The possibility of experimental determination of the Weinberg angle in the conversion channel of the μ^- muonic-atom $2s \to 1s$ transition

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We discuss the question of the accuracy with which it is possible to measure the parameter of the standard model of the electroweak interaction $\sin^2\theta_W$, where θ_W is the Weinberg angle, in an experiment on observation of the P-odd anisotropy α_e, due to mixing of the muon-atom states $|2s\rangle$ and $|2p\rangle$ by the weak neutral interaction of the muon and the nucleus, of the angular distribution $w(\theta) = 1 + \alpha_e \cos\theta$ of the conversion electrons of the $2s \to 1s$ muonic transition in light μ^- muonic atoms. For isolated muonic atoms of the region μHe–μSc we have estimated the intensities of the various accompanying background processes which hinder the theoretical interpretation of the value of α_e. According to the estimates, at the present time the muonic atoms which are most promising for determination of $\sin^2\theta_W$ with an accuracy scale of a few percent are μ^6Li, μ^7Li, and μ^9Be.

1. INTRODUCTION

1. The weak neutral interaction of the negative muon and the nucleus mixes the muon-atom states $|2s\rangle$ and $|2p\rangle$ of opposite parity. If at the initial moment the muon is in the $|2s\rangle$ orbit, this mixing leads to a number of observable P-odd effects:

1) to an angular distribution

$$w(\theta_\gamma)=1+\alpha_\gamma \cos\theta_\gamma \tag{1}$$

of the quanta $\hbar\omega_0$, radiated by a polarized muonic atom in the $(M1)$ γ transition $2s \to 1s$ (θ_γ is the angle between the spin-polarization vector of the initial $|2s\rangle$ state and the momentum of the quantum), or to a circular polarization P_γ of these quanta which does not depend on the value of the spin polarization of the initial $|2s\rangle$ state[1–6];

2) to an angular distribution

$$w(\theta_e)=1+\alpha_e \cos\theta_e \tag{2}$$

of the conversion electrons of the muonic $(E0+M1)$ $2s \to 1s$ transition in the electron shell of the muonic atom (θ_e is the angle between the spin polarization vector of the $|2s\rangle$ state and the electron momentum), or to a longitudinal polarization P_e of the conversion electrons which also does not depend on the value of the spin polarization of the $|2s\rangle$ state.[7]

A number of P-odd effects are possible also in pair conversion of the $(E0+M1)$ $2s \to 1s$ muonic transition,[7] but we shall not discuss them here.

Be measuring any of the P-odd correlations described above, it is possible to determine the weak neutral interaction constants of the muon and nucleon (or some linear combination of them) and the parameter of the standard electroweak interaction model $\sin^2\theta_W$, where θ_W is the Weinberg angle. In what follows we shall use an abbreviated notation: the weak μN interaction. Note that such experiments have not been carried out up to the present time. The parameter $\sin^2\theta_W$ is known at the present time with accuracy of the scale of 5% (Ref. 8):

$$\sin^2\theta_W=0.233\pm0.009\,(\pm0.005). \tag{3}$$

This result was obtained by a fit to a number of existing experimental data; the numbers in parentheses indicate the degree of theoretical uncertainty arising in analysis of the experimental results.

It shall be our goal to study the accuracy with which $\sin^2\theta_W$ can be determined experimentally in observation of the P-odd effects described above in light μ^- muonic atoms. For this purpose it is necessary first of all to analyze the accuracy of the theoretical interpretation of the observed P-odd effects, which we suppose to be determined as follows:

a) by the uncertainties which arise in establishment of the interaction constants and $\sin^2\theta_W$ on the basis of the observed P-odd correlations;

b) by the uncertainties due to the existence of accompanying background processes.

Point b) is due to the fact that in the experiment the detector of the informative particles (γ rays or electrons) has a finite energy resolution Δ and therefore together with the informative particles it records particles which arise in other decay channels of the initial muonic-atom $|2s\rangle$ state and actually carry no information at all on the weak interaction. The basic cause leading to appearance of accompanying background processes is the existence in the muonic atom of an electron shell (a number of specific effects are possible in the case of the lightest muonic atoms μH and μHe (Ref. 9)). In view of our insufficient knowledge of the dynamics of the process of reconstruction of the electron shell destroyed in the cascade of Auger transitions of the muon in population of the $|2s\rangle$ orbit, reliable calculations of the intensities of the accompanying background processes are apparently difficult at the present time. As a result it appears to us that at the present time in order to carry out the experiments described above it is necessary to use those muonic atoms in which the accompanying background processes are suppressed to the greatest possible degree and do not require a precise calculation.

2. The role of accompanying background processes for

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the $2s \to 1s$ $(M1)$ γ transition of the muon has been analyzed in detail in Refs. 9–12. As it turned out, in order to determine the constants of the weak μN interaction and $\sin^2 \theta_W$ in the optimal case μNa with accuracy of the order of a few percent, it is necessary to overcome a number of serious difficulties, both experimental and theoretical in nature. We assume that in this connection it is necessary to analyze the question of the possibility of using for experimental determination of the interaction constants and $\sin^2 \theta_W$ the process of $(E0 + M1)$ conversion of the muonic transition $2s \to 1s$ in the electron shell of the muonic atom. Although in this case the observable anisotropy α_e of the angular distribution of conversion electrons (2) is of the order of 10^{-6} (calculated for a spin polarization of the $|2s\rangle$ state $P_\mu = 1$), which is three orders of magnitude smaller than the quantities α_γ (1) and P_γ for μNa ($\sim 10^{-3}$), the conversion process can nevertheless serve as the most reliable source of experimental information on the weak interaction if the accompanying background processes are substantially less intense than the conversion process. Accordingly in Ref. 13 we carried out for muonic atoms of the region $2 \leqslant Z \leqslant 21$, where Z is the nuclear charge, in the framework of a relativistic variant of the Hartree-Fock-Slater (HFS) method, calculations, which were more realistic than those in Ref. 7, of the probabilities of conversion of the $2s \to 1s$ $(E0 + M1)$ transition of the muon in the electronic Ns orbits $(N = 1, 2, 3, 4)$ and of the characteristic values of the anisotropy α_e. Here the P-odd longitudinal polarization of the conversion electrons P_e was not considered, since in light muonic atoms $P_e \sim (Ze^2/\hbar c)^2 \alpha_e \ll \alpha_e$.[7]

In order to study the sensitivity of the value of α_e to variations of the population of the electonic orbits due to the processes of restoration of the electron shell of the muonic atom, calculations were made for some assumed chain of electron configurations. For fixed Z the anisotropy α_e turned out to be practically independent of the electron configuration and the quantum number N, which greatly simplifies the problem of its theoretical interpretation. Since the probability of the conversion process drops rapidly with increase of N (more rapidly than N^{-3} in view of the screening effect), the electron line most appropriate for measurement of the anisotropy α_e appears to us to be that which corresponds to conversion in the $1s$ electron orbit. In what follows we shall call this electron line the informative line. As an illustration we have given in the table probabilities $W(1s\frac{1}{2})$ calculated in the framework of the HFS method for electron configurations of neutral muonic atoms for the conversion process in the K shell, characteristic values of the anisotropy $\tilde{\alpha}_e$, and the electronic enhancement factor of the parity-nonconservation effect Φ, which is defined by the relation

$$\tilde{\alpha}_e = \Phi \delta(2s^1/_2,\ 2p^1/_2), \tag{4}$$

where $\delta(2s\frac{1}{2}, 2p\frac{1}{2})$ is the amplitude of mixing of the $|2s\frac{1}{2}\rangle$ and $|2p\frac{1}{2}\rangle$ muon-atom states by a weak μN interaction of the $\{\text{vector}\}_N \times \{\text{axial}\}_\mu$ type.[2,14]

3. For selection of the muonic atoms most appropriate for measurement of the anisotropy α_e it is necessary to compare the calculated values of $W(1s\frac{1}{2})$ with the probabilities of the various accompanying background processes. In the present work for isolated muonic atoms of the region $2 \leqslant Z \leqslant 21$ we considered three background processes leading to appearance in the conversion spectrum of fast electrons with energy close to the energy of the informative line of conversion of the $2s \to 1s$ transition of the muon in the K shell and which essentially contain no information on the weak μN interaction.

a) Discrete conversion line satellites due to the cascade of $E1$ transitions $2s\frac{1}{2} \to pj \to 1s\frac{1}{2}$ of the muon through virtual p states ($j = 1/2, 3/2$), where the second $E1$ transition gives and energetic conversion line, whereas in the first $E1$ transition there is excitation of the electron shell of the muonic atom (see Section 2 and the diagram in Fig. 1).

b) The conversion decay $2s\frac{1}{2} \to 1s\frac{1}{2} + 2e$ with ejection of two electrons in the continuum in a sequence of two $E1$ transitions of the muon through virtual p states of the muonic atom, in which one of the electrons has an energy close to the informative line (see Section 3 and the diagram in Fig. 2).

c) The decay $2s\frac{1}{2} \to 1s\frac{1}{2} + \gamma + e$ with emission of a soft γ ray and an energetic electron with energy close to the energy of the informative line (see Section 4 and the diagram in Fig. 3).

We shall denote by E_1 the kinetic energy of the informative electrons. In an experiment on measurement of the anisotropy α_e in the line E_1 an electron detector which has a finite energy resolution Δ (at the present time a realistic value is $\Delta \sim 100$–200 eV at $E_1 \sim 100$ keV according to Ref. 15) will accept into the band of the energy resolution in addition to the line E_1 also part of the spectrum of electrons emitted in processes (a), (b), and (c). As a result the observed anisotropy α_{obs} will be

$$\alpha_{\text{obs}} = \alpha_e \{1 + [W_c(\Delta) + W_{2e}(\Delta) + W_{\gamma e}(\Delta)] / W(1s^1/_2)\}^{-1}, \tag{5}$$

where α_e is the anisotropy value which is carried by the line E_1; $W_c(\Delta)$, $W_{2e}(\Delta)$, and $W_{\gamma e}(\Delta)$ are the probabilities of emission of electrons with energies in the interval from $(E_1 - \Delta)$ to $(E_1 + \Delta)$ corresponding to the background processes (a), (b), and (c). In the present work we shall present in essence estimates of the scale of the probabilities $W_c(\Delta)$, $W_{2e}(\Delta)$, and $W_{\gamma e}(\Delta)$. Therefore a measure of the theoretical uncertainties associated with the background processes considered and which arise in analysis of the observed anisotropy α_{obs} is the value of the ratio which occurs in the denominator of Eq. (5). In Sections 2–4 we shall discuss the individual terms entering into this quantity.

In order to investigate the sensitivity of the probabilities of the accompanying background processes to variations of

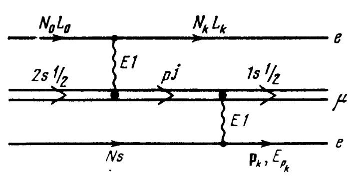

FIG. 1. Mechanism of occurrence of discrete conversion line satellites. In our estimate we take into account only the contribution of $|2pj\rangle$ intermediate states of the muon.

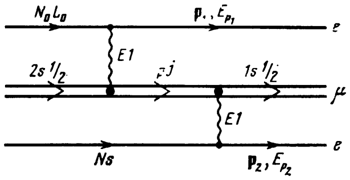

FIG. 2. Mechanism of two-electron decay of the muon-atom $|2s\rangle$ state. In our estimate we take into account only the contribution of intermediate $|2pj\rangle$ states of the muon.

the populations of the electron orbits due to the processes of restoration of the electron shell of the muonic atom, calculations were made for the following two limiting situations:

1. For the case in which in the shell of the muonic atom there are one or two electrons; corresponding estimates were made with use of the Coulomb functions of the discrete spectrum and the plane-wave approximation for the continuum states.

2. For the electron configurations of neutral muonic atoms; here we used the results published in Refs. 9–11 of numerical calculations carried out in the framework of the HFS method.

A discussion of the relative intensity of the informative electron line is given in Section 5.

2. DISCRETE CONVERSION LINE SATELLITES

1. The mechanism of occurrence of these lines is similar to the mechanism of the appearance of discrete satellites in the spectrum of quanta[9,10]; it is represented by the diagram in Fig. 1. For the electron shell we shall restrict the discussion to the model of a fixed average atomic field and shall not consider the processes of filling of the $(N_0L_0)^{-1}$ and $(Ns)^{-1}$ electron holes which are produced. For the electron orbits we shall use the following notation: N is the principal quantum number and L is the orbital angular momentum.

In the framework of this model, conservation of energy can be written as follows:

$$\varepsilon(2s^1/_2)-|E(N_0L_0)|-|E(Ns)|=\varepsilon(1s^1/_2)+E_{p_k}-|E(N_kL_k)|, \tag{6}$$

where $\varepsilon(nlj)$ is the energy of the muonic nlj orbit (n is the principal quantum number, l is the orbital angular momentum, and j is the total angular momentum of the orbit), $|E(NL)|$ is the binding energy of the electronic NL orbit, and E_{p_k} is the kinetic energy of the conversion electron with momentum $\hbar\mathbf{p}_k$. From Eq. (6) we obtain

$$E_{p_k}=E_1-\delta E_k, \tag{7}$$

where

$$E_1=\varepsilon(2s^1/_2)-\varepsilon(1s^1/_2)-|E(1s)| \tag{8}$$

is the energy of the informative line $2s\rightarrow 1s$ of conversion of the muon in the $1s$ orbit,

$$\delta E_k=|E(N_0L_0)|-|E(N_kL_k)|+|E(Ns)|-|E(1s)|. \tag{9}$$

Thus, it follows from Eqs. (7)–(9) that near the informative line E_1 there are discrete satellite lines shifted relative to the line E_1 by an amount δE_k. Since parity-nonconservation effects in the $pj\rightarrow 1s\frac{1}{2}$ conversion transitions of the muon for light muonic atoms are substantially suppressed, the satellite lines E_p contain essentially no information on the weak μN interaction; their angular distribution is practically isotropic. The contribution of these satellites to the valur of the observed anisotropy $\alpha_{\rm obs}$ given by Eq. (5) is determined by the ratio $W_c(\Delta)/W(1s\frac{1}{2})$, where $W_c(\Delta)$ is the combined probability of emission of discrete satellites E_{p_k} accepted by the detector of the informative electron line E_1:

$$W_c(\Delta)=\sum_{N_0L_0}\sum_{N_kL_kN}W(N_0L_0\rightarrow N_kL_k;\,Ns). \tag{10}$$

Here the quantity inside the summation sign is the probability of emission of an individual discrete satellite corresponding to the diagram of Fig. 1. The sum in Eq. (10) extends to all satellites whose energies E_{p_k} satisfy the condition

$$E_1-\Delta\leqslant E_{p_k}\leqslant E_1+\Delta. \tag{11}$$

2. Let us consider the quantity $W(N_0L_0\rightarrow N_kL_k;Ns)$. The operator of interaction of the electron shell of the muonic atom with the nucleus and with the muon we shall represent in the form

$$\hat{H}=\sum_{i=1}^{N_e}\hat{h}_i, \tag{12}$$

where $\hat{h}_i$ has for an individual electron the form

$$\hat{h}=-(Z-1)e^2/r_e+\hat{U}, \tag{13}$$

$$\hat{U}=e^2/|\mathbf{r}_e-\mathbf{r}_\mu|-e^2/r_e; \tag{14}$$

here N_e is the number of electrons in the shell; $\mathbf{r}_e$ and $\mathbf{r}_\mu$ are the radius vectors of the electron and muon. In accordance with Eq. (13) we shall consider the electron shell of the muonic atom to be the shell of an atom with nuclear charge $\tilde{Z}=Z-1$; characteristic dimensions of the electron and muon orbits are respectively as follows:

$$a_e=a_0/\tilde{Z},\quad a_\mu=(a_0/Z)(m_e/m_\mu), \tag{15}$$

where m_e and m_μ are the masses of the electron muon and $a_0=\hbar^2/m_e e^2=0.529\cdot10^{-8}$ cm is the atomic unit of length. The residual interaction operator $\hat{U}$ given by Eq. (14) produces transitions in the muon-electron subsysten. In addition, since the muon charge is distributed over a volume of

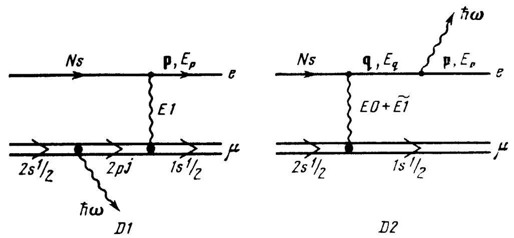

FIG. 3. Mechanism of the decay $2s\rightarrow 1s+\gamma+e$ with emission of an energetic electron. Diagram $D\,1$ leads to appearance in the hard part of the spectrum of electrons E_p of resonances at energies E_R (Ns; $2pj$) (31). In the bremsstrahlung diagram $D\,2$ the tilde denotes an $E\,1$ multipole due to mixing of $|2s\rangle$ and $|2p\rangle$ muonic-atom orbits.

finite radius of the order $a_\mu \ll a_\varepsilon$, the diagonal matrix elements of the operator $\hat{U}$ also are nonzero, which leads to an energy shift of the electron states of the muonic atom.

To estimate $W(N_0L_0 \to N_kL_k;Ns)$ we shall consider a situation in which the shell has two electrons in states Ns and N_0L_0 (see Fig. 1). In the framework of perturbation theory in the operator $\hat{U}$ (14) we obtain

$$W(N_0L_0\to N_kL_k;\ Ns) = {}^3/_4 W_\mu(E1;\ 2p\to 1s)\tilde{Z}^4 Z^{-2}(m_e/m_\mu)^2 \times KBK(E1;\ 2p\to 1s;\ Ns)\,(L_0 100|L_0 1 L_k 0)^2 |\langle N_kL_k|x_e^{-2}|N_0L_0\rangle|^2 \times \sum_{j=1/2,3/2}(2j+1)\frac{(e^2/a_0)^2}{[\Delta(2pj)-|E(N_0L_0)|+|E(N_kL_k)|]^2}, \quad (16)$$

where

$$W_\mu(E1;\ 2p\to 1s) = (2/3)^8(e^2/\hbar c)^5 Z^4(m_\mu c^2/\hbar) \approx 1.29\cdot 10^{11} Z^4\ [\mathrm{sec}^{-1}] \quad (17)$$

is the probability of radiation of a γ ray in the $2p \to 1s$ transition of the muon,[2,4] and

$$KBK(E1;\ 2p\to 1s;\ Ns) = (2^{11}/27\sqrt{3})\tilde{Z}^3 Z^{-7}(e^2/\hbar c)^{-3}(m_e/m_\mu)^{1/2}|\langle\lambda = 1|x_e^{-2}|Ns\rangle|^2 \quad (18)$$

is the coefficient of internal $E1$ conversion of the $2p \to 1s$ transition of the muon in the electronic Ns orbit calculated without taking into account time lag or interaction of the muon and electron currents,

$$\langle\lambda=1|x_e^{-2}|Ns\rangle = \int_0^\infty dx_e u_{\lambda=1}(x_e)u_{Ns}(x_e). \quad (19)$$

Here $x_e = r_e/a_e$; $u_{NL}(x_e)$ is the radial function of the electronic NL orbit; $u_\lambda(x_e)$ is the radial function of the continuum corresponding to orbital angular momentum λ. Asymptotically

$$u_\lambda(r_e\to\infty)\to\frac{1}{r_e}\sin\left[p_kr_e-\frac{\lambda\pi}{2}+\eta_\lambda(E_{p_k})\right], \quad (20)$$

where $\eta_\lambda(E_{p_k})$ is the phase shift. In Eqs. (18)–(20) we are neglecting the small difference of the electron momentum corresponding to $E1$ conversion of the $2p \to 1s$ transition of the muon in the Ns orbit from the quantity p_k. Then $(ABab|ABC\gamma)$ is the Clebsch-Gordan coefficient according to the notation of Condon and Shortley[16];

$$\langle N_kL_k|x_e^{-2}|N_0L_0\rangle = \int_0^\infty dx_e u_{N_kL_k}(x_e)u_{N_0L_0}(x_e), \quad (21)$$

$$\Delta(2pj) = \varepsilon(2s^1/_2) - \varepsilon(2pj). \quad (22)$$

In obtaining Eq. (16) we took into account as intermediate virtual states of the muon only the $|2pj\rangle$ states. The contribution of other discrete np and p states of the continuum is small, since

$$\Delta(2pj)-|E(N_0L_0)|+|E(N_kL_k)|\ll\varepsilon(2s^1/_2)-\varepsilon(np),\quad n\geqslant 3. \quad (23)$$

Since the characteristic velocity of the muon is $v_\mu \sim Z^2e/\hbar \lesssim c/6$ (for $Z \lesssim 20$) and $a_\mu \gtrsim 10$ Fm, which is several times larger than the radii of the nuclei $R_N \lesssim 4$ Fm, we used nonrelativistic Coulomb wave functions of the muon.

For an initial rough estimate of the radial integrals (19) and (21) we shall take as the electronic functions of the discrete spectrum the Coulomb functions in the field of a nucleus with charge $\tilde{Z}$. In view of the smallness of the parameter

$$\tilde{Z}e^2/\hbar v \sim 1/pa_e \sim (m_e/m_\mu)^{1/2} \ll 1 \quad (24)$$

(v is the velocity of the conversion electron), we shall use as the continuum wave function $u_{\lambda=1}(r_e)$ the function of the free motion. In what follows, estimates obtained in this approximation will be denoted by the superscript "Born". Taking into account Eq. (24), we obtain

$$\langle\lambda=1|x_e^{-2}|Ns\rangle\approx 2/N^{3/2}, \quad (25)$$

$$\mathrm{ICC}^{\mathrm{Born}}(E1;\ 2p\to 1s;\ Ns)\approx 3.5\tilde{Z}^3/N^3Z^7. \quad (26)$$

Each state N_0L_0 produces a series of discrete satellite lines due to $E1$ transitions of the electron, $N_0L_0 \to N_kL_k$. Following Refs. 9 and 10, for fixed N we shall divide all of these lines into two groups: A_N is the group produced by transitions $1s \to N_kp$ ($N_k \geqslant 2$), and B_N is the group due to all remaining series of transitions $N_0L_0 \to N_kL_k$. Since for fixed N_0L_0 the quantity $|\langle N_kL_k|x_e^{-2}|N_0L_0\rangle|^2$ drops rapidly with increase of N_k (as N_k^{-3} for large N_k), in each series of $N_0L_0 \to N_kL_k$ the most intense line is the one corresponding to the minimum possible value of N_k. Therefore we estimated the intensities of the lines due to the following transitions: $1s \to 2p$, $2s \to 3p$, $2p \to 3s$, $2p \to 3d$, $3d \to 4f$. The binding energies of the electron orbits were estimated with the Balmer formula for a nucleus with charge $\tilde{Z}$. The values of $\Delta(2pj)$ (Eq. (22)) used in the estimate are given in the table. For μHe–μCl they have been taken from Refs. 9, 10, 17, and 18, while for μAr – μSc they have been calculated on the basis of nonrelativistic approximate formulas without taking into account the fine-structure splitting[19]; here the radii of the nuclei Ar–Ca were taken from the tables of Ref. 20, and the radius of the Sc nucleus was calculated from the interpolation formula of Ref. 21. For μB we used in the estimate two values $\Delta(2p\frac{1}{2}) = 8.1$ eV and 13.6 eV, since in our rough estimate of the binding energies of the electron orbits the difference $|E(3p)| - |E(4f)|$ practically coincides with the value $\Delta(2p\frac{1}{2}) = 10.6$ eV (see the table). For estimation of the probability of $E0$ conversion of the $2s \to 1s$ transition of the muon in the electronic $1s$ orbit we used the plane-wave estimate obtained in the case of a single ($1s$) electron in the shell of the atom[9,11]:

$$W^{\mathrm{Born}}(E0;\ [Ns^1/_2]^1)\approx\frac{1.1\cdot 10^9}{N^3}\left(\frac{\tilde{Z}}{Z}\right)^3\ [\mathrm{sec}^{-1}]. \quad (27)$$

For the $2s \to 1s$ conversion transition of the muon in light muonic atoms, the contribution of the $M1$ multipole is suppressed by about a factor $(Ze^2/\hbar c)^2$ in comparison with the contribution of the $E0$ multipole.[7,13]

We note that it follows from Eqs. (16), (26), and (27) that the ratio

$$W^{\rm Born}(N_0L_0\to N_kL_k;\ Ns)/W^{\rm Born}(E0;\ [Ns^1/_2]^1)$$

does not depend on N. This is explained by the fact that the Born internal conversion coefficient ICC$^{\rm Born}$ $(E1;\ 2p\to 1s;\ Ns)$ and $W^{\rm Born}(E0;\ [Ns\frac{1}{2}]^1)$ are determined by the regions of r_e respectively of the order p_k^{-1} and $a_\mu \ll a_e$, i.e., by the value of the wave function of the electron Ns orbit for small r_e (the factor N^{-3} in Eqs. (26) and (27)).

3. We shall list briefly the results of the estimates described above. In the muonic atoms μLi and μBe the shift of the satellite $1s\to 2p$ of the group A_1, which is closest to the informative line and which is the most intense strongly shifted satellite, is $\delta E \lesssim 100$ eV. Therefore this satellite cannot be separated from the informative conversion line by existing electron detectors with energy resolution $\Delta\sim 100$–200 eV and leads to a value of the ratio $W_c(\Delta)/W(1s\frac{1}{2})$ of the order 10^{-3}. In the muonic atoms μB, μC, and μN the strongly shifted satellites already can be separated from the informative line, but here for a number of electron configurations there can be an enhancement of the satellites of the groups B_N due to the closeness of the splittings $\Delta(2pj)$ and the energy of certain electron transitions $N_0L_0\to N_kL_k$.[10,11] Therefore these muonic atoms are not desirable at present to use for measurement of the anisotropy α_e. In muonic atoms of the region $8\leqslant Z\leqslant 12$ the satellites of group A_1 can be separated from the informative line, while the satellites of group B_1 are 4–7 orders of magnitude less intense than the informative line[1)] E_1 with the exception perhaps of the muonic atom μO, in which we have for the value of the ratio due to the contribution of the satellites of the group B_1 $W_c(\Delta)/W(1s\frac{1}{2})\lesssim 10^{-3}$. In the muonic atoms μAl and μSi there is an enhancement of the satellites which are due to the electron transition $1s\to 2p$, which is a consequence of the closeness of the splittings $\Delta(2pj)$ and the energy of the $1s\to 2p$ transition.[10,11] The $1s\to 2p$ satellite of group A_1 is shifted with respect to the informative line by an amount of the order of 1.5 keV and can be separated. However, the $1s\to 2p$ satellite of group A_2 for certain electron configurations cannot be separated from the informative line, which leads to a value of the ratio $W_c(\Delta)/W(1s\frac{1}{2})\lesssim 10^{-2}$. In muonic atoms with $Z\geqslant 15$ the satellites of group A_1 can be reliably separated from the informative line E_1, while the contribution of the satellites of other groups to the ratio $W_c(\Delta)/W(1s\frac{1}{2})$ is $\lesssim 10^{-4}$.

3. TWO-ELECTRON DECAY OF THE $|2s\rangle$ STATE OF THE MUONIC ATOM

1. The mechanism of two-electron decay of the muonic-atom $|2s\rangle$ state with emission of an energetic electron is shown by the diagram in Fig. 2. In the approximation of a fixed mean atomic field we shall write the conservation of energy for such a process in the form

$$\varepsilon(2s^1/_2)-|E(N_0L_0)|-|E(Ns)|=\varepsilon(1s^1/_2)+E_{p_1}+E_{p_2},\qquad(28)$$

where E_{p_1} and E_{p_2} are the kinetic energies of electrons with momenta $\hbar\mathbf{p}_1$ and $\hbar\mathbf{p}_2$ and $E_{p_1}\ll E_{p_2}\approx E_1$. From Eq. (28) we have

$$E_{p_2}=E_1+|E(1s)|-|E(Ns)|-|E(N_0L_0)|-E_{p_1}.\qquad(29)$$

Thus, the energy E_{p_2} can take any value in the interval from zero to

$$E_1+|E(1s)|-|E(Ns)|-|E(N_0L_0)|.$$

Therefore a detector of the informative electron line E_1 which has a finite energy resolution Δ will cover, in addition to the line E_1, a portion of the continuous spectrum of electrons E_{p_2} near the high-energy edge of the spectrum

$$E_1+|E(1s)|-|E(Ns)|-|E(N_0L_0)|.$$

The corresponding contribution to the value of the observed anisotropy $\alpha_{\rm obs}$ (5) is determined by the ratio $W_{2e}(\Delta)/W(1s\frac{1}{2})$, where $W_{2e}(\Delta)$ is the combined probability of emission of electrons E_{p_2} accepted by a detector with energy resolution Δ, i.e., which satisfy the condition

$$E_1-\Delta\leqslant E_{p_2}\leqslant E_1+\Delta.\qquad(30)$$

2. In the process considered, the dominant contribution is from muonic $E1$ transitions through $|2pj\rangle$ states of the muonic atom. In the case in which the energy splitting of the muonic $|2s\frac{1}{2}\rangle$ and $|2pj\rangle$ orbits $\Delta(2pj)$ given by Eq. (22) is larger than the binding energy $|E(N_0L_0)|$ of the electronic N_0L_0 orbit, in the spectrum of electrons E_{p_2} there are resonances at energies

$$\begin{aligned}E_R(Ns;\ 2pj)&=\varepsilon(2pj)-\varepsilon(1s^1/_2)-|E(Ns)|\\&=E_1+|E(1s)|-|E(Ns)|-\Delta(2pj),\end{aligned}\qquad(31)$$

which correspond to real processes of $E1$ conversion of $2s\to 2pj$ and $2pj\to 1s$ transitions of the muon respectively in the electronic N_0L_0 and Ns orbits.[2)] For the condition

$$\Delta>|\Delta(2pj)+|E(Ns)|-|E(1s)||\qquad(32)$$

these resonances will lie in the band of the energy resolution of the detector for E_1 electrons. According to our estimates, in this case the resonances will make the main contribution to the value of

$$W_{2e}(\Delta)=\sum_R W_{2eR}(Ns;\ 2pj),\qquad(33)$$

where under the summation sign we have the contribution of the resonance E_R $(Ns;2pj)$ to the total probability of two-electron decay of the muonic-atom $|2s\rangle$ state; the sum in Eq. (33) extends over all resonances whose energies satisfy the condition (30). We have

$$\begin{aligned}W_{2eR}(Ns;\ 2pj)=&\Big[\sum_{N_0L_0}K(N_0L_0)\,W_e(E1;\ 2s\to 2pj;\ [N_0L_0]^1)\Big]\\&\times W_e(E1;\ 2p\to 1s;\ [Ns]^{K(Ns)})/W_\mu(E1,\ 2p\to 1s).\end{aligned}\qquad(34)$$

Here $K(N_0L_0)$ is the occupation number of the N_0L_0 orbit; the next factor is the probability of $E1$ conversion of the $2s\to 2pj$ transition of the muon in the N_0L_0 orbit in the case in which there is one electron in the orbit; in the numerator of the ratio we have the probability of $E1$ conversion of the

$2p \to 1s$ transition of the muon in the Ns orbit, where $K(Ns)$ is the occupation number of the Ns orbit after the $E\,1$ conversion of the $2s \to 2pj$ transition; in the denominator we have the probability of a radiative $2p \to 1s$ transition of the muon (17). The last two quantities are essentially independent of the quantum number j. In Eq. (34) it has been taken into account that the total width of the $2pj$ states of the muon is due mainly to the radiative transition $2p \to 1s$.

3. For evaluation of $W_{2eR}(Ns;2pj)$ we shall consider first the case in which there are only two electrons in the electron shell of the muonic atom: one in the state $|N_0L_0\rangle$, and the other in the state $|Ns\rangle$. Using perturbation theory in the operator $\hat{U}$ given by Eq. (14), we obtain

$$W_e(E1;\,2s\to 2pj;\,[N_0L_0]^1)=(e^2/\hbar a_0)\cdot 3\sqrt{2}\,N^{-3}\tilde{Z}^3Z^{-2}(m_e/m_\mu)^2$$

$$\times\left[(e^2/a_0)E_{p_j}^{-1}\right]^{1/2}(2j+1)$$

$$\times\sum_\lambda (L_0 100\,|\,L_0 1\lambda 0)^2\,|\langle\lambda|x_e^{-2}|N_0L_0\rangle|^2, \qquad (35)$$

where

$$E_{p_j}=\Delta(2pj)-|E(N_0L_0)|, \qquad (36)$$

$$\langle\lambda|x_e^{-2}|N_0L_0\rangle=\int_0^\infty dx_e u_\lambda(x_e)\,u_{N_0L_0}(x_e). \qquad (37)$$

The radial integrals (37) were evaluated by using for $u_\lambda(x_e)$ and $u_{N_0L_0}(x_e)$ the Coulomb functions in the field of a nucleus with charge $\tilde{Z}$ (we considered $N_0L_0 = 1s$ and $2s$). The binding energies of the electron orbits were calculated according to the Balmer formula, and in Eq. (35) we used the plane-wave estimate of the internal conversion coefficient (26). We note that in the approximation (26) the ratio

$$W_{2eR}^{\text{Born}}(Ns;\,2pj)/W^{\text{Born}}(E0;\,[Ns]^1)$$

does not depend on N (see Eqs. (27) and (35)).

A more realistic calculation is the one carried out by us in the framework of a relativistic version of the Hartree-Fock-Slater method with the Latter correction, Here the electron shell was treated as the shell of an atom with the charge of the muon-nucleus core, $\tilde{Z} = Z - 1$, the Coulomb field of which was approximated by the field of a uniformly charged sphere of radius $R_N = 1.2A^{1/3}$ Fm (A is the mass number of the nucleus; we did not take into account the fact that the muon charge is distributed over a finite volume of radius of the order $a_\mu \ll a_e$; see Eq. (15)). In this scheme we calculated the internal conversion coefficients of the $2p \to 1s$ transition of the muon in the electronic $1s$ orbit and the probabilities of the informative process of the $E\,0$ conversion $W(1s\tfrac{1}{2})$. The probabilities of $E\,1$ conversion of the $2s \to 2p$ muonic transition were taken from Ref. 11.

4. The quantity $W_{2e}(\Delta)$ (33) receives contributions from the resonances $E_R(Ns;\,2pj)$ ($N = 1, 2, 3, 4\ldots$), the energies of which satisfy the condition (30). The location of the resonances with respect to the informative electron line E_1 is shown schematically in Fig. 4. The most intense resonances are those at $E_R(1s;\,2pj)$ corresponding to $E\,1$ conversion of the $2pj \to 1s$ transitions of the muon in the K shell.

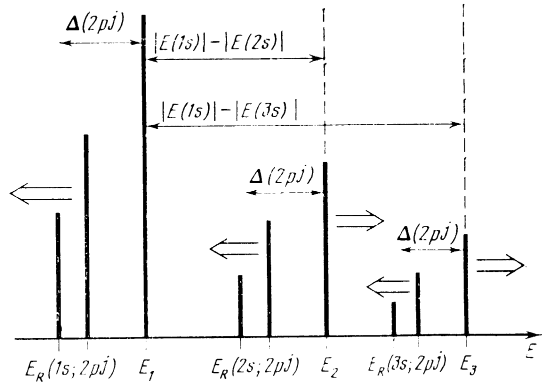

FIG. 4. Schematic arrangement of the resonances $E_R(Ns;\,2pj)$ ($N = 1, 2, 3;\ j = 1/2, 3/2$) relative to the informative electron line E_1 for muonic atoms of the region $Z < 12$. Here E_N ($N = 2, 3$) are the energies of the lines corresponding to conversion of the $2s \to 1s$ transition of the muon in electronic Ns orbits; $\Delta(2pj)$ ($j = 1/2, 3/2$) are the energy splittings of the $|2s\rangle$ and $|2pj\rangle$ muonic-atom orbits (22); $|E(Ns)|$ ($N = 1, 2, 3$) are the binding energies of the electronic Ns orbits. The double arrows show the directions of "motion" of the various lines relative to the line E_1 with increase of Z (see paragraph 4 of Section 3). In the region $Z = 12$ and 13 there is a crossing of the resonances $E_R(Ns;\,2pj)$ ($N = 2, 3$) and the line E_1.

The energy shifts of these resonances with respect to the informative line E_1 are equal to the quantities $\Delta(2pj)$ (see Eq. (31)). The intensity of resonances with $N \geqslant 2$ which are due to $E\,1$ conversion of $2pj \to 1s$ muon transitions in the electronic Ns orbits ($N \geqslant 2$) is substantially smaller than $W_{2eR}(1s;\,2pj)$, since the internal conversion coefficients ICC($E\,1;2p \to 1s;\,Ns$) drop rapidly with increase of N (faster than N^{-3}). We note that in view of the different dependence on Z of the splittings $\Delta(2pj) \propto Z^4$ and the differences in the electronic energies $[|E(1s)| - |E(Ns)|] \propto Z^2$ ($N = 2,3,\ldots$), the resonances $E_R(Ns;\,2pj)$ ($N = 2, 3, \ldots$), depending on Z, are located both to the right and to the left of the informative line E_1 (see Fig. 4).

The results of the estimates described above show that in the muonic atoms μC and μN the resonances $E_R(1s;\,2pj)$ are about an order of magnitude less intense than the informative line E_1. The energy shifts of these resonances with respect to the line E_1 do not exceed 100 eV (see Table I). Therefore the contribution of the resonances $E_R(1s;\,2pj)$ cannot be distinguished by a detector of the electrons E_1 with an energy resolution $\Delta \sim 100$ eV, which leads to a value of $W_{2e}(\Delta)/W(1s\tfrac{1}{2})$ of the order of 0.1. Thus, in the muonic atoms μC and μN the corrections due to the process of two-electron decay of the $|2s\rangle$ state, which must be introduced in analysis of the observed anisotropy α_{obs} (5), turn out to be significant and require more detailed investigation. In muonic atoms with $Z \geqslant 8$ the contribution of the resonances $E_R(1s;\,2pj)$ gives the following value:

$$\sum_j W_{2eR}(1s;\,2pj)/W(1s^1/_2)\approx {}^1/_8-{}^1/_4,$$

but for $Z \geqslant 8$ the splittings $\Delta(2pj)$ become greater than 100 eV and the contribution of the resonances $E_R(1s;\,2pj)$ can already in principle be separated. In the muonic atoms μNa, μMg, and μAl for certain electron configurations the energy

TABLE I.

Z	A	$\Delta(2p^1/_2)$, eV (22)	$\Delta(2p^3/_2)$, eV (22)	$W(1s^1/_2)\cdot 10^{-9}$, sec^{-1}	Φ	$\delta(2s, 2p)\cdot 10^3$	$\tilde{\alpha}_e\cdot 10^6$ (4)
2	4	−1,381	−1.528	0,152	−26,0	−0,51	+0,133
3	6	−1,1	−1,86	0,56	−24,8	−4,89	+1,21
3	7	−1,5	−2,26	0,56	−24,8	−4,86	+1,21
4	9	+2.2	−0,2	0,97	−24,0	+13.1	−3,13
5	10	+10,6±2.5	+4,7±2,5	1,27	−23,4	+6,22	−1,85
6	12	+33	+21	1,54	−22,8	+5,21	−1,19
7	14	+90	+67	1,79	−22,3	+4,13	−0,92
8	16	+162	+124	2,04	−21,7	+4,47	−0,97
9	19	+280	+218	2,28	−21,2	+5,22	−1,10
10	20	+460	+366	2,55	−20,6	+4,81	−0,99
11	23	+727	+588	2,83	−20,0	+5,38	−1.08
12	24	+1100	+905	3,14	−19,4	+5,00	−0,97
13	27	+1500	+1230	3,47	−18,8	+5,93	−1,12
14	28	+2100	+1740	3,84	−18.3	+5,66	−1,04
15	31	+2800	+2320	4.23	−17.7	+6.43	−1,14
16	32	+3600	+2980	4.68	−17.2	+6,44	−1,11
17	35	+4640	+3860	5,15	−16,7	+7,19	−1,20
18	40	5800	5800	5,65	−16,2	+8,9	−1,4
19	39	7100	7100	6,22	−15,7	+8.2	−1,3
20	40	8600	8600	6,86	−15,2	+8,3	−1,2
21	45	10700	10700	7,54	−14,8	+9,7	−1,4

Note. Z is the charge of the nucleus of the muonic atom, A is the mass number of the nucleus. The quantities $W(1s\frac{1}{2})$, Φ, and $\delta(2s, 2p)\equiv\delta(2s\frac{1}{2}, 2p\frac{1}{2})$ are defined at the end of paragraph 2 of Section 1.

shifts of the resonances E_R $(2s; 2pj)$ and E_R $(3s; 2pj)$ relative to the informative line E_1 turn out to be less than 100 eV; here we have a ratio $W_{2e}(\Delta)/W(1s\frac{1}{2})\sim 10^{-2}$. In the remaining muonic atoms of this region of $Z(8\leqslant Z\leqslant 10$ and $Z\geqslant 14)$ the resonances E_R $(Ns, 2pj)$ $(N=2, 3)$ do not give any contribution to the ratio $W_{2e}(\Delta)/W(1s\frac{1}{2})$.

5. In the muonic atom μLi the muonic $|2s\rangle$ orbit is located energetically below the $|2pj\rangle$ orbits $(j=1/2, 3/2)$ and we have $\Delta(2pj)<0$. Therefore in μLi the resonances E_R $(ns; 2pj)$ are not present. In μBe we have $\Delta(2p\frac{3}{2})<0$, and $\Delta(2p\frac{1}{2})>0$. However, $\Delta(2p\frac{1}{2})$ is smaller than the ionization potential of the neutral muonic atom μBe; $E_i(\mu\text{Be})\approx E_i(\text{Li})\approx 5.4$ eV.[22] Thus, in μBe, at least for the nutral muonic atom with an unexcited electron shell, the resonances E_R $(Ns; 2pj)$ also are not present.

Let us estimate the value of $W_{2e}(\Delta)$ for the case in which there are no resonances E_R $(Ns; 2pj)$. We shall again consider the process of ejection of two electrons from an initial configuration with one electron in the $|N_0L_0\rangle$ state and the other electron in a $|Ns\rangle$ state.

Since the energy $E_{p_2}\approx E_1$, the main contribution to $W_{2e}(\Delta)$ is from the diagram shown in Fig. 2, and it is sufficient to take into account just the contribution of intermediate $|2pj\rangle$ states of the muon. In the framework of perturbation theory, taking into account the plane-wave estimate (25), we obtain

$$W_{2e}(\Delta)=(e^2/\hbar a_0)N^{-3}(2^{18}/\pi\cdot 3^{10})\tilde{Z}^5Z^{-1}(m_e/m_\mu)^4 \times\sum_{j=1/2,\,3/2}(2j+1)\int_0^{E_{p_1}^{max}}dE_{p_1}(E_{p_1}E_{p_2})^{-1/2} \times\frac{(e^2/a_0)^2}{[\Delta(2pj)-|E(N_0L_0)|-E_{p_1}]^2} \times\sum_\lambda(L_0\,100\,|\,L_0\,1\lambda 0)^2\cdot|\langle\lambda\,|\,x_e^{-2}\,|\,N_0L_0\rangle|^2, \quad (38)$$

where

$$E_{p_1}^{max}=\Delta-|E(N_0L_0)|. \quad (39)$$

We shall set $N_0L_0=Ns=1s$; the radial integral $\langle\lambda=1|x_e^{-1}|1s\rangle$ entering into (38) will be evaluated with use of the Coulomb wave function of the electronic 1s orbit. As the continuum wave function we shall use both a plane wave (here in the approximation $E_{p_2}\approx E_1$ and $p_1a_b<1$ the integral over the E_{p_1} electron spectrum entering into (38) is calculated analytically) and a Coulomb function (here the integral over the E_{p_1} spectrum was calculated numerically). The results obtained in this way show that in μLi and μBe with a value of the energy resolution of the detector of the informative electron line E_1 lying in the interval $100\lesssim\Delta\lesssim 500$ eV, we have the ratio $W_{2e}(\Delta)/W(E0; [1s\frac{1}{2}]^1)\lesssim 10^{-3}$.

4. THE DECAY $2s\to 1s+\gamma+e$

1. Here we shall consider the decay of the muonic-atom $|2s\rangle$ orbit with knockout from a Ns state of a conversion electron with momentum $\hbar\mathbf{p}$, kinetic energy $E_p\approx E_1$, and radiation of a soft γ ray with energy $\hbar\omega\ll E_p$. We write the conservation of energy for this process in the form

$$\varepsilon(2s^1/_2)-|E(Ns)|=\varepsilon(1s^1/_2)+E_p+\hbar\omega. \quad (40)$$

From this we obtain

$$E_p=E_1+|E(1s)|-|E(Ns)|-\hbar\omega. \quad (41)$$

It follows from (41) that the energy E_p can take any value in the interval from zero to $E_1+|E(1s)|-|E(Ns)|\geqslant E_1$. Therefore a detector of the informative electrons E_1 with energy resolution Δ will accept in addition to the line E_1 also the part of the E_p electron spectrum lying in the interval

$$E_1-\Delta\leqslant E_p\leqslant E_1+\Delta. \quad (42)$$

The corresponding contribution to the value of the observed anisotropy α_{obs} (5) is determined by the ratio $W_{\gamma e}(\Delta)/W(1s\frac{1}{2})$, where $W_{\gamma e}(\Delta)$ is the probability of emission of electrons E_p in the interval (42).

2. For the muonic atoms μBe–μSc the muonic $|2s\rangle$ orbit is located energetically above the $|2pj\rangle$ orbits ($j = 1/2, 3/2$) and the values of $\Delta(2pj)$ (22) are greater than zero. (See the table; in μBe we have $\Delta(2p\frac{1}{2}) > 0$ and $\Delta(2p(3/2)) < 0$.) Therefore for these muonic atoms in the E_p electron spectrum there are resonances at energies $E_R(Ns; 2pj)$ (31) which correspond to real transitions $2s \to 2pj + \gamma$ and $2pj \to 1s + e$ (see diagram D1 in Fig. 3). These resonances will lie within the energy resolution of the detector of E_1 electrons if the condition (32) is satisfied. We estimated the intensities of the resonances to be the same as in the case of two-electron decay of the muonic-atom states (see Section 3, paragraphs 2 and 3), using the plane-wave estimate $\mathrm{ICC}^{\mathrm{Born}}(E1; 2p \to 1s; Ns)$ (26) and the expression given in Ref. 11 for the probability of a radiative $2s \to 2pj\ E1$ transition of the muon:

$$W_\gamma(E1;\ 2s\to 2pj)$$
$$=2.25\cdot 10^6(2j+1)Z^{-2}[\Delta(2pj)/(e^2/a_0)]^3[\mathrm{sec}^{-1}]. \quad (43)$$

The results obtained in this way show that the intensity of the strongest resonances $E_R(1s; 2pj)$ in the spectrum of the electrons E_p reaches values $(10^{-3}–10^{-2})W^{\mathrm{Born}}(E0; [1s]^1)$ only in the region $Z \geqslant 14$. However, here the energy shifts of these resonances relative to the informative line E_1, which are equal to the splittings $\Delta(2pj)$ (22), have values on a scale of several keV (see the table) and the resonances $E_R(1s; 2pj)$ can be separated by the electron detector from the line E_1. Thus, in the muonic atoms μBe–μSc the resonances in the hard part of the spectrum of E_p electrons due to the decay $2s \to 1s + \gamma + e$ impose essentially no limitations on the accuracy of the theoretical interpretation of the anisotropy α_{obs} observed in the line E_1.

3. Let us estimate the contribution to $W_{\gamma e}(\Delta)$ of the nonresonance part of the amplitude of the decay $2s \to 1s + \gamma + e$ with radiation of a soft γ ray $\hbar\omega$ and an energetic electron $E_p \approx E_1$. At energies $E_p \neq E_R(Ns; 2pj)$ the main contribution to the anisotropy α_e is from the bremsstrahlung diagram $D2$ with radiation of a photon (see Fig. 3). In the plane-wave approximation its contribution is

$$W_{\gamma e}{}^{D2}(\Delta) \approx (2^{23}\sqrt{2}/\pi 3^{13})N^{-3}(e^2/\hbar a_0)(e^2/\hbar c)^3$$
$$\times(m_e/m_\mu)^4\tilde{Z}^3 Z^{-4}[E_1/(e^2/a_0)]^{1/2}\ln(\Delta/\lambda), \quad (44)$$

where λ is the half-width of the muonic-atom $|2s\rangle$ orbit, which is introduced to the perturbation theory formulas to remove the infrared divergence at $\omega \to 0$. The diagram $D2$ leads to appearance of a P-odd anisotropy α_e of the angular distribution of the electrons E_p, and for the region $Z \leqslant 21$ considered, we have

$$\frac{\alpha_e-\hat{\alpha}_e}{\alpha_e} \sim \frac{\hbar\omega}{E_1} \leqslant \frac{\Delta}{E_1}. \quad (45)$$

As a result the correction $\delta\alpha_e$ to the value of α_e due to the process $D2$ turns out to be of the following order:

$$\frac{\delta\alpha_e}{\alpha_e} \sim \frac{W_{\gamma e}{}^{D2}(\Delta)}{W^{\mathrm{Born}}(E0;\ [Ns]^1)}\frac{\Delta}{E_1}. \quad (46)$$

Note that in view of (27) and (44), the ratio $\delta\alpha_e/\alpha_e$ is practically independent of the quantum number N.

We shall estimate $\delta\alpha_e/\alpha_e$, for example, for the muonic atoms μHe and μLi, in which the muonic-atom $|2s\rangle$ orbit is located energetically below the $|2pj\rangle$ orbits ($j = 1/2, 3/2$) and the resonances $E_R(Ns; 2pj)$ in the spectrum of electrons E_p do not appear. In the case $N = 1$, $\Delta \sim 100–200$ eV we have

$$W_{\gamma e}{}^{D2}(\Delta)/W^{\mathrm{Born}}(E0;\ [1s]^1)\sim 10^{-3}, \quad (47)$$

$$\Delta/E_1 \sim 10^{-2}, \quad (48)$$

$$\delta\alpha_e/\alpha_e \sim 10^{-5}. \quad (49)$$

With increase of Z the energy E_1 increases in proportion to Z^2, and $\ln\lambda$ depends only weakly on Z, so it follows from Eqs. (44), (27), and (46) that $W_{\gamma e}^{D2}(\Delta) \propto \tilde{Z}^3/Z$, and the ratio $W_{\gamma e}^{D2}(\Delta)/W^{\mathrm{Born}}(E0; [1s]^1) \propto Z^2$, and $\delta\alpha_e/\alpha_e$ is essentially independent of Z, i.e., for all muonic atoms of the region $2 \leqslant Z \leqslant 21$ and with $\Delta \sim 100–200$ eV the estimate (49) is valid. Note that with Δ of the order of several hundred eV the diagram $D2$ gives the main contribution to the value of $W_{\gamma e}(\Delta)$ up to values $Z \approx 20$.

5. RELATIVE YIELD OF THE INFORMATIVE CONVERSION LINE IN AN ISOLATED MUONIC ATOM

Since the anisotropy α_e of the angular distribution of the conversion electrons is small ($\alpha_e \sim 10^{-6}$), for its experimental determination it is desirable to use muonic atoms in which the relative yield $Y(E_1)$ of the informative electron line E_1 for the case with one muon in the $|2s\rangle$ state is rather large. The value of $Y(E1)$ is determined by the ratio

$$Y(E_1) = W(1s^1/_2)/W_{tot}, \quad (50)$$

where $W(1s\frac{1}{2})$ is the probability of $E0$ conversion of the $2s \to 1s$ transition of the muon in electrons of the K shell and W_{tot} is the total probability of decay of the muonic-atom $|2s\rangle$ state. The various decay channels of the $|2s\rangle$ state and the quantity W_{tot} have been discussed in detail in Refs. 9–11. We shall distinguish a number of regions in the charge Z of the nucleus.

1. For the muonic atoms μHe and μLi the muon-atom $|2s\rangle$ orbit is located energetically below the $|2pj\rangle$ orbits ($j = 1/2, 3/2$). Therefore the real process of $E1$ conversion of the $2s \to 2pj$ transition of the muon is forbidden here and for an isolated muonic atom the main decay channel of the $|2s\rangle$ state, which competes with the informative process of $E0$ conversion of the $2s \to 1s$ muonic transition in the K shell, becomes the decay $2s \to 1s + e + \gamma$ with radiation of an energetic photon with energy $\hbar\omega \approx \varepsilon(2s) - \varepsilon(1s)$. The mechanism of this decay is shown by the diagram in Fig. 5. We calculated the probabilities of this process for electronic configurations $\{(1s)^1\}$ and $\{(1s)^2\}$ of the muonic atoms μHe and μLi, using for the electron both nonrelativistic Coulomb wave functions and functions obtained in a relativ-

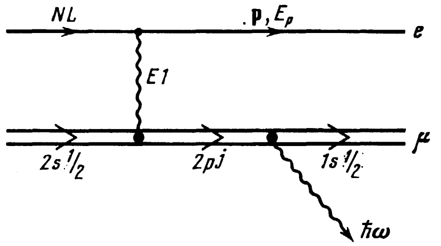

FIG. 5. Mechanism of the decay $2s \to 1s + e + \gamma$ with radiation of an energetic photon.

istic version of the Hartree-Fock-Slater method. The calculations show that for isolated μHe and μLi atoms the value of $Y(E_1)$ is respectively 0.7 and 0.6. In production of μLi in a metallic target the channel of radiative decay of the $|2s\rangle$ state with excitation of conduction-band electrons is opened.[9] The probability of this process is of the order $10^8\ \text{sec}^{-1}$, and therefore the yield $Y(E_1)$ is somewhat decreased but remains at the level of several tenths.

2. In the muonic atom μBe the muonic-atom $|2s\rangle$ orbit is located energetically above the $|2p\frac{1}{2}\rangle$ orbit but below the $|2p(3/2)\rangle$ orbit. The value of $\Delta(2p\frac{1}{2})$ (22) is 2.2 eV, which is less than the ionization potential of the neutral isolated muonic atom: $E_i(\mu\text{Be}) \approx E_i(\text{Li}) \approx 5.4$ eV.[22] Therefore in μBe the real process of E 1 conversion of the $2s \to 2p\frac{1}{2}$ transition of the muon is forbidden, at least, for electronic configurations $\{(1s)^k\}$ and $\{(1s)^2(2s)^k\}$, $k = 1, 2$. Calculations of the probability of the process $2s \to 1s + e + \gamma$ show that for an isolated μBe atom, depending on the electronic configuration, $Y(E_1) \approx 0.5$–0.6. However, in production of μBe in metal the real process of E 1 conversion of the $2s \to 2p\frac{1}{2}$ transition of the muon in conduction-band electrons becomes possible.[9] The probability of this process is of the order 10^{11} sec^{-1} and the yield $Y(E\,1)$ drops to the order of magnitude 10^{-2}, which makes it difficult to use a metallc target.

3. For the muonic atom μB the energy splitting of the muonic-atom $|2s\rangle$ and $|2p\frac{1}{2}\rangle$ orbits is known with a large error[17]:

$$\Delta(2p^1/_2) = (10.6 \pm 2.5)\ \text{eV}. \tag{51}$$

The ionization potential of the neutral isolated muonic atom μB lies in this interval: $E_i(\mu\text{B}) \approx E_i(\text{Be}) \approx 9.3$ eV.[22] The yield $Y(E_1)$ will depend substantially on the ratio between $\Delta(2p\frac{1}{2})$ and $E_i(\mu\text{B})$. Thus, in the case $\Delta(2p\frac{1}{2}) > E_i(\mu\text{B})$ the real process of E 1 conversion of the $2s \to 2p\frac{1}{2}$ transition of the muon in the electrons of the L shell becomes possible. The yield $Y(E_1)$ turns out in this case to be of the order 10^{-3}–10^{-2}. In the opposite case, in which $\Delta(2p\frac{1}{2}) < E_i(\mu\text{B})$, calculations of the probability of the process $2s \to 1s + e + \gamma$ give a value, depending on the electronic configuration, $Y(E_1) \approx 0.4$–0.6. Thus, the existing uncertainty in the value of $\Delta(2p\frac{1}{2})$ does not permit one at the present time to make a reliable estimate of the yield $Y(E_1)$ for an isolated μB atom. With formation of μB in a metal, the real process of E 1 conversion of the $2s \to 2pj$ ($j = 1/2, 3/2$) transitions of the muon in conduction-band electrons becomes possible.[9] Here $Y(E_1) \sim 10^{-2}$, which, as in the case of μBe, makes it difficult to use a metallic target.

4. For the muonic atoms μC–μNa the splittings $\Delta(2pj)$ are less than the binding energy of the K electrons even for the neutral muonic atoms, and therefore, if capture of electrons into the L shell does not occur during the lifetime of the muon in the $|2s\rangle$ orbit, the yield $Y(E_1)$ turns out to be of the order of several tenths. In the opposite case the main channel for decay of the muonic-atom $|2s\rangle$ state becomes the process of E 1 conversion of the $2s \to 2pj$ transitions of the muon in L electrons. Here $Y(E_1)$ drops to an order of magnitude 10^{-4}–10^{-3}.

5. In heavier muonic atoms with $Z \geqslant 12$ the channel of E 1 conversion of $2s \to 2pj$ transitions of the muon in K-shell electrons is open; in μMg it occurs only in the case of the electron configuration of a neutral muonic atom, while for μP it is possible also in the hydrogen-like case (with one electron in the K orbit). Here the yield $Y(E_1)$ is maintained at a level 10^{-4}–10^{-3}.

6. DISCUSSION OF RESULTS, AND CONCLUSIONS

As was noted in the Introduction, in the present work we present essentially estimates of the scale of intensities of the accompanying background processes. Therefore in analysis of the results, as a measure of the theoretical uncertainties due to these processes we shall use directly the value of the ratio

$$[W_c(\Delta) + W_{2e}(\Delta) + W_{\gamma e}(\Delta)]/W(1s^1/_2),$$

which enters into the denominator of the formula (5). In addition, in the estimate we assume that for all muonic atoms of the region $2 \leqslant Z \leqslant 21$ the residual spin polarization of the muonic atom in the $|2s\rangle$ state P_μ is of the order 0.1 (according to Refs. 19 and 23 for light muonic atoms in the $|1s\rangle$ state $P_\mu \sim 0.05$–0.2). Let us consider now the following groups of muonic atoms in order of increasing nuclear charge.

1. The muonic atom μHe. Here the characteristic value of the anisotropy of the angular distribution of the conversion electrons is $\tilde{\alpha}_e \sim 10^{-7}$ (see the table) and $\alpha_e \sim P_\mu \tilde{\alpha}_e \sim 10^{-8}$. The accompanying background processes considered by us lead to essentially no theoretical uncertainties in analysis of the observed anisotropy α_{obs}. For an isolated muonic atom the yield of the informative conversion line is $Y(E_1) \approx 0.7$.

2. The muonic atoms μLi and μBe. Here $\tilde{\alpha}_e \sim 10^{-6}$ and $\alpha_e \sim P_\mu \tilde{\alpha}_e \sim 10^{-7}$. With an electron detector energy resolution $\Delta \gtrsim 100$ eV the accompanying background processes considered (conversion line satellites of the group A_1 and two-electron decay of the muonic-atom $|2s\rangle$ state) lead to theoretical uncertainties of the order of several tenths of a percent in analysis of the observed anisotropy α_{obs}. For isolated muonic atoms the yield of the informative conversion line $Y(E_1) \approx 0.5$–0.6. With production of μBe in a metal $Y(E_1) \sim 10^{-2}$.

3. The muonic atoms μB, μC, and μN. For these muonic atoms $\tilde{\alpha}_e \sim 10^{-6}$ and $\alpha_e \sim P_\mu \tilde{\alpha}_e \sim 10^{-7}$. The resonances in the energetic part of the spectrum of conversion

electrons emitted in two-electron decay of the muonic-atom $|2s\rangle$ state lead to theoretical ambiguities of the order of 10% in analysis of the observed anisotropy $\alpha_{\rm obs}$, with an energy resolution of the informative-electron detector $\Delta \gtrsim 100$ eV. In addition, for these muonic atoms an enhancement of the conversion line satellites is possible. In this connection it appears to us that it is desirable at the present time to use μB, μC, and μN for determination of $\sin^2 \theta_W$.

4. Muonic atoms of the region $8<Z<21$. Here also $\tilde{\alpha}_e \sim 10^{-6}$ and $\alpha_e \sim P_\mu \tilde{\alpha}_e \sim 10^{-7}$. The conversion line satellites and resonances in the spectrum of electrons emitted in two-electron decay of the muonic-atom $|2s\rangle$ state, for an informative-electron detector energy resolution $\Delta \sim 100$ eV, lead to theoretical uncertainties in the value of $\alpha_{\rm obs}$ of the order 0.1% for μO and of the order 1% for μNa–μSi. In the remaining muonic atoms of this group in Z, the accompanying background processes considered lead to essentially no theoretical ambiguities in analysis of the observed anisotropy $\alpha_{\rm obs}$. However, for all muonic atoms of the region $8\leqslant Z\leqslant 21$ the yield of the informative electron line is $Y(E_1) \sim 10^{-4}$–10^{-3}.

Thus, our analysis of the accompanying background processes for isolated muonic atoms shows that at the present time the muonic atoms μLi and μBe (but not in metallic form) are the most promising for experimental determination in muonic atoms of the constants of the weak μN interaction and $\sin^2 \theta_W$ with an accuracy scale of a few percent.

[1)] The intensity of the satellites of group B_N ($N>2$) is still lower.

[2)] By "real" we mean a multistage process in which the energy of the system is conserved in each stage, of course, with accuracy corresponding to the width of the resonance.

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