What if a film conductivity exceeds the speed of light?

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We investigate the electrodynamics of a thin film with account taken of retardation effects. We show that for a two-dimensional conductivity \( \sigma > c/2\pi \) there exist weakly damped plasma waves at the lowest frequencies. We calculate the reflection, transmission, and absorption coefficients of obliquely incident light.

1. The conductivity \( \sigma \) of a film has units of velocity. The question in the title is therefore meaningful and calls for an exhaustive answer. Conductivity governs the character and rate of Maxwellian relaxation of the excess charges. In bulky conductors the excess charge density \( \rho(\mathbf{r},0) \) relaxes without changing the initial distribution \( \rho(\mathbf{r},0) \) with a decrement \( 4\pi \sigma \varepsilon_0 \)

\[ \rho(\mathbf{r}, t) = \rho(\mathbf{r}, 0) \exp(-t/\tau). \]

According to electrostatics\(^1\) charges relax by spreading with an effective velocity \( 2\sigma \). This velocity becomes comparable with that of light if the film sheet resistance is 188 \( \Omega \). An electrostatic approach to relaxation in films with so high a conductivity is inadequate, and account must be taken of retardation, with the field described by the complete set of Maxwell equations.

Maxwellian relaxation corresponds to dissipative dynamics of the charges at low frequencies, \( \omega \sigma \ll 1 \), where \( \tau \) is the carrier free-path time. Nondissipative dynamics pertains to plasma oscillations whose spectrum takes the form \( \omega_0(k) = (2\pi\varepsilon_0\varepsilon_\infty k/m)^{1/2} \) in the two-dimensional (2D) case (Ref. 3). We calculate in this paper the spectrum of the plasma oscillations in the dissipative and nondissipative regions, with allowance for retardation effects.

2. Consider a 2D layer with conductivity \( \sigma \) perpendicular to the \( z \)-axis. The complete set of Maxwell equations for the vector and scalar potentials \( \mathbf{A}, A \), and \( \varphi \) and the material equations are

\[ \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \mathbf{k} \right) \mathbf{A} = 4\pi \sigma \mathbf{j}, \]

\[ \nabla \times \mathbf{A} + \frac{1}{c^2} \frac{\partial \mathbf{A}}{\partial t} = 0, \]

\[ \frac{\partial \varphi}{\partial t} = \frac{1}{c} \frac{\partial \mathbf{A}}{\partial \mathbf{k}}. \]

The continuity equation

\[ \frac{\partial \varphi}{\partial t} + \nabla \cdot \mathbf{j} = 0 \]

follows from Eqs. (1) and (2).

We seek the natural oscillations in the system described by Eqs. (1)-(3) in the form of a wave \( \exp(ik \cdot r - i\omega t) \) (where \( k \) is a two-dimensional wave vector) propagating along the 2D layer of the wave and having a field localized near the layer. The potentials \( \mathbf{A}, A, \varphi \), and \( \mathbf{A} \) are therefore proportional to \( \exp(-x^2) \) where

\[ u = (k^2 - \omega^2/c^2)^{1/2}. \]

It follows from (1) that \( A_z = 0 \) and the vector potential can be sought in the form \( \mathbf{A} = k(z) \mathbf{A} + \{k \times \mathbf{l}, \mathbf{A} \} \). Expressing \( \varphi, \mathbf{A} \), and \( \mathbf{A} \) with the aid of (1), (2), and (3) in terms of \( \mathbf{A} \), we obtain the dispersion relation. The transverse-wave spectrum is given by the equation

\[ \left( k^2 - \frac{\omega^2}{c^2} \right)^2 - 4m \sigma \omega / c = 0, \]

in which account is taken of the frequency dispersion of the conductivity \( a_\omega = [\sigma(p/c)/m(1-i\omega\tau)] \).

The dispersion equation for the longitudinal waves is

\[ \frac{2m \sigma}{\varepsilon_0} \left( k^2 - \frac{\omega^2}{c^2} \right)^2 = 0. \]

Equations (5) and (6) contain a characteristic parameter \( x = \sigma/c^2 \). Analysis of Eq. (5) shows that the transverse mode is purely relaxational if \( x < 1 \) and corresponds as \( k \rightarrow \omega/c \) to ordinary plasma oscillations:

\[ \omega_0(k) - 1 \left( 1 - x^2/k^2 \right)^{1/2} \]

If \( k \ll x^2, \) the spectrum is purely relaxational:

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\[ \omega = \left( \frac{\omega_p^2}{m} \right)^{1/2} \frac{1}{2\pi} \]  

As \( x \rightarrow -1 \), the threshold value of \( k \), \( -0 \), and the plasma-oscillation spectrum at \( x = 1 \) and \( k < 1/cr \) assume the form

\[ \omega_0(k) = \left( \frac{c^2 k^2}{2\pi} \right) \exp(-\alpha x). \]

If \( 0 < k < 1 \), the spectrum (11) is conserved in the region \( (x-1)^{3/2}/cr < k < 1/cr \), and the longest-wavelength oscillations obey the dispersion law

\[ \omega = \omega(k) = \left( \frac{c^2 k^2}{2\pi} \right) \exp(-\alpha x). \]

For \( x > 1 \), the intermediate asymptotic value (11) is not realized and the plasma dispersion law can be described by Eq. 12 for all \( k < 1/cr \). It is seen from (12) that these long-wave-oscillations attenuate weakly. The reason is that the wave field is concentrated in a region of thickness \( \delta = 1/Re k < 1/\alpha x \) much larger than the thickness of the \( k^{-1} \) region in which the dissipation takes place. The transition from the purely relaxational spectrum (9) to the plasma oscillations (12) can be tracked by examining the character of the dispersion region near the layer. For \( x < 1 \), the damping rate is determined by the packet's wave front. The first term in the exponent describes the time evolution of the packet, viz., the packet contracts as it propagates along the surface. The energy density in the \( (x,z) \) plane is then transported along a vector \( v = (1, -\partial \sigma / \partial k) \) at an angle \( \theta \) to the normal:

\[ \cos \theta = \frac{1 + \delta v_x/v}{\delta v_x/v}. \]

The field distribution in a surface plasma wave Eq. (12) is shown for \( x > 1 \) in Fig. 2. The Poynting vector \( \vec{S} \) makes an angle \( \theta \) with the normal,

\[ \cos \theta = 2\alpha x. \]

The normal part of \( \vec{S} \) is connected with the transport of the field energy \( v \) to the 2D layer in which the dissipation takes place. The tangential component determines the energy transported by a plasmon with velocity

\[ v = \frac{x \beta}{\sin \theta - \cos \theta} \]

The phase velocity \( v = \alpha/k \) is determined by the velocity of the point where the phase front crosses the 2D-layer plane:

\[ v = \frac{x \beta}{\sin \theta - \cos \theta}. \]

The velocity (15b) is not observable as a signal-transport velocity, but the spectrum \( \omega = vk \) can apparently manifest itself in Raman scattering of light. At short wavelengths \( k \gg 1/cr \) the regime (12) is replaced by the usual dispersion law of plasma oscillations (10).

3. The velocity at which a signal can be propagated by a wave with a dispersion \( \omega(k) \) can be determined by investigating the evolution of the wave packet. For example, the wave intensity \( |\psi(x,t)|^2 \) is determined by the integral

\[ \psi(x,t) = \int \left( \frac{dk}{2\pi} \right) \Delta(k) \exp[ikx - \omega(k)t]. \]

Evaluating the integral, we find

\[ \psi(x,t) \sim \exp[ikx - \omega(k)t] \exp[-(x^2 - 2tk - \omega(k)t)^2/2], \]

from which it follows that \( v = \Delta \omega/k \).

A surface-wave packet evolves not only along the plane but also in a transverse direction, and is determined by the dispersion of \( \omega(k) \) and \( x(k) \). In the case of interest to us we have \( x = x_0 + \delta x \), with \( x_0 \ll x \ll x_0 \). Just as above

\[ \psi(x,x,t) = \int \left( \frac{dk}{2\pi} \right) \Delta(k) \exp[ikx - \omega(k)t - \omega(k)x] \exp \left( -\frac{1}{2} \Delta \frac{\partial \theta_\omega}{\partial k} \frac{\partial x}{\partial k} \right). \]

We need not fear the last term, which increases as \( |x| \to \infty \), in the argument of the exponential. It stems from the fact that far from the \( x = 0 \) plane the field is determined by the packet component that is least attenuated. No such growth occurs if \( |k - k_0| \) is bounded. The second term in the exponent is of second origin and causes the phase of the wave to be different for equal \( x \) and different \( x \). This implies bending of the packet's wave front. The first term in the exponent describes the time evolution of the packet, viz., the packet contracts as
FIG. 3. Absorption coefficient of a wave polarized in the incidence plane $P_1$ vs the angle of incidence or different values of the parameter $x$.

Figure 3 shows the $P_1(\beta)$ dependence for $\omega \tau = 0.1$ at various values of $x$. If a wave of frequency $\omega$ and wave vector $(k, q)$ is incident on the film, the amplitudes $r_1$ and $t_1$ have a pole at the unphysical values $q = \mp i(\omega/k)$, where $\omega$ and $k$ are related by Eq. (6). For $x > 1$ the angular dependence $P_1(\beta)$ has at $\cos \beta = 1/x$ a maximum that meets the condition that the incident wave be in resonance with the surface plasmon $\omega = \omega_p = \omega_{pl}(x) \sin \beta$.

In high-mobility samples, when $x > 1$, these resonance conditions are met for almost grazing incidence.

For another polarization of the incident wave, when $E$ is perpendicular to the plane of incidence, the quantities $r_1$ and $t_1$ can be obtained from (16) by following the rule

$r_1(x) = r_1(1/x), \quad t_1(x) = t_1(1/x), \quad P_1(x) = P_1(1/x)$.

Therefore when natural light is incident on the film the transmitted light is predominantly polarized in the incidence plane. For normal incidence of the wave we have $r_1 = r_1$ and $t_1 = t_1$. The results obtained from (16) with $\cos \beta = 1$ agree with the answer to problem 5 in §66 of Ref. 4.

5. It is natural to compare the electrodynamics of a thin film with the electrodynamics of a thin wire. The wire conductivity has the dimension of the diffusion coefficient, and at low frequency the relaxation spectrum is of the form $\omega = -2\pi k^3$ (Ref. 1). At high frequencies, without allowance for retardation, we have $\omega_p = (2\pi e^2/m)k$. If $x = 2\pi e^2/mc^2 > 1$, however, the complete set of Maxwell equations must be solved. Such scattering leads to a dispersion law

$\omega(k) = -\frac{k^3}{2\pi^2} (1 + \frac{x}{2})$.

In the low-frequency region we obtain for $k < k_0 = (1/2\pi)(1 + \frac{x}{2})$ the known result $\omega = -2\pi k^3$, while for $k > k_0$ we have $\omega(k) = \omega_p(1 + \frac{x}{2})^2$.

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