Neutrino electrodynamics and possible consequences for solar neutrinos

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The neutrino electromagnetic moment matrix and the possibility that some of the elements of this matrix are of the order of 10^{-10} of the Bohr magneton are discussed. Flavor oscillations and spin precession are examined for a neutrino in a magnetic field in the presence of matter. The interaction between solar neutrinos and the magnetic field in the interior of the convective zone of the Sun can lead, in this case, to the 11-year and semiannual variations in the neutrino flux, shown experimentally to be correlated with the magnetic activity of the Sun.

1. INTRODUCTION

There has been continuing interest in the mass matrix that determines the masses of neutrinos, in neutrino oscillations, and, when Majorana terms are present, in the probability of double β-decay without the emission of a neutrino. In this paper, we discuss a further class of static characteristics of neutrinos, namely, the electromagnetic moment (EMM) matrix μ that appears in the Lagrangian for the interaction between a neutrino and the electromagnetic field $F_{\mu\nu}$:

$$L_{int}={}^1/_2\mu_{ij}(\bar{\nu}_R)_i\sigma_{\mu\nu}F_{\mu\nu}(\nu_L)_j+\text{h.c.}={}^1/_2\mu_{ab}(\bar{\nu}_R)_a\sigma_{\mu\nu}F_{\mu\nu}(\nu_L)_b+\text{h.c.} \tag{1}$$

The subscripts L and R indicate left- and right-handed fields, the subscripts i, j label the current states of the neutrinos, $i, j = e, \mu, \tau, \dots$ (this will be referred to as the flavor basis), and a, b refer to the eigenstate basis of the neutrinos mass matrix, $a, b = 1, 2, 3, \dots$. In the latter basis, the real (imaginary) part of the diagonal elements of the matrix μ_{ab} are the magnetic (electric) dipole moments of the mass state $\nu_1, \nu_2, \nu_3, \dots$ (with masses $m_1 < m_2 < m_3$), and the off-diagonal elements of μ_{ab} describe the decay of neutrinos into lighter neutrinos and γ-rays, e.g., $\nu_2 \to \nu_1\gamma$.

We shall examine the possibility that some of the elements of the matrix μ may be of the order of $10^{-10}\,\mu_B$ ($\mu_B = e\hbar/(2m_e c)$ is the Bohr magneton), which may already have been seen experimentally in the form of the very specific variations in the solar neutrino flux recorded by the Davis group.[1] This question has been examined in our previous brief communications.[2–4] Here, we would like to give a more complete and closed presentation of the possible manifestations of the EMM in solar-neutrino experiments.

Briefly, the effect is that the EMM ensures that the helicity of the neutrinos is partially modified during their passage through the toroidal magnetic field **H** in the convective zone of the Sun, and the observed solar activity is ascribed to processes in this zone. This results in a reduction in the flux of left-handed neutrinos, as detected by the Cl–Ar method. The quantity $|\mathbf{H}|$ and, consequently, the reduction in the flux reach their respective maxima at the maximum of the 11-year cycle of solar activity. Moreover, **H** changes sign at the solar equator, so that, because of the inclination of the Earth's orbit to this equator, there should also be semiannual variation in the recorded neutrino flux.

In Section 2, we shall discuss electroweak interaction models, in which EMM values of the order of 10^{-11}–10^{-10} of the Bohr magneton can be obtained, and we summarize experimental and astrophysical limits on the matrix μ. In Section 3, we consider the behavior of the neutrino helicity when the EMM interact with the magnetic field in the presence of matter, which may be very significant for neutrino propagation in the solar interior. In Section 4, we give a brief review of aspects of the theory of the Sun that are relevant to this question, and estimate the time variation in the recorded neutrino flux. Finally, in Section 5, we discuss possible further studies of this effect, using the new solar-neutrino detectors that are being built at present.

2. THE NEUTRINO EMM MATRIX

1. In standard $SU(2)_L \times U(1)$ theory, if the right-handed neutrino ν_R is an $SU(2)$ singlet, the matrix μ can be found from the diagrams shown in Fig. 1, and turns out to be proportional to the neutrinos mass matrix m (Refs. 5, 6):

$$\mu=\frac{3eG}{8\sqrt{2}\pi^2}m=\frac{3m_eG}{4\sqrt{2}\pi^2}\mu_B m\approx 2.7\cdot 10^{-10}\mu_B\frac{m}{m_N}, \tag{2}$$

where G is the Fermi constant and m_N the nucleon mass. The EMM described by this formula are exceedingly small, e.g., $\mu_{ee} \sim 10^{-17}\mu_B$ for $m_{ee} = 30$ eV. The reason for the fact that μ and m are mutually proportional (and, hence, μ is small) in this scheme is that the W boson in the diagrams of Fig. 1 interacts only with left-handed currents. This means that the change in helicity demanded by an interaction such as (1) must occur on the external neutrino line, $\hat{p}\nu_L = m\nu_R$. Hence, it is clear that the neutrino EMM can be substantially increased if the theory contains right-handed currents. The change in helicity can then occur on the charged fermion line, so that the matrix μ will contain the charged lepton masses instead of the neutrino masses.

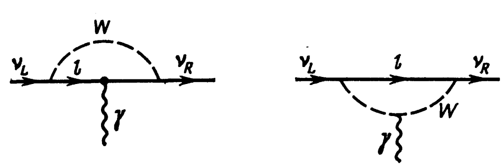

FIG. 1. Diagrams describing the origin of the neutrino EMM.

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2. As an example of a model with right-handed currents, let us examine the widely discussed scheme with left-right symmetry $SU(2)_L \times SU(2)_R \times U(1)$ (see, for example, Ref. 7). The mediator of the usual weak interaction in this scheme is the charged boson W_1 containing a small admixture of the right-handed boson W_R:

$$W_1 = W_L \cos\varphi + W_R \sin\varphi.$$

The neutrino EMM matrix is determined in this model by the masses of the charged leptons and the matrices U and V that mix the neutrinos in the left-handed (U) and right-handed (V) currents[6,8]:

$$\mu_{ij} = \frac{Gm_e}{\sqrt{2}\pi^2}\mu_B \sin 2\varphi \sum_l U_{il} m_l V_{lj}, \qquad (3)$$

where the sum is evaluated over the types of charged leptons. In the minimum model[7] of this kind with the flavor basis we have $U = V = 1$, so that μ_{ij} is diagonal:

$$\mu_{\nu_l} \approx 3.6 \cdot 10^{-10} \mu_B \sin 2\varphi (m_l/m_N).\ l = e, \mu, \tau, \ldots. \qquad (4)$$

In this scheme, the maximum EMM has a neutrino current state that appears in the doublet with the heaviest charged lepton. In the case of three generations, this is the ν_τ, for which the magnitude of μ corresponding to the maximum value $\sin 2\varphi = 0.1$ allowed by the experimental limit[9] ($\sin\varphi \lesssim 0.05$) is

$$\mu_{\nu_\tau} \approx 0.6 \cdot 10^{-10} \mu_B. \qquad (5)$$

For the mass states of the neutrinos, the EMM values in this case are obviously proportional to the fraction of the ν_τ present in them. We note that the matrix μ_{ab} is nondiagonal in the mass basis, i.e., decays of the form $\nu_2 \to \nu_1 \gamma$ should occur. It is also clear that, if there is a fourth generation, the neutrino appearing in this generation will have the highest EMM.

It is clear from the above example that matrix elements of the order of $10^{-10}\mu_B$ can be obtained in schemes involving an admixture of right-handed currents allowed by the corresponding experimental limits.[9] Of course, the mechanism for the appearance of the neutrino EMM may be quite different, and the structure of the matrix μ may be significantly different from that described by (3) and (4). In particular, in supersymmetric models, the appearance of fairly large EMM (including $\mu_{ee} \sim 10^{-10}\mu_B$) seems quite possible because these schemes include heavy fermions (winos and higgsinos), whose masses effectively turn over the helicity in graphs similar to those shown in Fig. 1.[1]

We emphasize that, strictly speaking, the EMM are not necessarily related to the masses of the neutrinos themselves. In principle, it is possible to consider the EMM of massless neutrinos without introducing contradictions although, from the point of view of the natural condition $\mu \neq 0$ for $m = 0$, this seems strange and requires special compensation by counterterms in the neutrino mass diagrams.

3. Let us now consider the limits on the matrix μ that follow from reactor experiments and astrophysical estimates. The interaction (1) produces an additional contribution to the neutrino-electron scattering cross section described by the graph of Fig. 2. The differential cross section for νe scattering due to this mechanism is the same for ν_e and $\tilde{\nu}_e$, and is given by[10]

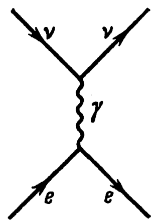

FIG. 2. Diagram showing the scattering due to the neutrino EMM.

$$\frac{d\sigma_{em}}{dT} = \left(\sum_j |\mu_{ej}|^2/\mu_B^2\right)\frac{\pi\alpha^2}{m_e^2}\left(\frac{1}{T} - \frac{1}{E_\nu}\right), \qquad (6)$$

where E_ν is the energy of the incident neutrino and T the kinetic energy of the recoil electron, which is actually the quantity recorded experimentally.

The sum over the neutrino current states in (6) corresponds to summation over the different final states of the neutrinos in the reaction $\tilde{\nu}_e e \to \tilde{\nu}_j e$.

Let us compare (6) with the well-known expression for the $\tilde{\nu}_e e$ cross section due to the weak interaction (see, for example, Ref. 11):

$$\frac{d\sigma_W}{dT} = \frac{G^2 m_e}{2\pi}\left[\left(1 - \frac{T}{E_\nu}\right)^2 (1 + 2\sin^2\theta_W)^2 + 4\sin^4\theta_W - (1 + 2\sin^2\theta_W)\sin^2\theta_W \frac{m_e T}{E_\nu}\right], \qquad (7)$$

where θ_W is the Weinberg angle. The electromagnetic and weak amplitudes do not interfere, so that the total cross section is obtained by combining the expressions given by (6) and (7). When $T \ll E_\nu$, the cross section given by (7) becomes constant, while that in (6) behaves as T^{-1}, and when $\sin^2\theta_W = 0.22$, it becomes comparable with the weak cross section for $T \simeq 0.3$ MeV if $\mu_{\text{eff}} \equiv (\Sigma |\mu_{ej}|^2)^{1/2} = 10^{-10}\mu_B$. Thus, the problem of reducing the experimental limit for μ_{eff} involves, above all, a reduction of the threshold for the detection of recoil electrons, which is complicated by the higher background at low electron energies. The experimental data reported in Ref. 12 have been used to show[13,14] that $\mu_{\text{eff}} \lesssim 2 \times 10^{-10}\mu_B$.

The best limitation on the matrix μ is obtained by considering the cooling of young white dwarfs due to the decay of plasmons γ^* to $\nu\tilde{\nu}$ pairs. The decay width due to the EMM is

$$\Gamma_\mu(\gamma^* \to \nu\tilde{\nu}) = \left(\sum_{i,j} |\mu_{ij}|^2/\mu_B^2\right)\frac{\alpha\omega_p^3}{24 m_e^2}, \qquad (8)$$

where ω_p is the plasma frequency in the star. Analysis of astrophysical data gives the following limit[15]:

$$\left(\sum_{i,j} |\mu_{ij}|^2\right)^{1/2} \leqslant 0.7 \cdot 10^{-10}\mu_B, \qquad (9)$$

which is actually the upper limit for the norm of the matrix

μ. The fact that (8) contains precisely this quantity is due to the summation over all the types of neutrino pairs. This summation, and the neglect of the neutrino masses, is justifiable if the masses are substantially smaller than $\omega_p/2$. For young white dwarfs, for which (9) was obtained, $\omega_p \simeq 30$–40 keV, so that the last assumption seems fully acceptable. It would be interesting to repeat the analysis given in Ref. 15 with more recent astrophysical data, and deduce the upper limit for $|\mu|$, which may well turn out to be more stringent than (9).

4. To conclude this section, let us consider a further aspect of the neutrino magnetic moment. The introduction of right-handed neutrinos ν_R increases the number of neutrino degrees of freedom by a factor of two, which may give rise to difficulties in explaining the primeval abundance of ^{4}He (see, for example, the review in Ref. 16 and the book by Okun'[11]). Leaving to one side the question of how serious these difficulties really are, we note that, if we allow the nonconservation of lepton number, we can avoid the introduction of the right-handed neutrino field and, instead, use the antineutrino field. The analog of (1) then assumes the "Majorana" form:

$$L_{int}={}^1/_2\tilde{\mu}_{ij}\,(\overline{\nu^c})_{Ri}\sigma_{\mu\nu}F_{\mu\nu}\nu_{Lj}+\text{h.c.} \tag{10}$$

The matrix $\tilde{\mu}_{ij}$ should then be antisymmetric because the part of the lepton operator in (10) that is symmetric in the neutrino types will then vanish. In other words, only the off-diagonal matrix moments are possible in this case. We shall not consider specific models leading to interactions such as (10), but it is not difficult to foresee that, in such models, the condition for the experimental absence of a double β-decay without the emission of neutrinos will be a very stringent limitation.

3. SPIN PRECESSION AND NEUTRINO OSCILLATIONS IN A MAGNETIC FIELD, INCLUDING THE EFFECT OF MATTER

1. We begin by considering the motion of a neutrino with energy $E \gg m$ in a vacuum, taking oscillations into account. The standard description of these oscillations follows from the fact that the momentum p in the neutrino wave function $\psi_\nu = \exp(-iEt + ipz)\psi_0$ is a matrix in the space of the neutrino types, and is related to the mass matrix by

$$p=(E^2-m^2)^{1/2}\approx E-m^2/2E \tag{11}$$

(we shall choose the phases of the ν_L and ν_R so that the matrix m is Hermitian). Let us extract the unimportant phase factor $\exp[iE(z-t)]$:

$$\psi_\nu=\exp\,[iE(z-t)]\nu(z).$$

According to (11), we then have $\nu = \exp[-im^2z/(2E)]\nu_0$, so that $\nu(z)$ satisfies the evolution equation.

$$i\frac{d\nu}{dz}=\frac{m^2}{2E}\nu. \tag{12}$$

This implies that ν is a vector (column vector) in the space of the neutrino types. To include the interaction with the magnetic field and with matter, we recall that each of the elements of the column is a spinor:

$$\begin{pmatrix}\nu_R\\ \nu_L\end{pmatrix} \tag{13}$$

(the direction of motion of the neutrino is taken to be the spin quantization axis).

2. When the electromagnetic field $F_{\mu\nu}$ is present, the interaction (1) leads to the following modification of (12):

$$i\frac{d\nu}{dz}=\mathscr{H}\nu, \tag{14}$$

where the "Hamiltonian' $\mathscr{H}$ is

$$\mathscr{H}=\frac{m^2}{2E}-\mu P_+\boldsymbol{\sigma}\cdot(\mathbf{H}-i\mathbf{E})P_- -\mu^+P_-\boldsymbol{\sigma}\cdot(\mathbf{H}+i\mathbf{E})P_+, \tag{15}$$

and $P_\pm = (1 \pm \sigma_3)/2$ are projectors onto the ν_R and ν_L states. We note that, since $P_\pm\sigma_3 P_\mp = 0$, the longitudinal components of $\mathbf{H}$ and $\mathbf{E}$ do not appear in $\mathscr{H}$. This is readily understood because the longitudinal field components remain unaltered under the transformation to the neutrino rest system, whereas the lateral components contain the factor $\gamma = E/m \gg 1$. The derivation of the Hamiltonian (15) assumes that the field $F_{\mu\nu}$ changes little over the length E^{-1}. It is also assumed that the neutrino is not deflected by field gradients, which is valid with high precision in all realistic situations.

We note that we deliberately retained the difference between μ and μ^+ in (15) because the phases of the mass matrix were fixed. If it were possible to observe the phase difference between neutrinos oscillations in the presence and absence of the field, it would be possible to find the relative phases of the elements of the matrices m and μ. Nonzero values of these phases would correspond to CP violation in the neutrino sector.

It is immediately clear from (14) and (15) that, for the mass states ν_a in a transverse magnetic field $\mathbf{H}_\perp$, there are oscillations between left- and right-handed components (spin precession) with frequency $\omega(z) = |\mu_{aa}||\mathbf{H}_\perp(z)|$. If the region in which $\mathbf{H}_\perp$ is present (its direction is independent of z) intercepts a beam of the ν_L, the numbers of the ν_L and ν_R are given by

$$N_L(z)=N_0\cos^2\left(\int^z \omega(z)\,dz\right),\quad N_R=N_0\sin^2\left(\int^z \omega(z)\,dz\right). \tag{16}$$

In the case of Dirac masses that we are considering, precession will occur in an arbitrarily weak transverse field because the left- and right-handed neutrinos are degenerate in energy. Oscillations between the mass states $\nu_{aL}\leftrightarrow\nu_{bR}$ are possible because of the diagonal terms in μ_{ab}. However, these oscillations have appreciable amplitudes only when the mixing element $|\mu_{ab}||\mathbf{H}_\perp|$ is comparable with the difference between the diagonal elements of the Hamiltonian (19):

$$|\mu_{ab}|\,|\mathbf{H}_\perp|\gtrsim\left|\frac{m_a^2-m_b^2}{2E}\right|. \tag{17}$$

To estimate the order of magnitude of the various quantities in this expression, we note that $\Delta m_{ab}^2 \lesssim 10^{-7}\,\text{eV}^2$ for $|\mathbf{H}_\perp| = 10^3$ G, $|\mu_{ab}| = 10^{-10}\mu_B$, and $E = 10$ MeV.

3. Finally, let us consider the general case where the

neutrino propagates in matter (it is precisely this case that is interesting from the point of view of solar neutrinos). Only the coherent interaction between the neutrinos and matter is significant because the noncoherent neutrino interaction cross section is exceedingly small. Coherent effects are conveniently examined directly at the level of the Lagrangian. The weak interaction neutrino Lagrangian is

$$L_W = -\frac{G}{\sqrt{2}}(\bar{\nu}_e\gamma_\mu(1+\gamma_5)\nu_e)(\bar{e}\gamma_\mu(1+\gamma_5)e) - \frac{2G}{\sqrt{2}}\sum_i \bar{\nu}_i\gamma_\mu(1+\gamma_5)\nu_i J_\mu^n - \frac{4G}{\sqrt{2}}\sum_i T_3^R\bar{\nu}_i\gamma_\mu(1-\gamma_5)\nu_i J_\mu^n. \quad (18)$$

The first term is due to the interaction between the charged currents of the electron-neutrino and the electron, and the second is due to the interaction with the neutral current

$$J_\mu^n = J_\mu^3 - \sin^2\theta_W J_\mu^{em} \quad (19)$$

(J^{em} is the electromagnetic current and J^3 is the current of the third component of weak isospin). Finally, the third term in (18) occurs when the third component of weak isospin of right-handed neutrinos, T_3^R, is not zero. The next step is to average the Lagrangian (18) over matter. For matter at rest, which does not have a macrosocopic spin magnetic moment, only the average of the time component of the vector part of the current is nonzero. We thus have

$$\langle\bar{e}\gamma_\mu(1+\gamma_5)e\rangle = \delta_{0\mu}n_e, \quad (20)$$

where n_e is the electron density. For electrically neutral matter, (19) shows that neutral currents cancel out for protons and electrons, and only neutrons provide a contribution to $\langle J_\mu^n\rangle$:

$$\langle J_\mu^n\rangle = -{}^1/_4\delta_{0\mu}n_n. \quad (21)$$

As a result, the neutrino equation of motion, obtained by averaging the Lagrangian over matter, corresponds to the following Hamiltonian in (14):

$$\mathcal{H} = m^2/2E - \mu P_+\boldsymbol{\sigma}(\mathbf{H}-i\mathbf{E})P_- - \mu^+ P_-\boldsymbol{\sigma}(\mathbf{H}+i\mathbf{E})P_+ + C_L P_- + C_R P_+, \quad (22)$$

where the matrices C_L and C_R for the interactions between left- and right-handed neutrinos have a diagonal form in the flavor basis:

$$(C_L)_{ij} = \delta_{ei}\delta_{ej}\sqrt{2}Gn_e - \delta_{ij}Gn_n/\sqrt{2}, \qquad (C_R)_{ij} = -\delta_{ij}\sqrt{2}Gn_n T_3^R. \quad (23)$$

4. Let us begin by considering the case where there is no electromagnetic field, so that the helicities are not reversed. The matrix C_L then describes the effect of matter on the oscillations of left-handed neutrinos, first considered by Wolfenstein.[17] The contribution of neutrinos to C_L is then the same for all neutrino types, and is unimportant for oscillations, whereas the contribution of electrons leads to a striking and potentially exceedingly important effect, discovered by Mikheev and Smirnov,[18] which can be summarized as follows. Let us suppose that $(m^2)_{ee}$ is smaller than $(m^2)_{\mu\mu}$ or $(m^2)_{\tau\tau}$ (to be specific, we shall consider ν_e and ν_τ). If the density n_e is such that

$$\sqrt{2}Gn_e = [(m^2)_{\tau\tau} - (m^2)_{ee}]/2E, \quad (24)$$

The difference $\mathcal{H}_{\tau\tau} - \mathcal{H}_{ee}$ between the diagonal matrix elements is then zero. This corresponds to "level crossing," well-known in quantum mechanics. If $n_e(z)$ varies sufficiently slowly and passes through the value given by (24), and if there is slight mixing ($\mathcal{H}_{e\tau}$), neutrinos of a given type will adiabatically transform into neutrinos of the other type. In our example, $\nu_1 \simeq \nu_e$ will transform to $\nu_2 \simeq \nu_\tau$. The adiabatic condition then requires that the characteristic length l for a change in n_e must satisfy the condition

$$l \gg 2E/(m^2)_{e\tau}. \quad (25)$$

Mikheev and Smirnov have analyzed numerically the solution of the neutrino evolution equation and found[18] that the mechanism they have discovered is effective for solar neutrinos in a very wide range of values of Δm^2 and mixing angles. (Typical intervals for $E \simeq 10$ MeV are 10^{-6} eV$^2 \lesssim \Delta m^2 \lesssim 10^{-4}$ eV^2, $\sin^2 2\theta \gtrsim 10^{-3}$).

5. When spin precession in a magnetic field is considered, the difference between C_L and C_R becomes significant because it lifts the degeneracy of the left- and right-handed components. Let us consider the simplest case of one neutrino type in a uniform magnetic field $|\mathbf{H}_\perp| = H =$ const in constant-density matter, so that $\Delta C = C_L - C_R =$ const. Discarding the unimportant phase factor, the solution of (14) can then be written in the form

$$\nu(z) = \exp\{i(\mu H\sigma_1 - \Delta C\sigma_3/2)\}\nu_0 = [\cos\omega z + i(\mu H\sigma_1 - \Delta C\sigma_3/2)\omega^{-1}\sin\omega z]\nu_0, \quad (26)$$

where $\omega = [(\mu H)^2 + (\Delta C/2)^2]^{1/2}$. If the initial state ν_0 is a pure left-handed state, the number of left- and right-handed neutrinos at the point z is then, respectively, given by

$$N_L = \cos^2\omega z + \cos^2 2\theta_H \sin^2\omega z, \quad N_R = \sin^2 2\theta_H \sin^2\omega z, \quad (27)$$

where $\tan 2\theta_H = 2\mu H/\Delta C$. These two expressions correspond to the usual picture of the oscillations with mixing angle θ_H. It is clear that the mixing angle is a maximum ($|\theta_H| = \pi/4$) for $\Delta C = 0$, and that this angle is always less than $\pi/4$ for $\Delta C \neq 0$. The necessary condition for effective modulation of the left-handed neutrino flux is therefore

$$2\mu H > \Delta C. \quad (28)$$

When $\mu = 10^{-10}\mu_B$ and $H = 10^3$ G, this condition demands that the electron density must be less than 10^{22} cm^{-3} in the case of the electron-neutrino, and ΔC is due to the interaction between charged currents. For the other neutrino types, the limit on the neutrino density for $T_3^R = 0$ is $n_n \lesssim 2\times 10^{22}$ cm^{-3}. If, on the other hand, $T_3^R = 1/2$, the left- and right-handed nonelectron neutrinos are degenerate even in the presence of matter of any density.

In the opposite limit from (28), i.e.,

$$\Delta C \gg 2\mu H \quad (29)$$

spin precession is strongly suppressed, and the beam consists mostly of the ν_L.

4. THE SUN AND POSSIBLE CONSEQUENCES FOR SOLAR NEUTRINOS

1. Neutrino spin precession cannot be observed under terrestrial laboratory conditions because, when $\mu = 10^{-10}\mu_B$, the field H necessary to turn the spin through an angle of the order of unity over a path length L is given by

$$HL \sim \mu^{-1} \approx 3 \cdot 10^{13} \text{ G}\cdot\text{cm}. \quad (30)$$

Searches for this phenomenon must therefore be concentrated on natural neutrino sources in which there are large-scale natural magnetic fields. The nearest source of this type is the Sun, and we therefore begin with a brief account of the relevant parts of the theory of the Sun that are significant for the spin precession of solar neutrinos.

The solar radius $R_\odot$ is about 7×10^{10} cm. The core, in which the nuclear reactions providing the Sun with its energy take place, accounts for about a quarter of the radius. We note, however, that neutrino generation occurs in this region mostly as a result of the reaction $pp \to d + e^+ + \nu_e$, in which the maximum energy of the ν_e is 0.42 MeV, and is below the detection threshold of the Cl–Ar method.[1] High-energy neutrinos from ^{7}Be and ^{8}B, recorded in Ref. 1, originate from the central, hottest part of the core, whose radius amounts to only $\sim 3\cdot10^9$ cm (see Ref. 19 for the flux calculations). The radiative transfer zone (so called because of the way in which heat is removed from the solar core) extends up to $R \simeq 0.7R_\odot$. Finally, the last 2×10^{10} cm of the solar radius correspond to the convective zone in which heat is transferred by turbulent convection. This zone contains the currents responsible for global magnetic fields with the 22-year (quasi) periodicity, and the modulus of the magnetic field has an 11-year cycle. During solar-active years, the magnetic field in the convective zone has a toroidal structure (it points in the azimuthal direction). The strength of this field can be judged from the sunspot field[20] ($H \simeq 2\times10^3$–4×10^3 G). The sunspots are regions in which the lines of force of the field that has "floated up" to the surface either leave or enter the Sun. It is therefore very likely that, during the years of solar activity maximum, the magnetic field in the convective zone is of the order of a few kG. The field H may increase to some extent between the surface and the bottom of the convective zone. The 22-year component of the field cannot penetrate the radiative heat-transfer zone because of the jump in the magnetic permeability (~ 1 in the radiative transfer zone and $\sim 10^{-5}$ in the convective zone).[21] While the Sun remains quiet at the minimum of the 11-year cycle, the field in the convective zone decreases by at least an order of magnitude.[2)] (A more detailed account of the structure of the Sun and its magnetic field can be found in the literature.[20,21])

2. Thus, the product of the average (within the convective zone) field H and the depth L of the zone may reach $HL \simeq 3\times10^{13}$–10^{14} G·cm. The estimate given by (30), therefore suggests that, for $\mu = 10^{-10}\mu_B$, the flux of left-handed neutrinos measured in Ref. 1 was effectively modulated during the 11-year cycle. If the product μHL does not reach $\pi/2$, the flux recorded for the active Sun should be a minimum; the maximum flux should, at any rate, be reached at the activity minimum. The possible presence of this type of anticorrelation first emerged as a result of the analysis of experimental data[1] in Ref. 22 (see Ref. 23 for a discussion of possible time variations in the data of the Davis group with other periods). Figure 3 shows a graph taken from Ref. 1, in which the neutrino flux data are compared with the number of sunspots characterizing the solar activity. There is a clear reduction in the neutrino flux during the solar activity maximum in 1979–1980. However, the experimental uncertainties are very large and the statistical significance of the anticorrelation effect is still not clear.

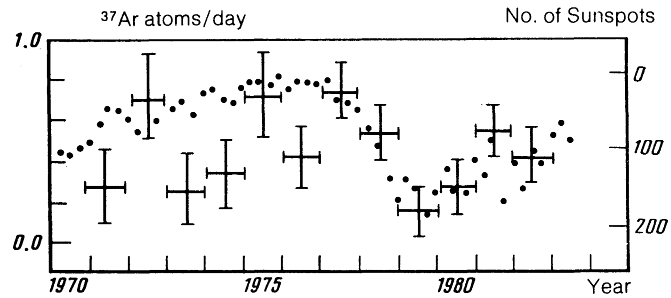

FIG. 3. Graph taken from Ref. 1 and showing the rate of production of ^{37}Ar as a function of time (annual averages are reproduced) and the number of sunspots (right-hand scale).

Another possible effect[4] requiring a shorter time of observation is the seasonal (semiannual) variation in the recorded flux of high-energy neutrinos. This effect is due to the fact that the sign of the magnetic field in the convective zone is different in the Northern and Southern Hemispheres, and is therefore zero at the equator. The transition region covers the latitude interval $\pm(5$–$7^\circ)$, which corresponds to distances of $\pm 6\cdot10^9$–8×10^9 cm from the equator. (The reduction in the field near the equator is reflected in the fact that there are no sunspots at these latitudes.) On the other hand, the plane of the Earth's orbit (the plane of the ecliptic) makes an angle of 7°15′ with the plane of the solar equator. This means that, when the Earth lies in the plane of the solar equator (at the beginning of June and the beginning of December), the central part of the core, whose diameter is 3×10^9 cm and in which the boron and beryllium neutrinos originate, is seen from the Earth through the equatorial "slit" in the magnetic field, and the flux of left-handed neutrinos should be a maximum. Conversely, at the beginning of March and the beginning of September (maximum distance from the plane of the solar equator), the strong-field region will lie in the field of view of the terrestrial neutrino detector during the years of the active Sun, and the recorded flux should change because the neutrino helicity is reversed. (When $\mu HL \lesssim \pi/2$, the minimum current should occur at the beginning of March and of September.) Seasonal variations should become weaker as the solar activity falls, and may disappear altogether during the years of the quiet Sun.

We emphasize that the last effect is not well-defined for pp-neutrinos because the size of the region in which these

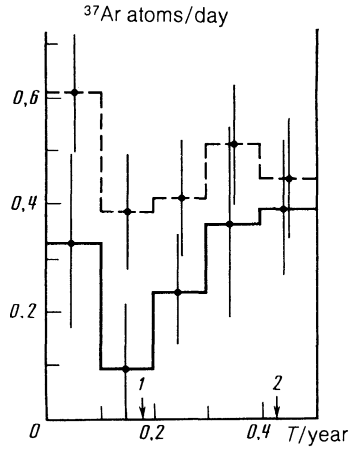

FIG. 4. Semiannual phase diagrams for the solar active years[1] 1979–1980 (solid line) and for 1975–1978 and 1983–1984 (dashed line). Arrow 1 shows the time of maximum distance of the Earth from the plane of the solar equator and arrow 2 shows the time when the Earth crosses this plane.

neutrinos originate ($R_\odot/4 \simeq 1.7\times10^{10}$ cm) is greater than the size of the equatorial gap in the magnetic field.

Figure 4 shows the semiannual phase diagrams for the 1975–1984 data.[1] It is clear that the flux was lower during the years of the quiet Sun near the beginning of March and of September. At other times, the reduction in the flux at these times is less well-defined (if is exists at all). We note, however, that the uncertainties in these data are large, and the existence of the seasonal variations cannot be deduced with a high degree of statistical confidence. Nevertheless, it is interesting that, if we remove from the 1979–1981 data, and from the data for the first half of 1982, all the series for which the dates of maximum sensitivity are closest to March 5 and September 5 (a total of 7 series), the average rate at which the ^{37}Ar atoms were created amounts to only 0.11 ± 0.08 atoms/day, whereas the average over all the counting-rate data was 0.45 ± 0.04 atoms/day. The difference between these two numbers is more than 3.5 standard deviations. (We note that the background estimated in Ref. 1 was 0.08 ± 0.03 atoms/day.)

3. At least three conditions must be met for the above effects to be present.

(1) The neutrinos entering the convective zone must have high enough magnetic moments so that $\mu HL \gtrsim 1$.

(2) The interaction with the matter present in the convective zone must not appreciably suppress the neutrino spin precession.

(3) The neutrinos must not be depolarized by the constant magnetic field in the solar interior (if it is present).

Let us consider condition (2) first. The density ρ of matter in the convective zone varies, according to the model proposed in Ref. 20, from about 0.2 g/cm^3 at the bottom of the zone to about 10^{-7} g/cm^3 at the surface. At half depth (10^{10} cm), it amounts to about 0.05 g/cm^3. About 75% of the mass in this region[3)] is accounted for by hydrogen and 25% by ^{4}He. Hence, near the bottom of the zone, the electron density n_e is about 10^{23} cm^{-3} and the neutron density is $n_n \simeq 10^{22}$ cm^{-3}. In view of (28), it is clear from the above estimates that the necessary condition for (2) to be satisfied in the case of the electron-neutrinos is that the magnetic field at the bottom of the zone must be of the order of 10 kG, whereas, for the ν_μ and ν_τ, the field can be weaker by an order of magnitude (all the estimates were obtained for $\mu = 10^{-10}\mu_B$).

If it turns out that variable fields of the order of 10 kG are not present near the bottom of the convective zone, or that the $\mu_{\nu e}$ is much smaller than $10^{-10}\mu_B$ (the latter may become clear as a result of a further increase in the precision of reactor experiments), the above effects may occur provided there is considerable mixing between ν_e and, for example, ν_τ ($\theta \simeq \pi/4$), and the oscillation length is substantially less than $R_\odot$. In that case, the convective zone will receive a noncoherent mixture of the mass states

$$\nu_1 = \nu_e \cos\theta + \nu_\tau \sin\theta, \qquad \nu_2 = -\nu_e \sin\theta + \nu_\tau \cos\theta,$$

for which (a) the interaction with electrons is suppressed (by the factors $\cos^2\theta$ and $\sin^2\theta \simeq 1/2$, respectively) and (b) their EMM may be of the necessary order of magnitude because of the presence of a large ν_τ admixture (see Section 2 for a discussion of the model with left-right symmetry), which ensures that condition (1) is satisfied.

Finally, it may turn out that the Mikheev-Smirnov mechanism[18] is working, and only the mass state ν_2 enters the convective zone. The probability of detecting such neutrinos in the experiment reported in Ref. 1 is then $\sin^2\theta$ and, when $\sin^2\theta < 1/3$, the estimated initial flux of solar neutrinos must be increased as compared with Ref. 19. On the other hand, conditions (1) and (2) with small θ are obviously more readily satisfied for ν_2 than for ν_e. We note that, since the effectiveness of the Mikheev-Smirnov mechanism[18] depends on the neutrino energy [see (24)], different relationships may, in general, be possible between the field modulation depth of the fluxes of high-energy and *pp*-neutrinos.

Finally, let us consider condition (3). The fact that the flux observed by the Davis group is so low as compared with the theoretical calculations[19] is sometimes explained[24] by supposing that the solar core contains a frozen-in (primeval) magnetic field of the order of 10^7 G, and that ν_e has an EMM of about $10^{-13}\mu_B$, so that the neutrinos are completely depolarized by the magnetic field (and the flux of left-handed neutrinos is reduced by a factor of two). The estimates given in Section 3 [inequalities (28) and (29)] show that the interaction between ν_e and matter then excludes depolarization even for $\rho \gtrsim 0.1$ g/cm^3, which is much lower than the density in the solar core (up to about 150 g/cm^3 at the center). The conclusions given in Ref. 24 are therefore invalid. If we adopt our value $\mu \simeq 10^{-10}\mu_B$, the condition for the absence of depolarization by the frozen-in field is more stringent (but can be satisfied). Actually, the random primeval field in the solar interior should be $H_{pr} \sim \rho^{2/3}$ (the condition for the frozen-in flux in matter). Hence, *to satisfy* condition (3), it is sufficient to satisfy the inequality given by (33) for the lowest density, i.e., on the surface of the radiative transfer zone (where, we recall, $\rho \simeq 0.2$ g/cm^3). If we take into account the estimates given above, this leads us to an upper limit for the frozen-in primeval field immediately under the surface of the radiative transfer zone: $H_{pr} \lesssim 10^4$ G for ν_e and $H_{pr} \lesssim 10^3$ G for ν_μ and ν_τ. If $H_{pr} \sim \rho^{2/3}$, the corresponding limits on the frozen-in field near the solar center are $H_{pr} \lesssim 10^6$ G and $H_{pr} \lesssim 10^5$ G. As far as we know, there is no evidence against these limits for H_{pr}.

5. MAGNETIC MOMENT AND DETECTORS OF SOLAR NEUTRINOS

It is clear from the foregoing discussion that, because the neutrino mass and electromagnetic matrices are not adequately known, there is a range of possibilities, and a choice must be made between them by using different detectors with different energy thresholds. For example, if the ν_e has an EMM $\leqslant \sim 10^{-10} \mu_B$, and oscillations are unimportant, the boron and beryllium neutrinos detected with the Cl–Ar detector should exhibit 11-year and semiannual flux variations, whereas for *pp*-neutrinos (Ga–Ge detectors[25] are being built for these neutrinos and superconducting indium detectors are being developed[26]), there are only the 11-year variations and the semiannual variations should be weak. If oscillations and the Mikheev-Smirnov effect are significant, and other types of neutrinos (e.g., ν_τ) are the only ones to have a magnetic moment, then, because of the energy dependence of the Mikheev-Smirnov effect,[18] the 11 year variation in the observed *pp*-neutrino flux can be either reduced or amplified in comparison with variations in the high-energy neutrino flux.

The liquid argon detector ICARUS,[27] which is sensitive to νe scattering at neutrino energies above ~ 5 MeV, is very promising for studies of solar neutrinos. This detector will give information on the flavor composition of the high-energy component of solar neutrinos, and will thus settle the question of the role of oscillations and the Mikheev-Smirnov effect.

From the point of view of the time varations in the solar neutrino flux, which we have examined in this paper, the new detectors must begin to acquire the necessary data by the end of the 1980s because the next solar activity maximum is expected about 1990. As noted above, semiannual variations in the flux during this period should be particularly well-defined.

Because the solar activity amplitude is irregular, it is at present difficult to forecast the modulation depth of the neutrino flux during the forthcoming solar activity maximum (the 1979–1980 peak was strong, judging by the number of sunspots). In this respect, it is very interesting to consider the possible solution of the converse problem, i.e., the problem of monitoring the magnetic field in the interior of the Sun by measuring the neutrino flux. If it were possible to establish independently the electromagnetic parameters of the neutino (e.g., by measuring the EMM of the ν_e in reactor experiments), this would enable us to perform quantitative studies of magnetic fields in the solar interior. At any rate, the time variation in the flux of solar neutrinos, regarded as a manifestation of the electromagnetic properties of these neutrinos, appears to us to deserve further theoretical and experimental study.

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