Ferromagnetic film on the surface of a superconductor: possible onset of inhomogeneous magnetic ordering

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A system consisting of a bulky superconductor and a thin ferromagnetic metallic film on its surface is considered. It is shown that under certain conditions the magnetic ordering in the film takes the form of a small-scale domain structure. This ordering corresponds to a minimum system energy if account is taken of the interaction between the magnetic moments and the superconducting electrons that penetrate into the film from the bulky superconductor.

I. INTRODUCTION

The strong exchange field (exceeding as a rule by several orders the superconducting critical temperature $T_c$) present in a ferromagnet suppresses superconductivity via the paramagnetic effect. For the same reason, the superconducting order parameter is radically decreased near the contact of a superconductor with a ferromagnetic metal. The principal role is played here not by the proximity itself to the normal metal, but precisely by the presence of a strong exchange field in the ferromagnet and by the electron penetration, due to the proximity effect, into this region of the strong exchange field.

The influence of the exchange field on the superconductivity is greatly weakened when a small-scale (compared with the superconducting correlation length $\xi_0 = 0.18 \frac{\hbar}{\Delta E_T}$) domain structure is produced in the ferromagnet—the exchange field is effectively averaged over Cooper-pair dimensions. Precisely such domain coexistence phases were predicted for ferromagnetic superconductors, and the experimental data agreed with this prediction.

For this reason, inhomogeneous magnetic ordering, rather than ferromagnetism, should likewise occur in a thin ferromagnetic-metal film of thickness $d$ (on the order of several dozen Angstroms) sputtered on the surface of a superconductor, provided that the Curie temperature $T_C$ of the film is parallel to the plane of the film (if the easy axis is perpendicular to the film plane a small-scale domain structure exists even in the absence of superconductivity). We assume that the film can have two magnetic states—ferromagnetic without domains (or with domains very large compared with $\xi_0$), and one with small-scale domain structure, see Fig. 1. We designate these states by F and $F^{-}\Delta S$. The details of the $F-\Delta S$ transition in a ferromagnetic film on the surface of a superconductor depend on many parameters of the system: on the homogeneity of the film, on the conditions of the contact on the ferromagnet-superconductor interface, on the density of the electronic states in the film and in the bulk, and others. To illustrate the physics of the phenomenon, we confine ourselves therefore to a patently greatly simplified model. We assume that the film can have two magnetic states—ferromagnetic without domains (or with domains very large compared with $\xi_0$), and one with small-scale domain structure, see Fig. 1. We designate these states by $F$ and $\Delta S$.

The considered transition in a ferromagnetic film on the surface of a superconductor can be observed by magneto-optic methods or with the aid of tunnel measurements. The use of a ferromagnetic film as the weak-coupling link of a Josephson structure would make feasible a Josephson junction with variable coupling—the transparency should increase jumpwise when the film goes over into the domain phase.

2. BASIC EQUATIONS

We consider the case when a superconductor surface is coated with a thin ferromagnetic-metal film of thickness $d$ that is small compared with the superconducting correlation length $\xi_0$, and assume that the easy axis of the ferromagnet is parallel to the plane of the film (if the easy axis is perpendicular to the film plane a small-scale domain structure exists even in the absence of superconductivity). We assume that the film can have two magnetic states—ferromagnetic without domains (or with domains very large compared with $\xi_0$), and one with small-scale domain structure, see Fig. 1. We designate these states by $F$ and $\Delta S$.

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pared with the exchange-field influence, and will be neglected here.

For the ferromagnetic state $F$ in the film, the magnetic induction on the superconductivity is shown in Ref. 1. If the DS phase is produced, the superconductivity of region near the film, we can use in lieu of the superconducting order parameter (under the condition to include this case is easy.

The solution of the equation that follows from (3) for $\Delta(x)$ is well known:

$\Delta(x) = \Delta_0 e^{i(n/\eta)}$, where $\ell^2 = \eta/4m|\tau|$, and the constant $C$ is determined by the condition (2), viz $\sinh(2C) = 2^{1/2}/\ell^2$ (we have taken it into account that $d \ll \ell$).

The solution for $F$ phase is

$\Delta(x) = \Delta_0 e^{i(n/\eta)}$.

The condensation-energy loss (per unit surface area)

$$\delta_{\text{cond}} = \frac{\hbar v_F}{2m} \left( 1 - \frac{1}{1 + C} \right)$$

due to the suppression of the superconductivity amounts in this case to

$$\delta_{\text{cond}} = \frac{\hbar v_F}{2m} \left( 1 - \frac{1}{1 + C} \right)$$

(5)

If the DS phase is produced, the superconductivity of the interface is not so strongly suppressed (in view of the averaging of the exchange field), and the properties of the DS phase (period, exchange-field amplitude) are determined just by the parameter $\gamma$ in (2). The condensation-energy loss is decreased and, as can be easily seen, equals

$$\delta_{\text{cond}} = \frac{\hbar v_F}{2m} \left( 1 - \frac{1}{1 + C} \right)$$

(6)

where $C = -\theta / (\theta + C)$, $\theta = 2\theta / \eta |\tau|$.

A contribution to the total energy of a system with a DS phase is made also by the inhomogeneity of the magnetic structure in the film: the presence of a domain wall entails an energy loss $E_{\text{w}} = \theta n_{\text{M}} a/(b d)$, where $b$ is the magnetic thickness (half the DS-structure period), $a$ is the magnetic correlation length and is of the order of the interatomic distance, and $\theta$ is of the order of the Curie temperature (the electron and magnetic-atom densities are assumed to be approximately equal). The expression for $E_{\text{w}}$ indicates that the energy loss per magnetic atom in a domain wall of approximately atomic thickness is ~ $\theta$, i.e., the magnetic anisotropy is regarded as strong—in the case of weak magnetic anisotropy the domain-wall energy is somewhat lower and a generalization to include this case is easy.

The total system energy in the presence of a DS phase in the film can be written in the form

$$E_{\text{total}} \approx \theta n_{\text{M}} a / (b d) + \frac{\hbar v_F}{2m} \left( 1 - \frac{1}{1 + C} \right)$$

(7)

The total energy for the ferromagnetic state in the film is

$$E_s = \frac{\hbar v_F}{2m} \left( 1 - \frac{1}{1 + C} \right)$$

(8)

The period of the domain structure is determined from the condition $E_{\text{w}} / \theta n_{\text{M}} a / (b d) = 0$, and the condition for the transition from the ferromagnetic to the DS structure is the equality $E_{\text{total}} = E_s$. The parameter $C$ in (7) is determined by the boundary condition (2), where $\gamma$ depends on $b$.

A complete solution of the problem calls thus for finding the boundary condition, i.e., for calculating the parameter $\gamma$ as a function of $b$.

3. BOUNDARY CONDITION

Assuming the mean free path of the electrons in the ferromagnetic film to be small, we use the Usadel equations to describe the superconductivity in the DS-phase region ($0 < x < d$, see Fig. 1).

In an approximation linear in the function $F$ (the anom-
alous Green's function integrated over the energies and velocity directions on the Fermi surface), the Usadel equation in the presence of the exchange field $H$ is

$$G(x, y) F^-(x, y) = \Delta(x, y),$$

(9)

where $\omega = \sigma/(2n + 1)$, $D = \nu F / 3$ is the diffusion coefficient, and the approximation linear in $F$ is used because we are considering the temperature region near $T_c$.

Rapid oscillations of the exchange field in the DS phase along the $y$ coordinate cause a weak dependence of $\Delta$ on $y$, which can be neglected, i.e., $\Delta \approx \Delta(x)$. Using for $F(x, y)$ the representation

$$F(x, y) = F(x) + \sum_{k} F_k(x) e^{iky},$$

(10)

where $k = k_x = \sigma n \hbar$, $n = 1, 2, \ldots$, and using the condition $\nu / k_x \gg \hbar / T_c$, we obtain for $F_0(x)$ the equation

$$\left( \frac{\nu}{k_x} + \frac{1}{\tau_c} \right) F_0 - \frac{D}{2} \frac{d F_0}{dx} = \Delta(x),$$

(11)

where

$$\frac{1}{\tau_c} - \sum_{k} 2\hbar k_x^2 / D^2 = 0.5 \frac{M_0^2}{\nu F}. \tag{12}$$

As seen from (11) the action of a DS structure on the superconductivity is analogous to that of magnetic scattering with a reciprocal time $\tau_c^{\pm}$ given by Eq. (12). This result is general for dirty superconductors, see Ref. 3.

From Eq. (1) and from the self-consistency condition we find that the behavior of the superconducting order parameter in the film is described by the equation

$$\Delta(x) = \frac{T_c}{T} \left[ \psi \left( \frac{1}{2} + \frac{1}{4\Delta(x)} \left( \frac{\nu}{k_x} - D \frac{d}{dx} \right) \right) - \psi \left( \frac{1}{2} \right) \right] \Delta(x),$$

(13)

where $\psi(x)$ is a digamma function and it follows from (3) that the solution of interest to us, which satisfies the condition $\Delta(x) = 0$ on the interface with the vacuum, is of the form

$$\Delta(x) = ch \chi x, \quad \chi = 2\pi D / \tau_c. \tag{14}$$

The boundary condition (2) takes thus the form

$$\gamma = \frac{\Delta(x)}{\chi} \bigg|_{x=d} = \vartheta \left( \frac{\gamma}{\tau} \right) = \vartheta \left( \frac{\gamma}{\tau_c} \right),$$

(15)

**4. F-DS TRANSITION IN A MAGNETIC FILM**

We determine the equilibrium energy in the case of a DS phase in the film. From the condition $d\Delta(x)/d\hbar = 0$, taking into account in (7) the dependence of $C$ on the period $b$ of the structure [Eqs. (12) and (15)], we get

$$\frac{d\gamma}{\tau_c} = n - 2\pi n \vartheta (2 \hbar \gamma C). \tag{16}$$

Using this condition, we find that at the transition point ($\Delta = \Delta_\ast$) the constant $C$ is given by

$$\vartheta (2\hbar \gamma C) = 0.24, \tag{17}$$

and since $\sinh (2\hbar \gamma C) = 2^{1/2} / \gamma C$, we have

$$b = 2^{1/2} / \gamma C = 2^{1/2} / \gamma C.$$