Complete analytic results for radiative-recoil corrections to ground-state muonium hyperfine splitting

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Analytic expressions are obtained for radiative corrections to the hyperfine splitting related to the muon line. The corresponding contribution amounts to $(Z^2\alpha)(Z\alpha)(m/M)[\frac{9}{2}\zeta(3) - 3\pi^2 \ln 2 + 39/8]$ in units of the Fermi hyperfine splitting energy. A complete analytic result for all radiative-recoil corrections is also presented

1. INTRODUCTION

Muonium provides an ideal testing ground for comparison of predictions of quantum electrodynamics with experiment. It is in this respect superior even to the hydrogen atom since the muon, in contrast to the proton, has no strong interactions and this makes possible exact calculations. At the same time muonium measurements are carried out at the highest level of accuracy and experimental technique, with the results being continuously improved.[1]

Currently the best measurements are those of the hyperfine splitting of the muonium ground state,[2] requiring an improvement in the accuracy of its theoretical value. In the first place one needs to calculate radiative-recoil corrections (RRC), which are the biggest among the various currently unknown contributions. The simplest RRC are connected with the contributions of vacuum polarization to the exchange photons, and their analytic calculation, which has its own history, was completed quite some time ago.[3] The RRC, due to radiative corrections to the electron line, were found in Ref. 4 with the help of numerical integration, and the corresponding analytic formulas appeared in our work.[5,6] Below we give analytic results for the RRC connected with the muon line, thus completing the program of analytic evaluation of all RRC in the ground state of muonium.

In Sec. 2 we describe the set of diagrams with radiative insertions in the muon line, which contribute to RRC, and obtain the basic formula for their evaluation. In Sec. 3 a convenient representation is derived for the muon factor and its main properties are investigated. The evaluation of the RRC, connected with the muon line, is given in Sec. 4. In the Conclusion a complete analytic formula is obtained for all RRC. The Appendices contain some of the unwieldy formulas and calculations.

2. ANALYSIS OF THE RRC CONNECTED WITH THE MUON LINE

We study the RRC in the framework of the two-particle formalism proposed by Gross,[7] and developed by Dul'yan and Faustov[8] and Lepage.[9] It can be shown that all the RRC connected with the muon line are described by the gauge-invariant set of diagrams shown in Fig. 1. They differ from the corresponding diagrams of Ref. 6 only in the fact that now the radiative insertions are in the muon, and not electron, line. The diagram selection was carried out in the Fried-Yenni (FY) gauge[10,11] for the radiative (attached at both ends to the muon line) photons. In this gauge the individual graphs are softest in the infrared region,[12] which simplifies substantially the analysis of possible contributions to the hyperfine splitting. The ultraviolet divergences manifestly cancel, and therefore the mass and vertex operators in Fig. 1 are renormalized, and moreover the vertex had the anomalous magnetic moment subtracted out. The subtraction has to do with the fact that in the FY gauge the behavior of the anomalous magnetic moment at low momenta is less soft than the remaining form factors and requires separate treatment. Moreover it is not hard to convince oneself that it does not contribute at all to the RRC.

The matrix elements for the diagrams shown in Fig. 1 should be evaluated under standard conditions (SC) between large components of free electron and muon spinors. By SC is meant that all external momenta are on the mass shell and their spatial components vanish. The corresponding RRC is given by the matrix elements, calculated under SC, times the square of the Coulomb wave function of the Schroedinger equation with reduced mass evaluated at the origin. The set of diagrams thus obtained is gauge-invariant under SC, and in what follows we use for all photons (radiative and exchange) the Feynman gauge, in which the photon propagator has its simplest form.

The RRC connected with the muon line are described by the relation

$$\delta E = \frac{Z\alpha}{\pi}\frac{m}{M}E_F\left(-\frac{3}{8}\right)\int\frac{d^4k}{\pi^2 i}\frac{1}{(k^2-\sigma^2+i0)^2}\langle\gamma_\mu\hat{k}\gamma_\nu\rangle_e \cdot\left(\frac{1}{k^2+2(m/M)k_0+i0}+\frac{1}{k^2-2(m/M)k_0+i0}\right)L_{\mu\nu}(k), \quad (1)$$

where m and M are the masses of the electron and muon respectively, α is the fine structure constant, E_F is the Fermi hyperfine splitting energy, $L_{\mu\nu}$ is the muon factor, and the

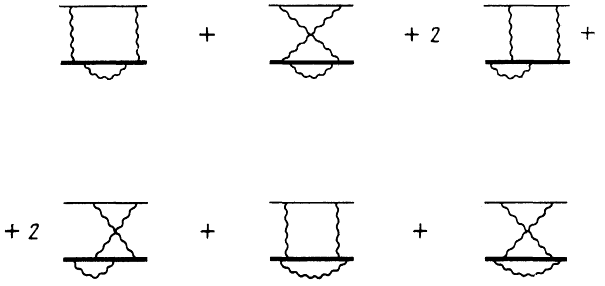

FIG. 1.

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angular brackets denote taking matrix elements between large components of free electron spinors. The mass σ of the exchanged photon, as well as its momentum k over which we are integrating, are made dimensionless using the muon mass. The constant Z, expressing the muon charge in terms of that of the electron, is equal to unity, but we keep it in the equations to indicate the origin of the various contributions.

3. THE MUON FACTOR

Substitution of the skeleton muon factor into Eq. (1) yields the leading logarithmic recoil correction, which contributes in the range of dimensionless momenta k between m/M and 1. The integrand in (1), with the muon factor ignored, diverges linearly at the lower limit, while the leading term in the skeleton muon factor is homogeneous in the momentum. Only its specific algebraic structure in this case decreases the degree of divergence to logarithmic.

Radiative corrections to the muon line enter expression (1) in the infrared regime, where they possess in the FY gauge additional softness compared to the skeleton.[12] This softness, combined with the fact that the characteristic momenta in the integral (1) are determined by the muon mass, ensures the absence of nonrecoil contributions. Such contributions are present in the case of radiative corrections to the electron line, where the momentum scale in the integral analogous to (1) is set by the mass of the electron.[6]

The algebraic structure of the muon factor, corresponding to the individual diagrams shown in Fig. 1, differs from the skeleton and its softness is just sufficient to make the integral in (1) logarithmically divergent, but it is not sufficient to achieve convergence. The divergence is cut off from below by the mass ratio m/M in the electron denominators of formula (1). The calculations are made substantially more complicated by the fact that the mass ratio cannot be ignored.

To overcome this difficulty we have obtained in the Feynman gauge an integral representation for the muon factors with insertions of the mass and vertex operators and the spanning photon in terms of the same Feynman variables (see Appendix 1). After summing the integrand expressions the full muon factor is brought to the form

$$L_{\mu\nu}=\frac{Z^2\alpha}{2\pi}\int\limits_0^1 dx\int\limits_0^x dy\Big[\langle\gamma_\mu\hat{k}\gamma_\nu\rangle\left(\frac{A_1}{\Delta}+\frac{k^2B_1}{\Delta^2}\right)+\langle\gamma_\mu\gamma_\nu\rangle\left(\frac{A_2}{\Delta}+\frac{k^2B_2}{\Delta^2}\right)+\langle\gamma_\mu\gamma_\nu\rangle k_0\left(\frac{A_3}{\Delta}+\frac{k^2B_3}{\Delta^2}\right)\Big],\qquad(2)$$

where

$$\Delta=-k^2y(1-y)+2k_0(1-x)y+x^2+\lambda^2(1-x)-i0\equiv y(1-y)(-k^2+2bk_0+a^2-i0),$$

x and y are Feynman parameters, λ is the dimensionless mass of the radiative photon, and the explicit expressions for the functions A_i and B_i are given in Table I.

The muon factor $L_{\mu\nu}$ describes the one-loop correction (with the contribution of the anomalous magnetic moment subtracted out) to the forward Compton scattering amplitude and is infrared-finite even for zero mass of the radiative photon. With the help of the representation (2) it is not hard to check directly the validity of the generalized low-energy theorem, which yields $L_{\mu\nu}(k)\sim O(k^2)$ at low momenta. In contrast to the usual low-energy theorem for Compton scattering the photon momentum in our case is off the mass shell, but on the other hand we consider only those spinor structures which contribute to hyperfine splitting. The integrals over the individual terms in expression (2), containing the funtions A_1 and B_i, are just as soft as the full factor $L_{\mu\nu}$. The behavior of the terms involving A_2 and A_3 is less soft, and it is only their sum that decreases like the square of the momentum.

The low-energy theorem indicates that the radiative corrections to the muon line, in contrast to the corrections to the electron line,[5,6] produce no contributions logarithmic in the mass ratio. Moreover, we may now omit in the expression for the energy shift (1) the mass ratio in the electron denominators. These have to do with corrections of the order of $(m/M)^2\alpha^2E_F$, in which we are not interested anyway, and the calculations with the electron mass left off are substantially simplified.

4. ANALYTIC CALCULATION OF THE RRC CONNECTED WITH THE MUON LINE

The actual calculation starts most conveniently with integration over the momentum in expression (1), rather than with the evaluation of the muon factor (2). The finite mass σ of the exchange photon in (1) is necessary to ensure convergence of that integral. The neglect of the electron mass, whose validity was demonstrated in the preceding section, means that we are considering radiative corrections to the scattering of a massless electron on a massive muon, regularized by the mass of the exchange photon. For technical reasons the role of the main infrared regularizer will be played below by the mass of the radiative photon, whose square is viewed as much larger than the mass of the exchange photon. We have explicitly verified that the results of the calculations are indeed independent of the relation between these masses.

To save on calculational labor it is convenient to combine the covariant (for $m=0$) denominators in formula (1) by means of the well-known trick

$$\frac{1}{(k^2-\sigma^2)^2k^2}=\int\limits_0^1 d\tau\frac{2\tau}{(k^2-\sigma^2\tau)^3}.\qquad(3)$$

All further calculations are carried out with the denominator $(k^2-\sigma^2\tau)^3$, the integration over the parameter τ being left for last. That integration is trivial, since the integrand expression turns out to be independent of the small mass of the exchanged photon as $\tilde{\sigma}^2\equiv\sigma^2\tau$ tends to zero.

We combine the denominators in expression (1) with the help of a new Feynman parameter z:

$$\frac{1}{(k^2-\tilde{\sigma}^2)^3}\frac{1}{\Delta}=-\frac{1}{y(1-y)}\int\limits_0^1 dz\frac{3(1-z)^2}{D^4},$$

$$\frac{1}{(k^2)^3}\frac{k^2}{\Delta^2}=\frac{1}{y^2(1-y)^2}\int\limits_0^1 dz\frac{6z(1-z)^2}{D^4},\qquad(4)$$

where $D=k^2-2k_0bz-a^2z-\tilde{\sigma}^2(1-z)$, and in the second relation $\tilde{\sigma}$ was set equal to zero since the corresponding mo-

TABLE I.

	A_1	A_2	A_3	B_1	B_2	B_3
Σ	$(1-x)^2\left[1-\frac{2y(1+x)}{x^2+\lambda^2(1-x)}\right]$	$1-x^2$	0	$y(x-2y)\left[1-\frac{x}{2}-\frac{2(1-x)(1+x/2)y}{x^2}\right]$	$\left(1+\frac{x}{2}\right)y(x-2y)$	0
2Λ	$-2(1-x)(x-y)+4\frac{y(1-x)(1-x-x^2/2)}{x^2+\lambda^2(1-x)}-2\frac{y(1-x)^2}{x}+2(1-x)(1-y)$	$-2(1-x)$	$2\frac{xy(1-x)^2}{x^2+\lambda^2}$	$(1-x^2)\frac{(x-2y)(x-y)y}{x^2}+4\frac{y^2(x-y)}{x^2}\left[\ln x+1-x+\frac{1-x^2}{4}\right]+\frac{(x-y)y^2(1-x)^2}{x^2}-2y\left[\ln x+1-x-\frac{y(x-y)}{x^2}\frac{1-x^2}{2}\right]$	0	$-\frac{(x-y)y^2(1-x)^2}{x^2}$
Ξ	$(x-y)(1-2y)-\frac{2}{x^2+\lambda^2(1-x)}\times\left(1-x-\frac{x^2}{2}\right)y-\frac{4y^2}{x}$	$2(x-y)(1-x)+\frac{2xy(1-x)}{x^2+\lambda^2(1-x)}+\frac{\lambda^2y(1-x)}{x^2+\lambda^2}$	0	$\frac{y^2(x-y)}{x^2}[-2+2x(1-y)-x^2]$	$(1-x)y\cdot(x-y)+\frac{2y^2(x-y)(1-x)}{x}$	0
T	$3\left(1-\frac{2xy}{x^2+\lambda^2}\right)-5x+3x^2+(8-4x)y+\left(2-\frac{4}{x}\right)y^2$	$(1-x)\left[-1+\frac{2xy}{x^2+\lambda^2(1-x)}+3x-2y+\frac{\lambda^2y}{x^2+\lambda^2}\right]$	$2\frac{xy(1-x)^2}{x^2+\lambda^2}$	$2y(\ln x+1-x)\times\left(\frac{2y}{x}-\frac{2y^2}{x^2}-1\right)+(1-x)y\left[1+x-\frac{y}{x}\left(4+3x\right)+\frac{y^2}{x^2}(5+3x)\right]+y\left[x\left(1-\frac{x}{2}\right)-2y-y^2+\frac{2y^3}{x}\right]$	$\left(2-\frac{x}{2}\right)xy-(1+2x)y^2-\frac{2(1-x)}{x}y^3$	$-\frac{(x-y)y^2(1-x)^2}{x^2}$

mentum integral is well-convergent. After integration over the momentum we obtain

$$\delta E=\frac{(Z^2\alpha)(Z\alpha)}{\pi^2}\frac{m}{M}E_F$$

$$\times\int_0^1 dx\int_0^x dy\int_0^1 dz\left\{-\frac{1}{2}\frac{c_1}{[z(a^2+b^2z)+\tilde{\sigma}^2(1-z)]^2}+\frac{c_2}{z(a^2+b^2z)+\tilde{\sigma}^2(1-z)}\right\}, \quad (5)$$

where

$$c_1=-\frac{3(1-z)^2}{y(1-y)}(-A_1b^2z^2+A_2bz+A_3b^2z^2)+\frac{6z(1-z)}{y^2(1-y)^2}(-B_1b^2z^2+B_2bz+B_3b^2z^2),$$

$$c_2=-\frac{3(1-z)^2}{y(1-y)}\left(-\frac{3}{4}A_1+\frac{1}{4}A_3\right)+\frac{6z(1-z)}{y^2(1-y)^2}\left(-\frac{3}{4}B_1+\frac{1}{4}B_3\right).$$

In view of the remark following formula (3) we omitted here the final integration over τ and the corresponding weight function.

The integration over z is carried out directly with the help of the formulas given in Appendix 2. We note that J_1 and J_2 contain only logarithmic infrared divergence, which is cut off by the mass of the exchanged photon, and the remaining integrals over z are infrared-finite.

It is still necessary to perform the parametric integral over x and y, which is so unwieldy that the full expression for it has been left to Appendix 3. The dependence on the mass of the exchanged photon disappears upon integration due to the remarkable identities

$$\int_0^1 dx\int_0^x dy\frac{A_1(x,y)}{a^2y(1-y)}=0,$$

$$\int_0^1 dx\int_0^x\frac{dy}{y(1-y)}\left(\frac{A_3}{a^2}-2\frac{bA_2}{a^4}\right)=0. \quad (6)$$

A careful inspection of the formulas of Appendix 3 shows that one of the expressions (6) always stands in front of the logarithm of the mass of the exchanged photon. Taking into account the difference from unity of the coefficient of $\tilde{\sigma}^2$ in the denominators of the right-hand side of expression (5) gives rise to corrections of order σ/λ relative to the integrals of the individual terms in (3.1). These integrals themselves contain up to linear infrared divergences, which are cut off by the mass λ of the radiative photon. By choosing values of the parameters such that $\sigma/\lambda^2\ll 1$, we avoid encountering any σ-dependent terms in the following. In evaluating the integrals of Appendix 3 it is important in the intermediate stages to retain the finite mass λ of the radiative photon, as is explained in Appendix 1. This finiteness is also needed for the validity of the identities (6).

The contributions to the hyperfine splitting, due to the various functions A_i and B_i, are collected in Table II. They were obtained as a result of extremely cumbersome and wearisome calculations, involving in particular the evaluation of many integrals over the standard functions $\mathscr{L}_i$. The weight functions in these integrals are often so singular that there are no simple algebraic relations between integrals with $\mathscr{L}_i$ and $\mathscr{L}_{i+1}$. Summing the results given in Table II we obtain the complete contribution to the hyperfine splitting energy, due to muon line radiative corrections:

$$\delta E=\frac{(Z^2\alpha)(Z\alpha)}{\pi^2}\frac{m}{M}E_F\left(\frac{9}{2}\zeta(3)-3\pi^2\ln 2+\frac{39}{8}\right), \quad (7)$$

where $\zeta(3)$ is the Riemann zeta function.

The expression in parentheses equals -1.0374, which is in beautiful agreement with the result of numerical integration,[4,13] -1.0372 ± 0.0091. The correction (7) equals 1.19 kHz, which is 7.5 times bigger than the error of the experimental data.[2]

TABLE II.

A_1	$-\frac{75}{4}\zeta(3)+\frac{15}{2}\pi^2\ln 2-\frac{145}{24}\pi^2+\frac{265}{8}$
A_2+A_3	$-\frac{11}{48}\pi^2+\frac{5}{4}$
B_1	$\frac{63}{4}\zeta(3)-\frac{15}{2}\pi^2\ln 2-\frac{109}{24}\pi^2+\frac{95}{4}$
B_2	$\frac{15}{8}\zeta(3)-\frac{3}{4}\pi^2\ln 2+\frac{23}{48}\pi^2-2$
B_3	$\frac{45}{8}\zeta(3)-\frac{9}{4}\pi^2\ln 2+\frac{5}{4}\pi^2-\frac{15}{4}$

5. CONCLUSION

Combining the above obtained result (7) with the contribution to RRC from vacuum polarization[3] and the insertions into the electron line[5,6] we obtain the complete analytic expression for all RRC:

$$\delta E_{rr}=\left(\frac{\alpha}{\pi}\right)^2\frac{m}{M}E_F\left(-2\ln^2\frac{M}{m}+\frac{13}{12}\ln\frac{M}{m}+\frac{21}{2}\zeta(3)+\frac{\pi^2}{6}+\frac{35}{9}+1.9(3)\right), \quad (8)$$

where we have taken into account the fact that for the muon $Z=1$, and where the last term in the round brackets arises from hadronic vacuum polarization.[3]

A typical answer for RRC consists usually of a linear combination of four characteristic terms: $\zeta(3)$, $\pi^2\ln 2$, π^2 and a simple fraction. As is seen from expression (7) in the muon line's RRC the term proportional to π^2 is absent, and in the complete RRC (8) the coefficient in front of the logarithm of 2 cancels out.

Formula (8) completes the program of analytic evaluation of order-$\alpha^2(m/M)E_F$ corrections to the hyperfine splitting. Numerically RRC (8) amounts to -3.6 kHz. From among the currently unknown contributions to the hyperfine splitting only the pure radiative correction of the form $\alpha^2(Z\alpha)E_F$ can reach a magnitude of the order of one kilohertz. Its evaluation is the next task of the theory.

APPENDIX 1

Integral representation for radiative insertions into muon line

The muon factor, corresponding to the sum of radiative corrections, has a softer behavior at low momenta of the exchanged photon than the contribution of each of the corrections individually. For this reason it is convenient for the proof of the low-energy theorem to sum up the contributions of radiative insertions into the muon factor prior to integration over the Feynman parameters. We obtain for them the representation (2), having expressed the contributions of individual diagrams in terms of the same Feynman parameters. The coefficients A_i and B_i are given in Table I, where Σ, Λ, Ξ and T denote the contributions from the mass operator, the vertex operator, the diagram with the spinning photon and their sum, respectively.

This integral representation for the sum was used in the actual calculations. In this connection we call particular attention to the need for accurate treatment of the infrared mass λ of the radiative photon. It usually enters the denominators of the expressions shown in Table I in the combination $x^2 + \lambda^2(1-x)$. The term proportional to λ^2 may be omitted in evaluating infrared-finite integrals of Sec. 4 with the functions B_i. The integrals with the functions A_1 and A_3 are logarithmically divergent, and the term with λ^2 must be kept, but one may ignore the fact that its coefficient differs from unity. Finally, integrals with the function A_2 are the most singular, even diverging linearly, and in this case it is necessary to take into account the difference from unity in the coefficient of λ^2, resulting in a finite contribution to the hyperfine splitting. The expressions for the functions A_i and B_i shown in Table I obtained in conjunction with the calculations of Sec. 4 and therefore contain the approximations just described.

APPENDIX 2

Standard integrals over *z*

The integration over z in expression (5) is performed directly and reduces to the evaluation of the following six simple standard integrals:

$$J_1 = \int_0^1 dz \frac{(1-z)^2}{(a^2+b^2z)z+\bar{\sigma}^2},$$

$$J_2 = \int_0^1 dz \frac{z(1-z)^2}{[(a^2+b^2z)z+\bar{\sigma}^2]^2} = -\frac{\partial}{\partial a^2} J_1.$$

TABLE III.

	$\mathscr{L}_{-1}$	$\mathscr{L}_0$	$\mathscr{L}_1$	$\mathscr{L}_2$
J_1	$\frac{1}{a^2}$	$-\frac{1}{a^2}-\frac{2}{b^2}$	$\frac{1}{b^2}$	0
J_2	$\frac{1}{a^4}$	$-\frac{1}{a^4}$	$-\frac{5}{a^2b^2}$	$-\frac{4}{a^4}$
J_3	0	$\frac{1}{b^2}$	$-\frac{1}{b^2}$	0
J_4	0	$-\frac{1}{b^4}$	$\frac{1}{a^2b^2}+\frac{2}{b^4}$	0
J_5	0	0	$\frac{1}{a^2b^2}$	0
J_6	0	$\frac{1}{b^4}$	$-\frac{2}{b^4}$	0

TABLE IV

	d_{-1}	d_0	d_1	d_2
A_1	$\frac{3}{4}$	$-\frac{3}{4}$	$\frac{1}{2}-\frac{9a^2}{4b^2}$	-2
A_2	$\frac{b}{2a^2}$	$-\frac{b}{2a^2}$	$-\frac{5}{2b}$	$-\frac{2b}{a^2}$
A_3	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}+\frac{3a^2}{4b^2}$	0
B_1	0	$\frac{1}{2}$	$\frac{1}{2}$	0
B_2	0	0	$\frac{b}{a^2}$	0
B_3	0	$\frac{1}{2}$	$-\frac{3}{2}$	0

$$J_3 = \int_0^1 dz \frac{1-z}{a^2+b^2z},$$

$$J_4 = \int_0^1 dz \frac{(1-z)^2}{(a^2+b^2z)^2}, \qquad \text{(A2.1)}$$

$$J_5 = \int_0^1 dz \frac{1-z}{(a^2+b^2z)^2} = -\frac{\partial}{\partial a^2} J_3,$$

$$J_6 = \int_0^1 dz \frac{z(1-z)}{(a^2+b^2z)^2} = -\frac{\partial}{\partial b^2} J_3.$$

All results are expressible in terms of the four standard functions:

$$\mathscr{L}_{-1} = \ln(a^2/\bar{\sigma}^2),\ \mathscr{L}_0 = \ln(1+b^2/a^2),$$
$$\mathscr{L}_1 = 1-(a^2/b^2)\mathscr{L}_0,\ \mathscr{L}_2 = {}^1/_2-(a^2/b^2)\mathscr{L}_1,$$

and are given in Table III.

APPENDIX 3

Two-dimensional integral for the energy shift

The contribution to the energy from the functions A_i and B_i takes the following form, after integration of expression (5) over the variable z:

$$\delta E_i = \frac{(Z^2\alpha)(Z\alpha)}{\pi^2}\frac{m}{M}E_F \cdot 3\int_0^1 dx \int_0^x dy \Big[\frac{d_k^{A_i}A_i}{y(1-y)a^2} - \frac{d_k^{B_i}B_i}{y^2(1-y)^2b^2}\Big]\mathscr{L}_k, \qquad \text{(A3.1)}$$

where $i = 1, 2, 3$; $k = -1, 0, 1, 2$; and the explicit formulas for the weight functions $d_k^{A_i(B_i)}$ are collected in Table IV.

The results of evaluating the corresponding integrals are shown in Table II, where a common dimensional factor $[(Z^2\alpha)(Z\alpha)/\pi^2](m/M)E_F$ was omitted.

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