Small-slope approximation in wave scattering by rough surfaces
A. G. Voronovich
P. P. Shirshov Institute of Oceanology, Academy of Sciences of the USSR
(Submitted 6 March 1985
Where the wave scattering amplitude of a rough surface can be calculated by perturbation theory (Bragg scattering), and the slopes of the boundaries can be assumed small, it becomes possible to construct an approximation which holds for an arbitrary roughness height. To first order in the slope, the scattering amplitude can be factored into a geometric factor and a kernel which describes Bragg scattering. To a large extent, this approximation solves the problem of describing the scattering by rough surfaces by a common approach combining perturbation theory with the tangent-plane approximation.

1. INTRODUCTION
A scattering of waves by an irregular boundary arises in many branches of physics. For example, the diffraction of thermal molecular beams by the surfaces of single crystals (a process used to determine the characteristics of the corresponding interaction potentials) may be thought of as a scattering by an irregular potential barrier of infinite height.\(^1\)\(^-\)\(^4\)

The interaction of surface plasmons in a metal with boundary irregularities gives rise to optical emission, which is exploited to measure the properties of the plasmons.\(^5\) A similar principle underlies the excitation of the plasmons which substantially enhance Raman scattering in a thin organic film.\(^6\)\(^-\)\(^8\)
The problem of wave scattering by irregular surfaces also arises in optics,\(^9\)\(^-\)\(^18\) radio astronomy,\(^9\) and radiophysics.\(^10\)\(^-\)\(^12\)

The practical importance of this problem has made it the subject of a special study in underwater acoustics.\(^13\)\(^-\)\(^15\) Finally, it is one of the classical problems of diffraction theory. Although the theory of wave scattering by irregular surfaces has been the subject of many studies, nearly all of the analytic work has been confined to two approaches,\(^16\)\(^-\)\(^17\) which employ essentially perturbation theory\(^13\)\(^-\)\(^15\) and a semiclassical approximation\(^16\)\(^-\)\(^18\), (either the Kirchhoff approximation or the tangent-plane method).

Not uncommonly, however, situations arise which cannot not be dealt with by either of these approaches alone. An example is the scattering of sound or electromagnetic waves by the wavy surface of the sea. In these cases, the so-called two-scale surface model is used\(^19\)\(^-\)\(^25\), the scattering calculations in this model are carried out by combining the two classical approaches given above. The boundary irregularities are put in two classes: the scattering by large-scale smooth components is described in the Kirchhoff approximation, while the small-scale irregularities are dealt with by perturbation theory. This arbitrary classification of the irregularities in two groups unavoidably introduces in the theory at least one parameter whose choice is somewhat arbitrary. This is an inconvenience; it complicates the calculation of the corrections to the two-scale model; and it acts as a noise in an analysis of the inverse problem, i.e., in determining the spectrum of irregularities from the characteristics of the scattered field.

There is another way to approach the problem of wave scattering by irregular boundaries. This other approach is free of the disadvantage discussed above. It requires only that a parameter associated exclusively with the geometry of the boundary—its slope—be sufficiently small. This parameter is indeed small in many cases of practical interest (for the wave motion of the seas, for example, it is \(\lesssim 0.1\)). This alternative procedure allows one to write an asymptotic expansion of the wave scattering amplitude in this parameter. This expansion has a rather simple structure and is easily calculated. Since the assumptions underlying this approach to the problem are quite general, it is possible to outline a universal procedure for deriving an expression for scattering in the small-slope approximation, as in the approach which uses an expression describing the Bragg scattering of waves by boundary irregularities. In the statistical case, this approach reduces to a simple redefinition of the correlation function of the elevations. The corresponding expression for Bragg scattering can then be used (the redefined correlation function admittedly depends on the parameters of the incident and scattered waves).

After introducing this approach we examine in more detail the very simple problem of the scattering of scalar waves, for which the boundary condition is a Dirichlet condition, and we offer a direct derivation of the expansion in small slopes which does not lean on any general assumptions. In particular, we find in the latter case that the high-frequency limit for the scattering amplitude found by using only the first two terms of this expansion agrees exactly with the corresponding expression derived in the Kirchhoff approximation.

2. SMALL-SLOPE APPROXIMATION: THE GENERAL APPROACH
We introduce a Cartesian coordinate system with \(z\) axis directed vertically downward, and we denote by \(r = (x, y)\) the horizontal components of the radius vector \(R = (x, z)\). We assume that the uniform half-space has an irregular boundary \(z = \eta(r)\) at its bottom, and we assume that a monochromatic plane wave of frequency \(\omega\) is incident in the upward direction on this boundary:
\[ \Psi_{\omega} = \psi_{\omega} \exp \{i(\mathbf{q}_{\perp} \cdot \mathbf{r} + \omega t)\}, \quad \mathbf{q}_{\perp} + i \mathbf{q}_{n} = \hbar \omega \mathbf{p} / \hbar, \]

where \( c = \text{const} \) is the propagation velocity (the normalization factor is chosen to normalize the energy flux density in the vertical direction). Here and below, \( x \) and \( v \) are the horizontal and vertical components of the wave vector, \( v = \nabla \chi = (k_z^2 - x^2)^{1/2}, \) \( l_1 \) is the index \( O \) specifies the incident wave. Mathematically, the problem reduces to one of solving the Helmholtz equation \( (\Delta + k^2) \Psi = 0 \) with the corresponding boundary condition and the radiation condition. We assume everywhere below that the boundary is noncompact and is, on the average, planar. In the region \( z < \min(y, t) \), the field can evidently be written as a superposition of plane waves, so that the total field \( \Psi \) is

\[ \Psi_{\omega} = \int S(\omega, \mathbf{x}) \exp \{i(\mathbf{q}_{\perp} \cdot \mathbf{r} + \omega t)\} d\mathbf{x}. \]  

The scattering amplitude \( S \) is completely analogous to the quantum-mechanical amplitude and has corresponding properties. In particular, a reciprocity theorem must hold \( [S(\omega, \mathbf{x}) = S(\omega, \mathbf{x} - \mathbf{v})] \), as must a corresponding unitarity condition. In principle, knowledge of the scattering amplitude is equivalent to knowledge of the Green’s function, into which it can easily be converted. Although finding the Green’s function is frequently the ultimate goal of the solution of a given diffraction problem, the scattering amplitude is considerably more convenient to work with for a theoretical analysis.

Our approach is based on the simple properties of the scattering amplitude. Specifically, if the irregular boundary \( z = \eta(r) \) is displaced a whole horizontally by a distance \( \delta \), \( \eta(r) \rightarrow \eta(r - \delta) + \delta \), the scattering amplitude evidently transforms in accordance with

\[ S(\omega, \mathbf{x}) \rightarrow S(\omega, \mathbf{x} + \delta) \exp \{-i(\mathbf{q}_{\perp} \cdot \mathbf{x} + \omega \delta)\}. \]

Analogously, lifting the boundary a distance \( h \), \( \eta(r) \rightarrow \eta(r + h) \), leads to the transformation

\[ S(\omega, \mathbf{x}) \rightarrow S(\omega, \mathbf{x} + h) \exp \{i(\mathbf{q}_{\perp} \cdot \mathbf{x} + \omega h)\}. \]

Working from these elementary comments, we see, the scattering amplitude in the form of the expansion

\[ S(\omega, \mathbf{x}) = \int \exp \{ -i(\mathbf{q}_{\perp} \cdot \mathbf{x} + \omega t) \} \eta(\mathbf{r}) \left| \Phi_{\mathbf{n}, \mathbf{m}} \right\rangle \langle \Phi_{\mathbf{n}, \mathbf{m}} \right| d\mathbf{r}, \]

where

\[ \Phi_{\mathbf{n}, \mathbf{m}} = \Phi_{\mathbf{n}, \mathbf{m}}(\mathbf{r}) + i \mathbf{q}_{\perp} \eta(\mathbf{r}) \left| \Phi_{\mathbf{n}, \mathbf{m}} \right\rangle \langle \Phi_{\mathbf{n}, \mathbf{m}} \right| d\mathbf{r}. \]  

Here the \( \eta(\mathbf{r}) \) are the Fourier components of the irregularities:

\[ \eta(\mathbf{r}) = \int \eta(\mathbf{r}) \exp \{i(\mathbf{q}_{\perp} \cdot \mathbf{r} + \omega t)\} d\mathbf{q}. \]

An integration is to be carried out over the variables \( \xi_1, \ldots, \xi_n \). The \( \Phi_{\mathbf{n}, \mathbf{m}} \) are functions, symmetric with respect to \( \xi_n \), which do not depend on the shape of the surface. Since \( \Phi \) is essentially an arbitrary functional of the elevations \( \eta \) which is written as an integral-power series, it is legitimate to seek \( S \) in the form (2). We see that a factor has been singled out in (2) which by itself provides the required transformation properties for \( S \). The reason for the \( \delta \)-functions in (2) is the law according to which \( S \) transforms under horizontal translations. Although, strictly speaking, this is not a consequence exclusively of the transformation property of \( S \) when the surface as a whole is raised, it is natural to assume (and this assumption is supported by the more specific calculations below) that the functional \( \Phi \) must not transform at all under the substitution \( \eta_0(\mathbf{r}) \rightarrow \eta_0(\mathbf{r}) + h \delta(\mathbf{r}) \). It follows immediately that the functions \( \Phi_{\mathbf{n}, \mathbf{m}} \) must vanish for \( \xi_n = 0, \ldots, n \). We can thus set

\[ \Phi_{\mathbf{n}, \mathbf{m}} = \Phi_{\mathbf{n}, \mathbf{m}}(\mathbf{r}), \]

where \( \Phi_{\mathbf{n}, \mathbf{m}} \) are the Fourier components of the irregularities.

The substituting this expression for \( \Phi_{\mathbf{n}, \mathbf{m}} \) into each other. Accordingly, the quantities \( \eta(\mathbf{r}) \) and \( \Phi_{\mathbf{n}, \mathbf{m}} \) [and also \( \Phi_{\mathbf{n}, \mathbf{m}}(\mathbf{r}), \) see below] are generally matrices.

It turns out that the kernels \( \Phi_{\mathbf{n}, \mathbf{m}} \) in expansion (3) can be found easily by taking the limit of ordinary perturbation theory, with \( \eta \rightarrow 0 \), if we assume that we know the expansion of the scanning amplitude in an ordinary integral-power series in the elevations \( \eta \) (the latter expansion is found from ordinary perturbation theory; usually, no difficulties will arise). However, we are forced to appeal to an additional assumption. Specifically, expression (2) for \( \Phi \) has some gauge arbitrariness, since we could add to \( \Phi_{\mathbf{n}, \mathbf{m}} \) any arbitrary function which vanishes for \( \mathbf{x} = \mathbf{x}_0 + \ldots + \xi_n \). The substitution

\[ \Phi_{\mathbf{n}, \mathbf{m}} = \Phi_{\mathbf{n}, \mathbf{m}}(\mathbf{r}) + \left| \Phi_{\mathbf{n}, \mathbf{m}} \right\rangle \left\langle \Phi_{\mathbf{n}, \mathbf{m}} \right|, \]

where \( \delta \) is arbitrary, does not change the magnitude of the scattering amplitude in (2). In other words, if we single out from \( \Phi_{\mathbf{n}, \mathbf{m}} \) a part which vanishes on this hypersurface, then an integration by parts will "correct" this part to a term of order \( n + 1 \). Using this circumstance, we can eliminate from \( \Phi_{\mathbf{n}, \mathbf{m}} \) on one of the integration variables (e.g., \( \xi_n \)), by setting \( \xi_n = \mathbf{x} - \mathbf{x}_0 - \ldots - \xi_n - 1 \) in \( \Phi_{\mathbf{n}, \mathbf{m}} \). Substituting this expression for \( \Phi_{\mathbf{n}, \mathbf{m}} \) into (2), we can perform in the resulting expression an operation which is the inverse of that which we have just described; specifically, we can integrate over the variable \( \xi_n \). As a result, we find a factor \(-i\nabla \eta\), which we can then eliminate by an integration by

A. G. Voronovich
parts. As a result, we find the following equation, which holds within quantities of order $(\mathcal{V}\eta)^s$.

$$\sum \exp \left[ -i(\mathbf{x}-\mathbf{x}_0) \cdot \mathbf{r} + i(\boldsymbol{\nu} + \mathbf{v}_0) \cdot \mathbf{r} \right]$$

\[
\times \frac{dr}{(2\pi)^3} e^{i\mathbf{q}\cdot(\mathbf{z} - \mathbf{z}_k - \ldots - \mathbf{z}_h)} 
\times \delta_{\eta}(\mathbf{z}, \mathbf{z}_k, \ldots, \mathbf{z}_h) + \sum \int \delta(\mathbf{x} - \mathbf{z}_k) \delta(\mathbf{x} - \mathbf{z}_h) d\mathbf{x} \cdot d\mathbf{z}_k \cdot d\mathbf{z}_h.
\]

We now seek the structure of the term of order $n - 1$ in (2), (3) make it possible to calculate $S$ with an accuracy up to $(n - 1)$st order in the parameter $\mathcal{V}\eta$. We now seek the structure of the term of first order in $\eta$, the scattering amplitude can be written in general as follows:

$$S(x, x_0) = \int_{\mathcal{V}\eta}^s d\mathbf{x} \cdot d\mathbf{z}_k \cdot d\mathbf{z}_h \delta(\mathbf{x} - \mathbf{z}_k) \delta(\mathbf{x} - \mathbf{z}_h)$$

\[+ \sum \int \delta(\mathbf{x} - \mathbf{z}_k) \delta(\mathbf{x} - \mathbf{z}_h) d\mathbf{x} \cdot d\mathbf{z}_k \cdot d\mathbf{z}_h.
\]

\[S_{\text{Bragg}} = \int_{\mathcal{V}\eta}^s d\mathbf{x} \cdot d\mathbf{z}_k \cdot d\mathbf{z}_h \delta(\mathbf{x} - \mathbf{z}_k) \delta(\mathbf{x} - \mathbf{z}_h) + \sum \int \delta(\mathbf{x} - \mathbf{z}_k) \delta(\mathbf{x} - \mathbf{z}_h) d\mathbf{x} \cdot d\mathbf{z}_k \cdot d\mathbf{z}_h.
\]

We now find $\Phi_0$. For this purpose we first write an expression for the scattering amplitude which holds within quantities of order $s$.

\[S_{\text{Bragg}} = \int_{\mathcal{V}\eta}^s d\mathbf{x} \cdot d\mathbf{z}_k \cdot d\mathbf{z}_h \delta(\mathbf{x} - \mathbf{z}_k) \delta(\mathbf{x} - \mathbf{z}_h) + \sum \int \delta(\mathbf{x} - \mathbf{z}_k) \delta(\mathbf{x} - \mathbf{z}_h) d\mathbf{x} \cdot d\mathbf{z}_k \cdot d\mathbf{z}_h.
\]

\[\Phi_0(x, x_0, \mathbf{v}) = \exp \left[ i(\mathbf{x} - \mathbf{x}_0) \cdot \mathbf{r} + i(\boldsymbol{\nu} + \mathbf{v}_0) \cdot \mathbf{r} \right]
\times \int \frac{dr}{(2\pi)^3} e^{i\mathbf{q}\cdot(\mathbf{z} - \mathbf{z}_k - \ldots - \mathbf{z}_h)} 
\times \delta_{\eta}(\mathbf{z}, \mathbf{z}_k, \ldots, \mathbf{z}_h) d\mathbf{x} \cdot d\mathbf{z}_k \cdot d\mathbf{z}_h.
\]

\[S_{\text{Bragg}} = \int_{\mathcal{V}\eta}^s d\mathbf{x} \cdot d\mathbf{z}_k \cdot d\mathbf{z}_h \delta(\mathbf{x} - \mathbf{z}_k) \delta(\mathbf{x} - \mathbf{z}_h) + \sum \int \delta(\mathbf{x} - \mathbf{z}_k) \delta(\mathbf{x} - \mathbf{z}_h) d\mathbf{x} \cdot d\mathbf{z}_k \cdot d\mathbf{z}_h.
\]
simplify the resulting expansion, by eliminating from (3) the $n = 2$ term which we have just calculated. According to (4), we find

$$\Phi(x, x_i) = \frac{s_i}{v + v_i} + \frac{s_j(v_{ij})}{v + v_j} B_i(x, x_i, x_j, x_k) + \frac{s_i B_i(x, x_i, x_j, x_k)}{v + v_i} + \frac{s_j B_i(x, x_i, x_j, x_k)}{v + v_j}. $$

We have thus derived a small-slope expansion with an accuracy of order $1/2$. This procedure can be continued to calculate structure functions of any order. Specifically, collecting terms of order $1/2$ and comparing the result with the known power-law expansion, we find $\Phi$. We transform it, in accordance with (4), into a term with $n = 2$; etc.

In the two examples mentioned above the kernel $B_i(x, x_i, x_j)$ can be put in the form$^1$

$$B_i(x, x_i, x_j) = -2v_{ij} B(x, x_j) B(x', x', x_i). $$

Expression (10) apparently holds for a wide range of scattering problems. In these cases, the small-slope expansion incorporate terms of order $1/2$ inclusively is

$$S_{1/2}(x, x_i) = S_i(v_{ij}) \int \frac{dr d\theta}{v + v_i} \left[ \exp \left( -i (x - x_i) r + i (v + v_i) \right) \right] \times \exp \left( -i (x - x_i) \right) \delta(\theta) + i (v + v_i) \eta(\theta) \right] \times (v + v_i + v_{ij} \right) B(x, x_i + x_j, x_k) + (v + v_i + v_{ij} \right) B(x, x_i, x_j, x_k) \eta(\theta) \right).$$

(11)

The expression in brackets vanishes if $z = 0$ and $z = -z_0$. We can therefore divide it by $\delta(\theta - x_i - x_j)$; we then immediately see that the second term in (11) transforms into a term of second order in (2) and (3). We do not find a singularity in $\Phi$, at $z_2 = 0$.

We now consider a spatially uniform Gaussian statistical ensemble of elevations, $\eta(\theta)$. The mean scattering amplitude can be written in general as

$$S_{1/2}(x, x_i) = P(x) \delta(\theta(\xi), \xi),$$

where $P(x)$ is the so-called mean reflection coefficient. The average in (11) can be carried out without difficulty; as a result we find

$$P(x) = S_{1/2} \int \left[ 1 - 2v_{ij} \right] B(x, x_i + x_j) \times B(x + x_k, x) - v_{ij} G(\xi) d\xi, $$

(12)

$$G(\xi) = \int W(\theta + \theta) \frac{d\theta}{(2\pi)^2}, $$

$$W(\theta) = \eta(\theta + \theta) \eta(\theta) \delta(\theta), $$

where $G(\xi)$ is the spectrum of irregularities, and $W(\theta)$ is the corresponding correlation function. The first term in this expression is the same as the expression for $P$ derived in the Kirchhoff approximation$^1$, while the correction term reflects scattering by small-scale components of the irregularities.

Writing the scattering amplitude as a sum of the mean and fluctuating parts,

$$S(x, x_i) = P(x) \delta(x - x_i) + \Delta S(x, x_i), $$

we can write

$$\Delta S(x, x_i) \approx \Delta S(x, x_i), $$

$$= Q(x, x_i) \delta(x - x_i - x + x_i + x_i + x_i), $$

Any second moment of the random fields can be expressed in terms of $Q$ by means of quadratures. In practice, we are interested in the scattering coefficients, which are related to $Q$ in the following way:

$$x = v_{ij} Q(x, x_i) - v_{ij} Q(x_i, x_i) \delta(x - x_i - x_i + x_i + x_i).$$

The expression for $m_i$ derived in the approximation $S = S_{1/2}$ is

$$m_i = m_i = \int \frac{d\theta}{(2\pi)^2} \left[ \exp \left( -i (x - x_i) \right) \eta(\theta) \right] \times \exp \left( -i (v + v_i) \right) \left( W(0) - W(\theta) \right) \right) \delta(\theta) \right) \right) \right) \right).$$

(13)

The integral containing the statistical characteristics of the elevations is of precisely the same form as that used in the Kirchhoff approximation, and it is essentially of purely geometric origin. The scattering properties of the surface, on the other hand, enter $m_i$ in the approximation $S = S_{1/2}$ in the form of a factor which has the same structure as when we used expression (5) for Bragg scattering:

$$m_i = \int \frac{d\theta}{(2\pi)^2} \left[ \exp \left( -i (x - x_i) \right) \eta(\theta) \right] \times \exp \left( -i (v + v_i) \right) \left( W(0) - W(\theta) \right) \right) \delta(\theta) \right) \right) \right) \right).$$

(14)

Comparing the latter expression with (13), we can say that the transformation from the small-perturbation method to the small-slope approximation reduces to a redefinition of the correlation function,

$$W(\theta) = W_{1/2}(\theta) = \frac{1}{v + v_i} \times \exp \left( -i (v + v_i) \right) \left( W(0) - W(\theta) \right) \right) \delta(\theta) \right) \right) \right) \right),$$

after which we should use expression (14), which follows from ordinary perturbation theory. An analogous rule can be formulated easily for $Q$ (and even for $\eta$ in (8)). This procedure would appear to be appropriate only when we are using the first approximation in the expansion in small slopes, in which case the geometric effects which stem from large-scale irregularities and the "actual" Bragg scattering, by irregularities with horizontal scale dimensions on the order of the wavelength, are factored out.

3. CASE OF SCALAR WAVES AND A FREE BOUNDARY

Since the arguments used in $m_i$ in the derivation of the small-slope expansion and in the calculation of the structure functions $\Phi$, were quite general in nature, we would like to
derive this expansion by some other methods, which does not make explicit use of those general considerations. In the present section of the paper we show how this can be done in the very simple scalar problem with a Dirichlet boundary condition (this situation corresponds to the scattering of sound waves by a free surface or to the scattering of TE electromagnetic waves by an ideal surface in the two-dimensional case). We employ the Rayleigh equation, which can be found by calculating the scattered field at \( z = \eta(x) \) directly from Eq. (1) and substituting the result into the boundary condition \( \eta_{\eta} - \eta_{x} = 0 \)

\[
\int S(x', \eta') \frac{1}{\sqrt{2\pi}} \exp \left[ i w(x' - iv \eta(t)) \right] dx' = -\frac{1}{\sqrt{2\pi}} \exp \left[ i w(x + iv \eta(t)) \right].
\]

This equation is valid if the Rayleigh hypothesis applies to the irregularities \( z^2 \). However, this is not an important point and in calculating the functions \( \Phi_n \) without discussing the convergence properties of the expansion.\(^{23}\) Multiplied the Rayleigh equation by \( \exp[ -i x r + iv \eta(x) ]/\sqrt{2\pi} \) and integrating over \( r \), we can easily put in the form

\[
\Phi_n(x, \eta) = -\left( \frac{\nu}{\nu_0} \right)^n \int \exp[-i(x - \eta_0 + x_0 + iv \eta_0)] \left[ \frac{d}{(2\pi)^2} \right] \times \nu \eta_0 \left( \frac{d}{(2\pi)^2} \right) dx'.
\]

Here we have used the following identity, which is easily found through integration by parts:

\[
\int \exp[-i(x - \eta_0 + x_0 + iv \eta_0)] \left[ \frac{d}{(2\pi)^2} \right] \times \nu \eta_0 \left( \frac{d}{(2\pi)^2} \right) dx' = \left( \frac{\nu}{\nu_0} \right)^n \int \exp[-i(x - \eta_0 + x_0 + iv \eta_0)] \left[ \frac{d}{(2\pi)^2} \right] \times \nu \eta_0 \left( \frac{d}{(2\pi)^2} \right) dx'.
\]

We see that the integral term in (15) contains the small parameter which we need—the slope—and that this equation can be solved by iteration. The result can easily be put in the form in (2), we find the following representation for the functional \( \Phi \):

\[
\Phi = \sum \Phi^{(n)} \to \Phi^{(n)}(x, \eta_0), E \eta \]
height of the irregularities. Specifically, this height need not be small in comparison with the length of the incident wave. This is done by introducing in the expression for the scattering amplitude a factor which imparts certain transformation properties to $S$. This factor is of essentially purely geometric origin; it reflects the circumstance that different rays in the incident wave travel unequal paths because of the boundary irregularities, and an originally plane phase front of the wave becomes distorted. It turns out that in a first approximation in the slope of irregularities the expression for the scattering amplitude can be factored into this quantity of geometric origin and a kernel which describes Bragg scattering, the resulting approximation [in particular, expression (11)] substantially solves the problem of describing the scattering by irregular surfaces in an approach which combines perturbation theory with the Kirchhoff approximation. The assumption of small slopes need not be a substantial restriction, since in the case of a surface of a general type, with slopes on the order of unity, a repeated reflection of waves will necessarily arise in the high-frequency limit, and the Kirchhoff approximation in its standard formulation will become inapplicable.

In particular, we find from these results the following experimental condition under which the slopes of the irregularities at the rough surface being probed are small [see (13), (14)]: The ratios of the second moments for the scattering amplitudes for waves of different types or of different polarizations, under the condition that the wave vectors of the incident and scattered waves are equal, must not depend on the spectrum of irregularities, and they must agree with the results calculated in first-order perturbation theory [i.e., in accordance with (14)].

I am deeply indebted to V. I. Tatarskii and V. I. Klyatskin for a useful discussion of this study.

Translated by Dave Parsons