Universality of the interrelationships of the components of the resistivity tensor in the integer quantum Hall effect

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The interrelationships between the resistivity ρ_{xx} and the deviation $\delta\rho_{xy}$ of the Hall resistivity are analyzed for various metal-insulator-semiconductor structures on silicon surfaces in the wing regions of the quantum-Hall-resistance plateaus. It is established that the coefficient of proportionality between $\delta\rho_{xy}$ and ρ_{xx} depends on the magnetic field, the dimensions of the sample, and the concentration and mobility of the carriers in the two-dimensional layer.

INTRODUCTION

It has been known since the experiment of von Klitzing, Dorda, and Pepper[1] that the Hall resistivity of a two-dimensional ($2D$) layer of carriers in a perpendicular magnetic field assumes quantized values $\rho_{xy} = h/\nu e^2$, while the diagonal components of the resistivity tensor simultaneously vanish: $\rho_{xx} \to 0$ (here ν is the number of filled Landau levels).

Despite the large number of studies of the quantum Hall effect, there is still some uncertainty concerning a number of key questions of importance for our understanding of this effect and its application in metrology. In particular, experiments have not completely cleared up the nature of the physical limitations on the precision of the quantization of ρ_{xy} and their relationship to the minimum value $\rho_{xx}^{\min}$. In previous measurements of the deviations $\delta\rho_{xy} = \rho_{xy} - h/\nu e^2$ the accuracy was limited by the capabilities of the apparatus and techniques, the resistivity in the contacts, the inhomogeneity of the charge distribution in the sample, etc. This conclusion is supported by the asymmetric or U-shaped form of the ρ_{xy} plateaus, the disagreement between the centers of the ρ_{xy} plateaus and the centers of the ρ_{xx} minima, etc. Experiment[2] reveals a quantitative disagreement between the measured (by means of potential contacts) values $(\rho_{xx}^{\min})_{\text{ext}} \sim 10^{-5}$–$10^{-3}$ Ω/square and the local values characteristic of the interior region of the $2D$ layer in the same sample:

$$(\rho_{xx}^{\min})_{int} < 10^{-12}\ \Omega/\square.$$

We have previously reported[3,4] the results of a study of the shape of the wings of the ρ_{xy} plateaus in a Si metal-insulator-semiconductor structure. It was established that a linear relationship exists between the deviations of the Hall resistivity $\delta\rho_{xy}$ and the diagonal component ρ_{xx}:

$$\delta\rho_{xy} = \alpha\rho_{xx}\,\mathrm{sign}(V_g^{(0)} - V_g). \tag{1}$$

Here V_g is the gate voltage and $V_g^{(0)}$ is the value of V_g corresponding to the center of the plateau. The measurements[3,4] were made in a fixed field with $H = 80$ kOe in the region of the $\nu = 4$ plateau and showed that, within the accuracy of the measurements, the coefficient $\alpha = 0.3 \pm 0.1$ is independent of the sample current J_x, the temperature T, and the location of the potential contacts.

The existence of linear relation (1) cannot be regarded as immediately obvious, since a number of theoretical considerations have been advanced[5] in favor of a quadratic relation: $\delta\rho_{xy} \propto \rho_{xx}^2$; moreover, it has been predicted[6] that the temperature dependence of $\delta\rho_{xy}$ and ρ_{xx} will be different at $T \to 0$, where the governing process is the tunneling conductivity.

The empirical result (1) was obtained by the authors using the phenomenological model of Ref. 7. This result is also consistent with scaling theory,[8] which predicts that relation (1) should have a universal character in the two cases for $T \to 0$, i.e., it should not depend on any external parameters such as the magnetic field, temperature, geometric dimensions, sample quality, etc.

The existence of a linear relation between $\delta\rho_{xy}$ and ρ_{xx} has been established[9,10] for the central region of the plateau:

$$\delta\rho_{xy} = -s\rho_{xx}^{\min}. \tag{2}$$

The corresponding measurements were made in MIS structure on Si,[9] for which a value $s \sim 0.1$ was found, and for GaAs heterojunctions.[10] Unlike Refs. 3 and 4, in Refs. 9 and 10 ρ_{xx} was varied not by changing the carrier density (i.e., V_g) but by changing the temperature. In these last two papers the linear relation (2) was satisfied over a range of changes $\rho_{xx}^{\min}/\rho_{xx} = 10^{-8}$–$10^{-4}$, but the coefficient $s = 0.015$–0.5 depended on the sample, on the location of the contacts on the sample, and on the direction of the magnetic field.

Thus, in the case when relation (1) actually describes some empirical invariant, this relation is important for the theory and also permits estimation of the precision to which

TABLE I.

Sample	Channel dimensions, mm²	$\mu^{\max}$, 10^4 cm²/V·sec ($T = 1$K)
1	2,5×0,25	3,0
2	1,2×0,4	1,7
3	5×0,8	4,6

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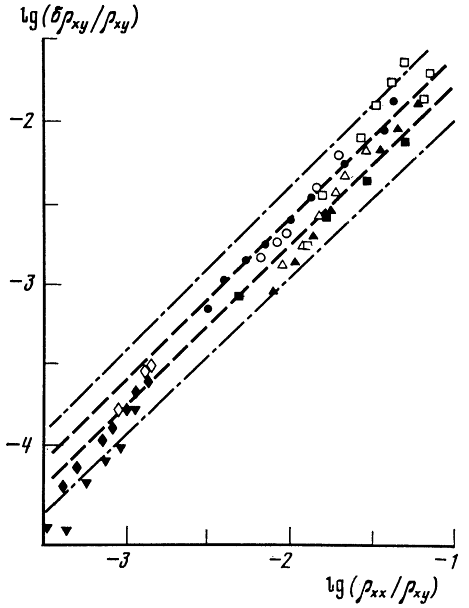

FIG. 1. Plot of $\log|\delta\rho_{xy}/\rho_{xy}|$ versus $\log(\rho_{xx}/\rho_{xy})$ for the regions of electron (filled symbols) and hole (open symbols) wings of the plateau. The legend for the symbols is given in Table II.

ρ_{xy} is quantized. However, as we shall see, the results of the experimental studies are not in complete agreement.[3,4,9,10]

PURPOSE OF STUDY AND THE SAMPLES USED

To clear up the uncertainties we carried out a more detailed analysis of the results of our experiments for three silicon MIS structures having different mobilities μ and different geometric dimensions as functions of the magnetic field, temperature, and concentration. All the samples were prepared on the (100) surface of p-type silicon and had an n-type inversion channel. The parameters of the samples are given in Table I.

MEASUREMENT REGION

To find the causes of the discrepancies noted above in the results of Refs. 3, 4, and 10, let us first ascertain the experimental conditions under which one can expect relation (1) to be satisfied (provided, of course, that such a universal relation exists at all).

The measured values of the components ρ_{ik} of the resistivity tensor are the averaged characteristics of a certain region of the sample between the corresponding potential contacts; these regions are obviously different for components ρ_{xx} and ρ_{xy}. The properties of the different regions can differ; the maximum difference will be observed near values of the filling coefficient ν which correspond to the centers of the plateaus of the $\rho_{xy}(V_g)$ curve. This difference will increase with decreasing temperature. Under these conditions the density of free states at the upper Landau levels attains a minimum, the screening of potential fluctuations in the 2D layer is a minimum, and, consequently, the variation in the potential in the 2D layer is a maximum.

For the reason stated we analyzed density regions which we believed to be more favorable—the wings of the ρ_{xy} plateau: the electron wing ($V_g > V_g^{(0)}$) and the hole wing ($V_g < V_g^{(0)}$). In these regions the density of states is not small even at low temperatures, and the distribution of charge in the 2D layer is more uniform. To eliminate the possible effect of distortions of the shape of the plateau on the results of the measurements, we processed only those $\rho_{ik}(V_g)$ curves for which the central part of the ρ_{xy} plateau did not have U-shaped variations of amplitude $|\delta\rho_{xy}/\rho_{xy}| > 10^{-5}$. Otherwise the sample was considered inhomogeneous, and it was heated to liquid-nitrogen temperature and again cooled.

RESULTS

Examples of the results of several measurements are given in Fig. 1. The results represent the dependence of $|\delta\rho_{xy}|$ on ρ_{xx} in the range $3\cdot10^{-5} \leqslant |\delta\rho_{xy}/\rho_{xy}| \leqslant 3\cdot10^{-2}$. The inner corridor defined by the dashed lines in Fig. 1 corresponds to values $\alpha = 0.17$–0.26, while the outer corridor corresponds to values $\alpha = 0.11$–0.49. As ρ_{xx} decreases one sees a tendency toward a steeper curve: $|\delta\rho_{xy}| \propto \rho_{xx}^{1.5}$.

For each series of measurements we found the average of α for the e and h wings of the plateau. The values found for α are compiled in Table II.

DISCUSSION

The data in Table II show that the values found for α are not correlated with the values of the magnetic field, temperature, mobility, or with the dimensions of the structure. Consequently, the arithmetic mean of all the results in Table II, $\langle\alpha\rangle = 0.21 \pm 0.06$, can be regarded as a single constant, at least for the lowest Landau level ($\nu \leqslant 4$).

The correctness of this statement has been established in the present study by the results of measurements in MIS structures for the interval $10^{-1} > |\rho_{xx}/\rho_{xy}| > 10^{-4}$, in which the value of ρ_{xx} was varied by changing V_g. We expect that relations (1) with the same numerical values of α will hold

TABLE II.

Sample	T, K	H,kOe	ν	α	Symbols in Fig. 1	Sample	T, K	H,kOe	ν	α	Symbols in Fig. 1
1	1,22	90	2	0,22±0,05	■□	2	0,4–2,0	80	4	0,3±0,1	–
	1,9	90	4	0,12±0,08	▼						
	2,81	90	4	0,18±0,08	◆◇	3	1,22	90	4	0,2±0,06	●○
	1,22	70	4	0,18±0,04	▲△		1,22	90	2	0,16±0,08	–
	1,22	60	4	0,26±0,04	–						

over a wider interval, and it would be interesting to check this for $2D$ layers in GaAs heterojunctions and also for smaller values of ρ_{xx} [attained near the center of the minimum of $\rho_{xx}(V_g)$]. However, to check these possibilities experimentally it will be necessary to establish an even more homogeneous state of the sample (and for GaAs heterojunctions the electrical leakage must also be made small) in order that the initial deviations from the ideal shape of the plateau on $\delta\rho_{xy}(V_g)$ be much smaller than the investigated values of $\delta\rho_{xy}$ and ρ_{xx}.

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