Interference contribution to the current correlation function in a microjunction at finite voltages

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Interference contributions to nonequilibrium current noise are considered under conditions when the inelastic lengths are much larger than the characteristic geometric dimensions. Expressions are obtained for the quantum shot noise due to elastic scattering by impurities, and for the contribution due to correlation via the medium with allowance for phonons. Nonlinear effects at low voltages are discussed.

If a finite voltage V is applied to the banks of a microjunction of two normal metals, a nonequilibrium noise current is produced in the junction. It is manifested by the fact that the current-fluctuation spectrum includes, besides the Nyquist fluctuations present at $V=0$, additional fluctuations of the current I. The intensity of the latter

$$S_V(\omega)=\int\{\langle I(t)I(0)\rangle-\langle I(t)\rangle\langle I(0)\rangle\}e^{i\omega t}dt \qquad (1)$$

as $V\to 0$ is proportional to V^2 and has at the lowest frequencies a spectrum $S_V(\omega)\propto 1/\omega$ (the angle brackets denote thermodynamic mean values). The present paper deals with the nonequilibrium-noise spectrum at frequencies ω comparable with reciprocal of the time $\tau_f=L^2/D$, of diffusion through the microjunction, a time assumed to be much shorter than the characteristic inelastic-scattering times of τ_{in} of the electrons and phonons (L is the characteristic dimension of the microjunction and D is the electron diffusion coefficient). The interference contribution $S^i_V(\omega)$ to such a noise, for samples with effective dimensionalities $d_{eff}=1, 2$, and 3, were considered in the limit $V\to 0$ in Refs. 1–3, where it was shown that $S^i_V(\omega)\propto V^2$. We consider here the noise at finite voltages in the microjunction for $d_{eff}=0$. For the interference contribution that is present in the case of pure elastic scattering of electrons by impurities [1,2] [we designate it $S^{el}_V(\omega)$] it turns out that at $eV>kT$ the quadratic dependence on the voltage gives way to a linear one. For the contribution due to the correlation via the medium[3] we designate it $S^m_V(\omega)$] the quadratic dependence on the voltage is also replaced at $V>V_c=\hbar/e\tau_f$ by a linear one. To consider the correlation via the medium it is necessary to take explicit account of the inelastic processes. We consider only the interaction between the electrons and acoustic phonons, but the conclusion that $S^m_V(\omega)$ is linear in the voltage at $V>V_c$ remains apparently correct also when electron-electron interaction is taken into account. It is impossible to calculate $S^i_V(\omega)$ for a specific disposition of the impurities, and we shall therefore calculate its average $\overline{S^i_V}(\omega)$ over the impurity positions. At finite voltages it is convenient to use the Keldysh diagram technique,[4,5] regarding the junction as a constriction of length L and cross section $S\ll L^2$. In this technique, the Green's function has the matrix form

$$\hat{G}=\begin{pmatrix} G^R & G^K \\ 0 & G^A\end{pmatrix}.$$

Here

$$G^{R(A)}(1,2)=\mp i\theta(\pm t_2\mp t_1)\langle\psi(1)\psi^+(2)+\psi^+(2)\psi(1)\rangle,$$
$$G^K(1,2)=-i\langle\psi(1)\psi^+(2)-\psi^+(2)\psi(1)\rangle,$$

while ψ^+ and ψ are creation and annihilation operators in the Heisenberg representation. Under thermodynamic-equilibrium conditions we have

$$G_\varepsilon^K=(G_\varepsilon^R-G_\varepsilon^A)[1-2n(\varepsilon)],$$

where $n(\varepsilon)$ is the Fermi distribution function.

We take inelastic relaxation into account only in calculations of the current correlators, so that G satisfies the equation

$$[\varepsilon+(\hbar^2/2m)\nabla^2-U(\mathbf{r})-e\varphi(\mathbf{r})]\hat{G}_\varepsilon(\mathbf{r},\mathbf{r}')=\delta(\mathbf{r}-\mathbf{r}'),$$

where $\varphi(\mathbf{r})$ is the electrostatic potential and $U(\mathbf{r})$ is a random potential of the impurities, assumed to have a Gaussian distribution and

$$\overline{U(\mathbf{r})}=0,\quad \overline{U(\mathbf{r})U(\mathbf{r}')}=2\pi\nu\tau\delta(\mathbf{r}-\mathbf{r}').$$

Here ν is the one-electron state density and τ is the mean-free-path time. Calculating the average Green's functions G in the $\varepsilon_F\tau/\hbar\gg 1$ approximation, we obtain

$$\overline{G}_\varepsilon^{R(A)}(\mathbf{r},\mathbf{r}')=\int dp\exp[i\mathbf{p}(\mathbf{r}-\mathbf{r}')]\{\varepsilon-p^2/2m\pm i/2\tau\}^{-1},$$

and $\overline{G}^K_\varepsilon(\mathbf{r},\mathbf{r}')$ obeys the diffusion equation

$$D\nabla^2\overline{G}_\varepsilon^K(\mathbf{r},\mathbf{r})=0. \qquad (2)$$

Taking into account the boundary conditions in the junction banks we have

$$\overline{G}_\varepsilon^K(\mathbf{r},\mathbf{r})|_{x=0,L}=2\pi i\nu[1-2n(\varepsilon\pm eV/2)], \qquad (3)$$

where $V=\varphi(L)-\varphi(0)$ is the potential difference across the junction. The current is expressed in terms of the Green's function with the aid of the equation

$$I=-i\frac{e\hbar}{2m}\int d\varepsilon\int dS(\nabla-\nabla')G_\varepsilon'(\mathbf{r},\mathbf{r}')|_{\mathbf{r}=\mathbf{r}'}. \qquad (4)$$

Averaging (4) over the random-potential distribution and using (2) and (3), we obtain for the total current I through the junction the usual expression

$$\overline{I}=DS\nabla\int\overline{G}_\varepsilon^K(\mathbf{r},\mathbf{r})d\varepsilon$$
$$=\frac{e\nu DS}{L}\int_{-\infty}^{+\infty}\left\{n\left(\varepsilon-\frac{eV}{2}\right)-n\left(\varepsilon+\frac{eV}{2}\right)\right\}d\varepsilon. \qquad (5)$$

The diagram corresponding to expression (4) is shown in

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Fig. 1e. It differs from the analogous Feynman diagram in that the field vertex is replaced by a stub corresponding to $\overline{G}^K_\varepsilon(\mathbf{r},\mathbf{r})$. This correspondence remains in force also in the more complicated diagrams corresponding to different contributions to $S^i_V(\omega)$. The value of $S^m_V(\omega)$ is given in the principal approximation in $(p_F l)^{-1}$ by a sum of diagrams shown in Fig. 1. We confine ourselves to the first order in λ_{e-ph} ($\lambda_{e-ph} = g^2$, g is the electron-phonon interaction constant), assuming that $\omega \gg \tau_{\rm in}^{-1}$.

The hatched rectangles on the diagrams correspond to two-particle Green's functions

$$P_{\varepsilon_1-\varepsilon_2}(\mathbf{r},\mathbf{r}')=\frac{1}{\pi\nu\tau}\overline{\langle G_{\varepsilon_1}{}^R(\mathbf{r},\mathbf{r}')G_{\varepsilon_2}{}^A(\mathbf{r},\mathbf{r}')\rangle}. \qquad (6)$$

In the diffusion approximation, $P_\omega(\mathbf{r},\mathbf{r}')$ obeys the equation

$$\left\{D\nabla^2+i\frac{\omega}{\hbar}\right\}P_\omega(\mathbf{r},\mathbf{r}')=-\frac{\delta(\mathbf{r}-\mathbf{r}')}{\tau} \qquad (7)$$

with boundary conditions $P_\omega(\mathbf{r},\mathbf{r}')\,|_{x=0,L}=0$, $\partial P_\omega(\mathbf{r},\mathbf{r}')/\partial\mathbf{r}\,|_s=0$. The thick points correspond to current vertices and the wavy lines to the factors $i\lambda_{e-ph}\cdot D^K_\varepsilon(\mathbf{r},\mathbf{r}')$, where D^K_ε is the phonon Keldysh Green's function

$$D^K(\omega,\mathbf{Q})=\left[\frac{\omega^2(\mathbf{Q})}{\omega^2(\mathbf{Q})-(\omega-iO)^2}-\frac{\omega^2(\mathbf{Q})}{\omega^2(\mathbf{Q})-(\omega+iO)^2}\right]\operatorname{cth}\left(\frac{\omega}{2T}\right).$$

For acoustic phonons we have $\omega(Q)=u(Q)$, where u is the speed of sound. We assume that the elastic constants in the microjunction, in the dielectric interlayer, and in the bulk conductor differ little and the phonons can propagate without reflection from the microjunction.

Let us calculate, for example, the contribution from diagram 1a, a contribution corresponding at $kT\ll eV$ to the expression

$$S_V^{m(a)}(\omega)=\iint\limits_{-eV/2}^{eV/2}d\varepsilon\,d\varepsilon'\left(\frac{eD}{\hbar L^2}\right)^2\sum_{n,n'=1}^{\infty}\frac{\lambda_{e-ph}}{D\pi^2n^2/L^2-i(\varepsilon'-\varepsilon)}$$
$$\times\frac{\int D'(\omega,\mathbf{Q})I(n,n',\mathbf{Q})\,d\mathbf{Q}}{[D\pi^2n'^2/L^2-i(\varepsilon'-\varepsilon-\omega)][D\pi^2n'^2/L^2-i(\varepsilon-\varepsilon'+\omega)][D\pi^2n^2/L^2-i(\varepsilon-\varepsilon')]} \qquad (8)$$

The factor $I(n,n',\mathbf{Q})$ stems here from the fact that the two-particle functions $P_\omega(x,x';y,y';z,z')$ are defined in the volume

$$0\leqslant x,\ x'\leqslant L,\quad 0\leqslant y,\ y'\leqslant S^{1/2},\quad 0\leqslant z,\ z'\leqslant S^{1/2},$$

while the phonon Green's function is defined in all of space. So long as we are interested in the frequencies $\omega\ll u/L$, we can assume that $I(n,n',\mathbf{Q})=\delta_{nn'}$. The value of $I(n,n',\mathbf{Q})$ for frequencies $\omega\ll uS^{-1/2}$ can be determined from the integral

$$\frac{4}{L^2}\iint\limits_0^L dx\,dx'\int\exp[i(x-x')Qx]$$
$$\times\, d\Omega_Q\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{n'\pi}{L}x\right)\sin\left(\frac{n\pi}{L}x'\right)$$
$$\times\sin\left(\frac{n'\pi}{L}x'\right). \qquad (9)$$

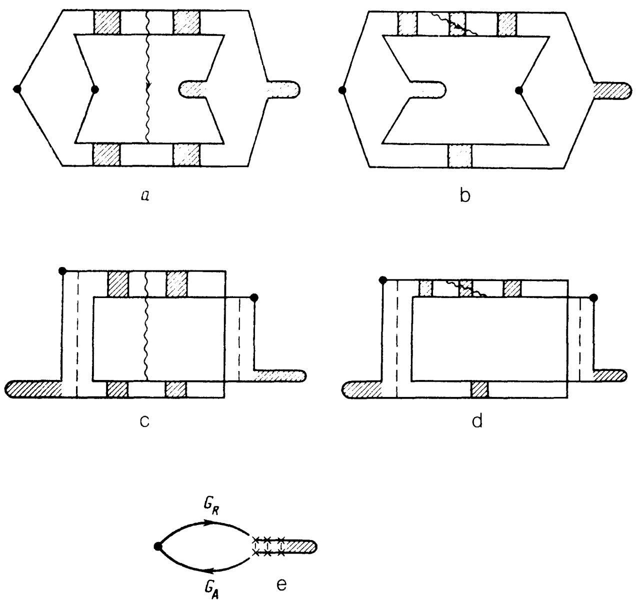

FIG. 1

We take, for the sake of argument, the microjunction to be a channel in a film. The integral with respect to $\mathbf{Q}$ in expression (8) is then two-dimensional and yields for $I(n,n',\mathbf{Q})=\delta_{nn'}$.

$$\frac{\omega^2}{2u^2S^{1/2}}\operatorname{cth}\left(\frac{\omega}{2T}\right). \qquad (10)$$

For the frequencies of interest to us the principal term in the sum over n will be the one with $n=1$; it remains now to calculate the integral over the energies in the expression

$$\iint\limits_{-eV/2}^{eV/2}d\varepsilon\,d\varepsilon'\left[\left(\frac{D\pi^2}{L^2}\right)^2+(\varepsilon-\varepsilon')^2\right]^{-1}\left[\left(\frac{D\pi^2}{L^2}\right)^2+(\varepsilon'-\varepsilon-\omega)^2\right]^{-1}. \qquad (11)$$

For $V\ll V_c$, the sum of all the diagrams yields

$$S_V{}^m(\omega)=8\frac{V^2}{V_c^2}\frac{e^2}{\hbar}\frac{D^2}{L^4}\frac{\lambda_{e-ph}\omega^2\operatorname{cth}(\omega/2T)}{S^{1/2}u^2(D^2\pi^4/L^4+\omega^2)}. \qquad (12)$$

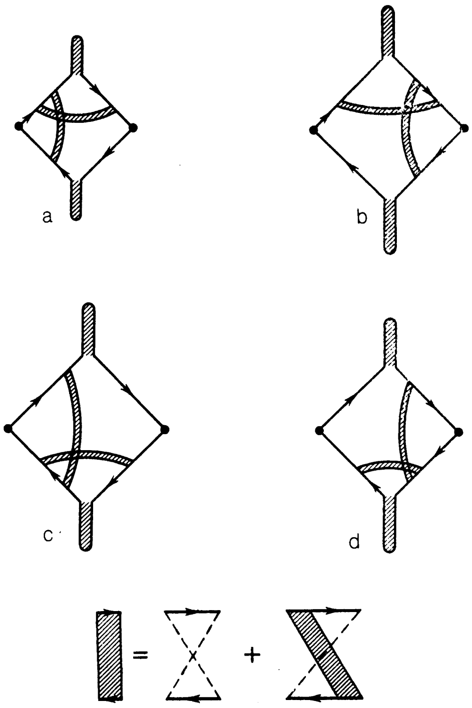

FIG. 2

For $V \gg V_c$ we have

$$S_V^{m}(\omega)=8\frac{V}{V_c}\frac{e^2}{\hbar}\frac{D^2}{L^4}\frac{\lambda_{e-ph}\omega^2\,\mathrm{cth}(\omega/2T)}{S^{1/2}u^2(4D^2\pi^4/L^4+\omega^2)}. \qquad (13)$$

We see that increasing the voltages changes not only the noise intensity but also the shape of the spectral line. Nothing like this occurs in another interference contribution so long as $\hbar\omega \ll \max(kT, eV)$. The value of $S_V^{el}(\omega)$ is given in the principal approximation in $(p_F l)^{-1}$ by the sum of the diagrams of Fig. 2 (Ref. 1).

At finite voltages $V \ll \hbar/e\tau$ and frequencies $\hbar\omega \ll \max(kT, eV)$ we have

$$S_V^{el}(\omega)=4T\left(\mathrm{cth}\frac{eV}{2T}\ln\left|\frac{e^{eV/2T}-1}{e^{-eV/2T}-1}\right|-1\right)$$
$$\times\frac{1}{(p_F L)^2R\tau^2[(D\pi^2/L^2)+\omega^2]}. \qquad (14)$$

Here $R = L/S\sigma_0$ is the microjunction resistance.

It can be stated in conclusion that the difference between the microjunction noise in the case $\tau_{in} \gg \tau_f$ and hence with $d_{eff} = 0$ and the cases $d_{eff} = 1, 2, 3$ is manifested by the fact that the contribution of the correlation via the medium turns out to be proportional to V^2 up to voltages V_c higher than in long samples. At the same time the considered contributions to the current correlation function $\langle\langle I(0)I(t)\rangle\rangle$ are more rapidly damped for longer times.

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[1] T. R. Kirkpatrick and J. R. Dorfman, Phys. Rev. Lett. **54**, 2631 (1985).
[2] R. O. Zaĭtsev, Zh. Eksp. Teor. Fiz. **90**, 1288 (1986) [Sov. Phys. JETP **63**, (1986)].
[3] V. V. Afonin and Yu. M. Gal'perin, *ibid.* **92**, 1875 (1987) [**65**, 1054 (1987)].
[4] L. V. Keldysh, *ibid.* **47**, 1515 (1964) [**20**, 1018 (1964)].
[5] A. I. Larkin and D. E. Khmel'nitskii, *ibid.* **91**, 1815 (1986) [**64**, 1075 (1986)].

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