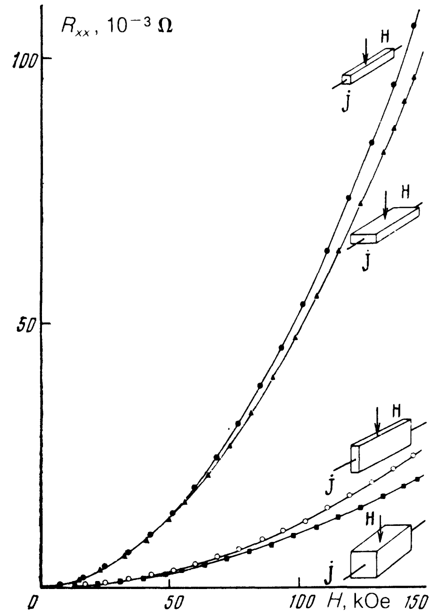

FIG. 1. Field dependence of the magnetoresistance $R_{xx}$ of tungsten samples 1V–3V with (110) faces at $T = 4.2$ K.

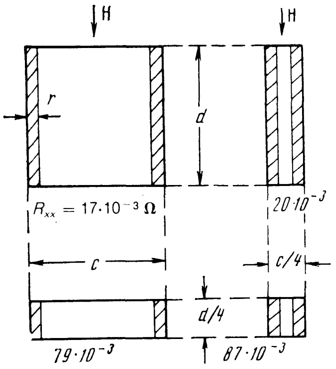

FIG. 2. Schematic diagram used to explain the results plotted in Fig. 1. It gives the relationship between cross-sectional areas of the samples and their magnetoresistances $R_{xx}$. The surface layers of thickness of the order of $r_H$ are shown shaded; $d$ and $c$ are the transverse dimensions of samples. $T = 4.2$ K, $H = 130$ kOe.

$R_{xx}$ (from $17 \times 10^{-3}$ to $20 \times 10^{-3}$ Ω), whereas a fourfold reduction in the cross-sectional area by reducing the size $d$ along the **H** direction increased $R_{xx}$ approximately by a factor of 4. The same tendency was observed with the plate 2V and the slab 3V.

We can thus see that reduction in the proportion of the carriers concentrated in the bulk of the sample and retention of a constant proportion of the electrons that interacted with the field **H** parallel to the surface had practically no effect on the magnetoresistance. We conclude from these results that a direct electric current in high-purity tungsten crystals ($l > d, c$) subjected to strong magnetic fields was concentrated in the surface layer. This was also true of the samples with the (100) faces.

The total electric current flowing through a sample consisted of the surface current $J_{\text{surf}}$ and bulk current $J_b$. It would be of interest to determine quantitatively how these currents were related. This could be done by calculating the electrical resistivities of the surface layer $\rho_{xx}^{\text{surf}}$ and of the bulk of the sample $\rho_{xx}^{b}$. The following method was used for this purpose. Assuming that the boundary between the surface and bulk currents $J_{\text{surf}}$ and $J_b$ was sharp and that the skin layer thickness was $r_H$ (according to Ref. 7), we derived an expression for the conductance of, for example, samples 1V and 2V, where the plane of sample 2V was parallel to **H**,

$$\frac{1}{R_{1\text{V}}} = \frac{1}{\rho_{xx}^{\text{surf}}} \frac{2r_H d_1}{L_1} + \frac{1}{\rho_{xx}^{b}} \frac{(c_1 - 2r_H) d_1}{L_1},$$
$$\frac{1}{R_{2\text{V}}} = \frac{1}{\rho_{xx}^{\text{surf}}} \frac{2r_H d_2}{L_2} + \frac{1}{\rho_{xx}^{b}} \frac{(c_2 - 2r_H) d_2}{L_2}. \qquad (1)$$

Here $L_1$ and $L_2$ represent the distance between the potential contacts with samples 1V and 2V, and c and d are the transverse dimensions of these samples (Fig. 2). We can calculate the $\rho_{xx}^{\text{surf}}$ and $\rho_{xx}^{b}$ by solving the above system of equations. A similar system can be derived also for another pair of samples 2V and 3V, but in the case of 2V the planes are perpendicular to **H**. Obviously, $\rho_{xx}^{\text{surf}}$ and $\rho_{xx}^{b}$ obtained for these two systems should differ, because in the case of thin samples scattering by surfaces perpendicular to **H** is manifested more strongly.

In fact, estimates of $\rho_{xx}^{\text{surf}}$ and $\rho_{xx}^{b}$ obtained for samples

1V and 2V with (110) faces in a field $H = 130$ kOe gave

$$\rho_{xx}^{surf} = (0.6\pm0.1)\cdot10^{-6}\ \Omega\cdot\text{cm},$$

$$\rho_{xx}^{b} = (1700\pm300)\cdot10^{-6}\ \Omega\cdot\text{cm},$$

whereas for samples 2V and 3V with (110) faces in $H = 130$ kOe, the corresponding estimates were

$$\rho_{xx}^{surf} = (0.5\pm0.1)\cdot10^{-6}\ \Omega\ \text{cm},$$

$$\rho_{xx}^{surf} = (700\pm300)\cdot10^{-6}\ \Omega\cdot\text{cm}.$$

Figure 3 shows the field dependences of $\rho_{xx}^{surf}$ for a sample with specularly reflecting [(110)] and diffusely reflecting [(100)] faces. We can see that, to a high degree of accuracy, the field dependences $\rho_{xx}^{surf}(H)$ are linear both in the specular reflection case ($q = 0.7$) and in the case of diffuse scattering ($q = 0.15$). According to Ref. 7, we have

$$\rho_{xx}^{surf} = \rho_0(1-q+\gamma)/\gamma = \rho_0[(1-q)l_{eff}/r_H+1],$$

where $\rho_0$ is the resistivity in the absence of the field and we can expect the slopes of the dependences $\rho_{xx}^{surf}(H)$ for the (110) and (100) faces (if we bear in mind that $l_{eff}^{-1} = l^{-1} + d^{-1}$, where $l = 3$ mm) to differ by a factor of 2.8. It follows from Fig. 3 that the difference is a factor of 2.6. Such a slight discrepancy is well within the limits of the experimental error. Hence, we may assume that the magnetoresistance of the surface layer $\rho_{xx}^{surf}$ is described well by the theoretical representations of Ref. 7.

The ratio $\rho_{xx}^{b}/\rho_{xx}^{surf}$ represents the ratio of the density of the surface current $j_{surf}$ to the density of the bulk current $j_b$ ($j_{surf}/j_b = \rho_{xx}^{b}/\rho_{xx}^{surf}$). Using the above values of $\rho_{xx}^{b}$ and $\rho_{xx}^{surf}$ at $H = 130$ kOe, we find that $j_{surf}/j_b \approx 2800$ for sample 1V and $j_{surf}/j_b \approx 2600$ for sample 3V. In the case of the diffusely reflecting faces of samples 4V and 6V, we have $j_{surf}/j_b \approx 560$ and 500, respectively. It follows clearly from these estimates that the distribution of the electric field in a sample is strongly inhomogeneous and that the magnetoresistance of the investigated tungsten crystals is governed primarily by the surface layer where conduction electrons are scattered on the surface.

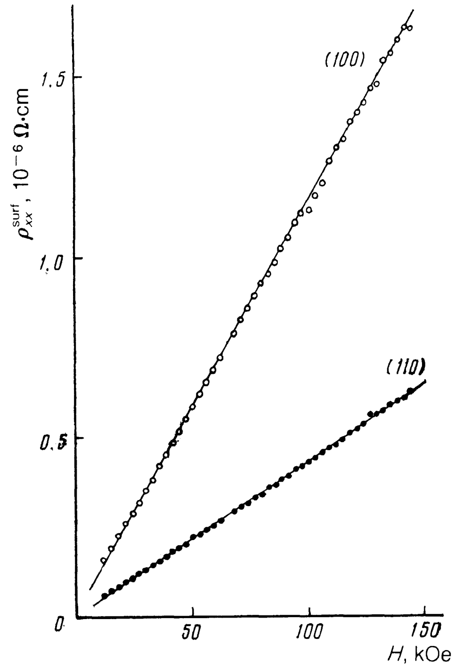


FIG. 3. Field dependence of the specific surface magnetoresistance $\rho_{xx}^{surf}$ of samples 1V–3V with (110) faces and of samples 4V–6V with (100) faces at $T = 4.2$ K. The method used in the calculation of $\rho_{xx}^{surf}$ is given in the text.

A confirmation of a significant inhomogeneity of the distribution of the current in strong magnetic fields was provided by the results of measurements of the magnetoresistance using the continuous hollow samples 10V and 10VP. Figure 4a shows the field dependences of the magnetoresistance obtained for these samples. Clearly, a reduction in the cross-sectional area by a factor of 1.3 because of the presence of a cavity inside the sample not only did not increase the resistance of the sample, but just the opposite: it reduced the resistance because of the appearance of internal surfaces on which conduction electrons were scattered creating an additional high-conductivity surface layer. In the case of a homogeneous distribution of the current in the absence of the static skin effect we found that in the case of the "dirty" samples 11V and 11VP, characterized by conduction electrons with a mean free path $l \approx 3\times10^{-2}$ mm, the appearance of a cavity increased the resistance of a sample by a factor of 1.5 (Fig. 4b). Figures 4a and 5 also show that with an inhomogeneous distribution of the current in the 10VP samples the anisotropy of the cavity shape results makes the magnetoresistance anisotropic.

Estimates of the values of $\rho_{xx}^{b}$ indicate that the bulk conductivity was fairly low. This could be checked directly by measurements of the magnetoresistance of pure samples with "infinite" dimensions, which of course can not be done in reality.

However, a good model of an infinite plate is a sample in the form of a Corbino disk in which there are no planes parallel to the magnetic field if the electric field distribution is radial, so that there is no electron scattering which would give rise to the static skin effect. Figure 6 shows the field dependence of the magnetoresistance of a Corbino disk scaled by the resistance at room temperature: $R_{xx}(H)/R_{293\,K}$, compared with the magnetoresistance of a plate of the same thickness and at the same frequency (samples 12V and 12VK). We can see that in a field $H = 80$ kOe the magnetoresistance of the Corbino disk was an order of magnitude higher than that of the plate. In contrast to metals with $n_e \neq n_h$ this increase was not due to closure of the Hall current lines, since the Hall field $E_H$ in tungsten was weak compared with the longitudinal field $E_x$: $E_x/E_H \sim 10^2$. A strong rise of the magnetoresistance exhibited by the Corbino disk was associated with the absence of the static skin effect and indicated a large bulk magnetoresistance.

The results obtained for the magnetoresistance of large tungsten single crystals ($l > d$) in strong magnetic fields were evidence of a considerable inhomogeneity of the distribution of the electric current in pure single crystals of compensated metals at right angles to **H** and **j**, and the dominant contribution of the surface scattering of conduction electrons to the magnetoresistance. Since the density of the surface current was several orders of magnitude higher than the density of the current in the bulk of a crystal, we could expect the Hall field arising under these conditions to vary strongly.

## 4. HALL EFFECT

A preliminary investigation of the Hall effect under the static skin effect conditions indicated that welding of con-

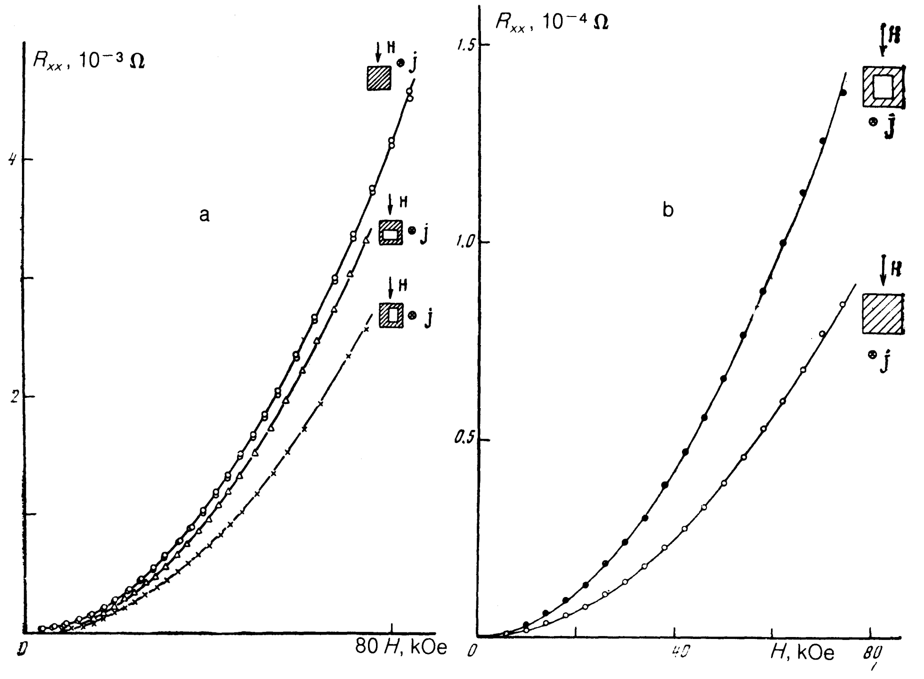


FIG. 4. Field dependence of the magnetoresistance $R_{xx}$ obtained for solid and hollow pure (10V and 10VP, Fig. 4a) and dirty (11V and 11VP, Fig. 4b) tungsten samples at $T = 4.2$ K.

tacts to a sample created a defective region on the surface near a contact, which altered the nature of the bulk and surface scattering of conduction electrons and distorted the distribution of the electric current near the surface. Our measurements of the Hall emf in accordance with the four-point method generally used for metals normally yielded a poorly reproducible value of $U_H$ which was several times smaller than that found by the five- and six-point methods.[14] For this reason we concluded that the four-point method for the determination of the Hall emf was unacceptable in the investigation of this effect under the static skin effect conditions. All the results of measurements of the Hall characteristics reported below were obtained by the five- and six-point methods.[14] They made it possible to compensate the contribution to $U_H$ which was even in **H** and due to the magnetoresistance, and reversal of the directions of **H** and **J** made it possible to eliminate the influence of the transverse even field and of stray thermomagnetic effects. The Hall coefficient $R_H$ was found from the expression

$$R_H = d[U_H(-H, +J) - U_H(+H-, +J) + U_H(+H, -J) - U_H(-H, -J)]/4JH, \quad (2)$$

where $J$ is the electric current in the sample, $d$ is the size of the sample in the **H** direction, and $U_H(\pm H, \pm J)$ is the emf at the Hall contacts for opposite directions of **H** and **J**.

Figure 1 in Ref. 6 shows the field dependence of the transverse emf $U_H$ for a tungsten sample 2V obtained for two opposite orientations of the magnetic field. Reversal of the field direction did not alter this dependence. Clearly, the emf $U_H$ was equal to the half-difference between these two functions: $U_H = [U_H(+H) - U_H(-H)]/2$, which increased nonlinearly with the magnetic field. Hence, the Hall

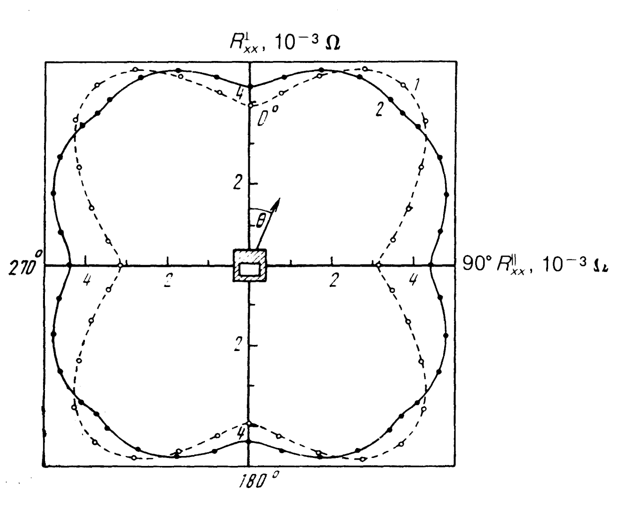


FIG. 5. Angular dependence of the magnetoresistance of a solid (continuous curve) and hollow (dashed curve) samples (10V and 10VP) at $T = 4.2$ K for $H = 80$ kOe. Here, $R^{\perp}_{xx}$ and $R^{\parallel}_{xx}$ are the values of the magnetoresistance in fields **H** perpendicular and parallel to the largest plane of the cavity. The points on the curves are separated by those of 10°.

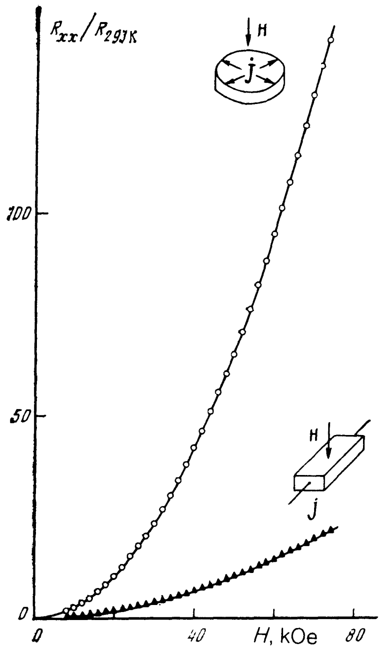


FIG. 6. Field dependence of the magnetoresistance of a plate (12V) and of a Corbino disk (12VK) at $T = 4.2$ K.

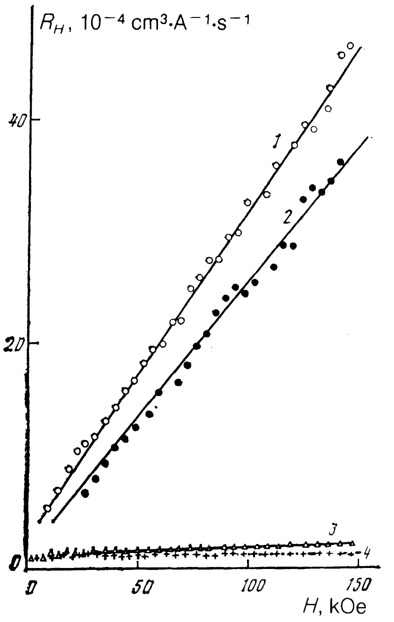


FIG. 7. Field dependence of the Hall coefficient $R_H$: 1) sample 4V with (100) faces at $T = 4.2$ K; 2) sample 1V with (110) faces at $T = 4.2$ K; 3) sample 1V at $T = 25$ K; 4) dirty sample 7V at $T = 4.2$ K.

coefficient calculated from Eq. (2) included a contribution which varied with the magnetic field so that the coefficient could be represented by $R_H = R_H^0 + R'_H(H)$. Here, $R_H^0$ is the value of $R_H$ in the limit $H \to 0$ and $R_H'(H)$ is the contribution dependent on the magnetic field. We can see from Fig. 7 that the value of $R_H$ in a field $H = 150$ kOe was tens of times higher than the values in the limit $H \to 0$ and had a field dependence which was nearly linear. We included in this figure the field dependences $R_H(H)$ for samples of rectangular shape (1V and 4V) with approximately similar transverse dimensions but with different faces (110) and (100). The condition $r_H \ll d < l$ was satisfied by these samples throughout the investigated range of magnetic fields.

We also plotted in Fig. 7 the field dependences of $R_H$ at $T = 25$ K (sample 1V, curve 3) and the field dependences $R_H$ for sample 7V which was "dirty" and had (100) faces, in which case the inequality $r_H \ll l < d$ was satisfied and the static skin effect was not observed. Clearly, in the case of this dirty sample (curve 4) the coefficient $R_H$ was independent of the magnetic field. A very weak field dependence of $R_H$ was observed also at $T = 25$ K when the inequality $l \ll d$ was obeyed because of the scattering by phonons. At this temperature the field dependence was practically identical with the curve for the dirty sample. In the case of pure samples 1V and 4V with $l > d$ at $T = 4.2$ K the coefficient $R_H$ increased linearly with the field and had different values for the specularly reflecting (110) and diffusely reflecting (100) faces. Extrapolation of the dependence $R_H(H)$ to zero magnetic field gave the values $R_H^0$, which agreed well with $R_H$ at temperatures $T > 100$ K characterized by high-temperature isotropic scattering of electrons by phonons.[4]

The magnetoresistance measurements (Figs. 1 and 3) indicated that in pure tungsten crystals the direct electric current was concentrated mainly near the surface. It was therefore of interest to determine how the Hall emf was affected by the formation of a cavity inside a sample. In the case of a homogeneous distribution of the current across a sample we would expect $U_H$ for a hollow sample to increase in proportion to the increase in the current density because of the formation of the cavity. When the current flowed mainly in the surface layer we would expect no change in $U_H$. Figure 8a shows the field dependence of the Hall emf $U_H$, normalized to the magnetic field $H$ and to the electric current through solid (10V) and hollow (10VP) samples. The results indicated that reduction in the cross-sectional area of the sample by creation of a cavity did not alter the Hall emf. As before, we found that $U_H/JH$ increased linearly with the magnetic field. Similar measurements carried out on dirty samples 11V and 11VP, when the static effect was absent, indicated that $U_H/JH$ for the hollow sample increased when the cross-sectional area was reduced, whereas $U_H/JH$ was independent of $H$ (Fig. 8b). This was further evidence that the main contribution to the measured Hall emf came from conduction electrons scattered by the surface under the static skin effect conditions. The bulk part of the sample played a much smaller role in the formation of $U_H$ and did not determine this emf.

## 5. DISCUSSION OF RESULTS

The results of our investigation of the magnetoresistance in "perfect" finite tungsten single crystals of different shapes and dimensions demonstrated convincingly that a considerable variation in the distribution of the direct electric current was established in strong magnetic fields (Figs. 1, 3, and 5), which increased with the field. The current density was several orders of magnitude higher near the surface than in the bulk. These results confirmed the calculations[7,8] predicting that in the case of pure compensated metals subjected to a transverse magnetic field the concentration of the direct electric current in the skin layer should occur in magnetic fields, no matter how strong. All the magnetoresistance data showed that in strong magnetic fields under static skin-effect conditions the scattering of conduction electrons on the surface of a sample is the dominant mechanism which governs the main features of the behavior of the magnetoresistance. The surface scattering is so strong that even the characteristic features of the Fermi surface topology become of secondary importance. In fact, the anisotropy of the magnetoresistance of tungsten (no data are given in the present

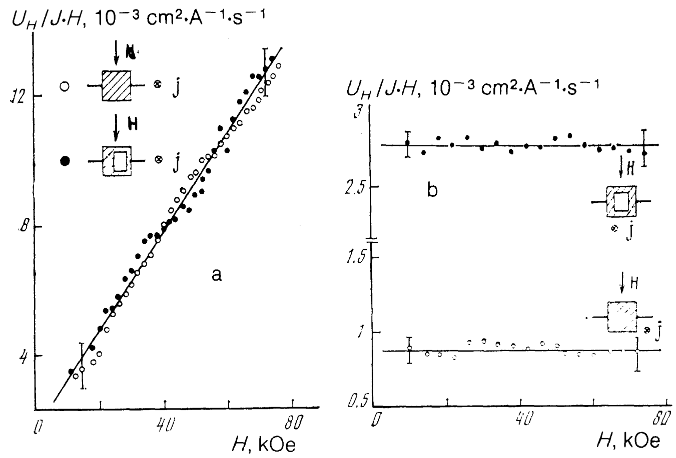


FIG. 8. Field dependence of the Hall emf $U_H$ normalized to the current in the sample $J$ and to the magnetic field $H$, shown for solid and hollow pure (10V and 10VP, Fig. 8a) and dirty (11V and 11VP, Fig. 8b) tungsten samples at $T = 4.2$ K.

paper and the reader is directed to Refs. 10 and 15) is entirely due to the shape of the sample and the nature of the scattering of electrons by its faces. It is also shown in Ref. 16 that even in cylindrical samples the magnetoresistance anisotropy observed under static skin-effect conditions is governed by the nature of crystallographic planes forming the surfaces of the sample.

In view of the dominant role of the surface scattering of conduction electrons in the static skin effect and magnetoresistance of tungsten, it is natural to assume that the anomalous linear rise of the Hall coefficient with the magnetic field at helium temperatures is related to this scattering mechanism. In the absence of the static skin effect there is no rise of the Hall coefficient. A quadratic field dependence of the Hall resistance had been reported earlier[17] for tungsten crystals with $\rho_{300\,K}/\rho_{1.3\,K} = 30\,000$ and it was suggested that the effect is due to residual dislocations and impurities in a sample which result in decompensation of light charge carriers. The results reported here do not support this conclusion, for the following reasons. Firstly, the anomalous rise of the Hall coefficient becomes stronger as the quality of a crystal improves and on increase in the mean free path of conduction electrons. Secondly, even if impurities and defects present in low concentrations could decompensate electrons and holes, so that $\Delta n = n_e - n_h \neq 0$, the value of $\Delta n$ should be independent of the external magnetic field $H$. Consequently, according to Ref. 5, carrier decompensation can only be manifested by a change in the Hall coefficient, but it cannot be responsible for the dependence of this coefficient on the magnetic field.

The role of the surface scattering in the low-temperature Hall effect in bounded crystals of compensated metals has been analyzed theoretically in Refs. 8, 9, and 18. The behavior of the Hall coefficient under the conditions of the diffuse static skin effect is considered in Ref. 9: this effect appears when the probability $W$ of intrasheet surface scattering is considerably greater than the probability of intersheet surface bulk recombination ($W_{surf}$ and $W_b$) of electrons and holes. In the case of tungsten there are no data on the intrasheet $\tau$ and recombination $\tau_{surf}$ and $\tau_b$ relaxation times, but we can assume that $\tau \sim \tau_{surf}$ for tungsten, by analogy with the majority of metals with the Fermi surface occupying a large part of the Brillouin zone and characterized by small momentum separations between the sheets. Therefore, the results of Ref. 9 cannot be applied here. Moreover, the theoretical results of Ref. 9 and the data obtained in the present study differ considerably in the influence of the surface scattering on the Hall coefficient. According to Ref. 9, in the case of the diffuse static skin effect the coefficient $R_H$ should decrease as a function of magnetic field proportionately to $H^{-1}$, whereas in the case of tungsten we find that the Hall coefficient rises nearly linearly with the field ($R_H \propto H$).

It is clear from Fig. 7, which gives the $R_H(H)$ dependence for samples with (100) and (110) faces, that the linear rise of $R_H$ is observed for the scattering by specularly reflecting (110) and diffusely reflecting (100) faces. The coefficient A in the dependence $R_H - R_H^0 = AH$ is different for the two faces:

$$A_{(100)}=2.9\cdot 10^{-8}\ \mathrm{cm^3\cdot A^{-1}\cdot s^{-1}\cdot Oe^{-1}},$$

$$A_{(110)}=2.4\cdot 10^{-8}\ \mathrm{cm^3\cdot A^{-1}\cdot s^{-1}\cdot Oe^{-1}}.$$

Bearing in mind the dominant role of the surface scattering in charge transport, we can assume that the rise of the Hall coefficient with the field depends on the nature of the surface scattering of electrons.

According to the theoretical investigation in Ref. 8, under static skin-effect conditions the off-diagonal component of the magnetoresistance tensor of a compensated metal plate in a magnetic field tilted by a small angle $\vartheta$ from its plane ($\vartheta \ll W_{surf}$) can be represented in the form[3)]

$$\rho_{xy} \approx \frac{p_F d}{e^2 n r_H^2}(1-W_{surf}) \propto H^2. \tag{3}$$

Here, $p_F$ is the Fermi momentum; $d$ is the thickness of the plate;, $e$ is the electron charge; $n$ is the carrier density given by $n = n_e = n_h$; $r_H$ is the Larmor radius; $W_{surf} = 1/N$; $N$ is the number of collisions of electrons with the surface needed to establish an equilibrium between electrons and holes. We can see that the Hall resistivity $\rho_{xy} = R_H H$ depends quadratically on $H$ and the coefficient in front of $H^2$ depends on the probability of intersheet transitions $W_{surf}$. This expression differs from the expression for $\rho_{xy}$ in Ref. 18 for the diffuse scattering case. It is pointed out in Ref. 8 that this difference is due to the fact that in Ref. 18 the contribution of the surface layers of thickness of the order of $r_H$ is not allowed for sufficiently accurately in the calculation of $\rho_{xy}$. In Ref. 8 it is this surface region, which carries the bulk of the electric current, that determines the existence of the antisymmetric contribution to the off-diagonal components of the magnetoresistance tensor which causes the Hall resistance to be an odd function of the magnetic field, $\rho_{xy}(H) = -\rho_{xy}(-H)$, and makes the Hall coefficient rise linearly with the magnetic field

$$R_H(H)=[\rho_{xy}(H)-\rho_{xy}(-H)]/2H \propto H.$$

The expression (3) for $\rho_{xy}$ is valid only in the symmetric case or for a spherical Fermi surface. Under these conditions we have $\rho_{xy} = 0$ if each collision of an electron with the surface is accompanied by its transfer to another Fermi surface sheet ($W_{surf} = 1$, diffuse scattering). In the specular scattering case we have $\rho_{xy} \neq 0$, since $W_{surf} < 1$. For real metals with nonspherical Fermi surfaces the value of $\rho_{xy}$ should not vanish (see Ref. 8). Therefore, in a study of the Hall effect in bounded samples of compensated metals with a long mean free path of conduction electrons ($l \gg d$) we should observe a quadratic field dependence of the Hall emf. However, in our experiments the degree of diffuseness of the scattering by the faces was not correlated with the values of the coefficient $A$ in the dependence $R_H - R_H^0 = AH$, because $A_{(100)} > A_{(110)}$ (see Fig. 7). According to Ref. 8, the opposite inequality should be obeyed.

This conflict may be due to the fact that a quantitative comparison of the value of $W_{surf}$ for different faces, obtained from the results of measurements of $R_H$ for samples with (100) and (110) faces, and the theoretical results of Ref. 8 cannot be carried out under our experimental conditions, because the measurements were made on bounded samples in the form of rectangular slabs and the calculations of Ref. 8 were carried out for an infinite plate. Moreover, in the case of samples with (100) and (110) faces the measurements of $R_H$ were made for different directions of **H**: in the case of (100) samples the field was $\mathbf{H}\|\langle 100\rangle$, whereas for (110) faces it was $\mathbf{H}\|\langle 110\rangle$. Therefore, if the parameters $W_{surf}^{(100)}$

and $W_{surf}^{(110)}$ are to be determined correctly, we must allow for the real areas of the extremal sections of the Fermi surface of tungsten for these directions of **H**. Nevertheless, in view of the reasonable qualitative agreement between the experimental results and the theory of Ref. 8, we may conclude that in the case of bounded tungsten crystals ($l > d$) the anomalous linear rise of the Hall coefficient as a function of the magnetic field is the result of the scattering of conduction electrons by the surface of a sample and of the concentration of the electric current in a thin surface layer roughly a Larmor radius thick. The difference between the Hall components of the magnetoresistance tensor near the surface (in a layer of thickness $\sim r_H$) and the Hall components in the bulk is clearly due to the fact that the electron paths in the surface layer are less symmetric than in the bulk of a crystal.

## 6. CONCLUSIONS

These low-temperature investigations of the galvanomagnetic properties of bounded tungsten single crystals ($l > d$) in magnetic fields up to 150 kOe, ensuring that the condition for the static skin effect ($r_H \ll d < l$) was satisfied reliably, can be used to draw the following conclusions.

1. In the case of tungsten we can show that for compensated metals with closed Fermi surfaces in strong magnetic fields, in which the Larmor orbit radius $r_H$ is several orders of magnitude smaller than the dimensions of a sample, these metals exhibit a strong static skin effect when the density of the electric current flowing near the surface of a crystal is considerably higher than the density of the current in the bulk ($j_{surf}/j_b \approx 5 \times 10^2 – 2 \times 10^3$ in fields $H > 100$ kOe). When the direct electric current in the sample varies strongly the magnetoresistance and the main features of its behavior are governed not so much by the features of the Fermi surface topology, as by the surface scattering of conduction electrons in a thin surface layer.

2. Under the conditions of the static skin effect in tungsten single crystals we observed for the first time an anomalously strong (by a factor of tens) linear rise of the Hall coefficient with the magnetic field, not previously reported for compensated metals. The experimental data and an analysis carried out using the results of published theoretical investigations enable us to establish that the anomalous field dependence of the Hall coefficient is due to the surface scattering of conduction electrons accompanied by intersheet umklapp processes and is manifested in strong magnetic fields when the direct electric current flows mainly in the surface layer of a sample.

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[1)]International Laboratory of Strong Magnetic Fields and Low Temperatures, Wrocław, Poland.

[2)]Measurements of the magnetoresistance and of the Hall emf in magnetic fields up to 150 kOe were carried out at the International Laboratory of Strong Magnetic Fields and Low Temperatures, Wrocław, Poland.

[3)]This expression is not given in Ref. 8, but (as communicated by the author of Ref. 8 personally) it is readily deduced from Eq. (30) in Ref. 8 which describes a conductivity $\sigma_{xy}$, if we assume that $\vartheta \ll W_{surf}$ and **H** is parallel to a high-symmetry axis.

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