Transverse electron focusing spectroscopy of the electron--phonon interaction

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It is shown theoretically that a new method for the spectroscopy of the electron--phonon interaction in metals can be based on transverse focusing of nonequilibrium electrons [V. S. Tsoi, JETP Lett. 19, 70 (1974)] that have experienced strong electron--phonon relaxation in the dirty vicinity of an emitter contact. The results are used to explain recent experiments that have revealed the phonon structure of a transverse focusing line [P. C. van Son, H. van Kempen, and P. Wyder, Phys. Rev. Lett. 58, 1567 (1987); V. V. Andrievskii, E. I. Ass, and Yu. F. Komnik, JETP Lett. 47, 124 (1988)].

1. Introduction

The method of trajectory focusing of electron fluxes in metals subjected to reverse' and longitudinal magnetic fields is essentially ballistic and extremely sensitive to carrier relaxation. The use of point contacts for the injection of electron beams into a metal makes it possible to focus high-energy carriers with a controlled amount of the excess energy. These two circumstances determine the possibilities of the focusing method in studies of the processes of relaxation of “hot” electrons in conductors. Already the early experiments on longitudinal focusing have revealed nonlinear dependences of the signal on the emitter current, attributed to the energy dependence of the mean free path of carriers' and to the thermal heating of electron fluxes. The possibility of investigating the electron-phonon interaction (EPI) by the method of transverse electron focusing was first pointed out in Ref. 5 and the influence of weak EPI processes on the transverse electron focusing was investigated theoretically in Ref. 6. An investigation of transverse focusing of a spatially localized energy distribution of nonequilibrium carriers, formed as a result of relaxation of a primary monoenergetic beam, was described in Ref. 7. If the trajectory radius is energy-dependent, this distribution is transformed into a spatially inhomogeneous one which can be determined by investigating the dependence of the collector voltage on the applied magnetic field. However, this procedure meets with difficulties due to the extremely short mean free paths of the high-energy electrons in the usual metals (\( \langle \tau \rangle \approx 10^{-11}\) cm). In semimetals of the bismuth type the low density of states and the high permittivity increase considerably the mean free paths. This is clearly the reason for the observation of nonlinear behavior in the case of transverse electron focusing reported in Ref. 9.

In spite of the obvious attraction of the method of transverse focusing in investigations of inelastic relaxation of accelerated electrons, the first major progress has been made in studies of the intrinsic electrical conductivity of extremely small point contacts, less than the characteristic length of the electron energy loss. Point-contact spectroscopy is now recognized worldwide as a simple and effective method for investigating relaxation processes in solids.

Very recently new experiments were reported which have continued the investigation of the transverse focusing of “hot” electrons begun in Ref. 5. A reduction in the amplitude of the electron focusing signal as the emitter injection current increases in silver and a maximum of the focusing signal at excess electron energies equal to a characteristic phonon energy were reported in Ref. 13. Such a dependence of the amplitude of the electron focusing line on the emitter voltage was convincingly explained in Ref. 13 by the role of the electron--phonon relaxation destroying the ballistic nature of the cyclotron motion of electrons. The most interesting results were obtained recently in a study of effects nonlinear in the emitter current and observed in the case of transverse focusing of electrons in bismuth. Andrievskii et al. observed a reproducible structure of maxima in the derivative of the electron focusing signal with respect to the emitter current, which could be compared readily with multiple and combination frequencies of relaxation phonons in bismuth. These experiments demonstrated convincingly the potentialities of the electron focusing method in studies of phonon relaxation in metals and semimetals.

We shall develop a theory of relaxation phenomena in the course of transverse focusing of electrons in a metal. Our formulation of the problem is distinguished by the need to allow for a strong spatial inhomogeneity of the contamination, giving rise to a difference between the rates of relaxation processes occurring in the emitter and in the cyclotron trajectory in the interior of a metal. Experiments reported in Ref. 15 yielded directly the \( l_e \), representing elastic scattering of electrons in an emitter contact, and it was shown that this length was three orders of magnitude less than the corresponding length in a bulk metal. It therefore follows that the two parameters \( L/l_e \) and \( b/L = b/(l_e/L) \) represent the role of the relaxation processes along a trajectory and at a contact can be in an arbitrary ratio, giving rise to two types of relaxation nonlinearities. We shall consider separately these relaxation mechanisms and show that the effects reported in Ref. 13 are largely due to the trajectory relaxation of electrons, whereas the phonon structure of the electron focusing signal observed in Ref. 14 is associated with the discrete nature of the phonon relaxation of carriers directly in an emitter contact. The relationships obtained in the present study demonstrate that it should be possible to reconstruct the EPI function from the dependence of the electron focusing signal on the emitter current, which would provide the basis for a new cyclotron method for the speci-
troscopy of the EPI in conductors. A special feature of this method, based on detection of the contribution of small electron groups focused in the collector, is the high directionality which makes it possible to study relaxation of specific electron states.

In studies of nonlinear effects in transverse focusing of electrons the problem of the nature of nonequilibrium electrons injected by a point contact becomes of primary importance. Investigations of the geometric nonlinearity effect\textsuperscript{16,17} have shown that electrons injected into bismuth are characterized by a considerable (of the order of the Fermi energy $E_F$) excess energy, but this energy is several times (and sometimes by an order of magnitude) smaller than the maximum possible energy $eV$ ($V$ is the voltage across the emitter). On the other hand, there is no thermal broadening of the peak. Investigations of the Shubnikov oscillations of the emitter resistance\textsuperscript{16} have demonstrated directly the low degree of thermal heating of an electron gas in a junction. When these data are taken as a whole, we find that the processes of electron-phonon relaxation in an emitter may be very significant, whereas reabsorption of the phonons emitted by electrons in a contact is weak.\textsuperscript{11} Consequently, we shall consider a model in which the emitter is a dirty channel (Fig. 1):

$$L < b < 2d$$

(1)

($b$ is the channel diameter and $2d$ is its length), where the electron-phonon relaxation length $\lambda_p$ is arbitrary and there are no phonon reabsorption processes. The emission of phonons in this channel determines the profile of the energy dependence of the distribution function of carriers injected from the dirty channel into the pure metal. The electron focusing signal makes it possible to determine this electron energy distribution and to reconstruct the energy dependence of the inelastic relaxation length $\lambda_p$.

The hypothesis of a low transparency $D$ of a tunnel barrier ($D = l/2d$) in the plane of contact between two conductors (heterojunction structure\textsuperscript{2,5,9,14,15,17,18}) has made it possible to postulate an abrupt change in the electrical potential in the heterojunction plane and thus simplify greatly the mathematical analysis of the problem without distortion of the principal features of the effect.

A calculation of the electrical potential $\phi$ at a measuring contact $P$ as a function of the magnetic field $H$ and the voltage $V$ applied to an emitter will be made using the standard geometry of the transverse electron focusing effect (Fig. 2).

2. FORMULATION OF THE PROBLEM AND THE COMPLETE SYSTEM OF EQUATIONS

The complete system of equations describing the problem consists of the kinetic equation for the nonequilibrium electron distribution function $n(r,p)$

$$\frac{\partial n}{\partial \tau} + \frac{\epsilon E}{c} + \frac{e V}{c} \frac{\partial n}{\partial p} = I_n(n) + I_{\nu}(n)$$

(2)

and the electrical neutrality equation

$$\int dp [n(x,p) - f_0(x)] = 0.$$  

(3)

Here, $\epsilon, e, r$, and $p$ are the charge, coordinate, and momentum of an electron; $c(r,p)$ and $v_0$ are the energy and velocity; $f_0(x)$ is the Fermi distribution function; $E$ and $H$ are the intensities of electric and magnetic fields. The collision integral $I_n(n)$ describing the elastic scattering of electrons by impurities, assumed to be isotropic, is

$$I_n(n) = \frac{1}{2\pi^2} \int \frac{d\mathbf{q}}{q^2} W_{\nu}(n(p) - n(p')),$$

(4)

where $A = 1$ and $W_{\nu}(q/2\pi)$ is the square of the modulus of the matrix element of the elastic scattering, governing the transport elastic relaxation time:

$$\tau_{\nu}(x) = \frac{1}{2\pi^2} \int \frac{d\mathbf{q}}{q^2} W_{\nu}(1 - \cos(\mathbf{q} \cdot \mathbf{x})).$$

(5)

Inelastic EPI processes are included in the collision integral

$$I_{\nu}(n) = \sum_q \frac{1}{2\pi^2} \int \frac{d\mathbf{q}}{q^2} \epsilon^* [n(p+q) (1 - n(p)) (N\epsilon + 1)]$$

$$- n(p) [1 - n(p+q)] [N\epsilon + 1] n(p-q) n(p') n(p-q) [1 - n(p-q)] (N\epsilon + 1)$$

$$+ [n(p-q) [1 - n(p)] N\epsilon + 1] n(p-q) [1 - n(p-q)] (N\epsilon + 1)$$

$$+ [n(p-q) [1 - n(p)] N\epsilon + 1] n(p-q) [1 - n(p-q)] (N\epsilon + 1).$$

(6)

Here, the summation is carried out over the numbers $\alpha$ of the branches of the phonon spectrum; $\epsilon^*$ is the square of the modulus of the matrix element of the EPI. The system of equations (2)-(3) should generally be supplemented by the kinetic equation for the determination of the phonon distribution function $N_{\nu}^\alpha$ of phonons with the dispersion law $\omega_{\nu}^\alpha$. However, we shall consider only the case of low temperatures and assume that the phonons are in equilibrium [at $T = 0$ we have to substitute $N_{\nu}^\alpha = 0$ in Eq. (6)].
The distribution functions of electrons \( n_{<0} \) incident on the surface of a metal at \( z = r \), and of electrons reflected by this surface \( n_{>0} \) are related by the boundary condition
\[
\frac{d}{dr} \left[ p, \frac{d}{dr} \left( p, n_{>0} \right) \right] = \left( p, \frac{d}{dr} \left( p, n_{<0} \right) \right),
\] (7)
which automatically ensures that there is no flow of the current \( (q, \frac{d}{dr} \left( q, n \right) \) normal to the surface). The momentum \( p \) and \( \hat{p} \) satisfy the conditions of specular reflection, which conserve the energy of a carrier and of the projection of its momentum \( p, = \hat{p}, \) on a plane which is in contact with the surface at the point \( x = r \). The integral operator describes the diffuseness of the elastic processes of the scattering of electrons by the outer surface of a metal.

If the inequality (1) is satisfied, the function describing the distribution of electrons \( n \) in the emitter can be regarded as dependent only on one coordinate \( z \), and without violating the generality of the solution of the problem, the condition which must be satisfied by the function \( n(z, p) \) on the surface of a contact between the metals \( z = 0 \) can be written in the form
\[
n_{>0}(z=0, p) = \left[ 1 - \frac{1}{D(z=0, p)} \right] n_{<0}(z=0, p) + D n_{<0}(z=0, p),
\] (8)
where \( D(p) \) is the probability of electron tunneling across the heterojunction at \( z = 0 \). The momentum \( p \) is related to the momentum \( \tilde{p} \) of electrons incident on the \( z = 0 \) boundary and of electrons transmitted by this boundary \( \tilde{p} \) under conditions that the energy \( e(p) = e(\tilde{p}) = e(\hat{p}) \) and the component of the momentum \( p, = \hat{p}, \) tangential to the \( z = 0 \) boundary be conserved.

The problem can therefore be separated into two parts: 1) determination of the distribution function \( n(z, p) \) for a dirty point contact; 2) calculation of the distribution function of electrons \( n(z, p) \) in a pure metal, satisfying the effective boundary condition in the \( z = 0 \) plane:
\[
n_{<0}(z=0, p) = \left[ 1 - D(z=0, p) \right] n_{<0}(z=0, p) + D n_{>0}(z=0, p),
\] (9)
where
\[
\left[ 1 - D(z=0, p) \right] n_{<0}(z=0, p) + D n_{>0}(z=0, p) = n(z, p).
\]
It is a two-dimensional vector in the \( z = s \) plane; \( S \) is the set of vectors lying in the emitter plane \( (z = d, x^2 + y^2 = b^2/4) \).

Knowing the function \( n(z, p) \), we can use the electrical neutrality equation to calculate the distribution of the potential on the surface of a sample.

If a tunnel barrier is characterized by a low transparency \( D \langle 1, d, \rangle \), then the Boltzmann equation (2) can be linearized with respect to a small (proportional to \( D \)) correction \( f \) to the Fermi distribution function \( f_\text{F} \).
\[
f(z, p) = \left[ f_\text{F} - f(z, p) \right],
\] (10)
The function \( f(z, p) \) averaged over the direction of the momenta satisfies the equation
\[
f(z, p) = \int f(z, p) \frac{d^3 p}{4\pi^2} = n(z, p) + \frac{1}{4\pi^2} \int df(z, p).
\]
(11)
where \( f(z, p) \) is the modulus of the electron velocity.

The boundary conditions at the heterojunction \( z = 0 \) and at the boundary between pure and dirty metals \( z = d \) can be obtained from Eq. (8) and from the condition of continuity of the distribution function of \( n(z, p) \) in the \( z = d \) plane by using a diffuse expansion of Eq. (10) and averaging over the electron momenta. These boundary conditions are of the form (see also Ref. 15)
\[
l_i \frac{df}{dz} \bigg|_{z=0} = l_i \frac{df}{dz} \bigg|_{z=d} = 0.
\] (13)
Using the diffuse expansion of Eq. (10), we can easily show that the boundary condition given in Eq. (13) in fact means that the current (proportional to the transparency \( D \)) injected in the \( z = 0 \) plane is independent of the elastic scattering length of electrons \( l_i \). This condition is physically self-evident in the limit of low transparency of a tunnel barrier \( D \langle 1, d, \rangle \) assumed above. It should also be noted that the boundary condition (14) ensures continuity of the electric current at the boundary between pure and dirty metals if we allow for the fact that in the pure metal this current is governed by the function \( f_\text{F} \), whereas in the dirty region the current is expressed in terms of the gradient combination \( l_i \frac{df}{dz} \).

It is not possible to obtain the solution of the problem for an arbitrary relationship between the lengths of elastic and inelastic electron scattering, and also between them and the geometric dimensions \( b, d, \) and \( L \) describing the experimental setup. Therefore, we shall consider the most interesting limiting cases.

3. TRAJECTORY RELAXATION IN THE CASE OF TRANSVERSE ELECTRON FOCUSING
Let us assume that the length \( L = \langle 1, \rangle \), representing the inelastic relaxation of carriers in a dirty metal is...
considerably greater than the emitter channel length and the value of \( l_e \) is comparable with the distance \( L \) between the emitter and collector. In this case we can simplify Eq. (11) by dropping the electron-phonon collision integral and the solution of this equation satisfying the boundary conditions of Eqs. (13) and (14) is

\[
f_s(x, e) = D \left( \frac{1}{2} - \frac{d-x}{L} \right).
\]

If in the case of a pure metal the elastic mean free path is \( l_e \gg L \), then in the emitter-collector region the carriers move along ballistic trajectories, which may have kinks due to the electron-phonon interaction. The main contribution to the value of \( \Delta R_{max}(e) \) is made by the group of effective electrons representing a small fraction \( n_0 \) of all the emitter-injected carriers \( I_0 \). Since in our problem the nonequilibrium part \( f \) of the carrier distribution function is a "sharp" function of the momentum, and since we are planning later to analyze the amplitudes of the maxima of an electron focusing line, we can ignore the incoming terms in the collision integrals of Eqs. (4) and (6) and write down the solution of Eq. (2) for a pure metal in the form

\[
f(t, p) = f(\tau - t(\tau)) \exp \left( -\int_{\tau}^{t} \frac{\partial f_s}{\partial e} d\tau' \right) - q(t) \frac{\partial f_s}{\partial e}
\]

where

\[
r(t) = \int r(\tau) d\tau',
\]

is the time of motion along a trajectory in a magnetic field, and \( dL_d(p) \) is the time at which an electron is last reflected by the surface of a sample;

\[
\eta = \eta' = \eta_0 = \eta_0(p),
\]

\[
\eta_0(p) = \sum I \int d\xi (u_{\eta_0} \delta(w-w_{\eta_0})) f_\eta_0 \psi_\eta_0.
\]

\( F(\tau - t(\tau)) \) is an arbitrary function of the characteristic which is found using the boundary condition of Eq. (9) (see, for example, Ref. 22). Without analyzing the procedure for solving the equation of electrical neutrality and separating the nonmonotonic part \( \eta(\tau) \) of the dependence of the potential at the measuring contact on the magnetic field (see Refs. 23 and 24), which describes the electron focusing line, we give the final result:

\[
\delta(R_{max}, e) = \sum \int d\xi (u_{\eta_0} \delta(w-w_{\eta_0})) f_\eta_0 \psi_\eta_0.
\]

where

\[
A_{\eta}(e) = \frac{1}{\sqrt{\pi}} \left( \frac{\Delta R_{max}(e) \approx \delta} {\Delta R_{max}(e) \approx \delta} \right) \left( 1 + \frac{\delta^2}{\Delta R_{max}(e)} \right).
\]
erity effect which shifts the focusing line (when the polarity of the emitter voltage is such as to retard the electrons) and also gives rise to an additional term of the emitter voltage is such as to retard the electrons) and also gives rise to an additional term of the peak and occurs on the right-hand side of Eq. (21). We shall consider the specific case when \( eV = e\nu + \nu' \). It is not possible to be separated on the basis of the signal amplitude beyond the ballistic trajectory is unimportant, that the function \( v(\varepsilon) \) that describes the change in the amplitude of the electron focusing line on the shape of a distribution function is substituted in the expression for the partial amplitude and contains the EPI function \( g(w) \). It should be pointed out that the function \( v(\varepsilon) \) does not contain the geometric form of the distribution function are strongest at lower energies. Therefore, the dependence of the jump on its number is \( n \). The exact expression for the discontinuity \( \Delta_0 = \Delta(0) \) is fairly cumbersome [see Eq. (A6)]. However, it can be simplified when the inelastic relaxation length of electrons in the emitter \( \lambda(\varepsilon) \) depends weakly on the energy. If the characteristic change \( \Delta \rho \approx \rho(\Delta(\varepsilon)/d) \) in the relaxation length \( \lambda(\varepsilon) \) satisfies the inequality \( \Delta \rho = \Delta(0) \), the ratio of the discontinuities of the distribution function at \( z = d \) is given by the expression
\[
\frac{\Delta \rho}{\Delta \rho_{F}} = \frac{\chi(\Delta(\varepsilon)/d, \Delta(\varepsilon))}{\chi(\Delta \rho_{F}, \Delta \rho_{F})} \lambda(\varepsilon) \Delta \rho_{F}.
\]
(29)

It is clear from Eq. (29) that the energy structure of the distribution function depends strongly on the parameter \( d/A \). In the limit of weak electron-phonon relaxation the largest discontinuity of the distribution function is \( \Delta \rho_{F} \) and is due to the energy edge of tunnel injection at \( \varepsilon_{F} = \varepsilon_{F}^* + \nu' \). Successive phonon replicas of the discontinuity are proportional to powers of the parameter \( d/A \rho_{F} \) [see Eq. (29)], which reflect the low probability of the processes of successive emission of phonons in a channel. The most probable are one-phonon scattering processes discussed in subsection 1 of the present section. In the case of strong electron-phonon relaxation (\( d/A > 1 \)) the number of nonequilibrium electrons falls exponentially with energy, so that discontinuities of the distribution function are strongest at lower energies. Therefore, the dependence of the jump \( \lambda(\varepsilon) \) on its number is an important qualitative criterion of the intensity of the electron-phonon relaxation process at a point contact.

Substituting the expression for the function \( f \) at \( z = d \) into Eq. (18), and differentiating with respect to the voltage \( V \), we obtain
\[
\frac{\delta \phi}{\delta V} = \sum_{n=1}^{\infty} A_n(\varepsilon) \Delta_0 \frac{e}{eV - n\nu_0}.
\]
(30)

If \( b/L \omega_0/eV < eV/\nu_0 < 1 \), the maxima of the derivative of Eq. (30), which appear in fields defined by the relationship where \( v(\varepsilon) \) is the density of electron states on the \( \sigma(p) = \varepsilon \) surface. Such a model describes well the process of, for example, inelastic relaxation of carriers by interaction with optical and intervalley phonons in semimetals, because in these materials the interaction of electrons with phonons is "localized" in small regions of the momentum space \( \Delta \rho_{F} \approx \Delta(\varepsilon)/d \). In this connection we should mention that experimental studies of the nonlinear effects in the case of electron focusing reported in Refs. 3, 9, 14, and 17 were carried out on bismuth.

Since the maximum energy of the electrons injected in the channel is \( \varepsilon_{F} = \varepsilon_{F}^* + \nu' \), it follows that the distribution function \( f(\varepsilon) \) satisfies the obvious relationship \( f(\varepsilon > \varepsilon_{F}) = 0 \). On the other hand, if \( \varepsilon < \varepsilon_{F} \), we have \( f(\varepsilon) \neq 0 \). Therefore, at the boundary of the nonequilibrium band, \( \varepsilon = \varepsilon_{F} \), there is an abrupt change in the distribution function \( \Delta \rho = \Delta(\varepsilon) / d \). The magnitude of the change is calculated in the Appendix [see Eq. (A2)] where it is also shown that a discrete structure of the energy relaxation process results in reproduction of this discontinuity at energies \( \varepsilon_{0} = \varepsilon_{F} - n\nu_0 \). The exact expression for the discontinuity \( \Delta \rho = \Delta(\varepsilon) / d \) is fairly cumbersome [see Eq. (A6)]. However, it can be simplified when the inelastic relaxation length of electrons in the emitter \( \lambda(\varepsilon) \) depends weakly on the energy. If the characteristic change \( \Delta \rho = \rho(\Delta(\varepsilon)/d) \) in the relaxation length \( \lambda(\varepsilon) \) satisfies the inequality \( \Delta \rho = \Delta(0) \), the ratio of the discontinuities of the distribution function at \( z = d \) is given by the expression
\[
\frac{\Delta \rho}{\Delta \rho_{F}} = \frac{\chi(\Delta(\varepsilon)/d, \Delta(\varepsilon))}{\chi(\Delta \rho_{F}, \Delta \rho_{F})} \lambda(\varepsilon) \Delta \rho_{F}.
\]
(29)

It is clear from Eq. (29) that the energy structure of the distribution function depends strongly on the parameter \( d/A \). In the limit of weak electron-phonon relaxation the largest discontinuity of the distribution function is \( \Delta \rho_{F} \) and is due to the energy edge of tunnel injection at \( \varepsilon_{F} = \varepsilon_{F}^* + \nu' \). Successive phonon replicas of the discontinuity are proportional to powers of the parameter \( d/A \rho_{F} \) [see Eq. (29)], which reflect the low probability of the processes of successive emission of phonons in a channel. The most probable are one-phonon scattering processes discussed in subsection 1 of the present section. In the case of strong electron-phonon relaxation (\( d/A > 1 \)) the number of nonequilibrium electrons falls exponentially with energy, so that discontinuities of the distribution function are strongest at lower energies. Therefore, the dependence of the jump \( \lambda(\varepsilon) \) on its number is an important qualitative criterion of the intensity of the electron-phonon relaxation process at a point contact.

Substituting the expression for the function \( f \) at \( z = d \) into Eq. (18), and differentiating with respect to the voltage \( V \), we obtain
\[
\frac{\delta \phi}{\delta V} = \sum_{n=1}^{\infty} A_n(\varepsilon) \Delta_0 \frac{e}{eV - n\nu_0}.
\]
(30)

If \( b/L \omega_0/eV < eV/\nu_0 < 1 \), the maxima of the derivative of Eq. (30), which appear in fields defined by the relationship

\[
\Delta \rho = \Delta(\varepsilon) / d \approx \rho(\Delta(\varepsilon)/d) \lambda(\varepsilon) \Delta \rho_{F}.
\]
(29)
are separated on the magnetic field scale and their positions make it possible to find the phonon frequency \( \omega_0 \). The relationship (31) reflects the spatial separation of the cyclotron orbits corresponding to different values of \( E_i \) (trajectories 1 and 2 in Fig. 2). The existence of specific phonon relaxation energies is the result of discrete selection of the magnetic fields \( H_{\omega_j} \) ensuring focusing of the relevant electrons in the collector. The ratio of the amplitudes of the derivative of the focusing signal at neighboring minima in the range \( \omega_0 \ll \omega \) is

\[
\left( \frac{\partial \sigma}{\partial V} \right)_{n=\omega_i} / \left( \frac{\partial \sigma}{\partial V} \right)_{n=\omega_{i+1}} = \frac{\Delta \omega}{\Delta \omega_i}, \tag{32}
\]

In accordance with Eq. (29), the approximate equality (32) can provide direct information on the energy dependence of the inelastic electron scattering length \( \lambda_i \).

Generalization of the result (30) to the case of two phonon frequencies \( \omega_i \) and \( \omega_j \) presents no fundamental difficulties, but it does lead to very cumbersome analytic expressions. We can show that the derivative \( \partial \sigma / \partial V \) has extrema in fields \( H_{\omega_{ij}} \) which are related to the combination frequencies \( \omega_{ij} + \omega_{ij} \) by

\[
k\Delta H_{\omega_{ij}}(x, E, H_{\omega_{ij}}) = L_i, \tag{31}
\]

It is this system of maxima of the derivative with respect to the emitter current representing the electron focusing signal that was reported in bismuth in Ref. 14.

We now consider the case of an extremely strong inelastic relaxation of electrons in the channel, when the following inequality is satisfied:

\[
\lambda_i < b < c d. \tag{34}
\]

In this case the processes of multiphonon relaxation are concentrated in the direct vicinity of the tunnel injector at \( z = 0 \). Electrons crossing the tunnel barrier acquire abruptly an excess energy \( eV \), which directly after the passage of the barrier falls to a value \( \Delta \epsilon = eV - N\omega \), because of phonon emission. Consequently, all the electrons injected into a band of energies \( [\epsilon_i, \epsilon_f + eV] \) are concentrated in an energy interval \( [\epsilon_i, \epsilon_f + eV - N\omega] \) where the threshold electron–phonon relaxation mechanism is impossible. Further transport of charge in the channel occurs subject to conservation of the electron energy and is described by the diffusion equation \( \partial \sigma / \partial x = 0 \). The boundary condition at \( z = 0 \) describing multiphonon relaxation reflects conservation of a partial flux of electrons with the specific energy as a result of a relaxation transition accompanied by the emission of a phonon of frequency \( \omega_{ij} \) (Ref. 26). This condition, in combination with the injection condition of Eq. (14), leads to

\[
- \epsilon(x) \sigma(x) d \frac{\partial \sigma}{\partial x} = \sum_{n=1}^{N} \int \epsilon(x + eV - N\omega_n) d\epsilon \epsilon(x + eV - N\omega_n) - \epsilon(x), \tag{35}
\]

The boundary condition at the other end of the channel, where \( z = d \), is given by Eq. (13), exactly as before. The solution of the problem is a function

\[
\sigma(z) = \left[ \frac{1}{2} + \frac{z}{d} \right] \int \epsilon(x) \sigma(x) d\epsilon \epsilon(x + eV - N\omega_n) - \epsilon(x), \tag{36}
\]

Using this function in Eq. (18) for the potential on the measuring contact and differentiating \( \phi(V) \) with respect to the emitter voltage, we obtain

\[
\frac{\partial \sigma}{\partial V} = \sum_{n=1}^{N} \epsilon(x + eV - N\omega_n) d\epsilon \epsilon(x + eV - N\omega_n) - \epsilon(x) \frac{\partial \sigma}{\partial x} d\epsilon \epsilon(x + eV - N\omega_n) - \epsilon(x), \tag{37}
\]

The derivative in Eq. (37) is a maximum in fields \( H_{\omega_i}(\epsilon_f + eV - N\omega_i) \) satisfying Eq. (20) when \( \epsilon = \epsilon_i + eV - N\omega_i \). An increase in the voltage shifts the maximum of \( \partial \sigma / \partial \omega \) toward higher magnetic fields right up to the value \( V = V^* \), such that \( eV^* - N\omega_i = \omega_0 \). At \( V = V^* \) the maximum disappears in a magnetic field \( H_{\omega_i}(\epsilon_f + eV - N\omega_i) \) in the magnetic field appears. As \( V \) is increased, maxima appear at \( H = H_{\omega_i}(\epsilon_f + eV - N\omega_i) \) and disappear at \( \partial \sigma / \partial V \) periodically.

5. TRANSVERSE FOCUSING UNDER CONDITIONS OF TRAJECTORY AND EMITTER ELECTRON–PHONON RELAXATION

In the preceding section we demonstrated that the electron–phonon relaxation process in the emitter results in
periodic rise of the signal \( \frac{\partial p}{\partial V} \) associated with redistribution of the electron energies. On the other hand, allowance for the electron relaxation on a cyclotron trajectory of motion from the emitter to the collector reduces the value of \( \frac{\partial p}{\partial V} \) as the excess carrier energy \( eV \) increases. Therefore, we can expect the simultaneous action of these two factors to give rise to a nonmonotonic dependence of the electron focusing signal on the emitter voltage. We shall consider the limiting case when

\[ l_\lambda > d, \quad l_\rho \ll L. \]  

These inequalities correspond to strong relaxation along a ballistic trajectory (see Sec. 3) and also make it possible to allow for the relaxation process in the emitter using perturbation theory (see Sec. 4.1). Calculations fully analogous to those in Secs. 3 and 4 yield

\[ \frac{\partial q_{s,\alpha}}{\partial V} = \frac{1}{2} D(s_e + \varepsilon V) A_\alpha(s_e + \varepsilon V) \int \frac{d\omega}{\omega} A_\alpha(\omega) \frac{1}{2 l_\rho(s_e + \varepsilon V)} \int d\omega g(\omega) \right] \]

\[ + \frac{3}{4} \frac{D(s_e + \varepsilon V)}{l_\rho(s_e + \varepsilon V)} \int \frac{d\omega}{\omega} \frac{g(\omega)}{2 l_\rho(s_e + \varepsilon V - \omega)} A_\alpha(s_e + \varepsilon V - \omega), \]  

where the function \( A_\alpha(\omega) \) is described by Eq. (19). The relationship (39) can be simplified greatly in the case of metals characterized by \( eV \ll \omega_0 \) (it should be noted that the experiments reported in Ref. 13 were carried out using silver). The dominant dependence on the voltage \( V \) in \( \frac{\partial p_{s,\alpha}}{\partial V} \) is governed by the energy dependence of the electron-phonon relaxation time \( \tau_\rho \). This dependence can be separated explicitly by measurement of a normalized quantity \( (\frac{\partial p_{s,\alpha}}{\partial V})/(\frac{\partial p_{s,\alpha}}{\partial V})(0)/(\partial V)^{-1} \). If \( eV \ll \omega_0 \), we obtain

\[ \frac{\partial q_{s,\alpha}}{\partial V} \left( V \right) = \frac{\partial q_{s,\alpha}}{\partial V} \left( 0 \right) \]

\[ \approx \exp \left[ \frac{L}{\omega_0(eV)} \left( 1 - \frac{3}{2} \frac{d}{2 l_\rho(s_e + \varepsilon V)} \right) \right] \]

\[ + \frac{3}{2} \frac{d}{2 l_\rho(s_e + \varepsilon V)} \exp \left[ -\frac{L}{\omega_0(eV - \omega)} \right], \]  

where

\[ l_\rho(\omega) = \int_{\omega_0}^{\omega} \frac{d\omega}{\omega} G_{\rho,\omega}(\omega), \]

\[ l_\rho(\omega) = \int_{\omega_0}^{\omega} \frac{d\omega}{\omega} g(\omega), \]

and the EPI functions \( G_{\rho,\omega}(\omega) \) and \( g(\omega) \) are defined by Eqs. (22) and (12).

The interpolation relationship (40) yields the correct asymptotic expressions in the cases when \( eV < \omega_0 \) and \( eV < \omega_0 \), where \( \omega_0 \) is the characteristic phonon frequency at which the EPI function \( g(\omega) \) has a maximum. We then have

\[ \psi'(eV) = \begin{cases} eV, & eV < \omega_0, \\ eV - \omega_0, & eV > \omega_0. \end{cases} \]

In the range of intermediate values of \( eV \), the relationship (40) describes qualitatively the dependence \( \frac{\partial p_{s,\alpha}}{\partial V}(V)/\partial V \) and approaches the exact description as \( g(\omega) \) in Eq. (39) approaches the \( \delta \)-function form. Only two terms in Eq. (40) contain exponential factors which depend on the relaxation length at shifted energies \( l_\rho(\omega + eV(1 - \eta)) \) and \( l_\rho(\omega) \). If \( l_\rho(\omega) \) is a decreasing function of the energy, we reach the conclusion that in spite of the weakness of the relaxation in the emitter \( (l_\rho < 1) \), the second term may be responsible for the considerable contribution to the EPI signal in the range of excess energies \( eV > \omega_0 \), where the relaxation-induced suppression of the first term is significant. This contribution depends nonmonotonically on the emitter bias \( V \), increasing at low voltages \( (eV < \omega_0) \) and falling exponentially in the range of high values of \( V \) \( (eV > \omega_0) \). The energy dependence of the signal \( \frac{\partial p_{s,\alpha}}{\partial V}(V)/\partial V \) is shown qualitatively in Fig. 4. The dependence of the EPI signal \( \frac{\partial p_{s,\alpha}}{\partial V}(V)/\partial V \) obtained in this way is in good agreement with the experimental results of Ref. 13.

6. CONCLUSIONS

Investigations of the nonlinear electrical conductivity of metallic point contacts in the presence of a magnetic field extend greatly the opportunities for studying the EPI in metals. Our results show that the method of transverse focusing of electrons makes it possible to study the electron-phonon relaxation processes. Such relaxation of high-energy electrons reduces the focusing signal representing the scattering of carriers by phonons in the course of motion on a cyclotron orbit. It is possible to study the EPI for selected electron groups characterized by extremal dimensions of the cyclotron orbits [Eq. (21)]. If we compare these results with the potentials of point-contact spectroscopy in the absence of a magnetic field, we reach the

FIG. 4. Schematic representation of the dependence of the amplitude of the electron focusing signal \( \frac{\partial p_{s,\alpha}}{\partial V} \) at its maximum on the emitter voltage \( V \) in the case of relaxation of electrons by interaction with phonons in the emitter and along a ballistic motion trajectory.
conclusion that the method of transverse relaxation is highly directional and stresses the contribution of specific electron states to the relaxation process.

The ability to use the method of electron focusing by a magnetic field is important in the study of the electron-phonon relaxation processes which accompany successive emission of more than one phonon (multiphonon relaxation). At first sight this possibility is in conflict with the ballistic nature of the method (even a one-phonon process removes an electron from the number of those that reach the collector) and is realistic under conditions of spatial localization of the relaxation phenomena occurring in a strongly contaminated vicinity of the emitter contact. When one-phonon processes are important, the second derivative of the focusing signal with respect to the emitter voltage is related in a simple manner to the thermodynamic EPI function [Eq. (26)], which differs from the corresponding function for point contacts [1-4] by the absence of the transport form factor of the electron-phonon scattering process. This is due to the fact that the focusing method is not based on the transport effects, but on the distribution of the density of nonequilibrium electrons injected from the point contact. Our analysis shows that strong relaxation of electrons in the emitter is accompanied by successive emission of more than one phonon by "hot" electrons, giving rise to discontinuities in the energy distribution of carriers. Discrete relaxation by interaction with phonons gives rise to a system of maxima in the case of the first derivative of the transverse focusing line with respect to the voltage. Analysis of the resultant structure makes it possible to determine the energy dependence of the inelastic relaxation length and of the frequency of relaxation phonons in metals with low carrier concentrations.

Point electrical contacts, which are small strongly contaminated regions, allow transverse focusing of electrons by a magnetic field to be utilized as a method for investigating spatially localized strongly nonequilibrium states of quasi-particles in a solid.

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APPENDIX: DETERMINATION OF "PHONON JUMPS" OF THE DISTRIBUTION FUNCTION OF ELECTRONS IN THE EMITTER

In the case of relaxation due to interaction with dispersion-free phonons [Eq. (27)] the integrodifferential equation (11) is replaced by a system of differential-difference equations:

\[ \frac{\partial f}{\partial z}(x) = \lambda_{-} f(x) (1 - \delta_{0}) + \lambda_{+} f(x + v_{T} \Delta_{0}) = 0 \quad (A1) \]

where

\[ \lambda_{\pm} = 3a v (v \pm a v_{T}) \mp v_{T} (v \mp a v_{T}) N (v \mp a v_{T}), \]

and

\[ N = [v / a v_{T}] \] is the maximum number of phonons which an electron can emit.

The magnitude of the first jump \( \Delta_{0} = \bar{n} \) is the boundary of the distribution function can be found by direct solution of Eq. (A1) and the boundary of a band and, after allowance for the boundary conditions of Eqs. (13) and (14), this magnitude is given by

\[ \Delta_{0} = \int_{0}^{\Delta_{0}} D(\epsilon_{s} - 0, z) \]

where

\[ D(\epsilon_{s} - 0, z) = \frac{\lambda_{+}}{\lambda_{-} + \lambda_{+}} \]

and

\[ \lambda_{\pm}(x) = 3a v (v \pm a v_{T}) \mp v_{T} (v \mp a v_{T}) N (v \mp a v_{T}), \]

which makes it possible (as is readily shown) to write down the solution of the recurrence equation of Eq. (A3) at \( z = d \) in the form

\[ \Delta_{0} = \frac{\lambda_{+}(x)}{2 \lambda_{+}(x)} \prod_{n=1}^{N} \frac{1}{\lambda_{n}(x)}, \quad n=1, 2, \ldots, N. \]

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The coefficients $B^{(s)}$ satisfy algebraic relationships

$$\begin{align*}
B^{(s)}_{n+1} &= \alpha_{n} B^{(s)}_{n} + k \omega_{n} \frac{d}{d\omega} B^{(s)}_{n}, \\
B_{1}^{(s)} &= 1.
\end{align*}$$

(A7)

It follows from Eq. (A8) that if $\gamma$ is independent of energy, then $\alpha_{s} \rightarrow \alpha$, and if $E_{F} \ll \omega_{n}$, then in the functions which occur in Eq. (A6) we can ignore the dependence on the relaxation frequency $\omega_{n}$ of $\Delta_{n}(\epsilon) \equiv \Delta_{n}(\epsilon) = \Delta_{n}$ and $\Delta_{n}^{'}$, is given by the expression

$$\begin{align*}
\Delta_{n} &= \frac{D(\epsilon_{n})}{2 \chi(\epsilon_{n})} \frac{d}{d\epsilon(\epsilon_{n})} \left( \frac{d}{d \epsilon} \Theta(\epsilon_{n}) \right)^{2}, \\
\epsilon_{n} &= \epsilon_{n} + \epsilon_{F}.
\end{align*}$$

(A8)

The possibility of a "bottleneck" in the processes of reabsorption of nonequilibrium phonons in semimetals is discussed in Ref. 19.

As analysis shows that allowance for these terms corresponds to the contribution of "ineffective" electrons to the amplitude of the focusing signal. This contribution determines the monotonic component of the electron focusing signal depending weakly on the magnetic field.


V. G. Peschanskii, Conductive Electrons [in Russian], Nauka, Moscow (1985), Chap. XII.


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