# Resonant spin-flavor precession of neutrinos and the solar neutrino problem

E. Kh. Akhmedov and O. V. Bychuk

*I. V. Kurchatov Institute of Atomic Energy and Institute of Nuclear Research, Academy of Sciences of the USSR*

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The resonant amplification of spin-flavor precession of neutrinos in solar matter and possible consequences of this phenomenon are discussed. It is shown that resonant spin-flavor neutrino precession may account for the deficit in solar neutrinos in the Davis experiment and for the anticorrelation between the neutrino count rate and solar activity. Experiments that could distinguish between spin-flavor neutrino precession and the Mikheev-Smirnov-Wolfenstein effect are examined. A new limit on the usual spin precession of solar neutrinos is derived.

## 1. INTRODUCTION

One of the most intriguing current problems in modern physics is that of the solar neutrinos. The essence of this phenomenon is that the neutrino flux of $2.1 \pm 0.3$ SNU, recorded in the experiment by Davis *et al.*,[1] is lower by a factor of about three than that predicted by the standard solar model,[2] i.e., $7.9 \pm 2.5$ SNU, where 1 SNU $= 10^{-36}$ captures per second per target atom. Another interesting feature of solar neutrinos is the observed anticorrelation between the count rate and solar activity, i.e., sunspot number. This anticorrelation was first noted in Ref. 3. Further evidence was obtained after a special analysis of the experimental results.[2,4] An additional argument in favor of anticorrelation has been provided by Davis' recent data,[5] obtained in five series of experiments in 1986–1987, in which the mean count rate was $5 \pm 1$ SNU, which is practically the same as the prediction of the standard solar model. We note that a solar activity minimum was observed in 1986–1987.

There are many published attempts to explain the anomalously low mean solar neutrino flux. An elegant solution of the problem was recently put forward by Mikheev and Smirnov,[6,7] who examined the effect of matter on neutrino oscillations. They generalized an earlier analysis by Wolfenstein[8] to the case of a medium of variable density and found that a resonant amplification of neutrino oscillations could occur under certain definite conditions. The width of the resonance layer is then significantly greater than the resonant length of the oscillations (i.e., the adiabatic condition is satisfied) and a practically complete transformation of $\nu_e$ into $\nu_\mu$ or $\nu_\tau$ that was not recorded in the Davis experiment becomes possible. The Mikheev-Smirnov-Wolfenstein effect (the MSW effect) depends significantly on the neutrino energy and should therefore lead to an appreciable distortion of the solar neutrino spectrum. On the other hand, it is unlikely that this effect could explain the anticorrelation between the solar neutrino count rate and the sunspot number, which is a measure of the magnetic field in the convective zone of the Sun.

A simple and attractive explanation of this anticorrelation has been put forward by Voloshin, Vysotskiĭ, and Okun',[9–12] who considered neutrino spin precession (NSP) in the toroidal magnetic field of the convective zone.

If the magnetic (or electric) dipole moment of the neutrino is different from zero, its spin should precess in a transverse magnetic field. Some of the left-handed neutrinos $\nu_{eL}$ then transform into right-handed neutrinos $\nu_{eR}$ that are sterile and are not recorded by the neutrino detectors. Since, at maximum solar activity, the magnetic field in the convective zone is at least an order of magnitude greater than that during the years of the quiet sun, the NSP hypothesis[9–12] provides a likely explanation of the anticorrelation between the neutrino count rate in the Davis experiment and the solar activity (11-year cycle). It also predicts a semiannual variation in the $\nu_{eL}$ flux due to the toroidal magnetic field and the inclination of the orbit of the Earth relative to the plane of the solar equator.[10,12,13] The effect of matter on NSP is discussed in Refs. 12 and 13, where it is shown that precession is suppressed by a medium and the suppression may be quite strong.[1)]

In contrast to the MSW effect, NSP is independent of the neutrino energy $E$, so that it should lead to an overall suppression of the $\nu_{eL}$ flux without distorting the spectrum of these neutrinos.

If the neutrinos are Majorana particles, they cannot have diagonal electromagnetic moments because of CPT invariance, but they can have off-diagonal (transition) moments that are responsible, in particular, for radiative transitions of the form $\nu_2 \to \nu_1 \gamma$ (for $m_2 > m_1$). The Dirac neutrino can also have both off-diagonal and diagonal electromagnetic moments. In a transverse magnetic field, the off-diagonal electromagnetic moments give rise to transitions of left-handed neutrinos of a given type into right-handed neutrinos (or antineutrinos) of another type, i.e., to neutrino spin-flavor precession (NSFP).[12,16]

It was shown in Refs. 17 and 18 (and independently, in Ref. 19) that resonant NSFP amplification analogous to the MSW effect in neutrino oscillations was possible in the presence of a medium. It was noted that resonant NSFP could explain the solar neutrino deficit and the anticorrelation between the neutrino count rate and solar activity. The adiabatic condition was also investigated in Refs. 17 and 18, and ranges of admissible parameter values for which resonant NSFP could explain the solar neutrino deficit were found.

In this paper, we give a detailed discussion of the resonant amplification of NSFP in solar matter for different configurations of the solar magnetic field. Possible consequences of this phenomenon are examined, and experiments that could distinguish between resonant NSFP and ordinary precession (without change of flavor) and the MSW effect are discussed. Some aspects of ordinary NSP are also discussed.

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## 2. THE MATRIX OF THE ELECTROMAGNETIC MOMENTS OF THE NEUTRINO

The interaction of the neutrino with an external field via its electromagnetic moments can be described by the effective Lagrangian

$$\mathscr{L} = \tfrac{1}{2}\tilde{\mu}_{ij}{}^{D}\bar{\nu}_{iR}\sigma_{\mu\nu}\nu_{jL}F^{\mu\nu} + \text{h.c.} \quad (1)$$

in the case of Dirac neutrinos, and by the Lagrangian

$$\mathscr{L} = \tfrac{1}{2}\tilde{\mu}_{ij}{}^{M}\bar{\nu}_{iR}{}^{c}\sigma_{\mu\nu}\nu_{jL}F^{\mu\nu} + \text{h.c.} \quad (2)$$

in the case of Majorana neutrinos. In the latter case, there is no need to introduce the sterile right-handed neutrinos because the Lagrangian (2) describes the transition of the left-handed neutrinos $\nu_{jL}$ into right-handed antineutrinos $\nu_{iR}^{c}$, which are also active. In (1) and (2), $(i,j = e,\mu,\tau,...)$ are the flavor indices and $\tilde{\mu}_{ij}$ are the electromagnetic moment matrices in the flavor basis, whose Hermitian parts $\mu_{ij}$ determine the magnetic dipole moments, and whose anti-Hermitian parts $\varepsilon_{ij}$ determine the electric dipole moments of the neutrinos. We note that nonzero off-diagonal electric dipole moments $\varepsilon_{ij}$ $(i\neq j)$ do not in themselves signify a breaking of CP invariance: this occurs only if both electric and magnetic off-diagonal moments were nonzero simultaneously.

Both magnetic and electric dipole moments should lead to NSP in a transverse magnetic field. In the ensuing analysis, we shall, for brevity, use the phrase "magnetic moment" although it has been shown[10-12] that it is the combination $(\mu^2 + \varepsilon^2)^{1/2}$ that appears in all the formulas for ultrarelativistic particles.

It is noted in Refs. 9–13 that NSP can lead to an appreciable reduction in the flux of left-handed solar neutrinos if their magnetic moment is $\mu_{ee} \sim (10^{-10}\text{–}10^{-11})\mu_B$, where $\mu_B = e/2m_e$ is the Bohr magneton. It is assumed that the average magnetic moment within the convective zone can reach $10^3$–$10^4$ G. Such neutrino magnetic moments are not inconsistent with existing experimental limits deduced from $\bar{\nu}e$ scattering[20,21]

$$\Big(\sum_j |\mu_{ej}|^2\Big)^{1/2} \leqslant 2\cdot 10^{-10}\mu_B, \quad (3)$$

and from astrophysical data on white dwarfs and helium stars[22-25]

$$\Big(\sum_{i,j} |\mu_{ij}|^2\Big)^{1/2} \leqslant 10^{-11}\mu_B. \quad (4)$$

More stringent limits on the magnetic moment of the electron neutrino have recently been obtained by analyzing neutrino events from the supernova SN 1987A (Refs. 26–28), but these limits are not absolute in character[20,30] and, at any rate, do not refer to the off-diagonal mangetic moments of the Majorana neutrino that are of particular interest here. We shall therefore base our discussion on the limit given by (4). Diagonal and off-diagonal neutrino magnetic moments of the order of $10^{-11}$ $\mu_B$ can be obtained theoretically, for example, in models with a charged $SU(2)_L$—singlet scalar.[31-36]

## 3. NEUTRINO SPIN PRECESSION WITHOUT CHANGE OF FLAVOR

Let us first briefly recall the relationships that describe NSP without change of flavor in the presence of matter.[12,13] We shall neglect possible neutrino mixing effects.

The evolution of the system $(\nu_{eL},\nu_{eR})$ in a magnetic field $B_\perp$ in the presence of matter is described by the equation

$$i\frac{d}{dt}\begin{pmatrix}\nu_{eL}\\ \nu_{eR}\end{pmatrix} = \begin{pmatrix}E+c_L & \mu_{11}B_\perp\\ \mu_{11}B_\perp & E\end{pmatrix}\begin{pmatrix}\nu_{eL}\\ \nu_{eR}\end{pmatrix}, \quad (5)$$

where $E$ is the neutrino kinetic energy, the mean potential energy of the interaction between $\nu_{eL}$ and matter is

$$c_L = 2^{1/2}G_F(N_e - N_n/2), \quad (6)$$

and 4 and $N_e$ are, respectively, the electron and neutron concentrations (the medium is assumed to be electrically neutral). The solution of (5) for a uniform field and constant-density medium can readily be obtained. If $\nu_{eL}$ beam is present at the origin, the probability of detecting $\nu_{eR}$ at the point $r$ is

$$P(\nu_{eL}\to\nu_{eR}; r) = \frac{(2\mu_{11}B_\perp)^2}{(2\mu_{11}B_\perp)^2 + c_L{}^2}\sin^2\Big\{\frac{1}{2}[(2\mu_{11}B_\perp)^2 + c_L{}^2]^{1/2}r\Big\}. \quad (7)$$

The corresponding probability in the absence of the medium can be deduced from (7) by setting $c_L = 0$.

It follows from (7) that the maximum fraction of $\nu_{eL}$ that can be transformed into $\nu_{eR}$ in the medium is always less than unity and depends on the ratio of $2\mu_{11}B_\perp$ to $c_L$, whereas, in the absence of the medium, all the left-handed neutrinos will be converted into right-handed neutrinos. It is thus clear that the medium will suppress NSP. The reason for this is that $\nu_{eL}$ and $\nu_{eR}$ are energy-degenerate in vacuum, but the medium lifts the degeneracy because $\nu_{eL}$ interacts with matter, which produces the average potential $c_L$ for them, whereas $\nu_{eR}$ are sterile and do not interact with the medium.

It is clear from (7) that NSP will efficiently convert $\nu_{eL}$ into $\nu_{eR}$ provided the following conditions are satisifed:

$$2\mu_{11}B_\perp \gtrsim c_L = (2)^{1/2}G_F(N_e - N_n/2), \quad \mu_{11}B_\perp L \gtrsim 1, \quad (8)$$

where $L$ is the size of the region in which the field is strong, i.e., where the first of the conditions (8) is satisfied. It has been shown[12] that the conditions given by (8) can, in principle, be satisfied in the convective zone of the Sun ($L \approx L_{\text{conv}} \approx 0.3R_\odot$ where $R_\odot \approx 7\cdot 10^{10}$ cm is the radius of the Sun and $\rho \lesssim 0.2$ g/cm$^3$). However, it is noted in Refs. 17 and 18 that the conditions given by (8) are necessary but, in general, not sufficient to ensure that NSP without change of flavor can explain the solar neutrino deficit and the anticorrelation between the neutrino count rate and solar activity. This will be discussed in greater detail in Sec. 9.8.

## 4. NEUTRINO SPIN-FLAVOR PRECESSION IN MATTER

We shall now consider NSFP due to the interaction between off-diagonal magnetic moments of the neutrino and the external magnetic field. For simplicity, we shall restrict our attention to two types of neutrino, assuming that $|\mu_{13}|,|\mu_{23}| \ll |\mu_{12}|$. The evolution equation for a set of ultrarelativistic neutrinos can be written in the form[17-19]

$$i\frac{d}{dt}\begin{pmatrix}\nu_{1L}\\ \nu_{2R}\end{pmatrix} = \begin{pmatrix}m_1^2/2E + c_L(t) & \mu_{12}B_\perp(t)\\ \mu_{12}B_\perp(t) & m_2^2/2E + c_R(t)\end{pmatrix}\begin{pmatrix}\nu_{1L}\\ \nu_{2R}\end{pmatrix}, \quad (9)$$

where $\nu_{1L} = \nu_{eL}$ and $\nu_{2R} = \nu_{\mu R} (\nu_{\tau R},...)$ for the Dirac neutrinos, and $\nu^c_{\mu R} (\nu^c_{\tau R})$ for Majorana neutrinos. In the latter case,

$$c_R = (2)^{1/2} G_F N_n/2,$$

where, in the Dirac case, $\nu_{2R}$ is sterile, so that $c_R = 0$. The coordinate dependence of $B_\perp$, $c_L$ and $c_R$ in (9) is shown as a time dependence because $r \approx t$ for the neutrinos.

If the neutrinos are Dirac particles, we have to consider both NSFP and ordinary precession, i.e., we must extend the basis of neutrino states and the set of equations given by (9). Majorana neutrinos have no diagonal magnetic moments and, for them, we can confine our attention to NSFP. We shall therefore mostly be concerned with the case of Majorana neutrinos. We note, however, that we can often neglect ordinary precession even in the case of Dirac neutrinos because the medium suppresses this precession, but, as will be shown later, it can amplify the flavor precession.

The eigenstates of the effective Hamiltonian of the neutrino system in an external magnetic field in the presence of a medium, $\nu_A$ and $\nu_B$, do not have definite helicity of flavor, and are superpositions of the states $\nu_{eL}$ and $\nu_{2R}$:

$$\nu_A = \nu_{eL} \cos \tilde{\theta} + \nu_{2R} \sin \tilde{\theta}, \quad \nu_B = -\nu_{eL} \sin \tilde{\theta} + \nu_{2R} \cos \tilde{\theta}. \tag{10}$$

For a uniform field and a constant-density medium, we can readily find the mixing angle $\tilde{\theta}$ and the probability of $\nu_{eL} \to \nu_{2R}$ conversion:

$$\operatorname{tg} 2\tilde{\theta} = 2\mu_{12} B_\perp / [c_L - c_R - (m_2^2 - m_1^2)/2E], \tag{11}$$

$$P(\nu_{eL} \to \nu_{2R}; r) = \sin^2 2\tilde{\theta} \sin^2 \left(\frac{\pi r}{l}\right) = \frac{(2\mu_{12} B_\perp)^2}{(2\mu_{12} B_\perp)^2 + (c_L - c_R - \Delta m^2/2E)^2} \sin^2 \left(\frac{\pi r}{l}\right) \tag{12}$$

where the precession length $l$ is given by

$$l = 2\pi/(E_A - E_B) = 2\pi/[(2\mu_{12} B_\perp)^2 + (c_L - c_R - \Delta m^2/2E)^2]^{1/2}. \tag{13}$$

It follows from (11) and (12) that, in the absence of matter ($c_L = c_R = 0$), NSFP is suppressed in comparison with ordinary precession: the fraction of $\nu_{eL}$ that can be converted into $\nu_{2R}$ is always less than unity.[12] However, resonant amplification of precession is possible in a medium if $\Delta m^2 \equiv m_2^2 - m_1^2 > 0$. The resonance condition $(c_L - c_R)_{\rm res} = \Delta m^2/2E$ for the Majorana neutrinos is

$$2^{1/2} G_F (N_e - N_n)_{\rm pes} = \Delta m^2/2E, \tag{14}$$

whereas in the Dirac case we have

$$2^{1/2} G_F (N_e - N_n/2)_{\rm pes} = \Delta m^2/2E. \tag{15}$$

These conditions differ from the resonance conditions for the MSW effect

$$2^{1/2} G_F (N_e)_{\rm res} \approx \Delta m^2/2E.$$

In general, when the density $\rho$ of the medium and the magnetic field $B_\perp$ are functions of the distance $r$ along the neutrino trajectory, the formulas given by (11)–(13) are no longer valid. However, if $\rho(r)$ and $B_\perp(r)$ varies sufficiently slowly (adiabatically), i.e., if the characteristic distances over which these quantities vary significantly are large in comparison with the precession length, the system will "follow" the variation in the external parameters. We can then use (11) and (13) as being approximately valid, assuming that $\tilde{\theta}$ and $l$ are functions of $r$.

Let us now provide a qualitative description of the neutrino evolution in the Sun under adiabatic conditions. The density of the medium is a maximum at the center of the Sun and, as follows from (11), the angle $\tilde{\theta}$ is small if

$$2^{1/2} G_F (N_e - N_n)_{r=0} - \Delta m^2/2E \approx 1.8 \cdot 10^{-12}\ \text{eV} - \Delta m^2/2E \gg 2\mu_{12} B_\perp(0).$$

Since the density of the medium decreases monotonically with distance $r$, we find that, for each value $\Delta m^2/2E < 2^{1/2} G_F (N_e - N_n)_{r=0}$, we can find a value $r = r_0 (0 < r_0 < R_\odot)$ for which the resonance condition (14) or (15) is satisfied. At resonance $\tilde{\theta} = \pi/4$, i.e., the mixing of $\nu_{eL}$ with $\nu_{2R}$ is a maximum. At resonance, $\operatorname{tg} 2\tilde{\theta}$ passes through a pole, i.e., it changes sign. As the solar surface is approached, the denominator in (11) tends to $-\Delta m^2/2E$ and the numerator decreases because the magnetic field decreases. This means that, as the distance from the Sun increases, $\operatorname{tg} 2\tilde{\theta}$ decreases in absolute magnitude, but remains negative, i.e., $\tilde{\theta} \to \pi/2$. Thus, if the neutrinos are created when the density is much greater than the resonant value, the mixing angle varies from $\tilde{\theta} \approx 0$ to $\tilde{\theta} \approx \pi/2$ along their trajectory.

It follows from (10) that, at the center of the Sun and for $\tilde{\theta} \approx 0$, the $\nu_{eL}$ state is practically identical with the eigenstate $\nu_A$ of the neutrino Hamiltonian. Under adiabatic conditions, the probability of transition from one adiabatic term to the other is exponentially small, and this means that the state $\nu_A$ will propagate in the medium and the field without passing to the state $\nu_B$, but its composition relative to $\nu_{eL}$ and $\nu_{2R}$ varies because the angle $\tilde{\theta}$ varies. With increasing distance from the Sun, $\tilde{\theta} \to \pi/2$, i.e., $\nu_A \approx \nu_{2R}$. It follows that, in the adiabatic state, resonant NSFP can lead to practically complete transformation of $\nu_{eL}$ into right-handed neutrinos (or antineutrinos) $\nu_{2R}$.

This phenomenon is analogous to the $\nu_{eL} \to \nu_{2L}$ conversion in the case of resonant neutrino oscillations.[6,7] When the resonant density is close to the maximum solar density, or

$$\mu_{12} B_\perp(0) \gg 2^{1/2} G_F (N_e - N_n)_{r=0},$$

we find that $\tilde{\theta} \approx \pi/4$ at the center of the Sun, i.e., the mixing angle $\tilde{\theta}$ varies along the neutrino trajectory between $\pi/4$ and $\pi/2$. Under adiabatic conditions, about half the $\nu_{eL}$ will be converted into $\nu_{2R}$ during this process.

## 5. ADIABATIC CONDITIONS

We now define the resonant NSFP region as the region in which $\sin^2 2\tilde{\theta} \geqslant 1/2$. It will be convenient to substitute

$$N_e(r) = \frac{\rho(r)}{m_N} Y_e(r), \quad N_n(r) = \frac{\rho(r)}{m_N} Y_n(r), \tag{16}$$

where $m_N$ is the nucleon mass, and $Y_e$ and $Y_n$ are, respectively, the mean number of electrons and neutrons per nucleon ($Y_e + Y_n = 1$). Since the density of matter in the Sun varies from $\rho(0) \approx 150$ g/cm$^3$ to $\rho(R_\odot) \approx 0$, whereas $Y_e$ varies only within the range $Y_e \simeq 0.67$–$0.86$, the rate of variation of $N_e(r)$ and $N_n(r)$ is actually equal to the rate of variation of $\rho(r)$. Expanding $B_\perp(r)$ and $\rho(r)$ near the resonance point $r_0$, we can readily show that the width $\Delta r$ of the resonance region is given by

$$\Delta r=\Delta r_-+\Delta r_+=2\alpha L_\rho[1-\alpha^2(L_\rho/L_B)^2]^{-1},$$
$$\Delta r_\pm=\alpha[L_\rho^{-1}\pm\alpha L_B^{-1}]^{-1},$$
$$\alpha\equiv 2\mu_{12}B_{\perp 0}/(\Delta m^2/2E),\quad B_{\perp 0}\equiv B_\perp(r_0), \tag{17}$$

where $\Delta r_-$ and $\Delta r_+$ are, respectively, the resonance half-widths for $r<r_0$ and $r>r_0$, and $L_\rho$ and $L_B$ are the characteristic distances over which there is a significant change in $\rho(r)$ and $B_\perp(r)$ in the resonance region:

$$L_\rho\equiv\left(-\frac{1}{\rho}\frac{d\rho}{dr}\right)^{-1}_{r=r_0},\quad L_B\equiv\left(-\frac{1}{B_\perp}\frac{dB_\perp}{dr}\right)^{-1}_{r=0}. \tag{18}$$

The adiabatic condition can also be formulated as the requirement that the resonant precession length be small in comparison with the width of the resonance region:

$$l_{\rm res}=\pi/\mu_{12}B_{\perp 0}\ll\Delta r. \tag{19}$$

It will be seen from the discussion given below that the adiabatic parameter is actually given by

$$\chi\equiv\frac{1}{\pi}\frac{l_{\rm res}}{\Delta r}=\frac{1}{8}\frac{\Delta m^2}{E}[1-(\alpha L_\rho/L_B)^2]\frac{1}{(\mu_{12}B_{\perp 0})^2L_\rho}$$
$$\approx\frac{2.5\cdot 10^{-12}\,{\rm cm}}{L_\rho}\left[1-\left(\alpha\frac{L_\rho}{L_B}\right)^2\right]\frac{\Delta m^2}{E}\left(\frac{{\rm eV}^2}{\rm MeV}\right)$$
$$\times\frac{1}{(\mu_{12}B_{\perp 0})^2({\rm eV}^2)}. \tag{20}$$

In a uniform magnetic field, the width of the resonant region is given by

$$\Delta r=2\alpha L_\rho\quad(\Delta r_-=\Delta r_+=\alpha L_\rho). \tag{21}$$

In general, the nonuniformity of the magnetic field has less effect on the adiabatic condition than the nonuniformity of matter. When $B_\perp(r)$ varies monotonically in the resonant region, the effects of the field variation are partially cancelled out. For example, when $dB_\perp/dr<0$, we have $\Delta r_->\alpha L_\rho$, $\Delta r_+<\alpha L_\rho$, but it follows from (20) that the sum $\Delta r=\Delta r_-+\Delta r_+$ differs from $2\alpha L_\rho$ by terms of second order in $(\alpha L_\rho/L_B)$. We note that the effects of resonant NSFP amplification are very clearly defined only for $\alpha\ll 1$, which means that the terms $(\alpha L_\rho/L_B)^2$ can be neglected in $\Delta r$ and $\chi$ for $L_B\gtrsim L_\rho$. We shall use this approximation in Sec. 7.

It is important to emphasize that, as the quantity $\Delta m^2/E$ (and, consequently, the resonant density) decreases, the adiabatic condition $\chi\ll 1$ is better satisfied provided the field $B_\perp(r)$ is almost uniform. Ths distinguishes NSFP from the MSW effect in which the opposite situation occurs. However, it is likely that the magnetic field varies quite rapidly in the solar interior.

Unfortunately, there is no direct experimental information on the radial dependence of the magnetic field $B_\perp$ inside the Sun.

## 6. THE SUN AND ITS MAGNETIC FIELD

Let us now briefly summarize some of the solar data[37–39] that will be useful below. Thermonuclear reactions that are the source of solar energy and are accompanied by the emission of $\nu_{eL}$ occur in the solar core ($x\equiv r/R_\odot\lesssim 0.25$), and the high-energy neutrinos from $^8$B and $^7$Be, recorded in the $^{37}$Cl–$^{37}$Ar experiments, are created in the very hot central part of the core ($x\lesssim 0.05$). The radiative transfer zone is present for $x\approx 0.7$ and is followed by the convective zone ($0.7\lesssim x\lesssim 1$).

The observed magnetic field on the surface of the Sun has a relatively complicated structure. The large-scale field has three main components, namely, the azimuthal (or subphotospheric toroidal) field, whose effect is seen in sunspots, the axisymmetric dipole-type poloidal field with a small admixture of a quadrupole component, and, finally, the monaxisymmetric sector field. The last two components are very weak ($\lesssim$ 1–2 G) and are of no interest to us. The toroidal field has opposite directions in the northern and southern solar hemispheres, and is zero near the equator. It has a 22-year period and its magnitude has an 11-year period. The maximum values of the toroidal field are observed at latitudes of $\pm$ 15–20°; the equatorial "gap" in which there is no field occurs at latitudes of $\pm$ 5°. The mean toroidal magnetic field on the solar surface is of the order of 10–100 G, but it can reach up to 5000 G in sunspots.

The Earth's orbit is at an angle of about 7° to the plane of the solar equator, so that the central part of the Sun that emits the boron and beryllium neutrinos is seen twice a year (in June and December) through the equatorial gap in the toroidal magnetic field. This means that, during the years of active Sun, NSP should produce semiannual variations in $\nu_{eL}$ count rates in $^{37}$Cl–$^{37}$Ar experiments. At the same time, the low-energy pp-neutrinos that provide the main contribution in $^{71}$Ga–$^{71}$Ge experiments are efficiently created throughout the solar core, so that the semiannual variations should be highly reduced for them.[10–13]

The magnetic field in the solar interior cannot be investigated experimentally because the solar material is opaque. Existing theoretical ideas are not, unfortunately, sufficiently unambiguous and reliable, so that our knowledge about processes occurring in the solar interior is very incomplete. It is thought that the toroidal magnetic field is a maximum near the bottom of the convective zone. Parker[40] has argued that it should be no more than about 100 G, since otherwise the relatively high buoyancy should rapidly bring the magnetic field to the solar surface, whch would be in conflict with observational data. However, this limitation has been removed by taking into account a number of important physical effects. According to existing data, the magnitude of $B_\perp$ at the bottom of the convective zone can reach $10^5$ G (Refs. 41 and 42).

The internal magnetic field of the Sun ($x<0.7$) has not been investigated theoretically to any great extent. It has been suggested[14,39] that a relatively strong remanent field is present in the solar interior, i.e., a field that has remained and become amplified during the contraction of the protostellar cloud. The creation of the azimuthal field from the poloidal field as a result of differential (nonrigid) rotation, by analogy with what happened to the convective zone, is examined in Ref. 43. As the magnetic lines of force wind on, they come closer to one another and this is responsible for the amplification of the field. As a result, the azimuthal field at the center of the Sun can reach $10^6$–$10^7$ G, but it falls by one or two orders of magnitude by the time the convective zone is reached.

In contrast to the toroidal magnetic field in the convective zone, the internal magnetic fields are frozen and are not subject to the 11-year variation. Their time constants are $\gtrsim 10^8$ y. The variable component of the magnetic field cannot penetrate from the convective zone to the radiative transfer zone because of the discontinuity in magnetic permeability: the permeability is $\mu \sim 1$ in the radiative transfer zone and $\mu \sim 10^{-5}$ in the convective zone.[37] We note that the jump in permeability can lead to a sharp rise in the field $B_\perp$ between the convective and radiative transfer zones. This follows from the fact that, when there are no currents on the separation boundary between the two zones, the tangential components $H_\perp = B_\perp/\mu$ must be continuous. We emphasize that, when the large-scale magnetic field is determined in the NSP problem, the field must be averaged over regions of space whose dimensions are small in comparison with the precession length.

It is clear from the foregoing that very little is known about the magnitude and form of the function $B_\perp(r)$ in the interior of the Sun. If the mass and magnetic moment of the neutrinos were known, we could try to solve the inverse problem, namely, investigate the magnetic field in the interior of the Sun by detecting the solar neutrinos. In the present situation, it seems reasonable to specify different "likely" magnetic field distributions in the Sun, and then use them to calculate the $\nu_{eL} \to \nu_{2R}$ conversion probabilities, which can then be compared with experimental results. The results of such numerical calculations will be given in Sec. 8.

## 7. THE SOLAR NEUTRINO PROBLEM AND THE NSFP PARAMETERS

We shall now make a few estimates and find the approximate ranges of parameter values for which resonant NSFP can resolve the solar neutrino problem.

As in the case of resonant neutrino oscillations, there are two basic solutions, namely, the adiabatic and the moderately nonadiabatic.

*7.1. Adiabatic solution.* In this case, in order to obtain a suppression factor for the $\nu_{eL}$ count rate in the $^{37}Cl$–$^{37}Ar$ experiment of the order of 1/3, we must assume that all $\nu_{eL}$ with energies $E > E_c \approx 6$ MeV undergo resonant precession and are almost completely transformed into $\nu_{2R}$, whereas $\nu_{eL}$ with energies $E < E_c$ leave the Sun without change because, for these neutrinos, the resonant density is greater than the maximum density of the medium at the center of the Sun. From now on, all estimates and numerical calculations will be performed for Majorana neutrinos. Moreover, we shall assume that $L_\rho \approx R_\odot/10$, whch is well satisfied for $0.2 \lesssim x \lesssim 1$ (Ref. 44). The adiabatic solution occurs when

$$\Delta m^2 \approx 4\cdot 10^{-5}\ \text{eV}^2, \quad \beta \equiv \left(\frac{\mu_{12}}{10^{-11}\mu_B}\right) B_{\perp 0}(\text{G}) \geqslant 10^8 \quad (22)$$

where the first condition is obtained from the requirement that $E_c \approx 6$ MeV and the second from the adiabatic condition[2)] for neutrinos with energy $E > E_c$. The predicted spectrum of solar $\nu_{eL}$ recorded on the Earth is practically the same as the corresponding prediction for the MSW effect under adiabatic conditions[6,7,45]: for $E < E_c$, the spectrum is given by the standard solar model; for $E \approx E_c$ it cuts off quite rapidly, and there are practically no electron neutrinos for $E > E_c$. This means, in particular, that the planned $^{71}Ga$–$^{71}Ge$ experiment should reveal only a slight ($\lesssim 10\%$) suppression of the $\nu_{eL}$ flux, since the main contribution to this experiment will be provided by low-energy neutrinos.

*7.2. Moderately nonadiabatic solution.* In this case, a large proportion of the $\nu_{eL}$ spectrum from $^8B$ and $^7Be$ is suppressed, but this suppression is weaker than in the adiabatic case.

The moderately nonadiabatic conditions are examined in Ref. 46 for the MSW effect, and the simple "resonant layer" model is put forward and satisfactorily reproduces the results of numerical calculations. This model can also be used for NSFP. According to it, precession is completely suppressed outside the resonant layer of thickness $\Delta r$ (for high densities by the medium and for low densities by the fact that $2\mu_{12}B_\perp$ is small in comparison with $\Delta m^2/2E$). Moreover, NSFP is assumed to occur with maximum amplitude throughout the resonant layer (it is assumed that the density of the medium in the resonant layer is constant and equal to the resonant value). The probability of detecting $\nu_{eL}$ on the surface of the Sun is then equal to the probability of finding it leaving the resonant region:

$$P(\nu_{eL}\to\nu_{eL}) \approx \cos^2\left(\pi\frac{\Delta r}{l_{\text{res}}}\right) = \cos^2\left[8\frac{E}{\Delta m^2}(\mu_{12}B_{\perp 0})^2 L_\rho\right]. \quad (23)$$

We note that the corresponding expression for the MSW effect is[46]

$$P(\nu_e\to\nu_e) \approx \cos^2\left[\frac{\sin^2 2\theta_0}{\cos 2\theta_0}\frac{\Delta m^2}{2E}L_\rho\right], \quad (24)$$

where $\theta_0$ is the vacuum mixing angle. The significant point here is that the arguments of the cosines in (23) and (24) have a different dependence on neutrino energy.

In the case of resonant NSFP, the moderately nonadiabatic solution occurs when the following conditions are satisfied:

$$\beta \approx 10^8(\Delta m^2[\text{eV}^2])^{1/2}, \quad 2\cdot 10^{-8} \lesssim \Delta m^2 \lesssim 10^{-5}\ \text{eV}^2. \quad (25)$$

The first of these is obtained by demanding that the argument of the cosine in (23) be equal to arccos $3^{1/2}$ for $E \sim 10$ MeV; the second is necessary to ensure that the resonant density lies between 150 and $1.5\times 10^{-2}$ g/cm$^3$ for neutrinos with energies $E \approx 0.8$–14 MeV. The density $\rho = 1.5\cdot 10^{-2}$ g/cm$^3$ is reached for $x_0 = r_0/R_\odot \approx 0.92$. Lower densities correspond to greater $x_0$, for which the magnetic field becomes too weak: $B_{\perp 0} \lesssim 100$ G. It follows from (13) and (4) that the resonant precession length is then comparable with the separation between the Sun and Earth, and precession can be neglected.

## 8. FLUX SUPPRESSION FACTORS

The solar neutrino flux suppression factors due to NSFP were calculated by numerical integration of the four differential equations obtained by separating (9) into real and imaginary parts. The integration was carried out on the segment $0 \leqslant x \leqslant 1$ for initial conditions $\nu_{eL}(0) = 1$, $\nu_{2R}(0) = 0$. The probability that the electron neutrino will "survive" was defined by

$$P \equiv P(\nu_{eL}\to\nu_{eL};\ x=1) = |\nu_{eL}(x=1)|^2. \quad (26)$$

The density distribution in the medium and its isotopic composition in the Sun were specified by numerical interpolation of data tabulated in Ref. 44. The magnetic field was taken in the following form:

Variant I

$$B_\perp(x)=\begin{cases} B_1\left(\dfrac{\gamma}{x+\gamma}\right)^k, & 0\leqslant x\leqslant 0.65 \\ B_0\left[1-\left(\dfrac{x-0{,}7}{0{,}3}\right)^2\right], & 0.65<x\leqslant 1\,. \end{cases} \qquad (27)$$

Variant II

$$B_\perp(x)=\begin{cases} B_1\left(\dfrac{\gamma}{x+\gamma}\right)^k, & 0\leqslant x\leqslant 0.65 \\ B_0/\mathrm{ch}[z(x-0{,}7)], & 0.65<x\leqslant 1\,. \end{cases} \qquad (28)$$

For $x\gtrsim 0.7$, this field simulates a toroidal magnetic field in the convective zone with maximum at the bottom of the zone $x=0.7$. The region $0.65\leqslant x\leqslant 0.7$ was regarded as the transition layer, which could be penetrated by the toroidal field. The azimuthal component of the internal magnetic field of the Sun was assumed to be given by a power-type formula. The value $\gamma=0.2$ was used in all the calculations. For $k=2$, this gave a reduction in the internal field between the center and bottom of the convective zone by a factor of about 20. The parameter $z$ in variant II was chosen so that the field on the surface of the Sun amounted to a few tens of G and decreased exponentially. In variant I, the field on the surface is zero. The off-diagonal magnetic moment $\mu_{12}$ of the neutrino is assumed to be $10^{-11}\,\mu_B$. Since the magnetic field always appears as the product $\mu_{12}B_\perp$, the results can readily be converted to any other value of $\mu_{12}$ by changing the scale of the magnetic field (in this case, the constants $B_1$ and $B_0$). In our calculations, $B_\perp$ was the azimuthal component of the solar magnetic field ($B_\perp=B_\varphi$). It was assumed that $B_\theta$ was small in comparison with $B_\varphi$.

Figures 1 and 2 show the calculated suppression factors for the $\nu_{eL}$ flux as functions of $E/\Delta m^2$. They are in good agreement with the estimates presented in Sec. 7. Comparison of curves 1–3 in Fig. 2 shows that the suppression factors depend significantly on the form of the function $B_\perp(r)$.

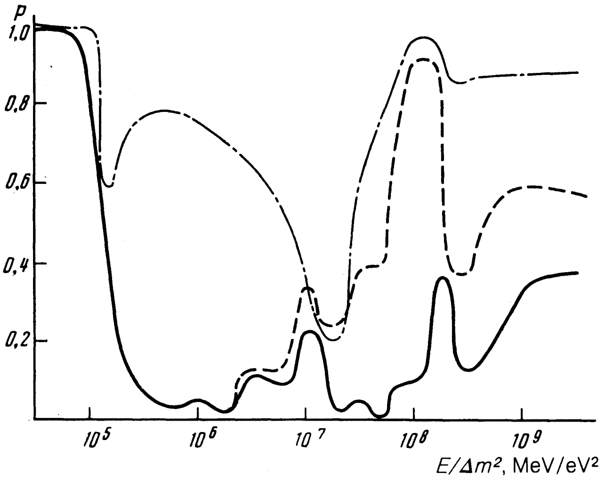


FIG. 1. Suppression factor $P$ for the solar neutrinos $\nu_{eL}$ as a function of $E/\Delta m^2$. The magnetic field is given by (27) (variant I). Solid line—$B_0=10^5$ G, $B_1=10^7$ G; dashed line—$B_0=10^4$ G, $B_1=10^7$ G; dot-dash line—$B_0=10^4$ G, $B_1=10^6$ G; $k=2$ in all cases.

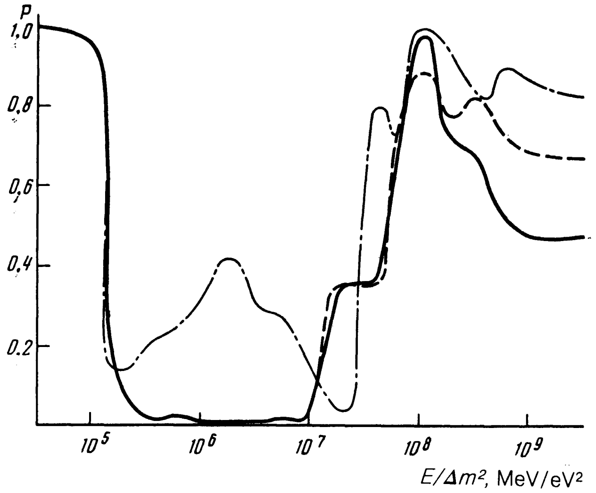


FIG. 2. Suppression factor $P$ as a function of $E/\Delta m^2$ for the $\nu_{eL}$ flux. Solid line—$k=2$, dot-dash line—$k=3$ (magnetic field—variant I); dashed line—$k=2, z=20$ (magnetic field—variant II). In all cases, $B_0=10^4$ G, $B_1=5\cdot10^6$ G.

## 9. RESULTS AND DISCUSSION

9.1. Resonant NSFP is, in many ways, analogous to the MSW effect, but there are significant differences between them. The principal difficulty in the analysis of NSFP is that the mixing of $\nu_{eL}$ and $\nu_{2R}$ is determined by the unknown function $\mu_{12}B_\perp(r)$, whereas in the case of the neutrino oscillations mixing is described by a single unknown parameter (the vacuum angle $\theta_0$). In a uniform magnetic field, a simple correspondence can be established between resonant NSFP and the MSW effect: all the formulas describing resonant precession can be obtained from the corresponding formulas for resonant oscillations by introducing the replacement

$$\mathrm{tg}\,2\theta_0\rightarrow\alpha\equiv2\mu_{12}B_\perp/(\Delta m^2/2E)$$

(for small $\theta_0$). Actually, it is clear from (11) that the parameter $\alpha$ plays the role of the tangent of twice the vacuum mixing angle for NSFP. The above replacement transforms the adiabatic parameter for the MSW effect, $\chi_{\rm osc}$, into the parameter $\chi$ given by (20), and the expression given by (24) into the expression given by (23).

The important difference between resonant precession and resonant oscillations of neutrinos is that $\alpha$ depends on the neutrino energy, whereas the angle $\theta_0$ does not. The consequence of this is that the adiabatic parameter $\chi$ and the corresponding parameter for resonant oscillations

$$\chi_{\rm osc}\,(\cos 2\theta_0/\sin^2 2\theta_0)(2E/\Delta m^2)L_\rho^{-1}$$

have opposite dependence on the neutrino energy. In the present case, resonant precession should suppress the high-energy part of the $\nu_{eL}$ spectrum to a greater extent than the low energy part, and the opposite phenomenon is expected for the MSW effect.

9.2. The above difference between the energy dependence of the suppression factors for the $\nu_{eL}$ flux could, in principle, be exploited for the experimental identification of resonant precession. However, strictly speaking, the difference occurs only in the case of a uniform (or almost uniform) magnetic field. The change in neutrino energy is accompanied by a change in the resonant density, i.e., in the position $r_0$ of the resonance inside the Sun. In a nonuniform magnetic field, there is also a change in the resonant value of the magnetic field $B_{\perp 0}$ and, conseqently, in the adiabatic parameter $\chi$. It follows that the resultant energy dependence of the parameter $\chi$ is relatively complicated and is determined by the coordinate dependence of the field $B_\perp$. There is a class of functions $B_\perp(r)$ for which the distortion of the solar neutrino spectrum by the NSFP effect is almost the same as that due to the MSW effect. For such magnetic field configurations, it will be very difficult or impossible to distinguish resonant precession from resonant oscillations by comparing Cl–Ar and Ga–Ge data, or even by measuring the neutrino spectra. Qualitative differences between the energy dependences of the factors for these two effects will arise only when $B_\perp^2(r)$ falls more rapidly as a function of than $\rho(r)$ [see (20) and (14)].

It is easier to distinguish resonant NSFP from ordinary NSP than from the MSW effect because precession without change of flavor does not, in general, depend on the neutrino energy. In principle, we could have a situation in which $B_\perp(r) \propto [\rho(r)]^{1/2}$ in the Sun. The $\nu_{eL}$ flux suppression factor due to NSFP will then also be independent of the neutrino energy. Although this possibility has a low probability, it cannot be excluded *a priori*.

9.3. However, there is a range of parameter values for this problem for which resonant NSFP can be unambiguously distinguished from the MSW effect and from ordinary NSP. When $\Delta m^2$ lies in the range $2\cdot 10^{-8} \lesssim \Delta m^2 \lesssim 2\cdot 10^{-7}$ $eV^2$ and $\beta \gtrsim 10^4$ or $10^5$ (the first and last values refer to the lower and upper limits of $\Delta m^2$, respectively), a large fraction of the boron and beryllium neutrinos will be in resonance in the convective zone of the Sun, whose magnetic field is subject to the 11-year variation. The $\nu_{eL}$ count rate in the Cl–Ar experiment should then exhibit the 11-year and semiannual variations. At the same time, all the pp-neutrinos, which have much lower energies, will be in resonance in the central part of the Sun, in which the field is frozen. For this reason, the $\nu_{eL}$ count rate in the Ga–Ge experiment should not undergo large temporal variations. Essentially, in the range of parameters that we are considering, resonant NSFP should lead to time-dependent variations in the shape of the solar neutrino spectrum. In this it differs from the MSW effect (for which there are no reasons to expect time variations in the $\nu_{eL}$ fluxes or spectra) and from the usual NSP (for whch 11-year variations should be observed in both Cl–Ar and Ga–Ge experiments).

9.4. There are also other possibilities whereby resonant NSFP could be distinguished from the MSW effect. Thus, if $\Delta m^2$ lies in the range $10^{-6}$–$5\times 10^{-6}$ $eV^2$, and the vacuum mixing angle is not too small ($\sin^2 2\theta_0 \gtrsim 0.1$), resonant oscillations should give rise to partial $\nu_{eL}$ regeneration in the material of the Earth.[47–49] This would lead to both diurnal and seasonal variations in the recorded solar neutrino flux. Such variations should be absent if the solar neutrino problem is due to NSP (ordinary or flavor).

9.5. In the case of neutrinos with Majorana masses, NSFP transforms $\nu_{eL}$ into $\nu_{\mu R}^c$ or $\nu_{\tau R}^c$, and resonant oscillations into $\nu_{\mu L}$ or $\nu_{\tau L}$. The result of conversion is thus the formation of left-handed neutrinos in one case and right-handed antineutrinos in the other. Both are active and can be detected in processes due to neutral currents. Unfortunately, they cannot be distinguished from one another. In $\nu d \to np\nu$ and $\nu A \to \nu A$ processes, the cross sections for $\nu$ and $\nu^c$ are practically the same in the energy range $E \lesssim 50$ MeV. This general rule applies to low-energy neutrino scattering. At first sight, the problem could be solved by using $\nu e$-scattering because the mass of the electron is small in comparison with the characteristic energy of the solar neutrinos. However, in the standard model and for $\sin^2\theta_W = \frac{1}{4}$, the $\nu_j e$ and $\nu_j^c e$ ($j \neq e$) scattering cross sections are exactly equal. For $\sin^2\theta_W \approx 0.22$, the difference between the cross sections is about 15% for energies $E \sim 10$ MeV. It is difficult to believe that such a small difference could be detected in solar neutrino experiments.

Reactions due to neutral currents can be used to distinguish between resonant NSFP and the MSW effect only if one of these phenomena transforms $\nu_{eL}$ into active neutrinos and the other into sterile neutrinos. This will occur when (1) the neutrinos have only Dirac masses and (2) they have both Dirac and Majorana masses, and oscillations into sterile states ($\nu_{eL} \to \nu_{eL}^c$ ($\nu_{\mu L}^c, \nu_{\tau L}^c, \ldots$). are resonantly enhanced. However, this would require a knowledge of the origin of the neutrino masses (Dirac or Majorana) that we do not have at present.

9.6. We have assumed in this paper that neutrino mixing is small and can be neglected. Strictly speaking, this is not inconsistent with sufficiently large off-diagonal neutrino magnetic moments because there is no direct connection between the off-diagonal moments and the off-diagonal masses. The latter can always be cancelled by suitable counterterms. We shall not consider here the question of naturalness which then arises. We merely note that the assumption $\theta_0 \approx 0$ is made for simplicity and is not a necessary condition for resonant NSFP. Our analysis is valid when $\theta_0 \ll \alpha$.

9.7. The resonant precession that we have examined in this paper can also play an important role in the dynamics of supernovas and in the early stages of the evolution of the Universe. If the neutrinos are Majorana particles, the presence of large off-diagonal magnetic moments of these particles would not be inconsistent with the observed abundance of $^4$He because the number of neutrino species is not then doubled (there are no sterile components).

9.8. In conclusion, we make a few remarks with regard to ordinary NSP (without change of flavor). For NSP to solve the problem of solar neutrinos and to explain the observed anticorrelation between their count rate and solar activity, we would have to satisify (8), obtained by replacing $B_\perp(r)$ with a field averaged over the depth of the convective zone.[12,13] However, it has been shown[17,18] that these conditions are not, in general, sufficient because field (and matter) nonuniformity effects are very significant for NSP. Let us examine this in further detail. The natural states of the neutrino Hamiltonian in the field $B_\perp$ in the presence of matter and for ordinary precession are given by (10) in which $\nu_{2R}$ is replaced with $\nu_{eR}$ and $\tilde{\theta}$ with the mixing angle $\theta$. In a uniform field and in uniform matter, this angle is given by

$$\operatorname{tg} 2\theta = \frac{2\mu_{11}B_\perp}{c_L} = \frac{2\mu_{11}B_\perp}{\sqrt{2}\,G_F(N_e - N_n/2)}. \tag{29}$$

At the center of the Sun, and if $B_\perp$ is not too large ($\mu_{11}B_\perp(0) \lesssim 10^{-12}$ eV), we have $\theta \approx 0$. When the field is strong and the density of matter is low (for example, in the convective zone), we have strong mixing of $\nu_{eL}$ and $\nu_{eR}$: $\theta \approx \pi/4$ (but always less than this value). At large distances from the center of the Sun, both matter density and the magnetic field fall rapidly and, if the field falls more rapidly than the density (or they decline according to the same law, but their numerical ratio is such that tg $2\theta \ll 1$), then $\theta \to 0$. This means that the mixing angle at first increases along the neutrino trajectory from $\theta \simeq 0$ to values close to $\pi/4$, and then returns to $\theta \simeq 0$. This distinguishes ordinary NSP from NSFP, for which $\tilde{\theta}$ varies from 0 to $\pi/2$. As a result, NSP will be suppressed in the central part of the Sun. It will occur with amplitude approaching unity in the region of the strong field, and will then become again very small at larger distances from this region. If the adiabatic condition is satisfied at the same time, about half the $\nu_{eL}$ will be converted into $\nu_{eR}$ in the strong-field region, but they will reconvert to $\nu_{eL}$ on leaving this region, i.e., a practically pure $\nu_{eL}$ beam will be present at both entry and exit. This means that ordinary NSP can solve the solar neutrino problem, but only if there is a departure from adiabaticity. This reduces to the requirement that

$$\frac{1}{\mu_{11}B_\perp} \gtrsim L_\theta \approx \frac{L_B L_\rho}{|L_B - L_\rho|} \approx \min\{L_B, L_\rho\}. \qquad (30)$$

It is reasonable to suppose that $L_B \gtrsim L_\rho \sim R_\odot/10$ (Sec. 6), from which we obtain the following upper limit for the magnetic field:

$$\mu_{11}B_\perp L_{\rm conv} \lesssim 3.$$

The lower limit follows from (8):

$$\mu_{11}B_\perp L_{\rm conv} \gtrsim 1.$$

We therefore conclude that ordinary NSP can be effective on the Sun only if the parameter $\mu_{11}B_\perp$ lies in a sufficiently narrow corridor of values. Hence, it follows at once that, when $\mu_{11}(B_\perp)_{\max}L_{\rm conv} \gg 1$ at maximum solar activity, the few count-rate oscillations that would have been seen in the absence of the restriction defined by (30) (Refs. 9–13) will not be observed during the 11-year period. Instead, there should be two minima of the $\nu_{eL}$ flux corresponding to $\mu_{11}B_\perp \sim (1\text{–}3)L_{\rm conv}^{-1}$: one before the activity peak and the other after this peak. The $\nu_{eL}$ flux should be practically free from suppression at the activity maximum itself.

It is interesting to note that matter can resonantly enhance ordinary precession (as in the case of NSFP) if $N_e \approx N_n/2$ [see (29)]. For stable nuclei, this relation cannot be satisfied but, in the neutrino-depleted matter of collapsing stars, resonant amplification of ordinary precession is possible, in principle.[18,19] It is shown in Ref. 29 that resonant amplification of NSP in collapsing stars removes the restriction on the magnetic moment $\mu_{ee}$, obtained by analyzing the neutrino signal from the supernova SN 1987A (however, see Ref. 30, in which it is shown that the $\nu_e\nu_e$ interaction can modify this conclusion).

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[1)]NSP in transverse magnetic fields was previously examined in relation to the Sun[14] and supernovas, neutron stars, and the interstellar medium.[15] However, neither the effects of the medium nor temporal variations in the neutrino fluxes were discussed in these papers.

[2)]We note that the parameter $\beta$ in (22) and thereafter is greater by an order of magnitude than the corresponding parameter $z$ introduced in Refs. 17 and 18, because, in these papes, $\mu_{12}$ was normalized to $10^{-10}\,\mu_B$.


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