Interaction of acoustic and electromagnetic waves in the anomalous skin effect

A. P. Kopasov

Scientific-Research Physicotechnical Institute at the N.I. Lobachevski State University, Gorki

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A solution is obtained of the problem of emission of electromagnetic radiation at combination frequencies $\omega_1 \pm \omega_2$ due to incidence of an electromagnetic wave of frequency ω_1 on a conductor in which a longitudinal acoustic wave of frequency ω_2 with a displacement vector normal to the surface is excited. The case of specular reflection of electrons by the conductor surface and isotropic dispersion law is considered. It is also assumed that the anomalous skin effect takes place. If the frequency of the acoustic wave is twice as high as the frequency of the electromagnetic wave, the difference frequency is $\omega_2 - \omega_1 = \omega_1$. This means that at $\omega_2 = 2\omega_1$ there is a correction to the surface impedance at the frequency ω_1 and this correction is linear in respect of the amplitude of sound. If the wavelength of sound is of the order of the skin depth at the frequency ω_1, this correction is no longer small compared with the linear impedance when the characteristic frequency of the oscillations of electrons trapped by the effective potential field of the acoustic wave becomes higher than the momentum relaxation frequency and the frequency ω_1. In the case of pure metals or semimetals at low temperatures this condition may be satisfied when the energy flux density of the acoustic wave is less than 1 W/cm^2. Consequently, an acoustic wave of moderate intensity may alter greatly the surface impedance.

When a conductor is subjected simultaneously to a transverse electromagnetic wave of frequency ω_1 and a longitudinal acoustic wave of frequency ω_2 with its displacement vector normal to the surface, the always-present nonlinearity gives rise to electromagnetic radiation at combination frequencies. If the waves are sufficiently weak, the nonlinear current at such combination frequencies can be represented as a sum of two terms. The first term, denoted by $\mathbf{j}^{(fs)}$, is the linear response to an electro-magnetic field of a nonequilibrium electron system perturbed by an acoustic wave. The second term, $\mathbf{j}^{(sf)}$, is a linear reaction to the field of a longitudinal acoustic wave in a nonequilibrium electron system perturbed by a transverse electromagnetic field.

Since the distribution function is anisotropic in the presence of an acoustic wave, a linear response to the magnetic field of the second wave is generated and under the anomalous skin effect conditions it dominates $\mathbf{j}^{(fs)}$. In its turn, the anisotropy of the distribution function induced by an electromagnetic field gives rise to a transverse component of the current $\mathbf{j}^{(sf)}$ created by a longitudinal acoustic wave. The two currents $\mathbf{j}^{(fs)}$ and $\mathbf{j}^{(sf)}$ may make comparable contributions to the electromagnetic radiation generated at the combination frequencies. This is manifested most strongly under the anomalous skin effect conditions when the skin depth is comparable with the acoustic wavelength. In this situation the two waves interact strongly with the same group of electrons moving almost parallel to the surface.

There is a special interest in the case when $\omega_2 = 2\omega_1$, because this gives rise to electromagnetic radiation resulting from the simultaneous effects of the acoustic and electromagnetic waves when the frequency of this interaction is identical with the frequency of the main electromagnetic wave ($\omega_2 - \omega_1 = \omega_1$). It means that when $\omega_2 = 2\omega_1$ a correction appears to the surface impedance at the frequency ω_1 and this correction is linear in respect of the amplitude of sound. We may assume (and this is confirmed by the calculations given below) that when the skin depth is comparable with the acoustic wavelength, the relative correction to the impedance is of the order of unity if the characteristic frequency of the oscillations of electrons trapped by the effective field of the longitudinal wave is of the order of or greater than the collision frequency. In the case of metals and semimetals at low temperatures this condition may be satisfied at moderate intensities of sound.[1] Therefore, an acoustic wave of moderate amplitude may alter considerably the surface impedance.

Electromagnetic radiation generated by a nonlinear current created by the simultaneous action of electromagnetic and acoustic waves on a conductor must obviously be taken into account in the interpretation of the results of recent investigations.[2,3] These investigations have revealed a very strong increase in the nonlinear reflection of an electromagnetic wave incident on bismuth, attributed to the excitation of acoustic waves. However, it should be pointed out that in Refs. 2 and 3 an important feature was the presence of a static magnetic field, whereas we shall consider only the case when the magnetic field is zero.

1. We shall consider a conductor occupying the half-space $z > 0$ on which an electromagnetic wave of frequency ω_1 is incident normally and in which a longitudinal wave of frequency ω_2 with its displacement vector parallel to the z axis is excited in some way. We shall determine the amplitudes of the waves emitted by such a conductor at the combination frequencies $\omega_1 \pm \omega_2$.

The reflection of electrons by the surface of the conductor will be assumed to be specular and the dispersion law of electrons will be regarded as isotropic. It is well known that in this situation the problem of finding the solution of the

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transport equation in the half-space by even continuation of the transverse electric fields and deformation fields to the region $z<0$ reduces the solution of this equation in unbounded space. We shall use this technique, which is valid also in the nonlinear case (when an allowance for the magnetic field of the wave must be made in the transport equation and, in accordance with the Maxwell equations, this field should have odd continuation).

We shall consider an unbounded conductor. The transport equation for the electron distribution function f can be written in the form

$$\frac{\partial f}{\partial t}+\frac{\partial \varepsilon}{\partial \mathbf{p}}\frac{\partial f}{\partial \mathbf{r}}+\left(e\mathbf{E}+\frac{e}{c}\left[\frac{\partial \varepsilon}{\partial \mathbf{p}}\mathbf{H}\right]-\frac{\partial \varepsilon}{\partial \mathbf{r}}\right)\frac{\partial f}{\partial \mathbf{p}}=I(f), \quad (1)$$

where $I(f)$ is the collision integral; $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic fields; $\mathbf{p}$ is the quasimomentum; t is the time; r is the radius vector; c is the velocity of light; e is the electron charge; ε is the Hamiltonian for an electron in a deformed crystal. The most general expression for ε was obtained recently by Andreev and Pushkarov.[4] However, we shall not consider here the effects which are quadratic or of higher order in respect of the amplitude of sound, so that we can employ an expression for ε which is linearized in respect of its displacement vector $\mathbf{u}$ and which was derived by Kontorovich in Ref. 5:

$$\varepsilon=\varepsilon_0(\mathbf{p})+(\lambda_{ik}(\mathbf{p})+p_iv_k)u_{ik}+(p_i-mv_i)\dot{u}_i, \quad (2)$$

where $\varepsilon_0(\mathbf{p})$ is the dispersion law of the undeformed crystal; $v_k=\partial\varepsilon_0/\partial p_k$; $\lambda_{ik}(\mathbf{p})$ is the deformation potential tensor; m is the mass of a free electron; u_{ik} is the strain tensor; $\dot{u}_i=\partial u_i/\partial t$. If the dispersion law is isotropic, then

$$\lambda_{ik}(\mathbf{p})=\lambda_1(p^2)\delta_{ik}+\lambda_2(p^2)p_ip_k/p^2.$$

Following Ref. 5, we shall write the collision integral $I(f)$ in the form

$$I(f)=-[f-f_0(\varepsilon-\mathbf{p}\dot{\mathbf{u}}-\delta\mu)]/\tau=-g/\tau, \quad (3)$$

where τ is the relaxation time; f_0 is the equilibrium distribution function; $\delta\mu$ is a nonequilibrium correction to the chemical potential which is found from the condition

$$\langle\!\langle g\rangle\!\rangle=\frac{2}{(2\pi)^3}\int d^3\mathbf{p}\, g(\mathbf{r},\mathbf{p},t)=0. \quad (4)$$

The transverse electromagnetic field in the conductor at the frequency ω_1 will be represented by

$$\mathbf{E}_1(z,t)=\int_{-\infty}^{\infty} dk_1\,\mathbf{E}(k_1,\omega_1)e^{ik_1z-i\omega_1t}+\text{c.c.},$$
$$\mathbf{H}_1(z,t)=\int_{-\infty}^{\infty} dk_1\,\mathbf{H}(k_1,\omega_1)e^{ik_1z-i\omega_1t}+\text{c.c.} \quad (5)$$

It follows from the conditions $\mathbf{E}_1(z,t)=\mathbf{E}_1(-z,t)$ and $\mathbf{H}_1(z,t)=-\mathbf{H}_1(-z,t)$, and from the Maxwell equations that

$$\mathbf{E}(k_1,\omega_1)=\mathbf{E}(-k_1,\omega_1),\quad \mathbf{H}(k_1,\omega_1)=-\mathbf{H}(-k_1,\omega_1),$$
$$\omega_1c^{-1}\mathbf{H}(k_1,\omega_1)=[\mathbf{k}_1\mathbf{E}(k_1,\omega_1)]. \quad (6)$$

In our case at the frequency ω_2 there is only one nonzero component of the stain tensor u_{zz}:

$$u_{zz}(z,t)=\int_{-\infty}^{\infty} dk_2\,u_{zz}(k_2,\omega_2)e^{ik_2z-i\omega_2t}+\text{c.c.},$$
$$u_{zz}(k_2,\omega_2)=u_{zz}(-k_2,\omega_2). \quad (7)$$

The acoustic wave described by the system (7) is accompanied by oscillations of the longitudinal electric field $E_z(z,t)$ of the same frequency and the Fourier component of this field defined by analogy with Eqs. (5) and (7) will be denoted by $E_z(k_2,\omega_2)$.

We shall assume that the amplitudes of both waves are sufficiently small and solve the transport equation (1) by the iteration method. We shall expand the function g as a series in terms of the field amplitudes:

$$g=g^{(1)}+g^{(2)}+\dots. \quad (8)$$

The function obtained in the linear approximation $g^{(1)}$ is described by

$$\frac{\partial g^{(1)}}{\partial t}+\mathbf{v}\frac{\partial g^{(1)}}{\partial \mathbf{r}}=-\frac{g^{(1)}}{\tau}-\frac{\partial f_0}{\partial\varepsilon}\{\lambda_{ik}\dot{u}_{ik}-\delta\dot{\mu}^{(1)}+e\tilde{\mathbf{E}}\mathbf{v}\},$$
$$\tilde{\mathbf{E}}=\mathbf{E}-\frac{1}{e}\nabla\delta\mu^{(1)}-\frac{m}{e}\ddot{\mathbf{u}}, \quad (9)$$

where $\delta\mu^{(1)}$ is the linear correction to the chemical potential. The function $g^{(1)}$ will be represented in the form $g^{(1)}=g^{(f)}+g^{(s)}$, where $g^{(f)}$ is the reaction to the electromagnetic wave of frequency ω_1 and $g^{(s)}$ is the reaction to the acoustic wave of frequency ω_2. Similarly, $\delta\mu^{(1)}$ is described by $\delta\mu^{(1)}=\delta\mu^{(f)}+\delta\mu^{(s)}$.

The transverse wave of Eq. (5) excites obviously acoustic vibrations of the same frequency. However, in the lowest approximation we can ignore the direct transformation of the electromagnetic wave into sound, because it is practically always weak (see Refs. 6 and 7). Bearing this in mind, we find from Eq. (9) that $\delta\mu(f)=0$ and

$$g^{(f)}(\mathbf{p},k_1,\omega_1)=\frac{ie\mathbf{E}(k_1,\omega_1)\mathbf{v}}{k_1v_z-\omega_1-i\tau^{-1}}\frac{\partial f_0}{\partial\varepsilon}, \quad (10)$$

where $g^{(f)}(\mathbf{p},k_1,\omega_1)$ is a Fourier component of the function $g^{(f)}(z,\mathbf{p},t)$. Then, $\delta\mu^{(s)}$ is described by[5]

$$\delta\mu^{(s)}=\frac{\langle\lambda_{zz}\rangle u_{zz}}{\langle 1\rangle}=\int d^3\mathbf{p}\frac{\partial f_0}{\partial\varepsilon}\lambda_{zz}u_{zz}\Big/\int d^3\mathbf{p}\frac{\partial f_0}{\partial\varepsilon}. \quad (11)$$

Substituting Eq. (11) into Eq. (9), we obtain

$$g^{(s)}(\mathbf{p},k_2,\omega_2)=\frac{\omega_2\Lambda_{zz}(\mathbf{p})u_{zz}(k_2,\omega_2)+ie\tilde{E}_z(k_2,\omega_2)v_z}{k_2v_z-\omega_2-i\tau^{-1}}\frac{\partial f_0}{\partial\varepsilon}, \quad (12)$$

where $\Lambda_{zz}(\mathbf{p})=\lambda_{zz}(\mathbf{p})-\langle\lambda_{zz}\rangle/\langle 1\rangle$. The effective longitudinal field $\tilde{E}_z$, which occurs in Eq. (12), can be found from the electrical neutrality condition which is equivalent to the condition $\langle\langle g^{(s)}\rangle\rangle=0$ (Ref. 5) if we describe $\delta\mu^{(s)}$ by Eq. (11).

Since we are considering the anomalous skin effect conditions, we shall assume that

$$\left|\frac{\omega_{1,2}+i\tau^{-1}}{k_{1,2}v_F}\right| \ll 1, \tag{13}$$

where v_F is the Fermi velocity. Using Eq. (13) and the condition $\langle\langle g^{(s)}\rangle\rangle = 0$, we find that

$$ieE_z(k_2,\omega_2) = -i\frac{\pi}{2v_F}\omega_2 \operatorname{sign}(k_2)\overline{\Lambda}_{zz}u_{zz}(k_2,\omega_2), \tag{14}$$

$$\overline{\Lambda_{zz}} = \Lambda_{zz}(p_z=0,\ p^2=p_F^2) = -{}^1/_3\lambda_2(p_F^2),$$

where p_F is the Fermi momentum. It follows from Eqs. (12) and (14) that

$$g^{(s)}(\mathbf{p},k_2,\omega_2) = \frac{\omega_2(\Lambda_{zz}(\mathbf{p})-(i\pi/2v_F)\operatorname{sign}(k_2)\overline{\Lambda_{zz}}v_z)u_{zz}(k_2,\omega_2)}{k_2v_z-\omega_2-i\tau^{-1}}\frac{\partial f_0}{\partial\varepsilon}. \tag{15}$$

We shall now find the function $g^{(2)}$ which is quadratic in respect of the field amplitudes. We shall be interested only in that part of the function $g^{(2)}$ which is proportional to the amplitude of the acoustic wave and to the amplitude of the electromagnetic wave, and we shall designate it $g^{(cr)}$. The function $g^{(cr)}$ can be described by

$$g^{(cr)} = g^{(fs)} + g^{(sf)} + \bar{g}, \tag{16}$$

where $g^{(fs)}$ is a linear reaction to the transverse electromagnetic field of Eq. (5) of a nonequilibrium system of electrons perturbed by an acoustic wave of Eq. (5), whereas $g^{(sf)}$ is a linear reaction to an acoustic wave in an electron system disturbed from equilibrium by the transverse electromagnetic field. The function $\bar{g}$ is due to the nonlinear term

$$(e/c)\,[\partial\delta\varepsilon/\partial\mathbf{p})\mathbf{H}_1]\,\partial f_0/\partial\mathbf{p}$$

in the transport equation. We shall not write down separately the equation for $\bar{g}$, but include it in $g^{(fs)}$, i.e., we shall understand $g^{(fs)}$ to be the sum $g^{(fs)} + \bar{g}$. We shall show later that the contribution of $\bar{g}$ to the current at the combination frequencies is negligible.

We therefore have the following equation for $g^{(fs)}$:

$$\frac{\partial g^{(fs)}}{\partial t} + v_z\frac{\partial g^{(fs)}}{\partial z} + \left(e\mathbf{E}_1 + \frac{e}{c}[\mathbf{vH}_1]\right)\frac{\partial}{\partial\mathbf{p}}\left(g^{(s)} + \frac{\partial f_0}{\partial\varepsilon}(\delta\tilde{\varepsilon}-\delta\mu^{(s)})\right) + \frac{e}{c}\left[\frac{\partial\delta\varepsilon}{\partial\mathbf{p}}\mathbf{H}_1\right]\frac{\partial f_0}{\partial\mathbf{p}} = -\frac{g^{(fs)}}{\tau}, \tag{17}$$

where $\delta\tilde{\varepsilon} = \delta\varepsilon - \mathbf{p}\dot{\mathbf{u}}$ and $\delta\varepsilon = \varepsilon - \varepsilon_0$. We then find that $g^{(sf)}$ obeys

$$\frac{\partial g^{(sf)}}{\partial t} + v_z\frac{\partial g^{(sf)}}{\partial z} + \frac{g^{(sf)}}{\tau} = -\left(eE_z - \frac{\partial\delta\varepsilon}{\partial z}\right)\frac{\partial g^{(f)}}{\partial p_z} - \frac{\partial\delta\varepsilon}{\partial p_z}\frac{\partial g^{(f)}}{\partial z}. \tag{18}$$

In writing down Eqs. (17) and (18) we have assumed that in our case the corrections to the chemical potential at the combination frequencies are $\delta\mu^{(cr)} = 0$. This is easily demonstrated. In fact, integrating Eqs. (17) and (18) with respect to quasimomenta and using Eqs. (10) and (15), we obtain $\langle\langle g^{(fs)}\rangle\rangle = \langle\langle g^{(sf)}\rangle\rangle = 0$.

Representing $g^{(fs)}$ in the form

$$g^{(fs)}(\mathbf{p},z,t) = \int_{-\infty}^{\infty} dk_1\,dk_2 \exp[i(k_1+k_2)z - i(\omega_1+\omega_2)t] \times g^{(fs)}(\mathbf{p},k_1,k_2,\omega_1,\omega_2) + \text{c.c.} \tag{19}$$

we find from Eq. (17) that

$$g^{(fs)}(\mathbf{p},k_1,k_2,\omega_1,\omega_2) = \frac{ieu_{zz}(k_2,\omega_2)}{(k_1+k_2)v_z-\omega_1-\omega_2-i\tau^{-1}} \times\left\{\left(\mathbf{E}(k_1,\omega_1)+\frac{1}{c}[\mathbf{vH}(k_1,\omega_1)]\right)\frac{\partial}{\partial\mathbf{p}} \times\frac{\omega_2(\Lambda_{zz}(\mathbf{p})-i(\pi/2v_F)\operatorname{sign}(k_2)\overline{\Lambda}_{zz}v_z)}{k_2v_z-\omega_2-i\tau^{-1}}\frac{\partial f_0}{\partial\varepsilon} + \mathbf{E}(k_1,\omega_1)\frac{\partial}{\partial\mathbf{p}}\left[\left(\Lambda_{zz}(\mathbf{p})+p_zv_z+\frac{mv_z\omega_2}{k_2}\right)\frac{\partial f_0}{\partial\varepsilon}\right] + \frac{e}{c}[\mathbf{vH}(k_1,\omega_1)]_z\frac{\omega_2}{k_2}\frac{\partial f_0}{\partial\varepsilon}\right\}. \tag{20}$$

The equation for $g^{(sf)}$ includes the longitudinal electric field

$$E_z = \tilde{E}_z + \frac{1}{e}\frac{\partial\delta\mu^{(s)}}{\partial z} + \frac{m}{e}\ddot{u}_z.$$

Using Eqs. (14) and (13), we find that $E_z = e^{-1}(\partial\delta\mu^{(s)}/\partial z)$. Therefore, it follows from Eq. (18) that

$$g^{(sf)}(\mathbf{p},k_1,k_2,\omega_1,\omega_2) = \frac{k_2[\Lambda_{zz}(\mathbf{p})+p_zv_z-(p_z-mv_z)\omega_2/k_2]u_{zz}(k_2,\omega_2)}{(k_1+k_2)v_z-\omega_1-\omega_2-i\tau^{-1}} \times\frac{\partial}{\partial p_z}\frac{ie\mathbf{E}(k_1,\omega_1)\mathbf{v}}{k_1v_z-\omega_1-i\tau^{-1}}\frac{\partial f_0}{\partial\varepsilon} - \frac{k_1u_{zz}(k_2,\omega_2)\{(\partial/\partial p_z)[\lambda_{zz}(\mathbf{p})+p_zv_z-(p_z-mv_z)\omega_2/k_2]\}}{(k_1+k_2)v_z-\omega_1-\omega_2-i\tau^{-1}} \times\frac{ie\mathbf{E}(k_1,\omega_1)\mathbf{v}}{k_1v_z-\omega_1-i\tau^{-1}}\frac{\partial f_0}{\partial\varepsilon}. \tag{21}$$

Equations (10), (12), (20), and (21) show that the functions $g^{(f)}$, $g^{(s)}$, $g^{(fs)}$, and $g^{(sf)}$ satisfy the specular reflection condition: $g(p_z,p_x,p_y,z=0) = g(-p_z,p_x,p_y,z=0)$. In general, the boundary condition should be satisfied not at $z=0$ but at $z=u_z(0,t)$ and, moreover, it should be modified, since the boundary moves at a finite velocity (we shall use the laboratory coordinate system not linked to the lattice). However, we can readily show that an allowance for the motion of the boundary is unimportant because the deformation (strain) is always very small: $|u_{zz}| \ll 1$.

[It is clear that the influence of the motion of a boundary can be neglected if $|u_z|$ is much smaller than the depth of the skin layer at frequencies $\omega_1 \pm \omega_2$ and ω_1, and the additional velocity acquired by an electron as a result of a collision with a moving boundary is $\dot{u}_z$, which is much less than the characteristic velocity of the active electrons $\tilde{v}_z$ along the normal to the surface. In the case of greatest interest to us when the acoustic wavelength is of the order of depth of the skin layer at frequencies ω_1 and $\omega_1 \pm \omega_2$, it follows from Eqs. (20) and (21) that $\tilde{v}_z$ is greater than or of the order of the velocity of sound and both these conditions are satisfied if $|u_{zz}| \ll 1$.]

Using Eqs. (20) and (21), we shall calculate the Fourier component of the nonlinear current at the frequency $\omega = \omega_1 + \omega_2$:

$$j_\alpha^{(cr)}(k,\omega) = j_\alpha^{(fs)}(k,\omega) + j_\alpha^{(sf)}(k,\omega),$$

$$j_\alpha^{(fs)}(k,\omega) = \frac{2}{(2\pi)^3}\int d^3\mathbf{p}\, v_\alpha \int_{-\infty}^{\infty} dk_1\, dk_2\, \delta(k-k_1-k_2) \times g^{(fs)}(\mathbf{p}, k_1, k_2, \omega_1, \omega_2), \tag{22}$$

$$j_\alpha^{(sf)}(k,\omega) = \frac{2}{(2\pi)^3}\int d^3\mathbf{p}\, v_\alpha \int_{-\infty}^{\infty} dk_1\, dk_2\, \delta(k-k_1-k_2) \times g^{(sf)}(\mathbf{p}, k_1, k_2, \omega_1, \omega_2).$$

We shall assume that in addition to the inequality (13), the following condition is satisfied:

$$|(\omega + i\tau^{-1})/kv_F| \ll 1. \tag{23}$$

After substitution of Eq. (20) into Eq. (22) and integration with respect to quasimomenta, we obtain

$$j_\alpha^{(fs)}(k,\omega) = \delta_{\alpha\beta}\frac{e^2p_F^2}{4\pi}\frac{\omega_2}{\omega_1}\int_{-\infty}^{\infty} dk_1\, dk_2\, \delta(k-k_1-k_2) \times \theta(-kk_2) E_\beta(k_1,\omega_1) \times \frac{k_1}{k|k_2|}\frac{\bar{\Lambda}_{zz}u_{zz}(k_2,\omega_2)}{\bar{\varepsilon}}\left(\frac{\omega_2+i\tau^{-1}}{k_2v_F} - \frac{\omega+i\tau^{-1}}{kv_F}\right)^{-2}, \tag{24}$$

where $\bar{\varepsilon} = [v_F^2/2(\partial v_z/\partial p_z)]|_{p_z=0,\ \varepsilon=\varepsilon_F}$ and ε_F is the Fermi energy and $\bar{\varepsilon} \sim \varepsilon_F$.

The main contribution to $j^{(fs)}$ comes from the first term in the braces in Eq. (20) describing the function $g^{(fs)}$ and in this term we can ignore the electric field of the wave. The appearance of the theta function $\theta(-kk_2)$ in Eq. (24) is due to the fact that the integrand in Eq. (22), which represents the function $g^{(fs)}$, has two poles of the variable $w = \cos Q$ (Q is the angle between the vector $\mathbf{v}$ and the z axis), which is located near the point $w = 0$ when the inequilities (13) and (23) are okeyed. A large contribution to this integral appears only when the poles are located on the opposite sides of the real axis, i.e., when $kk_2 < 0$.

We can similarly calculate $j_\alpha^{(sf)}(k,\omega)$:

$$j_\alpha^{(sf)}(k,\omega) = \delta_{\alpha\beta}\frac{e^2p_F^2}{4\pi}\int_{-\infty}^{\infty} dk_1\, dk_2\, \delta(k-k_1-k_2)\theta(-kk_1) \times E_\beta(k_1,\omega_1)\frac{k_2}{k|k_1|}\frac{\bar{\Lambda}_{zz}u_{zz}(k_2,\omega_2)}{\bar{\varepsilon}}\left(\frac{\omega_1+i\tau^{-1}}{k_1v_F} - \frac{\omega+i\tau^{-1}}{kv_F}\right)^{-2}. \tag{25}$$

It is clear from Eqs. (24) and (25) that the current at the combination frequencies $\mathbf{j}^{(cr)}(k,\omega)$ and, therefore, the field generated by the current has the same polarization as the electromagnetic wave incident on the investigated conductor. Let us assume that the electric field of the incident wave is polarized along the x axis. Then all the transverse fields and currents of interest to us will be also polarized along the x axis, so that we shall omit the relevant vector indices.

2. The amplitude of an electromagnetic wave emitted by a conductor at the combination frequency can be calculated by solving the Maxwell equations in the half-space where the external current is $j^{(cr)}(k,\omega)$ and the corresponding boundary condition for the fields applies at $z = 0$. This is easily done by continuing the Maxwell equations to the half-space $z < 0$. As a result, the amplitude of the electric field of the wave $E^{(cr)}(0,\omega)$ emitted at a frequency $\omega = \omega_1 + \omega_2$ is described by

$$E^{(cr)}(0,\omega) = -\frac{4\pi}{c}\zeta_L(\omega)\int_0^{\infty} dz\, j^{(cr)}(z,\omega)\mathscr{E}(z,\omega), \tag{26}$$

where $\zeta_L(\omega)$ is the linear surface impedance at the frequency ω: the function $\mathscr{E}(z,\omega)$ represents linear penetration of the electric field of frequency ω in the conductor $\mathscr{E}(z=0,\omega) = 1$. In the derivation of Eq. (26) we made allowance for the fact that under the anomalous skin effect condition we have $|\zeta_L| \ll 1$.

We shall now go over in Eq. (26) to the Fourier components

$$E^{(cr)}(0,\omega) = -\frac{4\pi^2}{c}\zeta_L(\omega)\int_{-\infty}^{\infty} dk\, j^{(cr)}(k,\omega)\mathscr{E}(k,\omega). \tag{27}$$

We shall represent the Fourier component $\mathscr{E}(k,\omega)$ of the function $\mathscr{E}(z,\omega)$ in the form

$$\mathscr{E}(k,\omega) = i\frac{\omega}{c}\frac{\chi^{-2}(\omega)}{\zeta_L(\omega)}e_\perp\left[\frac{k}{\chi(\omega)},\omega\right], \tag{28}$$

where $\chi^{-1}(\omega)$ is the depth of the skin layer at the frequency ω. It follows from Ref. 8 that in the case of specular reflection, we have

$$\chi(\omega) = \left|\frac{e^2p_F^2\omega}{c^2}\right|^{1/3},$$

$$\zeta_L(\omega) = \frac{4|\omega|\chi^{-1}(\omega)}{3\sqrt{3}c}\exp\left(-i\frac{\pi}{3}\operatorname{sign}\omega\right), \tag{29}$$

and the function $e_\perp(k,\omega)$ is of the form

$$e_\perp(k,\omega) = -\frac{1}{\pi}\left(k^2 - \frac{i\operatorname{sign}\omega}{|k|}\right)^{-1}. \tag{30}$$

Similarly, we find that

$$E(k_1,\omega_1) = i\frac{\omega_1}{c}\frac{\chi^{-2}(\omega_1)E(0,\omega_1)}{\zeta_L(\omega_1)}e_\perp\left[\frac{k_1}{\chi(\omega_1)},\omega_1\right], \tag{31}$$

where $E(0,\omega_1)$ is the amplitude of the electric field of frequency ω_1 on the surface of the conductor.

The explicit form of the Fourier component $u_{zz}(k_2,\omega_2)$ depends on the method of excitation of sound in the conductor. An acoustic wave may be excited at the $z = 0$ surface or at the second $z = d$ surface of a bulk conductor of thickness $d \gg \chi^{-1}(\omega_1), \chi^{-1}(\omega)$. We shall consider the most interesting case when the attenuation length of sound of frequency

ω_2 is much greater than $\chi^{-1}(\omega_1)$, $\chi^{-1}(\omega)$. We can then represent $u_{zz}(k_2,\omega_2)$ in the form

$$u_{zz}(k_2,\omega_2)=\frac{\bar{u}_{zz}(\omega_2)}{k_s(\omega_2)}e_\parallel\left[\frac{k_2}{k_s(\omega_2)},\omega_2\right], \qquad (32)$$

where $\bar{u}_{zz}(\omega_2)$ is the amplitude of the strain near the $z=0$ surface, $k_s(\omega_2)=\omega_2/s$, and s is the velocity of sound.

If the sound is excited at the $z=0$ surface, then

$$u_{zz}(z,t)=\bar{u}_{zz}(\omega_2)\exp\{ik_s(\omega_2)|z|-i\omega_2 t\}+\text{c.c.}$$

and we obtain

$$e_\parallel(k_2,\omega_2)=\frac{i}{\pi}\frac{1}{1-k_2^2+i0\,\mathrm{sign}\,\omega_2}. \qquad (33)$$

However, if sound is excited at the $z=d$ surface, whereas the $z=0$ surface is free (i.e., the conductor is in contact in vacuum on this side), then

$$u_{zz}(z,t)=u_{zz}(\omega_2)\exp(-i\omega_2 t)\sin k_s(\omega_2)|z|+\text{c.c.}$$

and $e_\parallel(k_2,\omega_2)$ is described by

$$e_\parallel(k_2,\omega_2)=e_\parallel(k_2)=\frac{1}{\pi}\frac{1}{1-k_2^2}. \qquad (34)$$

A singularity in Eq. (34) at $k_2^2=1$ should be integrated using the principal value.

It follows from Eq. (27) when an allowance is made for Eqs. (28)–(32), (24), and (25) that, after certain operations, we obtain

$$\frac{E^{(cr)}(0,\omega)}{E(0,\omega_1)}=\mathrm{sign}(\omega\omega_1)\frac{3\sqrt{3}}{4}\left|\frac{\omega}{\omega_1}\right|^{1/3}\exp\left(i\frac{\pi}{3}\,\mathrm{sign}\,\omega_1\right)$$
$$\times\left(\frac{\bar{\Lambda}_{zz}\bar{u}_{zz}(\omega_2)}{\varepsilon}\right)\left(\frac{v_F[k_s(\omega_2)\chi(\omega_1)]^{1/2}}{\omega_2+i\tau^{-1}}\right)^2$$
$$\times\left\{\frac{\omega_2}{\omega_1}I_1(\omega,\omega_1,\omega_2)+I_2(\omega,\omega_1,\omega_2)\right., \qquad (35)$$

where

$$I_1(\omega,\omega_1,\omega_2)=2\pi\left(\frac{\chi(\omega_1)}{k_s(\omega_2)}\right)^2\int_0^\infty dk_1\int_0^{k_1}dk_2\,e_\perp(k_1,\omega_1)$$
$$\times e_\perp\left[\frac{(k_1-k_2)\chi(\omega_1)}{\chi(\omega)},\omega\right]e_\parallel\left[\frac{k_2\chi(\omega_1)}{k_s(\omega_2)},\omega_2\right.$$
$$\times\frac{k_2(k_1-k_2)k_1}{\left(k_1+k_2\dfrac{\omega_1\tau}{\omega_2\tau+i}\right)^2}, \qquad (36)$$

$$I_2(\omega,\omega_1,\omega_2)=2\pi\left(\frac{\chi(\omega_1)}{k_s(\omega_2)}\right)^2\int_0^\infty dk_2\int_0^{k_2}dk_1 e_\perp(k_1,\omega_1)$$
$$\times e_\perp\left[\frac{(k_1-k_2)\chi(\omega_1)}{\chi(\omega)},\omega\right]e_\parallel\left[\frac{k_2\chi(\omega_1)}{k_s(\omega_2)},\omega_2\right]k_1(k_2-k_1)k_2$$
$$\times\left(\frac{\omega_2\tau}{\omega_2\tau+i}k_1+\frac{\omega_1\tau+i}{\omega_2\tau+i}k_2\right)^{-2}. \qquad (37)$$

It is therefore clear that Eqs. (35)–(37) solve the problem of emission of electromagnetic radiation at the combination frequencies. The amplitude of the wave at the sum frequency is obtained from Eq. (35) for $\omega_1,\omega_2>0$, whereas the amplitude at the difference frequency can be obtained from Eq. (35) replacing ω_1 with $-\omega_1$ (or ω_2 with $-\omega_2$). We shall assume that all frequencies ω_1, ω_2, and $\omega_1+\omega_2$ are of the same order of magnitude. In this case the values of I_1 and I_2 depend strongly only on one parameter which is $k_s(\omega_2)/\chi(\omega_1)$. If $k_s(\omega_2)/\chi(\omega_1)\ll 1$, it follows from Eq. (36) with a logarithmic precision that

$$I_1=-2\lambda\ln\left(\frac{\chi(\omega_1)}{k_s(\omega_2)}\right)\int_0^\infty dk_1\,e_\perp(k_1,\omega_1)e_\perp\left[\frac{k_1\chi(\omega_1)}{\chi(\omega)},\omega\right]$$
$$\sim\ln\left(\frac{\chi(\omega_1)}{k_s(\omega_2)}\right), \qquad (38)$$

where $\lambda=i$ even if $e_\parallel(k_2,\omega_2)$ is described by Eq. (33) and $\lambda=1$ for other methods of excitation of sound when Eq. (34) applies. In this case the main contribution to the integral of Eq. (36) comes from the region where $k_1\sim 1$ and $1\gg k_2\gg k_s(\omega_2)/\chi(\omega_1)$.

Obviously if $k_s(\omega_2)/\chi(\omega_1)\sim 1$, then the estimate $|I_1|\sim|I_2|\sim 1$ is valid; it should be noted that

$$\int_0^\infty dk_1 e_\perp(k_1,\omega_1)=-i\frac{2}{3^{3/2}}\exp\left\{-i\frac{\pi}{3}\,\mathrm{sign}(\omega_1)\right\}.$$

We can use Eq. (37) to show that $|I_2|\sim 1$ also when $k_s(\omega_2)/\chi(\omega_1)\ll 1$. If $k_s(\omega_2)/\chi(\omega_1)\gg 1$, it then follows from Eqs. (36) and (37)

$$|I_1|\sim|I_2|\sim\left(\frac{\chi(\omega_1)}{k_s(\omega_2)}\right)^2. \qquad (39)$$

In the case of metals and semimetals the condition $k_s(\omega_2)/\chi(\omega_1)\sim 1$ is satisfied at frequencies $\omega_{1,2}\lesssim 10^9$. We usually find that $\omega_{1,2}\tau\lesssim 1$. We can therefore see that for fixed values of $E(0,\omega_1)$ and $\bar{u}_{zz}(\omega_2)$, the amplitudes of the emitted combination harmonics are maximal in the frequency range where $k_s(\omega_2)/\chi(\omega_1)\sim 1$.

We shall now consider the case when $\omega_2=2\omega_1$. In this case the emitted difference harmonic is of the same frequency as the electromagnetic wave incident on the conductor. It follows that a nonlinear correction to the surface impedance appears at the frequency ω_1 and we shall denote this correction by $\Delta\zeta_{NL}(\omega_1)$. Using Eq. (35), we obtain

$$\frac{\Delta\zeta_{NL}(\omega_1)}{\zeta_L(\omega_1)}=\frac{E^{(cr)}(0,\omega_1)}{E(0,\omega_1)}=-\frac{3^{1/2}}{4}e^{-i\pi/3}\left(\frac{E^*(0,\omega_1)}{E(0,\omega_1)}\right)$$
$$\times\left(\frac{\bar{\Lambda}_{zz}\bar{u}_{zz}(\omega_2)}{\varepsilon}\right)\left(\frac{v_F[k_s(\omega_2)\chi(\omega_1)]^{1/2}}{2\omega_1+i\tau^{-1}}\right)^2$$
$$\times\{-2I_1(\omega_1,-\omega_1,2\omega_1)+I_2(\omega_1,-\omega_1,2\omega_1)\}. \qquad (40)$$

Obviously, this correction to the impedance $\Delta\zeta_{NL}(\omega_1)$ depends on the ratio of the phases of the acoustic and electromagnetic waves.

If $k_s(2\omega_1)/\chi(\omega_1)\sim 1$ and $\omega_1\tau\lesssim 1$, we find that

$$\left|\frac{\Delta\zeta_{NL}(\omega_1)}{\zeta_L(\omega_1)}\right|\sim\left|\frac{\bar{\Lambda}_{zz}\bar{u}_{zz}(\omega_2)}{\varepsilon}\right|[v_F k_s(2\omega_1)\tau]^2=(\omega_0\tau)^2. \qquad (41)$$

The quantity $\bar{\Lambda}_{zz}\bar{u}_{zz}(\omega_2)$ is the amplitude of the effective potential field acting on electrons and ω_0 is the characteristic frequency of the oscillations of electrons trapped by the acoustic wave.[1] In the case of metals and semimetals with long mean free paths the parameter $\omega_0\tau$ may be of the order of unity when the energy flux density in the acoustic wave is less than 1 W/cm^2. It follows from Eq. (41) that the condition of validity of perturbation theory in terms of the amplitude of the acoustic wave is $(\omega_0\tau)^2 \ll 1$. The amplitude of the electromagnetic wave can be assumed to be small if[9]

$$|H(0,\omega_1)| = \left|\frac{E(0,\omega_1)}{\zeta_L(\omega_1)}\right| \ll H_{cr} = \frac{m^*c}{ev_F\tau^2\chi(\omega_1)}, \qquad (42)$$

where m^* is the effective mass.

It therefore follows that an acoustic wave of moderate intensity may have a significant influence on the surface impedance. One could expect a static field $H_0 > H_{cr}$ parallel to the surface to reduce considerably this influence in the same way as it weakens the nonlinear effects in the absorption of sound.[1,10] The field H_{cr} may be very weak (less than or of the order of 1 Oe).

We considered only the longitudinal sound. We can easily show that the current at the combination frequencies due to a transverse acoustic wave and a transverse electromagnetic field is purely longitudinal in the case of an isotropic dispersion law and, consequently, it does not generate electromagnetic radiation (this may happen if a conductor has a fourfold or eightfold symmetry axis normal to the surface). In the case of strong anisotropy the radiation at the combination frequencies does not depend strongly on the polarization of sound and is of the same order of magnitude as in the case of the isotropic dispersion law and longitudinal sound investigated by us.

For the arbitrary ratio of the frequencies ω_1 and ω_2 there is no correction linear in the amplitude of sound to the surface impedance, but there is obviously a quadratic correction, which ceases to be small for $\omega_0\tau \gtrsim 1$. This effect will be considered in a separate communication.

[1] Yu. M. Gal'perin, V. L. Gurevich, and V. I. Kozub, Usp. Fiz. Nauk **128**, 107 (1979) [Sov. Phys. Usp. **22**, 352 (1979)].
[2] S. A. Vitkalov, V. F. Gantmakher, and G. I. Leviev, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 540 (1984) [JETP Lett. **39**, 660 (1984)].
[3] S. A. Vitkalov, V. F. Gantmakher, and G. I. Leviev, Solid State Commun. **54**, 1091 (1985).
[4] A. F. Andreev and D. I. Pushkarov, Zh. Eksp. Teor. Fiz. **89**, 1883 (1985) [Sov. Phys. JETP **62**, 1087 (1985)].
[5] V. M. Kontorovich, Usp. Fiz. Nauk **142**, 265 (1984) [Sov. Phys. Usp. **27**, 134 (1984)].
[6] M. I. Kaganov and V. B. Fiks, Zh. Eksp. Teor. Fiz. **62**, 1461 (1972) [Sov. Phys. JETP **35**, 767 (1972)].
[7] A. N. Vasil'ev and Yu. P. Gaĭdukov, Usp. Fiz. Nauk **141**, 431 (1983) [Sov. Phys. Usp. Fiz. Nauk **141**, 431 (1983) [Sov. Phys. Usp. **26**, 952 (1983)].
[8] G. E. Reuter and E. H. Sondheimer, Proc. R. Soc. London Ser. A **195**, 336 (1948).
[9] O. I. Lyubimov, N. M. Makarov, and V. A. Yampol'skiĭ, Zh. Eksp. Teor. Fiz. **85**, 2159 (1983) [Sov. Phys. JETP **58**, 1253 (1983)].
[10] V. I. Kozub, Zh. Eksp. Teor. Fiz. **68**, 1014 (1975) [Sov. Phys. JETP **41**, 502 (1975)].

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