Photoproduction of e^+e^- pairs near an extremal rotating black hole with current

V. I. Dokuchaev

(Submitted 1 March 1985)
Zh. Eksp. Teor. Fiz. **89**, 737–743 (September 1985)

It is shown that, near the event horizon of a rapidly rotating, almost extremal black hole through which an electric current flows, soft photons with trajectories close to spherical orbits can create e^+e^- pairs. The parameters of spherical orbits of uncharged particles near the horizon in the Kerr-Newman metric are determined. The rotating black hole can approach the extremal state by the inductive accumulation of electric charge when plasma is accreted in the presence of a regular magnetic field. Accreting supermassive black holes can be near the extremal state.

1. INTRODUCTION

In electrodynamic models of accretion, one considers the generation in the magnetosphere near black holes of a regular quasihomogeneous magnetic field frozen into the accreting plasma.[1–3] If the black hole rotates, then near it an electric field is also induced. An electric field also arises as a result of rotation of the accreting plasma. Under these conditions, the black hole behaves like a unipolar inductor, and through it an electric current can flow.[3,4] For brevity, we shall say that such a black hole is magnetized. It is hoped that models of accretion onto magnetized black holes will explain the giant jets of relativistic particles and electromagnetic radiation from quasars, the nuclei of active galaxies, and x-ray and gamma sources.

When a steady electric current flows through a black hole, a specific distribution of the electromagnetic field is formed around it.[5,6] Near the horizon of the black hole, in a locally nonrotating frame of reference,[7] this electromagnetic field has the configuration of a strong crossed field. We shall show that for certain parameters of the magnetized black hole even soft photons arriving from infinity or emitted by the accreting plasma are capable of producing e^+e^- pairs near the horizon of the black hole.

2. ELECTROMAGNETIC FIELD OF CURRENT THROUGH THE BLACK HOLE

We demonstrate the possibility of e^+e^- photoproduction by the example of the simplest axisymmetric model of a magnetized black hole in which a steady current I flows along its rotation axis, entering through its south pole and leaving through the north pole of the hole. The electric circuit is closed at infinity. The electromagnetic field produced by the steady current configuration in the gravitational field of the black hole can be calculated in the framework of the Newman-Penrose formalism by means of a multipole expansion.[8]

To elucidate the possibility of pair photoproduction, it is important for us to know the behavior of the electromagnetic field near the horizon of the hole, the radius r_+ of which is determined by the condition

$$\Delta \equiv r^2 - 2Mr + a^2 + q^2 = 0,$$

where M, a, and q are, respectively, the mass, specific angular momentum, and the electric charge of the hole (we use a system of units with $G = c = \hbar = 1$). In the locally nonrotating frame near the horizon ($\Delta \to 0$), the azimuthal component H_φ of the magnetic field and the polar component E_θ of the electric field that are produced by the current I have the form[9]

$$H \equiv H_\varphi \approx -E_\theta \approx 2I/\Delta^{1/2} \sin\theta, \tag{1}$$

where θ is the polar angle. In this expression, we have ignored the terms that behave in the limit $\Delta \to 0$ as $\ln(\Delta/M^2)$, or remain finite, or tend to zero on the horizon and are of no interest for our problem. The remaining components of the electromagnetic field tensor $F_{\mu\nu}$ are finite on the horizon. In accordance with (1), as the horizon is approached the electromagnetic field of the current flowing through the black hole tends to the configuration of a strong crossed field whose Poynting vector is directed into the black hole. The invariants of the field produced by the current remain finite on the horizon:

$$\max(|\mathbf{F}^2|, |\mathbf{F}\mathbf{F}^*|) \lesssim (I/M)^2.$$

Such behavior of the electromagnetic field near the horizon is a general property of any steady axisymmetric configurations of currents through a black hole.[5,6] The strength of the crossed field in the case of a general distribution of the currents is proportional to the total current through the horizon of the hole to the north of the latitude of the given position on the horizon.[4] The expression (1) is, in particular, valid in the region $0 < \pi/2$ near the horizon and with opposite sign in the region $\theta > \pi/2$ for the qualitatively more realistic model of a black hole magnetosphere in which equal currents I flow out from the north and south poles of the hole along its symmetry axis and flow back into the hole in the plane of the equator. Such a picture of the current distribution can arise if in the accreting plasma a quasihomogeneous magnetic field H_0 is generated that induces a current $I \lesssim H_0 M$ through the horizon of the black hole.

Because the behavior of the electromagnetic field near the horizon is universal, the conditions of e^+e^- photoproduction will not depend on the details of the distribution of the currents flowing through the black hole.

3. PRODUCTION OF e^+e^- PAIRS BY PHOTONS NEAR A MAGNETIZED BLACK HOLE

The probability of e^+e^- photoproduction in the steady crossed electromagnetic field of strength H depends on the

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magnitude of the dimensionless invariant parameter[10]

$$\chi \equiv \frac{e}{m^3}[(F_{\mu\nu}k^\nu)^2]^{1/2} = \frac{H}{H_{cr}}\frac{k_0+k_r}{m}, \quad (2)$$

where the critical field is $H_{cr} = m^2/e \approx 4.4\times 10^{13}$G; e and m are the electron charge and mass; k_0 and k_r are the local energy and the local radial momentum of the photon in the locally nonrotating frame. The probability of pair photoproduction is exponentially small for $\chi \ll 1$ and reaches its maximum value at $\chi \sim 1$, which can be simultaneously regarded as the threshold of pair production. In our case, the steady crossed field is produced near the horizon by the current I flowing through the black hole. It follows from (1) and (2) that under the realistic restriction $I/M \lesssim H_0 \ll H_{cr}$ (weak magnetic field) the threshold of pair production by photons with energy at infinity $E \lesssim m$ can be achieved if near the horizon there are photon trajectories with cuspidal points $k_r = 0$ or with $k_0 + k_r \sim k_0$ and with $k_0 \gtrsim E$. The last requirement is always satisfied. The minimal radius of trajectories of photons that come from infinity and have cuspidal points near the black hole is the radius of a spherical ($r =$ const) or circular ($r =$ const, $\theta =$ const) photon orbit, r_{ph}. We recall that for a Nordström-Reissner black hole with horizon radius $r_+ = M + (M^2 - q^2)^{1/2}$ the radius of a circular photon orbit is

$$r_{ph} = {}^3/_2 M + ({}^9/_4 M^2 - 2q^2)^{1/2}.$$

For all admissible values of the electric charge q, the difference $r_{ph} - r_+ \sim M$ and the threshold of e^+e^- pair production by soft photons is not attained. It can also be shown that for all photons coming from infinity $k_0 \sim E(\Delta/M^2)^{-1/2}$, but $k_0 + k_r \sim E(\Delta/M^2)^{1/2}$ as $\Delta \to 0$. Therefore, in accordance with (1) and (2), the threshold of e^+e^- production by soft photons near magnetized Nordström-Reissner black holes is not attained even near the horizon. It can be shown that the condition $\chi \sim 1$ for pair photoproduction by soft photons can be satisfied only near spherical photon orbits near the horizon of a rapidly rotating, almost extremal Kett-Newman black hole.

We introduce the dimensionless parameter $\delta \equiv (M^2 - a^2 - q^2)^{1/2}/M$ $(0 \leqslant \delta \leqslant 1)$, which characterizes the degree of extremality of the Kerr-Newman black hole. The radius of the horizon is $r_+ = M(1 + \delta)$. We find the conditions under which spherical photon orbits exist near the horizon of such a black hole. The equations describing the r and θ motion of an uncharged particle with mass μ in the Kerr-Newman metric[11] can be written in the form

$$\left(\frac{dr}{d\xi}\right)^2 = \frac{(r^2+a^2)^2 - a^2\Delta}{(r^2+a^2\cos^2\theta)^2}[E - V^+(r)][E - V^-(r)], \quad (3)$$

$$\left(\frac{d\theta}{d\xi}\right)^2 = \frac{a^2\cos^2\theta}{(r^2+a^2\cos^2\theta)^2}[E^2 - V^2(\theta)], \quad (4)$$

where $\xi = \tau/\mu$, τ is the proper time of the particle, and the radial, $V^+(r)$ and $V^-(r)$, and polar, $V^2(\theta)$, effective potentials are

$$V^\pm(r) = \frac{La(2Mr - q^2) \pm \Delta^{1/2}\{L^2r^4 + (\mu^2r^2 + Q)[(r^2+a^2)^2 - a^2\Delta]\}^{1/2}}{(r^2+a^2)^2 - a^2\Delta}, \quad (5)$$

$$V^2(\theta) = \mu^2 + a^{-2}[L^2\sin^{-2}\theta - Q\cos^{-2}\theta]. \quad (6)$$

In the expressions (3)–(6), E, L, and Q are integrals of the motion, with E the energy of the particle at infinity and L its angular momentum with respect to the symmetry axis. The quantity Q is a specific integral of the motion for the Kerr-Newman metric and is related to the total angular momentum of the particle. In the case $\mu = 0$, the parameter ξ in (3) and (4) is an affine parameter along the particle trajectory. The possibility of existence and the number of local extrema of the effective radial potential $V^+(r)$, which by means of the condition $E > V^+(r)$ determines the region of possible motions of classical particles in the Kerr-Newman metric outside the horizon of the black hole, depend on the values of the following dimensionless parameters of the particle trajectories:

$$\lambda \equiv L/|E|M, \quad \varkappa^2 \equiv Q/E^2M^2, \quad \gamma \equiv E/\mu$$

for given values of the black-hole parameters $\alpha \equiv a/M$ and δ. The effective radial potential $V^+(r)$ has the asymptotic behavior

$$V^+(\infty) = \mu, \quad V^+(r_+) = La/(r_+^2 + a^2).$$

For a particle with $\mu = 0$ in the region $r > r_+$, i.e., outside the horizon, this potential may not have local extrema at all or may have only one local maximum, corresponding to unstable spherical orbits. For $\mu \neq 0$, the effective radial potential $V^+(r)$ also may have no local extrema at all or may have only one local maximum and, at a greater distance from it, one local minimum, corresponding to stable spherical orbits. For $\lambda \leqslant 0$, spherical orbits exist only at coordinate distances $r_s - r_+ \geqslant 2^{1/2}M$ from the horizon for all admissible values of α and δ. For the existence of spherical orbits near the horizon, fulfillment of the conditions $\lambda > 0$ and $\delta \ll 1$ is necessary; we shall determine additional conditions below.

By means of the expression (5), using the conditions $E = V^+(r)$ and $[V^+(r)]' = 0$ for the radius r_s of the spherical trajectory nearest the horizon and the dimensionless impact parameter λ_s corresponding to it for $\lambda > 0$ and $\delta \ll 1$, we obtain

$$r_s \approx M[1 + 2\delta(4 - \alpha^{-2} - \gamma^{-2} - \varkappa^2)^{-1/2}], \quad (7)$$

$$\lambda_s \approx \alpha + \alpha^{-1}[1 + (4 - \alpha^{-2} - \gamma^{-2} - \varkappa^2)^{1/2}\delta]. \quad (8)$$

For validity of the expansions (7) and (8), which generalize the results of Ref. 12 for spherical trajectories to the case $q \neq 0$, $\delta \neq 0$, the restriction $\varkappa^2 + \gamma^{-2} < 4 - \alpha^{-2}$ must be satisfied. Under this restriction, the condition $[V^+(r)]'' < 0$ is satisfied for the spherical orbits nearest the horizon, and therefore they all correspond to the particle being at the maximum of the effective radial potential and are unstable. For these trajectories, the condition $\varkappa^2 \geqslant 0$ also follows from (4), (6), and (8). The combination of the two restrictions on $\varkappa^2$ leads to the condition $\alpha > \frac{1}{2}$, which is necessary for the existence of cuspidal points near the horizon of an almost extremal Kerr-Newman black hole. We shall say that such black holes are rapidly rotating.

By means of the additional condition $[V^+(r)]'' = 0$, which for spherical orbits near the horizon corresponds to the relation

$$\varkappa^2+\gamma^{-2}=4-\alpha^{-2}+o(\delta^{1/3}),$$

we find the stability limit that determines the limit parameters of stable spherical orbits with the smallest possible radius:

$$r_{ms}\approx M\left[1+\left(\frac{2\delta^2}{2-\alpha^{-2}-\varkappa^2}\right)^{1/3}\right], \tag{9}$$

$$\lambda_{ms}\approx\alpha+\alpha^{-1}[1+3\cdot2^{-1/3}(2-\alpha^{-2}-\varkappa^2)^{1/3}\delta^{4/3}], \tag{10}$$

$$\gamma_{ms}^{-2}\approx4-\alpha^{-2}-\varkappa^2-3\cdot2^{4/3}(2-\alpha^{-2}-\varkappa^2)^{2/3}\delta^{2/3}. \tag{11}$$

All stable spherical orbits are situated in the region $r > r_{ms}$. The expressions (9)–(11) hold provided $\gamma^{-2} - 2 > 0$, which by means of (11) and with allowance for the restriction $\varkappa^2 \geqslant 0$ can be written in the form $0 \leqslant \varkappa^2 < 2 - \alpha^{-2}$. For $\varkappa^2 = 2 - \alpha^{-2}$ we obtain for the stability limit of the spherical orbits

$$r_{ms}\approx M[1+(2\delta^2)^{1/4}].$$

When $\varkappa^2 > 2 - \alpha^{-2}$, there are no stable spherical orbits near the horizon. In particular, for photons ($\mu = 0$) stable spherical orbits are altogether absent everywhere outside the horizon. Particles moving in trajectories close to spherical orbits with $Q > 0$ precess with respect to the plane of the equator, rising above it to a maximum height with polar angle θ_0 determined by means of the expression (6) from the condition $E^2 = V^2(\theta)$. In accordance with the equation of motion (4), this equation determines the cuspidal point of the particle trajectory with respect to the angle θ. From the expressions (6) and (8), it follows that for $Q = 0$ the spherical particle trajectories degenerate into circular orbits lying entirely in the equatorial plane. The expressions (7)–(11) generalize the results of Ref. 13 for circular equatorial orbits with $\delta \ll 1$ to the case of spherical orbits with $Q \neq 0$ and $q \neq 0$.

Thus, if particles from infinity are to have cuspidal points near the horizon, the following conditions must be satisfied:

$$\delta\ll1,\quad \alpha>1/2,\quad 0\leqslant\varkappa^2<4-\alpha^{-2}-\gamma^{-2},\quad \lambda\approx\alpha+\alpha^{-1}.$$

Closest to the horizon will be the equatorial circular photon orbits with r_{ph} ($\gamma = \infty, \varkappa^2 = 0$). Note that despite the coordinate proximity of the spherical orbits to the horizon, their proper distance from the horizon is $l_s - l_+ \sim M$ even when $\delta = 0, \alpha > \frac{1}{2}$.

In the case of a magnetized black hole, photons on trajectories with parameters close to the parameters of spherical orbits have the conditions that are the best for e^+e^- pair production. For photons that arrive at the horizon from infinity with impact parameters λ appreciably less than λ_s in the expression (8), we find

$$k_0\sim E(\Delta/M^2)^{-1/2},\quad k_0+k_r\sim E(\Delta/M^2)^{1/2}\quad \text{as}\quad \Delta\to0.$$

However, for photons with $\lambda \approx \lambda_s$ when $\alpha > \frac{1}{2}$, $\delta \ll 1$, and $\Delta^{1/2} \gtrsim M\delta$ the relations $k_0 \sim k_0 + k_r \sim E/\alpha$ hold, this being due to the effect on the photons of the centrifugal forces near a rapidly rotating, almost extremal black hole. We find the condition under which the threshold for pair production is attained for such photons near a black hole with current. By means of the relations (1), (2), and (7), omitting numerical factors of order unity, and taking into account the fact that $I \lesssim H_0 M$, we obtain

$$\frac{E}{m}\frac{H_0}{H_{cr}}\gg\delta. \tag{12}$$

The right-hand side of this inequality, in the absence of a black hole and under otherwise equal conditions, would be equal to unity. It follows from the condition (12) that near the horizon of a rapidly rotating ($a > M/2$) nearly extremal ($\delta \ll 1$) black hole with current induced by a weak ($H_0 \ll H_{cr}$) magnetic field in the accreting plasma soft ($E \ll m$) photons, for example, photons of the microwave background, are capable of generating e^+e^- pairs for appropriate choice of their trajectories. But in the case of an extremal ($\delta = 0$) rapidly rotating magnetized black hole photons with arbitrarily low energy at infinity are capable of producing pairs.

If the relation (12) is satisfied, photons with local radial momentum $k_r \ll k_0$ near the horizon travel a distance $l \gtrsim M$ in the superstrong crossed electromagnetic field. At the same time, the mean free path of such photons with respect to pair photoproduction is determined by the electron Compton wavelength $\lambda\!\!\!^- \ll M$. Therefore, practically all photons that pass near the horizon of the magnetized black hole and satisfy the condition (12) are necessarily transformed into e^+e^- pairs. For the same reason, the approximation of a homogeneous steady crossed field is applicable for describing the process of pair photoproduction near the black hole.

We note that the description given above of e^+e^- photoproduction by soft photons in the steady superstrong crossed field near the horizon of the black hole applies only to a locally nonrotating frame. In a frame of reference falling freely into the black hole, the picture of e^+e^- photoproduction will look different. In this frame of reference, all components of the electromagnetic field tensor are finite on the horizon: $|F'_{\mu\nu}| \lesssim H_0 \ll H_{cr}$. However, photons with trajectories having cuspidal points sufficiently close to the horizon will, near these points in the freely falling frame of reference, move from the black hole with energies $k'_0 \sim E(\Delta/M^2)^{-1/2}$, as a result of which the invariant condition $\chi \sim 1$ for pair production can again be satisfied.

In the problem we are considering, e^+e^- photoproduction near a rapidly rotating, almost extremal magnetized black hole, the possibility of pair production by soft photons is the most interesting case. In this case, however, neither the resulting pairs themselves nor any significant fraction of their synchrotron electromagnetic or neutrino radiation can escape to infinity and avoid falling into the black hole. The point is that in the case of pair photoproduction in a crossed electromagnetic field the quantity $k_- \equiv k_0 + k_r$ is conserved. If $E \ll m$, the $k_- \ll m$ as well and the produced electron and positron have momenta directed almost straight into the black hole. For such electrons and positrons, there are no cuspidal points with $k_r = 0$ when only the gravitational field of the black hole is taken into account. Besides this, in the crossed electromagnetic field the charged particles are accelerated mainly along the Poynting vector of this field,[14] which in the case under consideration also points into the black hole. Only a small fraction of the synchrotron radiation, directed into the backward hemisphere of the charged particles falling into the black hole, can escape to infinity. In the case of pair photoproduction by photons with energy $E \gtrsim m$, the produced electron and positron are also necessarily drawn into the black hole. However, in this case the Penrose effect has the consequence that the certain fraction of the synchrotron radiation, emitted by the produced particles while their momenta were directed away from the black hole, can escape to infinity without a strong red shift. For the photons and neutrinos of the synchrotron radiation that escape to infinity without strong red shift, the following relations are satisfied:

$$\lambda \approx \lambda_s, \quad 0 \leqslant \varkappa^2 < 4 - \alpha^{-2}.$$

By means of the condition $E^2 \geqslant V^2(\theta)$, which follows from the equation of motion (4) and the expression (6) for the effective polar potential, we find that this synchrotron radiation will escape to infinity within a solid angle with

$$|\cos\theta| < \cos\theta_0 = [2(2+\alpha^2)^{1/2} - 3]^{1/2}/\alpha. \quad (13)$$

For $\alpha = 1$, we obtain

$$\theta_0 = \cos^{-1}(2\cdot 3^{1/2} - 3)^{1/2} \approx 46^\circ.$$

4. POSSIBLE EXISTENCE OF ALMOST EXTREMAL BLACK HOLES

We now discuss the conditions under which an accreting black hole can be close to the extremal state with $\delta = 0$. In the case of disk accretion, a black hole can be spun up by matter falling into it to a state with specific angular momentum $a \leqslant a_{\text{can}} = 0.9982M$. The presence of this limit is due to the back-reaction on a black hole of photons emitted by the hot accretion disk and absorbed by the hole.[15] The further approach of the black hole to the extremal state can be helped by a regular magnetic field that arises in the accreting plasma. As is shown in Ref. 16, a black hole rotating in a homogeneous magnetic field of strength H_0 tends to acquire from the external medium the electric charge

$$q = 2H_0 Ma. \quad (14)$$

Such behavior of the black hole is analogous to the behavior in a magnetic field of a rotating conducting sphere, which also tends to acquire an electric charge corresponding to a minimum of the total electromagnetic energy of the system and proportional, like the change in the expression (14), to the magnetic field strength and the angular velocity of the rotation of the sphere.[17] By means of the expression (14), we find the strength of the regular magnetic field in the accreting plasma at which the electric charge accumulated on a black hole with $a = a_{\text{can}}$ carries it to the extremal state:

$$H_0 \gtrsim 10^5 (M/10^{12} M_\odot)^{-1} \text{ G}. \quad (15)$$

The existing theories of accretion onto a black hole do not enable us to find the strength H_0 of the regular magnetic field in the accreting plasma. It is at least evident from (15) that for realistic (not too alarming) values of H_0 only very massive black holes (quasars) can in principle be close to the extremal state by virtue of accumulation of electric charge. The arguments we have given for the "extremalization" of a black hole through the inductive accumulation of electric charge are qualitative. The process of approach of the magnetized black hole to the extremal state with $\delta = 0$ requires more detailed study. If magnetized black holes can indeed be almost extremal, then near them even soft photons traveling along trajectories with impact parameters close to those of spherical trajectories will produce e^+e^- pairs.

I thank V. S. Berezinskiĭ and A. I. Nikishov for helpful discussions of the questions considered in the paper and for critical comments.

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Translated by Julian B. Barbour