Low-energy supergravity and the light *t*-quark

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A number of realistic SU(3)×SU(2)×U(1) models of low-energy spontaneously broken $N=1$ supergravity (SUGRA) with different Kähler potentials, including superstring SUGRA with the Witten potential, are considered. As a rule, the models suffer from two defects; (a) their Lagrangians depend on a number of unknown parameters (gravitino mass $m_{3/2}$, gaugino mass $\widetilde{M}_{1/2}$, etc.) and (b) they give rise to a cosmological constant $\Lambda = U_{\min}$ which is greater than the astrophysically admissible limit by tens of orders of magnitude. This paper presents a solution of the renormalization-group equations which determine the mass of the Higgs scalars and their superpartners, i.e., the quarkino, leptino ($s=0$), and Higgs guagino ($s=1/2$), in terms of unknown parameters of the theory. A class of no-scale SUGRA models is considered in which the parameter $m_{3/2}=m^0_{3/2}$ is determined by minimizing the potential $U_{\min}(m_{3/2})$. The only reasonable one among them is the superstring variant of the theory, which leads to a unique set of parameters for which $\Lambda = U_{\min}(m^0_{3/2})=0$. Numerical results are reported for a similar but more approximate model in which $\Lambda = U_{\min}(m^0_{3/2})=0$.

1. INTRODUCTION

In recent years, models based on supersymmetry (SUSY) have occupied a dominant position in the theory of elementary particles (see the review given in Ref. 1). They provide a natural solution of the problem of quadratic divergences in the Weinberg-Salam theory, and of the problem of "hierarchy" and "fine tuning" in grand unification theories (GUT). Supergravity models (SUGRA), in which SUSY is spontaneously broken[2] (as is, indeed, the case in the real world), are aesthetically particularly attractive because of the appearance of the vacuum average $z_0 = \langle z(x)\rangle$ of a scalar field $z(x)$ that is not observable physically because of its interaction with the graviton and gravitino fields. This is the so-called super-Higgs effect[2] in the "hidden" sector of the theory,[3] which appears on an energy scale of the order of Planck's mass $M_P\sim 10^{19}$ GeV.

The supersymmetric Lagrangian of chiral, $s^1=(z^i, \chi^i)$, and vector (gauge), $V_0=(V_a, \lambda_a)$, superfields have the following general form[1]:

$$\mathscr{L} = \int \Phi(s^i \exp(g_a V_a), \times \exp(g_a V_a) s_i^+) d^4\theta + \left\{\int [f(s^i) + \chi(s^i) W_a W_a] d^2\theta + \text{h.c.}\right\},$$

where $\Phi(z^i, z_i^+)$, $f(z^i)$, $\chi(z^i)$ are unknown functions [$f(z^i)$ is called the superpotential] and W_a are the chiral intensities[1] of the vector superfields. The chiral superfields $s^0=(z,\chi^0)$, where $z=z^0(x)$ for $i=0$, constitutes the "hidden" sector of the theory, whereas the other $s^i=(y^i,\chi^i)$, $i=1, 2, \ldots$ correspond to two physically observable fields [quark, lepton, or Higgs fields; for them, $z^i\to y^i(z)$].

The Lagrangian $\mathscr{L}$ expressed in terms of the superfield components turns out[1] to depend, apart from $\chi(s^i)$, on the single function

$$G(z^i, z_i^+) = -3\ln\Phi(z^i, z_i^+) + \ln|f(z^i)|^2,$$

which is called the Kähler potential. Thus, the part of the Lagrangian corresponding to the sector of scalar fields $z^i \equiv (z, y^i)$ has the form

$$\mathscr{L}_S = M_P^2 G_i^j \partial_\mu z^i \partial_\mu z_j^+ - U(z^i),$$

where

$$U(z^i) = M_P^4 (\exp G)[G_i (G_i^j)^{-1} G^j - 3] + D\text{-terms} \quad (1)$$

is the potential of the scalar fields z^i, where $G_i = \partial G/\partial z^i$, $G^j = \partial G/\partial z_j^+$, $G_i^j = \partial^2 G/\partial z^i \partial z_j^+$, and the D-terms [of the form $(g_a^2/4) D_a^2$] are due to gauge extensions of the derivatives. The function $G(z, z^+)$ also determines the interaction between the field $z(x)$ and the gravitino field ($s=3/2$), as well as the mass of the latter in the form

$$m_{3/2} = M_P [\exp(G/2)]_{\substack{z=z_0 \\ y^i=0}} = M_P |f(z_0)| / \Phi^{3/2}(z_0, z_0).$$

We shall now list certain variants of the theory that correspond to different choices of $G(z^i, z_i^+)$.

(a) The "minimal" choice is $-3\ln\Phi = z^i z_i^+/M_P^2$, which corresponds to $G_i^j = \delta_i^j/M_P^2$, i.e., the canonical form

$$|\partial_\mu z|^2 + \sum_{i=1}^n |\partial_\mu y^i|^2$$

of the kinetic term in $\mathscr{L}$. We then have

$$G(z^i, z_i^+) = z^i z_i^+/M_P^2 + \ln|f(z^i)|^2,$$

where $M_P^3 f(z) = h(z) + \widetilde{W}(y^i)$, where $h(z) \gg \widetilde{W}(y^i)$, since $h(z)\sim m_0 M_P^2$ and $\widetilde{W}(y^i)\sim (y^i)^3 \sim m_0^3$; here and in what follows, m_0 is a quantity on the scale of the mass of the W (or Z) boson of electroweak theory ($m_0\sim 100$ GeV, $M_P\sim 10^{19}$ GeV). When $h\gg\widetilde{W}$, we see from (1) that $U(z^i)\simeq V_0(z) + V(y^i)$, where

$$V_0(z) = \exp\left|\frac{z}{M_P}\right|^2 \left\{\left|\frac{\partial h}{\partial z} + \frac{z^+ h(z)}{M_P^2}\right|^2 - 3\left|\frac{h(z)}{M_P}\right|^2\right\}$$

and

$$V = V_{\text{SUGRA}}(y^i) = \sum_i \left|\frac{\partial W}{\partial y^i} + m_{3/2} y_i^+\right|^2 + (A-3) m_{3/2}(W+W^*). \quad (2)$$

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In these expressions, $A=\sqrt{3}z_0/M_P\sim 1$, $\widetilde{W}(y^i)=(m_{3/2}/m_0)\widetilde{W}(y^i)$ is the superpotential of the fields y^i, $m_{3/2}=[\exp(A/2)]m_0$, and z_0 is the value of the field $z=z(x)$ for which the potential $V_0(z)$ is a minimum. The value of $V_0(z)$ at the minimum determines (in the classical approximation) the size[1)] of the cosmological constant: $\Lambda=U(z_0^i)\simeq V_0(z_0)$.

(b) "No-scale" SU(1, 1) supergravity corresponds to the Kähler function

$$G=-3\ln(z+z^+)/M_P+y^iy_i^+/M_P^2+\ln|[h_0+\widetilde{W}(y^i)]/M_P^3|^2, \quad (3)$$

where $h_0=m_0M_P^2=\text{const}$ ($m_0\ll M_P$), for which $V_0(z)\equiv 0$, i.e., $U(z^i)\equiv V(y^i)$ has exactly the form given by (2) with $A=3$, and $m_{3/2}\simeq(M_P/2z_0)^{3/2}m_0$ [and, as in (2), $\widetilde{W}=(m_{3/2}/m_0)\widetilde{W}(y^i)$]. In this form, the function $G(z^i, z_i^+)$ given by (3) is not very different from the usual function[4,5] in which the first two terms in (3) are replaced with the SU(*N*, 1) symmetric term $3\ln(z+z^+-y^iy_i^+/3M_P)/M_P$, which leads to the potential (2), from which the terms proportional to $m_{3/2}$ are absent (although $m_{3/2}\neq 0$), i.e.,

$$V=V_{\rm SUSY}=\sum_i\left|\frac{\partial W}{\partial y_i}\right|^2+D\text{-terms}.$$

We shall not, in the ensuing analysis, use the potential in the form $V=V_{\rm SUSY}$ because the absence of the parameter $m_{3/2}$ substantially reduces the possibilities of the theory.

With the Kähler function given by (3), the depth of the minimum of the potential $V=V_{\rm SUGRA}$ (y_0^i) corresponding to the Higgs fields is shown by (2) to depend on the quantity $m_{3/2}=(M_P/3z_0)^{3/2}m_0$, i.e., on the vacuum expectation value $z_0=\langle z(x)\rangle$ of the hidden-sector field. This, in fact, defines z_0 through the condition that $V_{\min}=V(y_0^i)=f(m_{3/2})$ should be a minimum as a function of $m_{3/2}$, i.e., as a function of z_0 (see Section 5 below and Fig. 1).

(c) In the low-energy SU(3)×SU(2)×U(1) theory, which arises[6] without the intermediate SU(5) during the compactification of the 10-dimensional SUGRA "matched" to the superstring theory,[7] there is, as noted by Witten,[8] a Kähler function similar to (3) and containing not only $z(x)$ but also the further hidden-sector field $s(x)$:

$$G=-3\ln(z+z^+)/M_P+y^iy_i^+/M_P^2-\ln(s+s^+)/M_P+\ln|[h_0w_0(s)+\widetilde{W}(y^i)]/M_P^3|^2, \quad (4)$$

where $h_0=m_0M_P^2=\text{const}$ [as in (3)] and $w_0(s)=1+\beta_0\exp(\gamma s/M_P)$; β_0, γ are numbers of the order of unity. In the usual form of the theory, the first two terms are replaced with $-3\ln[(z+z^+-y^iy_i^+)/3M_P)/M_P]$, but this SU(*N*, 1) symmetric variant in which $V_{\rm SUGRA}\to V_{\rm SUSY}$ will not be considered here. The essential point is that the superpotential $W(y^i)$ was obtained for this case in Ref. 8 in the form of a purely cubic function of physical fields y^i: $\widetilde{W}(y^i)=\lambda_{ijk}y^iy^jy^k$. If we substitute (4) in (1), we obtain $U(z^i)=V_1(s)+V(y^i)$, where $V(y^i)=V_{\rm SUGRA}$ which is the same potential as (2) with $A=3$, $W=(m_{3/2}/m_0)\widetilde{W}(y^i)$, and $V_1(s)=|2s\,dw_0/ds-w_0(s)|^2$. At the minimum, the potential $V_1(s)$ is equal to zero for $s=s_0=\langle s(x)\rangle$, where $2s_0(dw_0/ds)_{s_0}=w_0(s_0)$. Hence, here again, $U(z_i)\equiv V(y^i)$ and everything subsequently reduces to (3) with the only difference that, now, $m_{3/2}=(M_P/2z_0)^{3/2}m_0'$ and $m_0'=(M_P/2s)^{1/2}m_0$.

We now proceed as follows. Assuming that the unification of all the interactions takes place in a unified SUGRA theory (GUT) on the scale $Q^2=M_{\rm GUT}^2$ (where $\alpha_1=\alpha_2=\alpha_3=\alpha_0$, $M_{\rm GUT}\sim 10^{-3}M_P$), we set the composition of the SU(3)×SU(2)×U(1) and, having solved the renormalization-group equations, we obtain the masses of the Higgs scalars and the superpartners of ordinary particles (i.e., leptino-quarkino and Higgs-gaugino) as functions of the "input" parameters, i.e., $m_{3/2}$ and [in version (a) of the theory] the constants $A=\sqrt{3}z_0/M_P$ and B. The latter constant is a factor in the part of the potential (2) that is quadratic in the field, where $B=2$ if $A=3$ and $\widetilde{W}(y^i)$ is almost quadratic [or $B=3$ if $\widetilde{W}(y^i)$ is cubic]. Moreover, because of the term $\chi(s^i)W_aW_a$ in the Lagrangian $\mathscr{L}$, the theory has one further parameter, namely, $\widetilde{M}_{1/2}=\tilde{\gamma}m_{3/2}$, $\tilde{\gamma}\sim 1$, which determines the masses M_1, M_2, and M_3 of the gau-

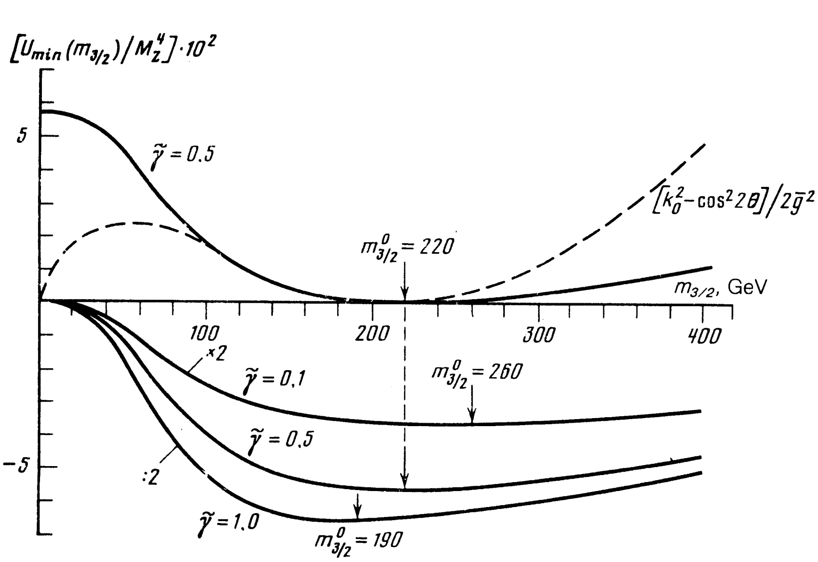

FIG. 1.

ginos, i.e., spinor partners of gauge fields. When $Q^2 \simeq M^2_{\rm GUT}$, all the M_a coincide and are equal to $\tilde{M}_{1/2}$. We also recall that a SUSY theory must have at least[9,10] two doublets of Higgs scalars, $y^1 = H_1$ and $y^2 = H_2$, in order to break the SU(2) symmetry of electroweak theory. If the part of the superpotential that contains them is bilinear, $W(y^i) \to W_2 = \mu_0(H_1, H_2^c)$, a further unknown parameter will appear, namely, $\mu_0 = m_{3/2}, x \sim 1$. If W in (3) or (4) is trilinear in the fields (evidently, this is required[8] for matching to superstring theory), then

$$W(y^i) \to W_3 = \lambda N(x)\,(H_1(x) H_2^c(x)),$$

where $N(x)$ is a new scalar field and, instead of μ_0, we have $\mu = \lambda N_0$, where $N_0 = \langle N(x) \rangle$ is determined by the vacuum expectation values of the Higgs fields H_1 and $(H_2^c)^i = \varepsilon^{ij} H_2^j$.

In addition to the mass parameters $m_{3/2}$, $M_{1/2}$, and μ_0 (or μ), one of which is determined by the mass of the Z boson, $M_z = 94$ GeV, and the parameters A, B [which appear in the early form of the theory (a)], the Yukawa coupling constant of the t-quark $h_{t_0} = h_t(M^2_{\rm GUT})$ plays an important part here and determines the evolution of all the quantities with Q^2, i.e., the square of the virtual momenta of the particles. We shall choose it so that the mass of the t-quark has the value obtained by the UA1 group at CERN: $m_t = (40 \pm 10)$ GeV.

In models (3) and (4), the scale $m_{3/2}$ is generated dynamically by the minimal condition applied to $V_{\min} = f(m_{3/2})$ in the form[4,5] $m_{3/2} = M_{\rm GUT} \exp(-1/h^2_{t_0})$, and $A = 3$, $B = A - 1$ (or $B = A$ for $W \to W_3$). For given m_t and M_z, we can therefore determine all the parameters theoretically, except for μ_0. In the superstring form of SUGRA, this quantity is determined because, instead of μ_0, we have the constant λ (or $\mu = \lambda \langle N \rangle$), which is uniquely determined by the condition that the cosmological constant is $\Lambda = V_{\min}(m_{3/2}) = 0$.

2. SU(5) MODEL OF SUGRA GUT AND THE PARAMETER *B*

Let us now consider an example of the grand unification theory (GUT), which is "immersed" in supergravity (SUGRA) with a hidden sector and leads (as a result of supersymmetry breaking by the super-Higgs effect) to the low-energy effective Lagrangian used below at energies less than the unification scale. Thus, first, we shall construct a simple realistic SUGRA GUT and, second, demonstrate the origin of the parameter B in the case of the "minimal" Kähler potential.[2)]

We confine our attention to the sector of Higgs scalars because matter multiplets (quarkinos-quarks) and gauge fields are introduced in a standard manner. For the simplest SU(5) theory, this sector consists of the 24-plet $\hat{\Phi} = \hat{t}_a \hat{\Phi}_a$, the 5-plet $H(x)$, and the anti-5-plet $H'(x)$ (the lowest components of these 5-plets form the doublets H_1^i and $(\varepsilon H_2^*)_i = \varepsilon_{ij} H_2^j$, $i, j = 1, 2$). The superpotential made up of these fields has the following form in the most general case of renormalizable theory:

$$W = \mathrm{tr}\left[\frac{c_0}{3}\hat{\Phi}^3 + \frac{M}{2}\hat{\Phi}^2\right] + \lambda(H'\hat{\Phi}H) + M'(H'H),$$

where M and M' are mass operators of the order of $M_{\rm GUT}$, and c_0 and λ are numerical coupling constants that are small in comparison with unity.

The interaction between the scalar components y^i of these fields (in the case of the minimal kinetic term in the Lagrangian) is determined in the standard form (2), where the parameter $A = \sqrt{3} z_0/M_P$ is given by the vacuum expectation value of the hidden-sector field $z(x)$. Using (2), we obtain

$$V = \Big\| c_0[(\hat{\Phi}^2)_{\beta\alpha} - {}^1/_5\,\delta_{\beta\alpha}\,\mathrm{tr}\,\Phi^2] + (M + m_{3/2})\,\Phi_{\beta\alpha} + \lambda\left[H_\alpha' H_\beta - \frac{\delta_{\alpha\beta}}{5}(H'H)\right]\Big\|^2 + |\lambda(\hat{\Phi}H)_\alpha + m_{3/2} H_\alpha^*|^2 + |\lambda(H'\hat{\Phi})_\alpha + m_{3/2} H_\alpha'^*|^2 + (A-3)\,m_{3/2}(W + W^*) + D\text{-terms}, \qquad (5)$$

where the first line takes into account the fact that the derivative $\partial W/\partial \Phi_{\alpha\beta}$, $\Phi_{\alpha\beta} = \Phi_a(\hat{t}_a)_{\alpha\beta}$ must have zero trace.

If we consider the vacuum expectation values of fields of the form

$$\langle\hat{\Phi}\rangle_0 = \begin{vmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{vmatrix} V_0, \quad \langle H \rangle_0 = \langle H' \rangle_0 = 0,$$

which correspond to the SU(5)→SU(3)×SU(2)×U(1) breaking, and minimize the potential with respect to $V_0 = c_0^{-1}(M + m_{3/2})x$, we obtain

$$x = 1 + (A-3)\,m_{3/2}^2/M^2 + O(m_{3/2}^3/M^3).$$

It is clear that the correction proportional to $A - 3$ is highly suppressed by the factor $m^2_{3/2}/M^2$ (it is absent altogether in unbroken SUSY, i.e., for $m_{3/2} = 0$). Because of this, the high-energy scale $M \sim M_{\rm GUT}$ does not penetrate into the low-energy potential of the doublets H_1, H_2.

If we choose M' so that $\mu_0 = M' - 3\lambda V_0$ is of the order of M_W (i.e., negligible in comparison with M'), we obtain from (5) the following potential for the Higgs doublets:

$$V(H_1, H_2) = (\mu_0^2 + m_{3/2}^2)(|H_1|^2 + |H_2|^2) + B m_{3/2}\mu_0 (H_1 \varepsilon H_2^* + \text{h.c.}) + \frac{g'^2}{8}(|H_1|^2 - |H_2|^2) + \frac{g^2}{8}(H_1^* \tau H_1 + H_2^* \tau H_2)^2, \qquad (6)$$

where $\varepsilon = \begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix}$, τ are the Pauli matrices, and

$$B = A - 1 + \frac{3\lambda}{c_0}\frac{m_{3/2}}{\mu_0}(A-3) \qquad (7)$$

is the coefficient in front of the term representing the interaction of the doublets H_1, H_2. As can be seen, when $A \neq 3$, the numerical value of B depends significantly on the parameters λ, c_0 of the grand unification theory. The potential $V(H_1, H_2)$ in the second line in (6) includes standard D terms. The right-hand side of (5) also includes the quartic interaction $\lambda^2(H_1, H_2)^2$ of the Higgs fields, but this is exactly canceled by the Born contribution of the pole graph corresponding to the exchange of the 24-plet $\hat{\Phi}$.

The result given by (7) is related to the SU(5) model of grand unification, and a different model would give a different value of B. For example, direct SUGRA generalization of the $SU(3)\times SU(2)\times U(1)$ model without SU(5) unification would yield $B=A-1$ instead of (7).

As can be seen, the entire effect of the grand unification scheme on the low-energy $SU(3)\times SU(2)\times U(1)$ Lagrangian reduces to the addition of the parameters B and μ_0 to the parameters $m_{3/2}$, $\widetilde{M}_{1/2}=\widetilde{\gamma} m_{3/2}$, and A, determined by the form of the theory on the scale M_P. A general proof of this is given in Ref. 11.

We conclude this section by determining the numerical values of $M_{\rm GUT}$ and the gauge coupling constants $\alpha_i=g_i^2/4\pi$, where, in the usual notation, $\alpha_3=g_s^2/4\pi$, $\alpha_2=g^2/4\pi$, and $\alpha_1=(3/5)g'^2/4\pi$ [the last of these is valid for the SU(5) unification model]. According to the renormalization-group equations, these constants depend on the virtuality of the momenta, P^2, as follows:

$$\tilde{\alpha}_i(P^2)=\tilde{\alpha}_i(M_0^2)[1-b_i\tilde{\alpha}_i(M_0^2)\ln(Q^2/M_0^2)]^{-1}, \tag{8}$$

where $\tilde{\alpha}_i=\alpha_i/4\pi$ and M_0 is arbitrary. If we take $M_0=M_{\rm GUT}$, we obtain $\tilde{\alpha}_i(M_0^2)=\alpha_0/4\pi$. For the minimal choice of the SUSY multiplets (3 lepton-quark generations and 2 Higgs doublets), we have $b_1=11$, $b_2=1$, $b_3=-3$. The grand unification condition

$$\alpha_0=\alpha_3(M^2_{\rm GUT})=\alpha_2(M^2_{\rm GUT})={}^5/_3\alpha_1(M^2_{\rm GUT})$$

yields

$$\ln(M^2_{\rm GUT}/M_W^2)=(b_1+b_2-{}^8/_3b_3)^{-1}[\tilde{\alpha}^{-1}(M_W^2)-{}^8/_3\tilde{\alpha}_3^{-1}(M_W^2)],$$
$$\tilde{\alpha}^{-1}=\tilde{\alpha}_2^{-1}+{}^3/_5\tilde{\alpha}_1^{-1}. \tag{9}$$

Substituting $\alpha(M_W^2)=1/128$ and $\alpha_3(M_W^2)\simeq 1/10$, we obtain $\ln(M^2_{\rm GUT}/M_W^2)\simeq 64$, $M_{\rm GUT}=6\times 10^{15}$ GeV, and $\alpha_0=\alpha_2(M^2_{\rm GUT})=1/24$.

3. LOW-ENERGY LAGRANGIAN AND SU(2) SYMMETRY BREAKING. THE MASSES OF HIGGS BOSONS

The Lagrangian of the low-energy theory contains both the field of chiral supermultiplets and the vector (gauge) supermultiplets. The chiral multiplets are the fields of quarks and quarkinos (scalar quarks) of all three generations, and also the field of Higgs scalars and their spinor partners, the higgsinos. Vector multiplets include the $SU(3)\times SU(2)\times U(1)$ gauge bosons and their partners of spin 1/2, the gauginos.

Let us now introduce the superpotential of the low-energy theory, confining our attention to the lepton-quark (super) family of the third generation, which has the highest masses. The contribution of particles in the first two generations is unimportant for the ensuing analysis because the corresponding Yukawa coupling constants are small. By analogy with (5), the superpotential in the old SUGRA form[3-5] contains the cubic (W_3) and quadratic (W_2) terms[10]:

$$W=W_3+W_2=h_t(Q_LH_2)T_R^{+}+h_b(Q_LH_1)B_R^{+}$$
$$+h_\tau(L_LH_1)\tau_R^{+}+\mu_0(H_1H_2^c),$$
$$W_2=\mu_0(H_1H_2^c),\quad H_2^c=\varepsilon H_2,\quad \varepsilon=\begin{pmatrix}0&1\\-1&0\end{pmatrix}, \tag{10}$$

where, in the modern "superstring" form (4) of the theory, $W_2\to W_3'=\lambda N(x)(H_1H_2^c)$ and N is a new scalar field. The upper-case symbols in these expressions represent chiral superfields, for example, the superfield $T_R^{+}=(\bar{t}_R,\tilde{t}_R^{*})$ includes the field of the t-quark t_R and the scalar field of the quarkino $\tilde{t}_R$. It is convenient to work with fields of a given, for example, left-hand, chirality. Instead of T_R, we shall therefore use the left-hand field $T_R^{+}=(T_R)^{+}$. The weak SU(2) doublet

$$Q_L=\begin{pmatrix}T_L\\B_L\end{pmatrix},\quad L_L=\begin{pmatrix}N_L\\E_L\end{pmatrix}$$

contains superfields in each component. For example, $N_L=(\nu_L,\tilde{\nu}_L)$, where $\tilde{\nu}_L$ is the scalar neutrino (neutralino) field. The Higgs-higgsino fields are doublets of this kind:

$$H_1=\begin{pmatrix}H_1^{+}\\H_1^{0}\end{pmatrix},\quad H_2=\begin{pmatrix}H_2^{0}\\H_2^{-}\end{pmatrix},\quad H_2^c=\begin{pmatrix}H_2^{-}\\-H_2^{0}\end{pmatrix}.$$

Their scalar component will be indicated below by the same symbol as the entire superfield (and the spinor-doublet fields of the higgsino will be denoted by $\widetilde{H}_1$, $\widetilde{H}_2$).

The interaction between the spinor components of chiral multiplets is the same as in global supersymmetry. It is specified, first, by the (super) gauge extension of the kinetic energies of chiral multiplets and, second, by the superpotential (10).

For the scalar components of chiral multiplets, the interaction consists of terms corresponding to the extension of the kinetic energies and including the gauge fields, and terms that follow from the superpotential (10) when it is substituted in (2). The part of the latter terms that is proportional to $m_{3/2}$ and $m_{3/2}^2$

$$V'=m_{3/2}^2\sum_i|y^i|^2+[Am_{3/2}W_3(y^i)+Bm_{3/2}W_2(y^i)+\text{h.c.}], \tag{11}$$

explicitly breaks global SUSY, where y^i represents [as in (2)] the scalar components of the fields.

The potential energy of the Higgs scalars that corresponds to the superpotential $W_2=\mu_0(H_1\varepsilon H_2)$ has the form given by (6). The vacuum expectation values of the doublets H_1, H_2

$$\langle H_1\rangle_0=\begin{vmatrix}0\\v_1\end{vmatrix},\quad \langle H_2\rangle_0=\begin{vmatrix}-v_2\\0\end{vmatrix},\quad \langle\varepsilon H_2\rangle_0=\begin{vmatrix}0\\v_2\end{vmatrix}, \tag{12}$$

must correspond to the minimum of this potential $V(\langle H_1\rangle_0, \langle H_2\rangle_0)=V(v_1,v_2)$. The second term in (6) must then be negative, i.e., for $v_1>0$ we must have $v_2>0$ if the constant B is positive. Substituting (12) in (6), we obtain

$$V(v_1,v_2)=\mu_1^2v_1^2+\mu_2^2v_2^2-2\mu_3^2v_1v_2+(\bar{g}^2/8)(v_1^2-v_2^2)^2, \tag{13}$$

where $\bar{g}^2=g^2+g'^2$, $\mu_1^2=\mu_2^2=\mu_0^2+m_{3/2}^2$, and $\mu_3^2=Bm_{3/2}\mu_0$. The values v_1 and v_2 must be found from the condition for the minimum of the potential: $\partial V/\partial v_1=0$, $\partial V/\partial v_1=0$. We note that, if

$${}^1/_4\|\partial^2V/\partial v_i\partial v_k\|=\mu_1^2\mu_2^2-\mu_3^4>0,$$

the minimum of $V(v_1,v_2)$ occurs at $v_1=v_2=0$ and corresponds to the state with unbroken SU(2)-symmetry. If, on the other hand, $\mu_1^2+\mu_2^2<2\mu_3^2$, this minimum is reached for

$v_1 = -v_2 \to \infty$, in which case $V \to -\infty$. The minimum of $V(v_1, v_2)$ is thus seen to be reached for finite nonzero v_1, v_2 if

$$\mu_1^2\mu_2^2 < \mu_3^4, \text{ but } \mu_3^4 < [(\mu_1^2+\mu_2^2)/2]^2. \tag{14}$$

When $\mu_1 \neq \mu_2$, these conditions are readily satisfied because $(\mu_1^2+\mu_2^2)/2 > \mu_1\mu_2$ but, for $\mu_1^2 = \mu_2^2$ [which occurs in (13) when all the coefficients are determined on the scale $Q^2 = M^2_{\rm GUT}$], these conditions are contradictory.

An elegant way of resolving this dilemma in the case of broken SUSY (which, in fact, occurs for $m_{3/2} \neq 0$) was found in Refs. 12 and 13 a few years ago. The point is that the parameters μ_1^2 and μ_2^2 are equal for $Q^2 = M^2_{\rm GUT}$ but, in the broken-supersymmetry theory, they are renormalized by radiative corrections (even in the single-loop approximation) and begin to depend (as do $\bar g^2$ and μ_3^2) on the quantity Q^2, i.e., on

$$l = \ln[M^2_{\rm GUT}/(Q^2+m^2_{3/2})],$$

where the second term in the denominator is important only for $m^2_{3/2} \gg M_z^2$. Since the t-quark is the heaviest (i.e., $h_t \gg h_b \gg h_\tau$) and, with the superpotential (10), its mass arises from the doublet H_2, the quantity μ_2^2 is renormalized more strongly than μ_1^2 (and becomes relatively smaller) during the transition to $Q^2 \simeq M^2_W$. It is then precisely in the required region $Q^2 \simeq M^2_W$ that we have the possibility of satisfying (14) in a range of values of the parameters $m_{3/2}$, $\tilde M_{1/2}$, A, B. The renormalization-group equations for the parameters of the low-energy Lagrangian were first obtained by the Japanese group in Ref. 12 and are given and solved in the Appendix (see also Ref. 14).

If we now assume that these equations have been solved, and the inequalities (14) are satisfied, we can find v_1 and v_2 from the conditions for minimum $V(v_1, v_2)$. Differentiating the potential (13), we obtain

$$\sin 2\theta = \frac{2v_1v_2}{v^2} = \frac{2\mu_3^2}{\mu_1^2+\mu_2^2},$$

$$\frac{\bar g^2 v^2}{2} = \frac{\mu_1^2-\mu_2^2}{\cos 2\theta} - (\mu_1^2+\mu_2^2),$$

where $v^2 = v_1^2 + v_2^2$ and we have introduced the convenient variables θ through the substitutions

$$v_1/v = \sin\theta, \quad v_2/v = \cos\theta, \quad v = (v_1^2+v_2^2)^{1/2},$$

where

$$\cos 2\theta = R/(\mu_1^2+\mu_2^2), \quad R = [(\mu_1^2-\mu_2^2)^2 - 4(\mu_3^4-\mu_1^2\mu_2^2)]^{1/2}. \tag{15}$$

Equation (15) determines the vacuum expectation values of v_1 and v_2 in terms of the parameters μ_1, μ_2, μ_3 of the potential whose minimum value is

$$V_{min} = -\frac{1}{2\bar g^2}(\mu_1^2-\mu_2^2-R)^2 = -\frac{M_Z^4}{2\bar g^2}\cos^2 2\theta, \tag{16}$$

where

$$M_Z^2 = \frac{\bar g^2 v^2}{2} = \frac{\mu_1^2-\mu_2^2-R}{\cos 2\theta} = \frac{\mu_1^2-\mu_2^2}{\cos 2\theta} - (\mu_1^2+\mu_2^2) \tag{17}$$

is the mass of the Z boson. As can be seen, at the minimum, the potential $V(v_1, v_2)$ is negative and of the order of M_z^4.

In the above minimal SUSY variant of the theory, we have one charged and three neutral Higgs bosons. Their masses are determined[10] by the potential (6), in which $\mu_0^2 + m^2_{3/2}$ is replaced with μ_1^2 (for $|H_1|^2$) or with v_2^2 (for $|H_2|^2$), and we have introduced the deviations $H_1' = H_1 - \langle H_1\rangle_0$, $H_2' = H_2 - \langle H_2\rangle_0$ of the fields from their vacuum expectation values (12). This yields the charged-boson mass

$$M_{H^\pm}^2 = \mu_1^2+\mu_2^2+M_W^2, \tag{18}$$

which is greater than $M_W^2 = g^2v^2/2 = (\bar g^2 v^2/2)\cos^2\theta_W$, since $\mu_1^2+\mu_2^2 > 0$. The neutral-particle masses are then given by

$$M^2_{H_a^0} = \mu_1^2+\mu_2^2, \quad M^2_{H^0_{b,c}} = \frac{1}{2}\{(M^2_{H_a^0} + M_Z^2) \mp [(M^2_{H_a^0}+M_Z^2)^2 - 4M^2_{H_a^0}M_Z^2\cos^2 2\theta]^{1/2}\}. \tag{19}$$

As can be seen, when $\cos 2\theta \ll 1$, which is readily satisfied (by having $|v_1/v_2| \simeq 1$), the theory predicts the existence of a light neutral Higgs boson.

4. MASSES OF SUPERSYMMETRIC PARTNERS

We shall now write down the scalar-particle potential that follows[3)] from (2) and (10), confining our attention, for simplicity, to the scalar partners of quarks and leptons (Q_L, U_R, D_R, L_L, E_R) of a particular generation:

$$V = V_4+V_3+V_2,$$

where the quartic V_4, cubic V_3, and quadratic V_2 terms are respectively given by

$$\begin{aligned} V_4 = {}& h_u^2|H_2\tilde Q_L|^2 + h_d^2|H_1\tilde Q_L|^2 + h_e^2|H_1\tilde L_L|^2 + |h_uH_2\tilde U_R^* + h_dH_1\tilde D_R^*|^2 \\ & + h_e^2|H_1\tilde E_R^*|^2 + h_u^2|\tilde Q_L\tilde U_R^*|^2 + |h_d\tilde Q_L\tilde D_R^* + h_e\tilde L_L\tilde E_R^*|^2 \\ & + \frac{g^2}{8}[H_1^*\tau H_1 + H_2^*\tau H_2 + \tilde Q_L^*\tau\tilde Q_L + \tilde L^*\tau\tilde L_L]^2 \\ & + \frac{g'^2}{8}\Big[H_2^*H_2 - H_1^*H_1 - \tilde L_L^*\tilde L_L + 2\tilde E_R^*\tilde E_R + \frac{1}{3}\tilde Q_L^*\tilde Q_L - \frac{4}{3}\tilde U_R^*\tilde U_R + \frac{2}{3}\tilde D_R^*\tilde D_R\Big]^2, \end{aligned} \tag{20}$$

$$\begin{aligned} V_3 = {}& h_u(m_{3/2}A_uH_2 + \mu H_1^*)\tilde Q_L\tilde U_R^* + h_d(m_{3/2}A_dH_1 + \mu H_2^*)\tilde Q_L\tilde D_R^* \\ & + h_e(m_{3/2}A_eH_1 + \mu H_2^*)\tilde L_L\tilde E_R^* + \text{h.c.}, \end{aligned} \tag{21}$$

$$\begin{aligned} V_2 = {}& \mu_1^2H_1^2 + \mu_2^2H_2^2 - \mu_3^2(H_1^*\varepsilon H_2 + \text{h.c.}) \\ & + m_Q^2|\tilde Q_L|^2 + m_u^2|\tilde U_R|^2 + m_D^2|\tilde D_R|^2 + m_L^2|\tilde L_L|^2 + m_E^2|\tilde E_R|^2; \end{aligned} \tag{22}$$

and the single-loop corrections ensure that all the quartic and Yukawa interaction constants h_a^2, h_aA_a, $a = u, d, e$ are the "running" constants like the masses μ, μ_1, μ_2, μ_3, i.e., they depend on $l = \ln[M^2_{\rm GUT}/(Q^2 + m^2_{3/2})]$. As Q^2 is reduced from $M^2_{\rm GUT}$ to M^2_W (by 28 orders of magnitude!), the quantity l changes from 0 to $l_0 \simeq 64$. All the quarkino and leptino masses in the potential V_a also become l-dependent (m_Q^2, m_U^2, m_D^2, m_L^2, m_E^2).

When $Q^2 = M^2_{\rm GUT}$, i.e., when $l = 0$, we have

$$m_Q^2 = m_U^2 = m_D^2 = m_L^2 = m_E^2 = m_{3/2}^2, \quad \mu = \mu_0,$$

$$A_u(0) = A_d(0) = A_l(0) = A,$$

$$\mu_1^2 - \mu^2 = \mu_2^2 - \mu^2 = m_{3/2}^2, \quad \mu_3^2 = m_{3/2}\mu_0 B. \tag{23}$$

Since the Yukawa constants are proportional to the masses of the quarks (and leptons), $m_t > m_b > m_\tau$, and so on, we can confine our attention to the contribution (to V_4 and V_3) of third-generation terms that are proportional to h_t. Because of the difference between the Yukawa constants, the quantities $A_u \to A_t$, $A_d \to A_b$, $A_e \to A_\tau$ are found to vary with increasing l in different ways, and are not equal for $Q^2 \quad M_W^2$.

As Q^2 varies, the mass parameters of the spinor-gaugino superpartners of gauge fields are also found to be the "running" parameters: the mass $M_3 = M_3(l)$ of the gluino $\tilde{g}$ and the parameters M_2, M_1 defining the masses of the $W^\pm$ and Z bosons and the photino. When $l = 0$, i.e., $Q^2 = M_{\rm GUT}^2$, they are all equal: $M_3(0) = M_2(0) = M_1(0) = \widetilde{M}_{1/2}$, where the parameter $M_{1/2}$ is determined by the hidden sector of the theory. The renormalization-group equations obtained by the Japanese[12] and Madrid[10] groups are written down and solved for $h_\tau^2, h_b^2 \ll h_t^2$ in the Appendix. The solution corresponds to the boundary conditions (23).

A. Quarkino and leptino. If we substitute $H_1 = \langle H_1 \rangle_0 + H_1'$, $H_2 = \langle H_2 \rangle_0 + H_2'$ in (20)–(22), we find that the quarkino and leptino masses consist of the following parts: (1) mass terms determined by the potential V_2 and (2) terms proportional to g^2 and g'^2 representing the gauge interaction in the potential V_4. Moreover, for the partners of heavy quarks (t-quarks), there are contributions due to: (3) off-diagonal mass terms in the potential V_3, which mix the fields due to the superpartners of right-hand quarks (right-hand quarkinos) with left-hand quarks, and are proportional to the quark masses, and (4) terms in V_4 that are proportional to the squares of the quark masses.

If we take into account the first two types of term, we obtain[10] the following values for the masses of the quarkinos and leptinos, i.e., the superpartners of light quarks ($\tilde{u}_{L,R}$, $c_{L,R}$ or $\tilde{d}_{L,R}$, $\tilde{s}_{L,R}$) and leptons ($\tilde{e}_{L,R}$, $\tilde{\mu}_{L,R}$ or $\tilde{\nu}_e$, $\tilde{\nu}_\mu$, $\tilde{\nu}_\tau$):

$$\begin{aligned}
m_{\tilde{u}_L}^2 &= m_Q^2 + M_Z^2(-1/2 + 2/3 \sin^2\theta_W)\cos 2\theta,\\
m_{\tilde{d}_L}^2 &= m_Q^2 + M_Z^2(1/2 - 1/3 \sin^2\theta_W)\cos 2\theta,\\
m_{\tilde{u}_R}^2 &= m_u^2 - 2/3 M_Z^2 \sin^2\theta_W \cos 2\theta,\\
m_{\tilde{d}}^2 &= m_u^2 + 1/3 M_Z^2 \sin^2\theta_W \cos 2\theta,\\
m_{\tilde{e}_L}^2 &= m_L^2 - M_Z^2(1/2 - \sin^2\theta_W)\cos 2\theta,\\
m_{\tilde{\nu}_L}^2 &= m_L^2 - 1/2 M_Z^2 \cos 2\theta,\\
m_{\tilde{e}_R}^2 &= m_E^2 + M_Z^2 \sin^2\theta_W \cos 2\theta,
\end{aligned} \qquad (24)$$

where $m_Q^2, m_U^2, m_d^2, m_L^2, m_E^2$ are the coefficients in (23), given by Eqs. (A22) and (A23) in the Appendix.

If we take into account the mixing of the left-hand and right-hand t-quarkinos [due to the off-diagonal term $(A_t m_{3/2} + v_1/v_2) m_t$ in V_3], we obtain the following matrix for their masses

$$\begin{bmatrix} m_{\tilde{u}_L}^2 + m_t^2, & (A_t m_{3/2} + \mu/\operatorname{tg}\theta) m_t \\ (A_t m_{3/2} + \mu/\operatorname{tg}\theta) m_t, & m_t^2 + m_{\tilde{u}_R}^2 \end{bmatrix},$$

whose diagonalization determines the masses m_{t_h} and m_{t_l} of the heavy and light t-quarkinos:

$$m_{t_h, t_l}^2 = m_t^2 + 1/2(m_{\tilde{u}_L}^2 + m_{\tilde{u}_R}^2) \pm [1/4(m_{\tilde{u}_L}^2 - m_{\tilde{u}_R}^2)^2 + (A_t^2 m_{3/2} + \mu \operatorname{ctg}\theta)^2 m_t^2]^{1/2}, \quad (25)$$

where $m_{\tilde{u}_L}^2$, $m_{\tilde{u}_R}^2$ are defined in (24) and $m_t \simeq 40$ GeV is the mass of the t-quark.

We note that the minimum $\langle H_1 \rangle_0$, $\langle H_2 \rangle_0 \neq 0$, $\langle \tilde{Q}_L \rangle_0 = \langle L_L \rangle_0 = \ldots = 0$ of the total scalar-field potential V, found in Section 3, is stable under small field perturbations. However, the presence of the cubic terms in the potential V may give rise to additional (and deeper) minima at nonzero quarkino and (or) leptino fields. The condition for the absence of these minima leads to the upper bound for the parameters A_a.

Let us now consider the behavior of the potential V in the region $\langle H_1 \rangle_0 = \langle \tilde{D}_R \rangle_0 = \langle \tilde{D}_L \rangle_0 = v$ when all the other fields are zero:

$$V = 3h_d^2 v^4 - 2|A_d| m_{3/2} h_d v^3 + (\mu_1^2 + m_Q^2 + m_D^2) v^2.$$

When the inequality

$$|A_b| < [3(\mu_1^2 + m_Q^2 + m_D^2)]^{1/2}/m_{3/2}$$

is satisfied at the minimum $v \approx m_{3/2}/h_b$, the potential becomes $V \approx -m_{3/2}^4 h_b^2$, which is much deeper than the above minimum (17) corresponding to $V \sim m_{3/2}^4/g^2$ (Ref. 9). A similar examination of the region $\langle H_1 \rangle_0 = \langle \tilde{e}_R \rangle_0 = \langle \tilde{e}_L \rangle_0$ and $\langle H_2 \rangle_0 = \langle U_R \rangle_0 = \langle U_L \rangle_0$ leads to the inequalities

$$|A_\tau| < [3(\mu_1^2 + m_L^2 + m_E^2)]^{1/2}/m_{3/2},$$

$$|A_t| < [3(\mu_2^2 + m_Q^2 + m_U^2)]^{1/2}/m_{3/2}.$$

The initial value $A_a(0) \equiv A$ must be chosen so that all these inequalities are satisfied. When $\mu_0 \ll m_{3/2}$, they lead to the condition $|A| \leqslant 3$ at the point $Q^2 = M_{\rm GUT}^2$.

B. Gluino and higgsino-wino. When $l_0 = 64$, we have $\lambda_3(l_0) = 0.33$. Hence, the mass of the gluino is given by

$$\tilde{m}_g = M_3(l_0) = \widetilde{M}_{1/2}/\lambda_3(l_0) \approx 3\widetilde{M}_{1/2} = 3\tilde{\gamma} m_{3/2}. \quad (26)$$

It is determined by the choice of the parameter $\widetilde{M}_{1/2}$. In principle, this parameter can be taken to be very small ($\widetilde{M}_{1/2} \ll m_{3/2}$), but the most natural choice corresponds to $\tilde{\gamma} \simeq 1$, i.e, $\widetilde{M}_{1/2} \simeq m_{3/2} \sim M_z$.

The spinor partners of the Higgs-higgsino scalars have the same quantum numbers as the spinor partners of the vector fields (wino for $W^\pm$). The SUGRA Lagrangian contains four left-hand Weyl spinors $\widetilde{H}_{1L}^+$, $\widetilde{H}_{3L}^+$, $\widetilde{W}_L^-$, $\widetilde{W}_L^+$. Its terms that are quadratic in these fields have the form

$$\Delta \mathscr{L}^\pm = M_2 \widetilde{W}^- \widetilde{W}^+ - \mu \widetilde{H}_{1L}^- \widetilde{H}_{2L}^+ + g(v_2 \widetilde{W}^- \widetilde{H}_{2L}^+ + v_1 \widetilde{H}_{1L}^- \widetilde{W}_L^+).$$

The mixing of these fields produces two Dirac particle fields. The mass matrix

$$|\widetilde{W}_L^-, \widetilde{H}_{1L}^-| \begin{vmatrix} M_2 & g v_2 \\ g v_1 & -\mu \end{vmatrix} \begin{vmatrix} \widetilde{W}_L^+ \\ \widetilde{H}_{2L}^+ \end{vmatrix}$$

corresponding to $\Delta \mathscr{L}^\pm$ is non-Hermitian (it is the product of a unitary matrix and a Hermitian matrix). Instead of this matrix, it is convenient to diagonalize the Hermitian matrix $\widetilde{M}\widetilde{M}^+$, where

$$\hat{M}=\begin{vmatrix} M_2, & gv_2 \\ gv_1, & -\mu \end{vmatrix},$$

which gives

$$M^2(\tilde{H}^{\pm}_{a,b}) = \frac{1}{2}\{g^2v^2+M_2^2+\mu^2\pm[(g^2v^2+M_2^2+\mu^2)^2 -4(g^2v_1v_2-\mu M_2)^2]^{1/2}\}, \quad (27)$$

where $v^2 = v_1^2 + v_2^2$. The first term on the right is the mass of the W boson $M_W^2 = g^2v^2/2$, $g^2v_1v_2 = M_W^2 \sin 2\theta$, and $M_2 = M_{1/2}/\lambda_2(l_0)$.

For small $\mu < M_Z$ and small $\sin 2\theta$, one of the two charged particles is light, $M^2_{\tilde{H}_b} \ll M^2_{\tilde{H}_a} \sim M^2_W$. The fact that this particle has not been seen means that the parameter μ_0 cannot be very small.

In the neutral-particle sector, the Lagrangian again contains four left-hand Weyl fields, $\tilde{W}_3$, $\tilde{B}$, $\tilde{H}_1^0$, $\tilde{H}_2^0$:

$$\Delta\mathcal{L}^0 = \frac{1}{2}M_1(\tilde{B}\tilde{B}) + \frac{1}{2}M_2(\tilde{W}_3\tilde{W}_3) + \mu(\tilde{H}_2^0\tilde{H}_1^0)$$

$$+\frac{g}{\sqrt{2}}[v_1(\tilde{W}_3\tilde{H}_1^0) - v_2(\tilde{W}_3\bar{\tilde{H}}_2^0)]$$

$$+\frac{g'}{\sqrt{2}}[v_1(\tilde{B}\tilde{H}_1^0) - v_2(\tilde{B}\bar{\tilde{H}}_2^0)] + \text{h.c.}$$

The matrix describing their mixing is

$$|\tilde{B}, \tilde{W}_3^0, \bar{\tilde{H}}_1^0, \tilde{H}_2^0| \begin{vmatrix} M_1 & 0 & \frac{g'v_1}{\sqrt{2}} & \frac{-g'v_2}{\sqrt{2}} \\ 0 & M_2 & \frac{gv_1}{\sqrt{2}} & \frac{-gv_2}{\sqrt{2}} \\ \frac{g'v_1}{\sqrt{2}} & \frac{gv_1}{\sqrt{2}} & 0 & \mu \\ -\frac{g'v_2}{\sqrt{2}} & \frac{-gv_2}{\sqrt{2}} & \mu & 0 \end{vmatrix} \begin{vmatrix} \tilde{B} \\ \tilde{W}_3 \\ \tilde{H}_1^0 \\ \bar{\tilde{H}}_2^0 \end{vmatrix}$$

and determines the four Majorana masses of the neutral fermions. In this matrix, $M_1 = \tilde{M}_{1/2}/\lambda_1(l_0)$ and $M_2 = \tilde{M}_{1/2}/\lambda_2(l_0)$. Its eigenvalues $\mathfrak{M}_H$ are the solutions of the equation

$$\Delta = M_Z^2(\mathfrak{M}_H - \mu\sin 2\theta)(\mathfrak{M}_H - M_{\tilde{\gamma}}) - (\mathfrak{M}_H - M_1)(\mathfrak{M}_H - M_2)(\mathfrak{M}_H^2 - \mu^2) = 0,$$

$$M_{\tilde{\gamma}} = [e^2(l_0)/e^2(0)]\tilde{M}_{1/2}, \quad e^2(l) = e^2(0)\Big/\left[1+\frac{e^2(0)}{4\pi}\frac{l_0}{3\pi}\right], \quad (28)$$

where $e^2(l)$ is the square of the electric charge: $(e^2)^{-1} = (g^2)^{-1} + (g'^2)^{-1}$. One of these four particles has the smallest mass. When μ_0, $\tilde{M}_{1/2} < M_Z$, this particle is the photino and its mass is given by the first term in (28) in the form

$$\mathfrak{M}_{H_1^0} = M_{\tilde{\gamma}} = [e^2(l_0)/e^2(0)]\tilde{M}_{1/2} \approx 0.5\tilde{M}_{1/2}.$$

If, on the other hand, μ_0, $M_{1/2} > M_Z$, none of the roots in (28) is close to $M_{\tilde{\gamma}}$.

As they decay, all the superpartners form a lighter particle, namely, $\tilde{\gamma}$ or H^0. This particle is stable and its existence may have important cosmological consequences.

5. CHOICE OF PARAMETERS, NO-SCALE SOLUTION CORRESPONDING TO $\Lambda = U_{\min} = 0$

1. For given $m_{3/2}$, $\mu_0 = xm_{3/2}$, A, B, and given masses m_t and M_Z (defining, respectively, h_t^0 and $M_{1/2} = m_{3/2}$), the formulas given by (18)–(19) and (24)–(28), together with the solutions of the renormalization-group equations [see Ref. 14 and Eqs. (A12)–(A16) in the Appendix], enable us to calculate the masses of the Higgs bosons and all the superpartners. The question is: What governs the choice of the four basic parameters of the theory? We recall that, if $m_{3/2}$ is given together with one of the quantities $x = \mu_0/m_{3/2}$ or $\gamma = \tilde{M}_{1/2}/m_{3/2}$, the other is determined by the condition $M_Z = 94$ GeV in (17), i.e., by the equation $m_{3/2}^2\gamma_Z^2 = M_Z^2$, where [see (A19)–(A21)]

$$\gamma_Z^2 = (\gamma_1^2 - \gamma_2^2)/\cos 2\theta - (\gamma_1^2 + \gamma_2^2),$$

$$\gamma_i^2 = \mu_i^2/m_{3/2}^2 = x^2q^2(l) + a_i(l) + \tilde{\gamma}^2 b_i(l) + \tilde{\gamma}\delta_i(l), \quad i = 1, 2. \quad (29)$$

As $m_{3/2}$ and $\tilde{M}_{1/2}$ (or $m_{3/2}$ and μ_0) increase, the masses of all the superpartners are also found to increase. This is illustrated in Table I in which the columns correspond to $m_t = h_t(M_Z^2)v\cos\theta \simeq 40$ GeV ($h_t^0 \simeq 0.085$) and small[4] $\cos 2\theta \ll 1$ (i.e., $\sin\theta \simeq \cos\theta$). Only the photino and neutral Higgs masses remain small, i.e., of the order of some tens of GeV.

2. We shall now consider no-scale SUGRA with the planar Kähler potential (3) (for which $A = 3$, $B = 2$) and the superpotential (10), so that, for given m_t and H_Z, the only unknowns are $m_{3/2}$ and $\tilde{\gamma} = \tilde{M}_{1/2}/m_{3/2}$ (or $m_{3/2}$ and $x = \mu_0/m_{3/2}$). We now fix γ and, for each $m_{3/2}$, take $\mu_0 = xm_{3/2}$, so that $M_Z = \gamma_Z m_{3/2} = 94$ GeV [with γ_Z taken from (29)], and for the resulting μ_0 we use (10) and (15) to construct the quantity

$$U_{\min}(m_{3/2}) = V_{\min} = -[(m_{3/2}^2\gamma_Z^2)^2/2\bar{g}^2]\cos^2 2\theta$$

as a function of $m_{3/2} = m_0(M_P/2z_0)^{3/2}$.

The result is shown in the form of the three curves in the lower part of the figure for the three values $\tilde{\gamma} = 1$, 0.5, and 0.1 (of the two solutions of $\gamma_Z^2 = M_Z^2/m_{3/2}^2$ for μ_0 and each $m_{3/2}$, we choose one, namely, that corresponding to small $\mu_0 = xm_{3/2}$, i.e., $x \simeq 0.2$–0.3). The curves in the figure have minima at $m_{3/2} = m_{3/2}^0$ in the region $m_{3/2}^0 \simeq 200$–260 GeV. The masses of all the new particles of these three minima are listed in the three columns of Table II, in accordance with (18)–(19) and (24)–(28). They are all of the order of a few hundred GeV, except for the lighter photinos and the Higgs boson H_b^0, and are not too sensitive to the choice of the parameter $\tilde{\gamma} = \tilde{M}_{3/2}/m_{3/2}$.

This early form of the theory suffers from one major defect: according to the figure, it leads to a negative cosmological constant $\Lambda = V_{\min}$ of the order of M_Z^4, which exceeds the admissible astrophysical value $\Lambda \lesssim (0.01\text{ eV})^4$ by 50 orders of magnitude(!). We also note that it is sufficient for the condition $M_Z = \gamma_Z m_{3/2} = 94$ GeV to be satisfied only at the minima of the curves in the figure, i.e., for $m_{3/2} = m_{3/2}^0$ (and not along the entire curves, as demanded above).

3. A totally different situation arises in the modern "superstring" variant of SUGRA with the Kähler potential giv-

TABLE I. Increase in the mass of superpartners with increasing $m_{3/2}$, $\tilde{M}_{1/2}$, and μ_0.

Input parameters	$m_{3/2}$	30	80	80
	$\tilde{M}_{1/2}$	20	20	60
	μ_0	39	88	107
Parameters of $v(H_1, H_2)$	μ_1	63.5	146.4	173.6
	μ_2	57.2	135.2	156.1
	μ_3	60.4	140.8	164.7
	$\cos 2\theta$	0.047	0.065	0.091
Scalars	$m_{H^\pm}$	119	216	248
	$m_{H_a^0}$	227	220	251
	$m_{H_b^0}$	3.0	5.5	8.0
	$m_{H_c^0}$	85	199	233
	$m_{\tilde{\nu}_L}$	29	79	86
	$m_{\tilde{E}_R}$	32	81	82
	$m_{\tilde{E}_L}$	34	82	90
	$m_{\tilde{U}_R}$	56	83	164
	$m_{\tilde{U}_L}$	59	90	174
	$m_{\tilde{D}_R}$	61	96	175
	$m_{\tilde{D}_L}$	62	92	176
Spinors	$m_{\tilde{g}}$	60	60	176
	$m_{N_1^0} = m_{\tilde{\gamma}}$	10	10	30
	$m_{N_2^0}$	62	40	13
	$m_{N_3^0}$	130	176	203
	$m_{N_4^0}$	54	121	147
	$m_{\tilde{W}_L}$	78	141	156
	$m_{\tilde{W}_H}$	230	226	627

Note. All the masses in Tables I and II are in GeV; $M_Z = 94$, $m_t = 40$, $A = B + 1 = 3$, $h_t^0 \simeq 0.085$.

en by (4) (for which $A = 3$) and cubic superpotential (10), in which

$$W_2 \to W_3' = \lambda N(x)\,(H_1(x) H_2^c(x)), \quad H_2^c = \varepsilon H_2(x). \quad (30)$$

In the potential $V(H_1, H_2)$ [see (6)], the quantity μ_0 is then replaced with $\lambda N(x)$, B is replaced with 3, and (this is very important) $V(H_1, H_2)$ acquires the additional positive terms

$$\Delta V_N = m_N^2 N^2(x) + \lambda^2 |(H_1 H_2^c)|^2, \quad (31)$$

where $m_N \lesssim m_{3/2}$ is a mass close to $m_{3/2}$ (as $Q^2 \to M^2_{\rm GUT}$, $m_N \to m_{3/2}$).

When $v_1, v_2 \neq 0$, the field $N(x)$ acquires a vacuum expectation value that can be found by minimizing the sum $U = V + \Delta V_N$ with respect to N_0, where $\Delta V_N = m_N^2 N_0^2 + \lambda^2 v_1^2 v_2^2$ and $V = V(v_1, v_2)$ is the potential (13) with $\mu_3^2 = 3m$, $\mu = \lambda N_0$. This gives

$$N_0 = 3 m_{3/2} \lambda v_1 v_2 / (m_N^2 + \lambda^2 v^2), \quad (32)$$

where the minimization of $U = V + \Delta V_N$ with respect to v_1, v_2 leaves (17) still valid except that, now,

$$\sin 2\theta = 6 m_{3/2} \mu / (\mu_+^2 + \lambda^2 v^2), \qquad \mu_+^2 = \mu_1^2 + \mu_2^2 = 2\mu^2 + \Delta\mu_+^2$$

[$\Delta\mu_+^2 = \mu_+^2 - 2\mu^2$ is given by Eqs. (A19) and (A20) in the Appendix]. Bearing this in mind, and adding ΔN_N to the potential (13), we find that

$$U_{min} = (V + \Delta V)_{min} = (\bar{g}^2 v^4/8)\,(\varkappa_0^2 \sin^2 2\theta - \cos^2 2\theta)$$
$$= [(m_{3/2}^2 \gamma_Z^2)^2 / 2\bar{g}^2]\,[\varkappa_0^2/(1+\varkappa_0^2) - \cos^2 2\theta], \quad (33)$$

where $\varkappa_0^2 = (2\lambda/\bar{g}^2)(9a^2 - 1)$ and $a = m_{3/2}^2 / \times (m_{3/2}^2 + \lambda^2 v^2)$. The constant λ can be chosen so as to ensure that (33) is zero at the minima of the curves in the figure: $\Lambda = U_{\rm min}(m_{3/2}) = 0$. This requires that $\varkappa_0^2 = \cot^2 2\theta$. For example, when $\tilde{\gamma} = 0.5$, so that (see Table II) $(\cos 2\theta)^2_{\rm max} = 0.08$, $a \simeq 1$, we must have $\varkappa_0^2 \simeq 16\lambda^2/\bar{g}^2 \simeq 0.08$, i.e., $\lambda^2 \simeq \bar{g}^2/200$ must be very small ($\bar{g}^2 \simeq 0.65$). The shape of the function (33), $U_{\rm min} = U_{\rm min}(m_{3/2})$, is then as shown by the solid curve in the upper part of the figure (for $\tilde{\gamma} = 0.5$), which differs from the lower curve only through the shift by the constant amount $\sim \varkappa_0^2$. If the parameter μ_0 is fixed by the condition $m_{3/2}\gamma_Z = M_Z = 94$ GeV only at the point $m_{3/2} = m_{3/2}^0 = 230$ GeV at the minimum of the curve, without demanding that it be satisfied for all the $m_{3/2}$, the shape of the curve becomes different and can be approximately represented by the dotted curve in the figure, which leads to $U_{\rm min}(0) = 0$ and a rapid rise in $U_{\rm min}(m_{3/2})$ as $m_{3/2} \to \infty$.

6. CONCLUSION

The above approach is interesting because it leads to a theory with zero cosmological constant $\Lambda = U_{\rm min}(m_{3/2}^0) = 0$. It would be correct (i.e., self-consistent) if the quantity $\mu = \mu_0 q(l)$ were to be close to $\mu = \lambda N_0$, where $q(l)$ is the renormalization factor (A14), determined as in the early theory by the condition $m_{3/2}\gamma_Z = M_Z = 94$ GeV.

However, for $\tilde{\gamma} = 0.5$, Table II shows that $\mu = 77$ GeV, whereas (32) predicts a value lower by a factor of more than one hundred because λ is small:

$$\lambda N_0 = (3\lambda^2 v^2 / 2 m_{3/2})\, a \sin 2\theta \approx 0{,}6 \text{ GeV}$$

($a \sim 1$, $\lambda^2 v^2 = 2\lambda^2/\bar{g}^2 M_Z^2 \sim 10^{-2} M_Z^2$). The quantity $m_{3/2}^0$ decreases with increasing $\tilde{\gamma} = M_{1/2}/m_{3/2}$ (see the figure), whereas λ^2 increases, and the discrepancy is rapidly reduced. If the theory can be made self-consistent by increasing the parameter $\tilde{\gamma} = \tilde{M}_{1/2}/m_{3/2}$, this will determine it unambiguously and the shape of the potential $U_{\rm min} = U_{\rm min}(m_{3/2})$ will be similar to that shown by the broken curve in the figure (with $m_{3/2}^0$ smaller than indicated in the figure). As a result, the parameters $m_{3/2} = m_{3/2}^0, \tilde{M}_{1/2}$ will be determined dynamically by the theory, but the cosmological constant Λ will not vanish automatically[15]: this will require a choice of the constant λ in (30). This variant of the theory will be examined later.

TABLE II. Masses of superpartners at the minima for $m_{3/2} = m^0_{3/2}$ (see figure).

$m^0_{3/2}$	260	230	190
$\widetilde{\gamma}=\widetilde{M}_{1/2}/m_{3/2}$	0.1	0.5	1.0
$\mu_0 = x\, m^0_{3/2}$	110.2	77.2	61.1
μ_1	301.3	266.5	249.1
μ_2	255.8	197.2	152.2
μ_3	277.8	230.0	196.9
$(\cos 2\theta)_{max}$	0.154	0.270	0.413
$m_{H^\pm}$	404	342	303
$m_{H^0_a}$	406	344	304
$m_{H^0_b}$	14	24	37
$m_{H^0_c}$	395	331	292
$m_{\widetilde{\nu}_L}$	259	239	222
$m_{\widetilde{E}_R}$	261	232	198
$m_{\widetilde{E}_L}$	261	243	228
$m_{\widetilde{U}_R}$	234	336	488
$m_{\widetilde{U}_L}$	251	259	515
$m_{\widetilde{D}_R}$	253	361	518
$m_{\widetilde{D}_L}$	268	366	513
$m_{\widetilde{g}}$	72.6	323	539
$m_{N^0_1} = m_{\widetilde{\gamma}}$	13	60	94
$m_{\widetilde{N}^0_2}$	30	2.0	214
$m_{\widetilde{N}^0_3}$	200	190	12
$m_{\widetilde{N}^0_4}$	151	107	86
$m_{\widetilde{W}_L}$	166	111	87
$m_{\widetilde{W}_H}$	285	1195	1967

We note that, when the above $SU(3)\times SU(2)\times U(1)$ model is generalized to a unified theory, we encounter a difficulty due to the fact that the proton decays too rapidly, which can be traced to the operators of dimensionality 5. However, in the superstring variant (4) of the theory, which arises during the compactification of the 10-dimensional SUGRA, this difficulty can evidently be avoided.[6]

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APPENDIX

(a) Renormalization-group equations.[12,10] The coupling constants and masses of scalars and fermions vary with Q^2 because of (a) interactions due to the exchange of gauge fields and (b) the Yukawa interactions. The Yukawa coupling constants do not contribute to the derivatives of the gauge constants (8)

$$d\ln\tilde{\alpha}_j(l)/dl = -b_j\tilde{\alpha}_j(l),$$

$$j=1,\,2,\,3 \quad (b_1=11,\ b_2=1,\ b_3=-3).$$

Similarly,[12]

$$d\ln\widetilde{M}_j(l)/dl = -b_j\tilde{\alpha}_j(l).$$

Hence, the gaugino masses

$$\widetilde{M}_j(l)=\widetilde{M}_{1/2}\alpha_j(l)/\alpha_j(0)=\widetilde{M}_{1/2}/\lambda_j(l),\quad \lambda_j(l)=1+b_j\tilde{\alpha}_j(0)l,\quad j=1,\,2,\,3 \tag{A1}$$

vary in proportion to $\alpha_j(l) = \alpha_j(0)/\lambda_j(l)$ and are equal to $M_{1/2}$ for $l=0$, i.e., $Q^2 = M^2_{\rm GUT}$. We recall that $l = \ln[M^2_{\rm GUT}/(Q^2+m^2_{3/2})]$. We also recall that

$$\tilde{\alpha}_3(0)=\tilde{\alpha}_2(0)={}^5/_3\tilde{\alpha}_1(0)=\tilde{\alpha}_0=\alpha_0/4\pi,\qquad \alpha_0\approx{}^1/_{24}.$$

The remaining quantities, i.e., the Yukawa constants $Y_a(l) = (h_a(Q^2)/4\pi)^2$, the constants $A_a(l)$, $a = t, b, \tau$, the parameters $\mu(l)$, $\mu_1^2(l)$, $\mu_2^2(l)$, $\mu_3^2(l) = \widetilde{B}(l)\mu(l)m_{3/2}$ of the Higgs potential, and the masses m_i^2 of the scalar particles, are found to vary, because of both factors, in accordance with the following renormalization-group equations[12,10]

$$\begin{aligned} dY_t/dl&=[{}^{16}/_3\tilde{\alpha}_3(l)+3\tilde{\alpha}_2(l)+{}^{13}/_9\tilde{\alpha}_1(l)-6Y_t-Y_b]Y_t(l),\\ dY_b/dl&=[{}^{16}/_3\tilde{\alpha}_3+3\tilde{\alpha}_2+{}^{7}/_9\tilde{\alpha}_1-Y_t-6Y_b-Y_\tau]Y_b(l), \end{aligned}\tag{A2}$$

$$\begin{aligned} dY_\tau/dl&=(3\tilde{\alpha}_2+\tilde{\alpha}_1-3Y_b-4Y_\tau)Y_\tau(l);\\ dA_t/dl&=[{}^{16}/_3\tilde{\alpha}_3(l)M_3(l)+3\tilde{\alpha}_2(l)M_2(l)\\ &\quad+{}^{13}/_9\tilde{\alpha}_1M_1(l)]/m_{3/2}-6Y_tA_t-Y_bA_b,\\ dA_b/dl&=[{}^{16}/_3\tilde{\alpha}_3M_3+3\tilde{\alpha}_2M_2\\ &\quad+{}^{7}/_9\tilde{\alpha}_1M_1]/m_{3/2}-Y_tA_t-6Y_bA_b-Y_\tau A_\tau, \end{aligned}\tag{A3}$$

$$\begin{aligned} dA_\tau/dl&=[3\tilde{\alpha}_2M_2+3\tilde{\alpha}_1M_1]/m_{3/2}-4Y_\tau A_\tau-3Y_bA_b;\\ d\mu^2(l)/dl&=(3\tilde{\alpha}_2-\tilde{\alpha}_1-3Y_t-3Y_b-Y_\tau)\mu^2(l), \end{aligned}\tag{A4}$$

where, if $\mu_3^2 = m_{3/2}\mu(l)B(l)$, the

$$d\widetilde{B}(l)/dl=-(3\tilde{\alpha}_2M_2+\tilde{\alpha}_1M_1)/m_{3/2}+(3Y_tA_t+3Y_bA_b+Y_\tau A_\tau). \tag{A5}$$

Moreover,

$$\begin{aligned} d(\mu_1^2-\mu^2)/dl&=3\tilde{\alpha}_2M_2^2+\tilde{\alpha}_1M_1^2(l)-3(\mathfrak{M}_b^2+A_b^2m_{3/2}^2)Y_b\\ &\quad+(\mathfrak{M}_\tau^2+A_\tau^2m_{3/2}^2)Y_\tau,\\ d(\mu_2^2-\mu^2)/dl&=3\tilde{\alpha}_2M_2^2+\tilde{\alpha}_1M_1^2-3(\mathfrak{M}_t^2+A_t^2m_{3/2}^2)Y_t(l), \end{aligned}\tag{A6}$$

where

$$\mathfrak{M}_t^2=m_Q^2+m_U^2+(\mu_2^2-\mu^2),\quad \mathfrak{M}_b^2=m_Q^2+m_D^2+(\mu_1^2-\mu^2),\quad \mathfrak{M}_\tau^2=m_L^2+m_E^2+(\mu_1^2-\mu^2) \tag{A7}$$

and the masses m_k^2 on the right-hand sides of the last two equations satisfy the equations

$$\begin{aligned} dm_Q^2/dl&={}^{16}/_3\tilde{\alpha}_3M_3^2(l)+3\tilde{\alpha}_2(l)M_2^2(l)+{}^{1}/_9\tilde{\alpha}_1(l)M_1^2(l)\\ &\quad-(\mathfrak{M}_t^2+A_t^2m_{3/2}^2)Y_t-(\mathfrak{M}_b^2+A_b^2(l)m_{3/2}^2)Y_b(l), \end{aligned}\tag{A8}$$

$$\begin{aligned} dm_U^2/dl&={}^{16}/_3\tilde{\alpha}_3M_3^2+{}^{16}/_9\tilde{\alpha}_1M_1^2-2(\mathfrak{M}_t^2+A_t^2m_{3/2}^2)Y_t(l);\\ dm_D^2/dl&={}^{10}/_3\tilde{\alpha}_3M_3^2+{}^{4}/_9\tilde{\alpha}_1M_1^2-2(\mathfrak{M}_b^2+A_b^2m_{3/2}^2)Y_b(l),\\ dm_L^2/dl&=3\tilde{\alpha}_2M_2^2+\tilde{\alpha}_1M_1^2-(\mathfrak{M}_\tau^2+A_\tau^2m_{3/2}^2)Y_\tau(l),\\ dm_E^2/dl&=4\tilde{\alpha}_1(l)M_1^2(l)-2(\mathfrak{M}_\tau^2+A_\tau^2m_{3/2}^2)Y_\tau. \end{aligned}\tag{A9}$$

The coupled equations (A2)–(A9) must be solved subject to the boundary conditions (23) for $l=0$:

$$A_t(0)=A_b(0)=A_\tau(0)=A, \quad \tilde{B}(0)=B,$$
$$(\mu_1^2-\mu^2)_{l=0}=(\mu_2^2-\mu^2)_{l=0}=m_Q^2(0)=m_U^2(0)$$
$$=m_D^2(0)=m_L^2(0)=m_E^2(0)=m_{3/2}^2. \qquad (A10)$$

The Yukawa constants are determined for small $Q^2=M_W^2$, i.e., near the mass surface of the corresponding quarks. For example,[5)] $m_t\simeq h_t(M_W^2)v\sin\theta$, since $M_W=g(M_W^2)v/\sqrt{2}$, so that

$$h_t(M_W^2)\approx(m_t/M_W)g(M_W^2)/\sqrt{2}\sin\theta,$$

or

$$Y_t(l_0)=(m_t/M_W)^2(\tilde{\alpha}_2(l_0)/2\sin^2\theta)\approx(m_t/19{,}2M_W)^2$$
$$\approx 6.8\cdot 10^{-4} \qquad (A11)$$

for $m_t\simeq 40$ GeV and $\tilde{\alpha}_2(l_0)=g^2(M_W^2)/(4\pi)^2\simeq(19.2)^{-2}$ for $2\sin^2\theta=1-\cos 2\theta\simeq 1$, i.e., for $\cos 2\theta\ll 1$, where $\sin\theta\simeq\cos\theta$. Similarly, $Y_b(l_0)$ and $Y_\tau(l_0)$ are proportional to m_b^2 and m_τ^2, respectively, i.e., they are much less than $Y_t(l_0)$. Here and below, $l_0=\ln(M_{\rm GUT}^2/M_W^2)\simeq 64$.

(b) Solution of the renormalization-group equations for Y_τ, $Y_b\ll Y_t$. Equations (A2)–(A10) can readily be solved analytically in the realistic case[6)] Y_τ, $Y_b\ll Y_t$, neglecting Y_τ and Y_b on the right-hand sides as compared with Y_t (in principle, they can readily be taken into account to first order in the small quantities).

Thus, substituting $\varphi_t(l)=(16/3)\tilde{\alpha}_3(l)+3\tilde{\alpha}_2+(13/9)\tilde{\alpha}_1$, we obtain from the first equation in (A2) the equation $dY_t/dl=\varphi_t(l)-6Y_t^2(l)$, whose solution

$$Y_t(l)=Y_t^0E(l)/D(l), \quad D(l)=1+6Y_t^0F(l), \qquad (A12)$$

where

$$E(l)=\int_0^l\varphi_t(l)\,dl, \quad F(l)=\int_0^l E(l_1)\,dl_1$$

enables us to express the "bare" value $Y_t^0=Y_t(0)$ in terms of the "physical" constant (A11):

$$Y_t^0=Y_t(l_0)/[E(l_0)-6Y_t(l_0)F(l_0)]\approx Y_t(l_0)/12,$$
$$h_t^0=h_t(M_{\rm GUT}^2)\approx h_t(M_W^2)/\sqrt{12}, \qquad (A13)$$

since $E(l_0)\simeq 13$, $F(l_0)=290$, and $6Y_t^0F(l_0)\simeq 1.18$, according to (A11).

In the same approximation, Eq. (A4) (without the last two terms on the right) and Eq. (A3) (without the last term on the right) have a trivial solution and yield

$$\mu=\mu_0q(l), \quad q(l)=[\lambda_2(l)]^{3/2b_2}[\lambda_1(l)]^{1/2b_1}/D^{1/4}(l),$$
$$A_t(l)=A/D(l)+\tilde{\gamma}D^{-1}(l)\int_0^l H_1(l_1)D(l_1)\,dl_1, \qquad (A14)$$

where $\mu(0)=\mu_0$, $A_t(0)=A$, $\tilde{\gamma}=M_{1/2}/m_{3/2}$, and $\lambda_j(l)$ and $D(l)$ are introduced[10] in (A1) and (A12), where

$$H_1(l)=({}^{16}/{}_3\tilde{\alpha}_3M_3+3\tilde{\alpha}_2M_2+{}^{13}/{}_9\tilde{\alpha}_1M_1)/\tilde{M}_{1/2}.$$

Moreover,

$$\int_0^l H_1(l_1)D(l_1)\,dl_1=H_2(l)D(l)-6Y_t^0H_3(l),$$

where

$$H_2(l)=\int_0^l H_1(l_1)\,dl_1=\tilde{\alpha}_0[{}^{16}/{}_3h_3(l)+3h_2(l)+{}^{13}/{}_{15}h_1(l)],$$
$$H_3=\int_0^l E(l_1)H_2(l_1)\,dl_1=lE(l)-F(l).$$

Here and below, we use the notation introduced by Ibañez and Lopez[10]:

$$h_j(l)=\int_0^l\lambda_j^{-2}(l_1)\,dl_1=l/\lambda_j(l),$$
$$f_j(l)=2\int_0^l\lambda_j^{-3}(l_1)\,dl_1=[1-\lambda_j^{-2}(l)]/b_j\tilde{\alpha}_j(0),$$

where $j=1, 2, 3$ and the quantities $f_j(l)$ will be required below. Thus,[14]

$$A_t(l)=A/D(l)+\tilde{\gamma}[H_2(l)-6Y_t^0H_3(l)/D(l)]. \qquad (A15)$$

Equations (A6) and (A5) can be solved (for Y_τ, $Y_b\ll Y_t$) by evaluating the single integrals and, subject to condition (A10), they yield

$$\mu_1^2=\mu_0^2q^2(l)+m_{3/2}^2+\tilde{M}_{1/2}^2({}^3/{}_2f_2(l)+{}^3/{}_5f_1(l)),$$
$$\mu_3^2=m_{3/2}\mu(l)\tilde{B}(l), \qquad (A16)$$
$$\tilde{B}(l)=B+3Y_t^0F(l)A/D(l)+[3Y_t^0H_3(l)/D(l)-H_7(l)]\tilde{\gamma},$$

where[14] $H_7(l)=\tilde{\alpha}_0[3h_2(l)+(3/5)h_1(l)]$.

The other renormalization-group equations can be solved using the above values of $Y_t(l)$ and $A_t(l)$. To calculate $\mu_2^2-\mu^2$, m_Q^2, m_U^2, we combine the corresponding equations [the second in (A6) and (A8)]. This gives

$$d\mathfrak{M}_t^2/dl=[2\tilde{H}_1(l)\tilde{M}_{1/2}^2-6Y_t(l)A_t^2(l)m_{3/2}^2]-6Y_t(l)\mathfrak{M}_t^2,$$

where

$$\tilde{H}_1(l)=[{}^{16}/{}_3\tilde{\alpha}_3M_3^2(l)+3\alpha_2(l)M_2^2(l)+{}^{13}/{}_9\tilde{\alpha}_1(l)M_1^2]/\tilde{M}_{1/2}^2$$

is a function analogous to $H_1(l)$ in (A14) and $\mathfrak{M}_t^2$ is given by (A7), where $\mathfrak{M}_t^2(0)=3m_{3/2}^2$. The solution of this equation has the same form as the equation for $A_t(l)$ in (A14):

$$\mathfrak{M}_t^2(l)=3m^2/D(l)+2D^{-1}(l)$$
$$\times\int_0^l[\tilde{H}_1(l_1)\tilde{M}_{1/2}^2-3Y_t(l_1)A_t^2(l_1)m_{3/2}^2]D(l_1)\,dl_1. \qquad (A17)$$

From (A8) and the equation for $\mu_2^2-\mu^2$, given by (A6), it also follows that

$$d\mathfrak{M}_-^2/dl=8[{}^4/{}_3\tilde{\alpha}_3(l)M_3^2(l)+{}^1/{}_9\tilde{\alpha}_1(l)M_1^2(l)],$$
$$\mathfrak{M}_-^2=m_Q^2+m_U^2-(\mu_2^2-\mu^2), \quad \mathfrak{M}_-^2(0)=m_{3/2}^2,$$

i.e.,

$$\mathfrak{M}_-^2(l)=m_{3/2}^2+2\tilde{\alpha}_0[{}^8/{}_3f_3(l)+{}^2/{}_{15}f_1(l)]\tilde{M}_{1/2}^2. \qquad (A18)$$

Evaluating the integrals on the right in (A17), and taking $\mu_2^2-\mu^2=(\mathfrak{M}_t^2-\mathfrak{M}_-^2)/2$, we obtain

$$\mu_2^2(l)=\mu_0^2q^2(l)+m_{3/2}^2a_2(l)+\tilde{M}_{1/2}^2b_2(l)+m_{3/2}\tilde{M}_{1/2}\delta_2(l), \quad (A19)$$

where

$$a_2(l)=[3/D(l)-1]/2-3Y_t^0F(l)A^2/D^2(l),$$

$$\delta_2=-6Y_t^0H_3(l)/D^2(l),$$

$$b_2(l)=\left(\frac{3}{2}f_2+\frac{3}{10}f_1\right)\tilde{\alpha}_0+18Y_t^{02}H_3^2(l)/D^2(l)$$

$$-\frac{3Y_t^0}{D(l)}\int_0^l E(l)\,[\bar{H}_2(l_1)+H_2^2(l_1)]\,dl_1, \qquad \text{(A20)}$$

$$\bar{H}_2(l)=\int_0^l \bar{H}_1(l_1)\,dl_1,$$

in which the last integral in $b_2(l)$ can be transformed by integrating by parts,[14] or can be determined numerically. Obviously, the value of $\mu_1^2(l)$ given by (A16) can also be written in the same form as (A19):

$$\mu_1^2(l)=\mu_0^2q^2(l)+m_{1/2}^2a_1(l)+\bar{M}_{1/2}^2b_1(l)+m_{1/2}\bar{M}_{1/2}\delta_1(l), \qquad \text{(A21)}$$

with $a_1\equiv1$, $\delta_1\equiv0$, $b_1(l)=\tilde{\alpha}_0(\frac{3}{2}f_2(l)+\frac{3}{5}f_1(l))$.

Subtracting (A6) from (A9), and multiplying by 1/3 or 2/3, we readily see that the quantities

$$\frac{d}{dl}\left[m_Q^2-\frac{1}{3}(\mu_2^2-\mu^2)\right],\quad \frac{d}{dl}\left[m_U^2-\frac{2}{3}(\mu_2^2-\mu^2)\right],$$

for Y_τ, $Y_b \ll Y_t$ are given by an expression similar to the right-hand sides of (A9), which does not contain $\mathfrak{M}_t^2(l)$. Consequently, they can all be integrated directly and give

$$m_Q^2={}^2/_3m_{1/2}^2+{}^1/_3(\mu_2^2-\mu^2)+({}^8/_3f_3+f_2-{}^1/_{15}f_1(l))\tilde{\alpha}_0\bar{M}_{1/2}^2,$$
$$m_U^2={}^1/_3m_{1/2}^2+{}^2/_3(\mu_2^2-\mu^2)+({}^8/_3f_3-f_2+{}^1/_3f_1)\tilde{\alpha}_0\bar{M}_{1/2}^2; \qquad \text{(A22)}$$

$$m_D^2=m_{1/2}^2+({}^8/_3f_3+{}^2/_{15}f_1)\tilde{\alpha}_0\bar{M}_{1/2}^2,$$
$$m_L^2=m_{1/2}^2+({}^3/_2f_2+{}^3/_{10}f_1)\tilde{\alpha}_0\bar{M}_{1/2}^2,\quad m_E^2=m_{1/2}^2+{}^6/_5f_1(l)\tilde{\alpha}_0\bar{M}_{1/2}^2. \qquad \text{(A23)}$$

The quantities m_Q^2, m_U^2, m_D^2, and so on determine the masses of the scalar superpartners of quarks and leptons (the quarkinos and leptinos), as indicated in the text.

[1)]As noted by J. Polonyi (Budapest, 1977), even in the simplest, i.e., linear, superpotential $h(z)=(\beta+\zeta)m_0M_P^2$, $\zeta=z/M_P$, a potential $V_0(z)$ of the above form has a sharp minimum equal to zero: $\Lambda=V_0(z_0)=0$ for $\beta=\sqrt{2}(2-\sqrt{3})$ at the point $\zeta=\zeta_0=z_0/M_P=\sqrt{2}(\sqrt{3}-1)$. Hence, the scalar $z=z(x)$ is sometimes referred to as the Polonyi field.

[2)]For the "planar" potential, $A=3$, $B=2$.

[3)]We have in mind here a superpotential of the form (10), written as a sum over the generations. Similarly, V_4, V_3, and V_2 include, whenever necessary, the sum over the contributions of all three generations, or the contribution of one of them. As in (10), we must substitute $h_u\to h_t$, $h_d\to h_b$, $h_e\to h_\tau$ in V_4, V_3, and V_2 for the third generation.

[4)]$\cos 2\theta\simeq1$, i.e., $\sin 2\theta\ll1$, is obtained merely by introducing small $\mu_0\sim1$ GeV for $|B|\sim1$, or by taking $|B|\ll1$. According to (29), we then have $M_Z^2\simeq-2\mu_2^2$, i.e., physically reasonable solutions correspond to negative μ_2^2, which arise for $h_t^0\gtrsim1$, i.e., for heavy t-quarks ($m_t\gtrsim100$ GeV).

[5)]Similarly, $m_b\simeq h_b(M_W^2)v\cos\theta$, $m_\tau\simeq h_\tau(M_W^2)v\cos\theta$.

[6)]The authors are indebted to M. V. Burova and A. V. Losev for assistance in construcitng the solution in this approximation. The idea of using this approximation was also introduced in the CERN preprint[14] by Ibañez, Lopez, and Muñoz. This preprint was received as this paper was being completed, and the two communications partially overlap. To facilitate comparison between them, we used the notation of Ref. 14 whenever possible (see also Ref. 10).

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