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Excitation of plasma waves by an electromagnetic wave packet

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Nonlinear excitation of longitudinal Langmuir waves in a plasma by a short electromagnetic wave packet is considered. The possibility of accelerating particles by using fast plasma waves excited by a short laser pulse in a low-density plasma is discussed.

The feasibility of Čerenkov and transition radiation from a packet of electromagnetic waves was first discussed more than twenty years ago in Refs. 1 and 2. Čerenkov radiation from a femtosecond laser pulse was relatively recently recorded experimentally in a nonlinear electro-optical medium, and the pertinent theory is presented in Ref. 4.

No Čerenkov radiation is possible in an isotropic plasma, since the phase velocity of the transverse waves exceeds the speed of light. A wave packet, however, can emit longitudinal plasma waves. This question has attracted attention relatively recently in view of the development of new particle-acceleration methods. A computer experiment has shown that the efficiency with which plasma waves are excited by a short wave packet can be quite high.

The present paper is devoted to the theory of excitation of plasma Langmuir waves by a packet of electromagnetic radiation. We obtain the amplitude of the excited waves and the energy loss due to radiation. We obtain the amplitude of the excited waves and the energy loss due to radiation. The conditions under which the packet can be spread out by dispersion and diffraction are formulated. The possibility is discussed of using plasma waves excited by a laser pulse to accelerate electrons, as well as in diagnostic methods.

Conceivably, a phenomenon similar to the emission of plasmons by photons, considered in the present paper, is feasible also in other nonlinear material media.

1. ONE-DIMENSIONAL CASE: BASIC EQUATIONS

We consider first emission of plasma waves in the one-dimensional case. This approximation is justified if the length of the packet (in the direction of propagation) is much smaller than its cross section. This condition indeed holds in experiments with femtosecond laser pulses. A pulse of duration 10^{-14} s is 3 \mu m long, whereas its transverse dimension, determined by the focusing system, is usually tens or even hundreds of microns.

We describe the wave packet by using a system consisting of the Maxwell equations and the hydrodynamic equations for the electrons, disregarding the thermal motion and collisions of the latter. The plasma ions are assumed fixed.

From these equations we obtain for the electron momentum component \( p_x \), the electric field intensity \( E \), and the magnetic induction \( B \) all perpendicular to the \( X \) axis along which the packet propagates:

\[
\frac{\partial p_x}{\partial t} + v_x \frac{\partial p_x}{\partial x} = eE - \frac{e}{c} \rho B_x, \tag{1.1}
\]

\[
\frac{\partial E}{\partial t} = \frac{1}{c} \frac{\partial B}{\partial x}, \tag{1.2}
\]

\[
\frac{\partial B}{\partial t} = \frac{4\pi}{c} \frac{\partial \rho}{\partial x} + \frac{\partial}{\partial t} \left( \frac{m_n v_{xf}^2}{2} \right), \tag{1.3}
\]

where \( v_x \) and \( p_x \) are the transverse and longitudinal components of the electron velocity and are connected with \( p_x \) by the relation \( p_x = n_0 \left[ 1 - \left( c^2 + v_x^2 / c^2 \right)^{-1/2} \right] \), and \( n \) is the electron density. If the condition \( v_x \gg v_t \) is met, Eqs. (1.1)-(1.3) lead in the weakly relativistic case \( (c^2/v_x^2) \ll 1 \) to the following equation for \( v_x \):

\[
e^{2} \frac{\partial v_x}{\partial t} - \frac{e}{m} \frac{\partial E}{\partial x} - \omega_p^2 v_x = \frac{4\pi}{c} \rho n_0, \tag{1.4}
\]

where it is assumed that the electron density is equal to \( n_0 + \delta n \), \( n_0 \) is the density unperturbed by the packet, \( \delta n \) is the density perturbation due to the packet, and \( \omega_p = (4\pi\rho n_0/m)^{1/2} \) is the plasma frequency.

The terms in the right-hand side of (1.4) take into account nonlinear effects due to the relativistic transverse motion of the electrons, and also to the density perturbation \( \delta n \) produced by the packet. This perturbation is due to the electron motion along the packet propagation direction, and the corresponding system of equations takes the form

\[
\frac{\partial \delta n}{\partial t} + \frac{v_x}{c} \frac{\partial \delta n}{\partial x} = \frac{4\pi}{m} \frac{\partial \rho}{\partial x}, \tag{1.5}
\]

\[
\frac{\partial \rho}{\partial t} + v_x \frac{\partial \rho}{\partial x} = n_0 \frac{\partial v_x}{\partial t} + \frac{e}{m} \frac{\partial E}{\partial x} + \frac{\epsilon}{m} v_x B_x, \tag{1.6}
\]

\[
\frac{\partial E}{\partial t} = -4\pi e \epsilon \delta n, \tag{1.7}
\]

where \( \varphi \) is the charge-separation potential. In the weakly relativistic limit we obtain from (1.5)-(1.7) and (1.1) for small perturbations of the electron density:

\[
\frac{\partial \delta n}{\partial t} + v_x \frac{\partial \delta n}{\partial x} = n_0 \frac{\partial v_x}{\partial t} - \frac{e}{m} \frac{\partial E}{\partial x} - \frac{e}{m} v_x B_x. \tag{1.8}
\]

The system (1.4) and (1.8) determines the interrelated transverse and longitudinal motions of the electron in the wave packet.

The collisionless absorption, due to nonlinear forces, of the packet energy by a plasma is considered in a number of papers, (see, e.g., Ref. 8). We consider this question as applied to our statement of the problem.

In the nondissipative case, the total packet energy is, of course, conserved. This follows from the equations for the energy density, which follow from Eqs. (1.1)-(1.3) and (1.5)-(1.7)

\[
\frac{\partial \delta E}{\partial t} + \frac{\partial}{\partial x} \left( \delta E + B^2 \right) = -en_0 \varphi, \tag{1.9}
\]

\[
\frac{\partial \delta \varphi}{\partial t} + \frac{\delta v_x}{\partial t} \left( \frac{m_n v_{xf}^2}{2} \right) + \frac{\delta}{\partial x} \left( \frac{m_n v_{xf}^2}{2} + \frac{\delta E}{\partial x} \right) = -en_0 v_x. \tag{1.10}
\]
Adding (1.9) to (1.10) and using the continuity equation (1.6), we obtain on the right-hand side
\[ \frac{\partial}{\partial t} (\rho u_p) + \frac{\partial}{\partial x} (\rho u_p^2) = 0. \]
Thus, with allowance for the transverse and longitudinal particle motions, and also for the transverse and longitudinal fields, the equation for the total energy density is reduced to a divergent form, so that the energy-conservation law holds. Individually, however, the transverse and longitudinal energies are not conserved.

If we take the packet energy to mean only its transverse part, we can speak of energy loss. In our approximation, in which the density perturbations are small and relativistic effects are weak, we find from (2.9) that the transverse-energy density is
\[ \rho_\perp = \frac{m_e}{\rho_0} \frac{d}{d\xi} \left( \sin k_p (\xi - L) \right), \]
and its change per unit time amounts to
\[ \frac{\delta \rho_\perp}{\delta t} = \frac{\rho_0}{4 \nu_0} q = \frac{\rho_0}{4 \nu_0} \frac{d}{d\xi} \left( \sin k_p (\xi - L) \right). \]

2. EXCITATION OF PLASMA WAVES

Consider the excitation of plasma waves by a one-dimensional packet having a specified permanent form. The conditions under which this approximation is valid will be discussed below.

Assume that the transverse electron velocity is
\[ v_\perp(x,t) = \frac{1}{2} \left( \alpha (\xi) e^{-i \omega t} + \alpha^* (\xi) e^{i \omega t} \right), \]
where \( \omega \) and \( k \) are respectively the high carrier frequency and the wave number, which are connected by the dispersion relation \( \omega^2 = k^2 c^2 + \omega_0^2 \) (Ref. 7), and \( \alpha \) is a slowly varying envelope that depends on the combination of variables \( \xi = x - v_\perp t \), where \( v_\perp \) is the group velocity.

Substituting (2.1) in (1.8) we find that the packet produces high- and low-frequency density perturbations. The high-frequency perturbations occur at the second harmonic and if the plasma is transparent (for \( 2 \omega > \omega_p \)), they are localized only in the region of space where the packet is present:
\[ \delta n_\perp = \frac{m_e}{\sqrt{\pi}} \frac{1}{4 \nu_0^2} \left( \sin k_p (\xi - L) \right) \left( \begin{array}{c} 1 + \frac{2 \nu_0}{\pi} \end{array} \right), \]
where \( k_p = \omega_p / \nu_\perp \). According to (1.7), the perturbations of the potential are given by
\[ q = -\frac{m_e}{4 \nu_0^2} \int d\xi' \left( \sin k_p (\xi' - L) \right) \left( \begin{array}{c} 1 + \frac{2 \nu_0}{\pi} \end{array} \right). \]

Specifying the envelope \( \alpha (\xi) \), we can easily find with the aid of (2.3) and (2.4) the density and potential perturbations. By way of the simplest example, we consider a packet of rectangular shape:
\[ \alpha (\xi) = \begin{cases} 1 & \text{if } \xi > -L/2, \\ 0 & \text{otherwise}, \end{cases} \]
where \( \theta \) is the Heaviside step function.

From (2.3) we get
\[ \frac{\delta n_\perp}{n_0} = \begin{cases} \frac{2 \nu_0}{4 \nu_0} \left( \frac{2}{\pi} \right) & \text{if } \xi < -L/2, \\ \frac{2 \nu_0}{4 \nu_0} \left( \frac{2}{\pi} \right) & \text{if } \xi > L/2. \end{cases} \]

It can be seen from (2.5) that even inside the packet there exists, besides the terms that duplicate the form of the envelope, also a periodic perturbation of the density. What remains behind the packet is only this oscillating perturbation, and its amplitude depends on \( k_p L \). The maximum perturbations occur at \( k_p L = \pi (2l + 1) \), where \( l = 0,1,2, \ldots \).

The low-frequency perturbations of the density of the potential can therefore be plasma waves of length \( \lambda_p = 2 \pi v_\perp / \omega_0 \), and can exist outside the region within which the transverse field of the packet is localized. This makes emission of plasma waves by the packet meaningful.

The plasma-wave potential far behind the packet can be obtained from (2.4) without specifying the actual shape of the envelope
\[ q (\xi) = \varphi_0 \sin (k_p \xi + \varphi), \]
where \( \varphi_0 = (m \nu_\perp / \sqrt{\pi}) R \),
\[ R = -\left( \int \sin k_p (\xi - L) \right) \left( \frac{2}{\pi} \right), \]
\[ \varphi = \left( \int \sin k_p (\xi - L) \right) \left( \frac{2}{\pi} \right). \]

If the packet dimension \( L \) is small compared with the length of the excited plasma wave, then
\[ \varphi_0 = -\frac{m \nu_\perp}{4 \nu_0^2} \left( \sin k_p (\xi - L) \right). \]

The order of magnitude of the integral in (2.8) is \( L \nu_\perp^2 \), where \( \nu_\perp \) is the amplitude of the velocity of the oscillatory motion of the electrons in the transverse field. It can be seen that in the case of a small packet the electron energy in a longitudinal wave is approximately \( L \nu_\perp^2 \) times smaller than the average energy of the transverse oscillatory motion (called high-frequency potential) \( m \nu_\perp^2 / 4 \). If the packet dimensions are commensurate with the wavelength \( \lambda_p \), the potential energy of an electron in a plasma wave is of the same order as the high-frequency potential.
We consider one more example that permits a better understanding of the physical excitation of plasma waves, viz., a packet of Gaussian form

$$n(x) = \frac{a_0}{4\pi^{2/3}} \exp\left(-\frac{1}{2} \left| \frac{x}{L} \right|^2 \right),$$

where $L$ is the length of the packet. From (2.3) we get

$$\frac{\delta n}{n} = \frac{a_0}{4\pi^{2/3}} \exp\left(-\frac{1}{2} \left| \frac{x}{L} \right|^2 \right) \cdot \left( -\frac{1}{2} \frac{x}{L^2} \right) \cdot \frac{\partial}{\partial x} \exp\left(-\frac{1}{2} \left| \frac{x}{L} \right|^2 \right) + \ldots$$

(2.9)

where $\Phi$ is the probability integral. Figure 1 shows the function (2.9) at $k_pL = 1$. It can be seen that at this ratio of the plasma-wave and packet lengths the electron density increases in the region of the leading front and falls off at the trailing edge of the packet. This density distribution is due to the joint action exerted on the electrons by the high-frequency field and the charge-separation field. The high-frequency pressure forces on the leading and trailing edges of the packet are equal in size and opposite in direction. The charge separation field, on the contrary, is directed in the same direction at the packet location, and accelerates the electrons in the negative direction. This gives rise to a significant perturbation of the density behind the packet and to formation of a plasma wave.

3. ENERGY LOSS. DISTORTION OF PACKET SHAPE

The assumption that the shape of the packet remains unchanged is valid if the time during which energy is lost to plasma-wave emission and the packet shape changes is much longer than the plasma period (or takes place over a length greatly exceeding the plasma wavelength).

We consider first the energy loss and find the conditions under which it can be neglected. The expression given in Sec. 1 for $\delta W$, allows us to find the total energy loss per unit time:

$$\delta W = \int 4\pi \frac{dn}{n} \frac{\partial n}{\partial t} \left( \frac{d\phi}{dt} \right)^2.$$ 

With the aid of relations (1.8), (2.1), (2.3), and (2.6) this equation is transformed into

$$\delta W = \frac{m c^4}{2n_0} \left( \frac{d\phi}{dt} \right)^2 + \frac{1}{k_p^2} \left( \frac{d\phi}{dt} \right)^2 \frac{\partial}{\partial t},$$

(3.1)

Using the expression for the amplitude of the electric field in the plasma wave $E_p = k_p\phi$, we can represent Eq. (3.1) in the form $\delta W = (E_p^2/8\pi) \sigma_{\gamma_1}$. A packet propagating at the group velocity leaves a trail in the form of a plasma wave having an energy density $E_p^2/8\pi$.

We obtain the characteristic time during which the packet energy

$$W_0 = \frac{m c^4}{8\pi} \int d\phi \mid d\phi \mid^2$$

is altered by the emission of plasma waves by dividing $W_0$ by the (3.1):

$$T_{\text{rad}} = \frac{16c^6n_0^2}{u_p^2 k_p^2} \left( \frac{d\phi}{dt} \right)^2,$$

This leads to the following for a small packet:

$$T_{\text{rad}} = \frac{8n_0^2 m c^4}{u_p^2 k_p^2} \left( \frac{d\phi}{dt} \right)^2.$$

Note that the distance traversed by the packet before its energy is decreased by radiation of plasma waves is equal approximately to $T_{\text{rad}} = T_{\text{rad}} c$.

The condition $T_{\text{rad}} u_p = 1$ under which the radiation can be neglected is met if the right-hand side of (3.2) is large. This leads to a constraint on the packet energy:

$$W_0 = \frac{8n_0^2 m c^4}{u_p^2 k_p^2} \left( \frac{d\phi}{dt} \right)^2.$$

In addition to losing energy, the packet changes shape. This is due not only to spreading by the usual linear dispersion, but also to nonlinear effects. To take the packet shape change into account, we assume that the envelope in Eq. (2.1) is not only a function of the variable $\xi$, but also a slowly varying function of the time. Substituting (2.1) in (1.4) and using (2.2) and (2.3), we obtain the equation for the envelope:

$$2n_0 \frac{d^2}{dt^2} \left| \frac{d\phi}{dt} \right| - \frac{1}{k_p^2} \left( \frac{d\phi}{dt} \right)^2 = \frac{m c^4}{2n_0} \left| \frac{d\phi}{dt} \right|^2 \sin k_p(\xi - L).$$

(3.4)

Note that the first term in the right-hand side of (3.4) is the result of allowance for weak relativistic effects, and also for the high-frequency (second-harmonic) and low-frequency perturbations of the electron density.

The packet spreading due to linear dispersion can be easily shown, by comparing the first and second terms in the left-hand side of (3.4), to take place in a characteristic time $T_L$ and over a distance $L_L$:

$$T_L = 2n_0 \frac{d^2}{dt^2} \left| \frac{d\phi}{dt} \right|, L_L = 2 \left( \left| \frac{d\phi}{dt} \right| \right)^2 = \frac{1}{2n_0 \left( \frac{d\phi}{dt} \right)^2}.$$ 

(3.5)

The time of the nonlinear distortion of the packet shape depends on the ratio of the plasma wavelength $\lambda_p$ to the packet dimension $L$. For small packets the integral in the right-hand side of (3.4) is approximately equal to $\int_0^L \frac{d\phi}{dt}$, where $a_0$ is the maximum of the envelope. From a comparison of the linear terms it follows that the last is (Lau) times larger than the first and therefore determines the time and length of the nonlinear distortion of the packet:

$$T_a = a_0 = \frac{1}{k_p^2 L_p^2 u_p}, L_a = \frac{1}{2a_0^2 k_p^2 L_p^2 u_p}$$

(3.6)

Comparing (3.2) and (3.6) we conclude that the most-
substantial nonlinear effect is the change of the packet shape. To corroborate this conclusion we point out that to obtain the energy loss (3.1) it is necessary to retain in the left-hand side of (3.4) the small discarded term proportional to $\partial^2 U/\partial q^2$.

Let us formulate finally the conditions under which it is valid to assume a packet with given properties. If the packet energy is low ($W_0 < 2L_0 N_p n_c (k_p L_0)^{-4}$), the principal role is played by the spreading of the packet on account of linear dispersion. For this spreading to be negligible, the packet must be larger than $(c/\omega_p) (c_0/\omega_p)^{1/2}$. If the packet energy is high (the opposite inequality holds), the change of the packet shape by nonlinear effects is decisive, and this change can be neglected by satisfying the inequality $W_0 < 2L_0 N_p n_c (\omega/\omega_p)^{1/2} (k_p L_0)^{-4}$.

Thus, the packet spreads faster than it loses energy. It follows therefore that as the packet propagates and its initial shape is distorted, the excitation of plasma waves continues and the amplitude of the radiator waves changes and depends on the ratio of the packet size to $k_p$.

4. THREE-DIMENSIONAL PACKET

We consider in this section a more realistic problem—emission of plasma waves by a three-dimensional cylindrical-symmetry wave packet. We assume that the high-frequency pressure (or high-frequency potential) connected with the packet is characterized by longitudinal ($L_1$) and transverse ($L_2$) scales. Besides the previously discussed longitudinal shape distortion, such a packet diffuses also in the transverse direction by diffraction. If the diffractive broadening extends to distances greatly exceeding the plasma wavelength, it can be neglected. This condition leads to the following constraint on the transverse dimension of the packet:

$$L_2 > (1/k_2) (\omega/\omega_p)^{1/2}. \quad (4.1)$$

For a sufficiently short packet ($L_2 > L_1$) the distortion of the packet shape can therefore be neglected under the conditions discussed above, in which $L_1$ must be taken to mean $L_2$, and also if the inequality (4.1) holds.

To determine the electron-density perturbations produced by the packet one can use the hydrodynamic equations and also if the inequality (4.1) holds.

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We assume that $U$ depends on the variable $\xi = x - v_0 t$ that characterizes the longitudinal structure of the packet, and also on the transverse variable $\rho$. To be specific we consider a packet with a Gaussian dependence on this variable:

$$U(\xi, \rho) = V(\xi) \exp \left[ -\rho^2/2L_1^2 \right]. \quad (4.3)$$

From (4.2) we get

$$c_n(\xi, \rho) = \frac{m e}{n c^2} \exp \left[ -\rho^2/2L_1^2 \right] V(\xi) \frac{1}{L_1^2} \sin k_1 (\xi - \xi'). \quad (4.4)$$

The expression for the potential follows from the Poisson equation

$$\psi(\xi, \rho) = \frac{m e}{\omega_p} \int \frac{1}{2L_1^2} \sin k_1 (\xi - \xi') \, d\xi' V(\xi'). \quad (4.5)$$

In addition to the one-dimensional equation (2.4), in which the high-frequency potential $\psi$ should be taken to mean the quantity $m \sqrt{\omega/\omega_p}$, Eq. (4.5) contains a factor that determines the radial dependence. In particular, the potential far behind the packet is of the form

$$\psi(\xi, \rho) = \frac{m e}{\omega_p} R \int \frac{1}{2L_1^2} \sin k_1 (\xi + \rho). \quad (4.6)$$

where $R$ and $\phi$ are defined by Eqs. (2.7) and (2.7') in which $\omega/\omega_p$ must be replaced by $4\pi/\omega m$. Figure 2 shows lines of constant values of the function (4.3), and also the equipotentials for a packet that has a Gaussian dependence on the variable $\xi$.

An already noted, the oscillations of the electron density and of the potential set in not behind the packet, but right after its leading front passes. This explains the result of the two-dimensional numerical simulation of the passage of the front of a laser beam through a low-density plasma, where a pattern of equipotentials similar to that of Fig. 2 was observed.

5. CONCLUSION

The plasma-wave "wake" produced behind the packet can be recorded by using Raman scattering of a probing wave of frequency $\omega_0$, propagating at a small angle $\theta$ to the direction of the packet propagation. The dispersion laws for the probing, scattered, and Langmuir waves lead to a connection between the frequencies and the incidence angle (which is practically equal to the scattering angle):

$$\omega' = \frac{n_c}{n_e} (\omega/\omega_p) (\omega - \omega_0 \pm \omega_0 \pm \omega_0), \quad (5.1)$$

where $\omega$ is the packet carrier frequency, and the $\pm$ signs correspond to the anti-Stokes and Stokes scattered radiation with frequencies $\omega_0 \pm \omega_0 \pm \omega_0$. Given the frequencies, Eq. (5.1) determines the incident and scattering angles of the probing radiation. In particular, for $\theta = 0$ Eq. (5.1) leads to the connection $\omega_0 = \omega \pm \omega_0$ between the frequency of the probing radiation and the carrier frequency of the packet.

In a low-density plasma the longitudinal wave excited by the packet has a velocity close to that of light. At sufficiently high amplitude, it can be used to accelerate charged particles. The acceleration mechanism is the following.
particle is injected into the plasma, in the pocket-propagation direction, with an initial velocity $v_0$ and an energy $E_0 = mc^2 \gamma$; where

$$\gamma = \sqrt{1 - (v_0/c)^2}$$

A particle is accelerated if it lands in a plasma-wave region in which the electric field is directed along the wave motion. The maximum energy that such a resonant particle can acquire before it reaches that plasma-wave region in which the field reverses sign and begins to decelerate the particle is given by

$$E_{\text{max}} = \frac{e}{c^2} \omega_0 \sqrt{(1 + 2k_0 v_0/c)}.$$ 

In this case, the particle negotiates, together with the wave, a path $L = \Delta \epsilon/c \epsilon_0$. It is assumed that the plasma dimensions are also close to $L$. We estimate now the field intensity in a plasma wave produced by a laser pulse of duration $10^{-13}$ s and a frequency $\omega = 2 \times 10^{15} \text{ s}^{-1}$ (the wavelength is $0.\mu\text{m}$). In a plasma of density $n_0 = 10^{17} \text{ cm}^{-3}$, used in experiments on laser acceleration of articles, the plasma wavelength is $\sim 100 \mu\text{m}$. According to (2.8), the longitudinal-field intensity is approximately equal to

$$E_\parallel = \frac{e n_0 \omega_0 \gamma}{4 \epsilon_0}.$$ 

At an intensity $10^{13} \text{ W/cm}^2$ this yields $E_\parallel = 8 \times 10^9 \text{ V/cm}$. We indicate for comparison that in experiments in which the plasma wave is excited by a laser-produced plasma pulse, the laser intensity is approximately equal to $10^{13} \text{ W/cm}^2$.

The model considered by us is restricted by a number of assumptions. One is that the ions are immobile. Consideration of the ion motion introduces a possibility of parametric instabilities of the plasma wave, which the growth rate of the most rapidly developing instability is approximately $\omega_0 (m/m_i) \gamma^3$, where $m_i$ is the ion mass. It can therefore be assumed that over distances $\sim (c/\omega_0) (m_i/m) \gamma^3$ behind the packet the available time is insufficient for instability to develop.

Allowance for the thermal motion of the particles leads to two effects: first, damping of the Langmuir wave, which will apparently be small because the phase velocity of the wave is close to the speed of light, and second, to radial spreading of the "wake" of the plasma waves, which becomes substantial over a distance $\sim L_\parallel (\omega_0/c)/(\gamma v_0)$ behind the packet, where $v_\parallel$ is the electron thermal velocity.

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