Surface and contact thermoelectric effects due to phonon drag in superconductors

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It is shown that, in the case of a superconducting film deposited on a substrate in which thermal conduction is due to phonons (a dielectric or a superconductor with $T_c \gg T$), the contribution of phonon drag to the thermoelectric current of normal excitations may exceed the diffusion contribution by several orders of magnitude. Specific surface thermoelectric effects in a superconductor with $T_c \gg T$ in a magnetic field, which are due to the presence of quasiparticles localized near the surface, are discussed. In particular, a description is given of a surface analog of an effect investigated previously for the spatially homogeneous situation in thin films. The effect consists of the appearance of an imbalance in the population of the excitation spectrum branches and of a gauge-invariant potential proportional to $v_s \nabla T$. Thermoelectric effects near thermal contacts between the superconductor and a medium, whose thermal conductivity is due to phonons, are investigated. The phonon flux introduced into the specimen under these conditions produces a considerable drag effect in the contact region. It is shown that, when the thermally conducting medium is a superconductor with $T_c \gg T$ and both superconductors form a closed thermoelectric circuit, the contact contribution to the "thermoelectric" magnetic flux Φ_T may exceed the bulk diffusion contribution by two or three orders of magnitude. It is pointed out that these conclusions may be relevant for real thermoelectric experiments in which values of Φ_T are often found to exceed theoretical estimates by several orders of magnitude.

There has been considerable interest in recent years in thermoelectric phenomena in superconductors whose specific feature (first noted by Ginzburg[1]) is the balancing of the thermoelectric current of normal excitations by the current of the superconducting condensate. The first step was to examine the appearance of the unquantized thermoelectric addition to the magnetic flux threading the closed superconducting thermoelectric ring. This effect was investigated in both theory (see, for example, Refs. 2 and 3 and the review given in Ref. 4) and experiment.[5–8] It was shown subsequently, both theoretically and experimentally, that, when a temperature gradient and a superconducting current are simultaneously present in superconducting films, a difference is established between the populations of the electron-like and hole-like branches of the quasiparticle spectrum, which gives rise to the appearance of a thermoelectric contribution U_T to the gauge-invariant potential.[9–12] At the same time, whereas measurements of U_T showed adequate agreement with the theory, several experiments[6–8] concerned with the thermoelectric addition to the magnetic flux Φ_T showed the presence of temperature-dependent magnetic fluxes that exceeded theoretical estimates[2,3] by several orders of magnitude. The temperature dependence of these fluxes was much more pronounced than predicted by the theory.[8] One possible reason for this behavior is the presence of a masking "redistribution effect" due to the temperature dependence of the depth of penetration of the magnetic field and the sensitivity to "background" magnetic fields. This was pointed out in Refs. 13 and 14. However, it would appear that not all the observed anomalies are due to this effect. For example, the authors of Ref. 8 reported control measurements, made on a homogeneous superconducting circuit, in which no evidence was found for an appreciable contribution of the redistribution effects. The overall picture of phenomena occurring in a real thermoelectric circuit is thus still not entirely clear. Further studies of thermoelectric phenomena in superconductors would therefore appear to be important from two points of view. Firstly, it would be desirable to have new experiments capable of yielding additional information on the transport properties of superconductors. Secondly, it would be desirable to perform an analysis of the wide range of phenomena that can be observed in real thermoelectric experiments.

In a previous paper,[14] we drew attention to the fact that the electrodynamics and kinetics of superconductors suggest that, in some cases, the surface region may play an important role in thermoelectric effects. In particular, thermoelectric currents localized for one reason or another in the surface region may provide a contribution of the same order as that due to bulk currents of the same density. However, Ref. 14 was largely restricted to the analysis of thermoelectric currents associated with the motion of the condensate (and proportional to $v_s \nabla T$, where v_s is the superfluid velocity). It was assumed that the corresponding addition to the distribution function relaxes only on phonons; it was subsequently shown[15] that, in reality, this addition relaxed on impurities and was therefore negligible. In this paper, we shall examine a different class of surface phenomena due to the drag of quasiparticles by phonons. We shall show that this drag may often give rise to an anomalously large contribution to the thermoelectric effect in surface and contact regions.

The influence of drag processes on thermoelectric effects in superconductors was first investigated by Gurevich

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and Krylov.[16,17] They showed that the contribution of phonon drag to the thermoelectric current may be of the same order as or greater than the usual "diffusion" contribution. The size of the effect was determined, in particular, by the fact that the excitation-dragging phonon flux was limited by scattering on the excitations. We note, however, that this limitation (unavoidable in the spatially homogeneous situation examined in Refs. 16 and 17) may be lifted when the quasiparticles occupy a relatively small volume as compared with the entire volume in which the phonons propagate, and when the size of the corresponding region is smaller than the phonon mean free path l_{ph}. This situation may occur, in particular, when the semiconducting film of thickness $d < l_{ph}$ (here, we have in mind values of l_{ph} characteristic for a metal) is deposited on a dielectric substrate. The nonequilibrium phonon distribution function in the dielectric is determined by the relaxation time τ_{ph}, which is much greater than the corresponding time in the metal. Since, for temperatures less than or of order T_c, the phonon-phonon processes are "frozen out," the relaxation time τ_{ph} in a reasonably clean crystal dielectric is controlled by scattering by the boundaries, and may reach values of 10^{-6}–10^{-5} s. If the acoustic contact between the film and substrate is good enough, and the coefficient k representing the influx of phonons into the films is not too small, the phonons entering the film produce a substantial drag effect. We shall show that, when $k \sim 1$, the ratio between the corresponding contribution to the thermoelectric current and the diffusion contribution is determined by the parameter $\tau_{ph}/\tau_{e-ph} \gg 1$, where τ_{e-ph} is the escape relaxation time of electrons on phonons.

We note that Zavaritskiĭ and Zavaritskiĭ, in their interesting paper,[18] were the first to emphasize the contribution of drag in the situation where electrons occupied a small portion of the volume of the specimen. These authors[18] investigated the thermoelectric effects in quasi-two-dimensional n-type semiconducting systems on cleavage surfaces and in Ge bicrystals. Our treatment is essentially analogous to that put forward in Ref. 18.

It is obvious that, in the system that we are considering, the role of the dielectric can be played by a superconductor whose T_c is appreciably lower than the T_c of the film. Quasiparticle excitations in this superconductor are then "frozen out" and the phonon mean free path is large. It is probable that it is easier to achieve good acoustic contact with the film in this system. An interesting and unusual physical picture can be realized in a massive homogeneous superconductor when $T_c \gg T$ when a superconducting current is flowing over its surface. It is well known that, in this case, there is a group of quasiparticles localized in the surface layer, due to the additional term $\mathbf{p}\cdot\mathbf{v}_s$ in the total quasiparticle energy ($\mathbf{p}$ is the quasimomentum) and, for $pv_s \gg T$, the density of these particles is exponentially large in comparison with the corresponding value within the body of the specimen. In this situation, we are dealing with a peculiar layered system which, in many respects, is analogous to the above dielectric-superconductor system and, as will be shown later, it also exhibits the anomalous contribution of the drag effect. It turns out that the thermoelectric current of surface particles is then very sensitive to the nature of surface scattering, and the main contribution is provided by a small group of particles moving almost parallel to the surface. The associated thermoelectric magnetic flux is appreciable and accessible, at least in principle, to experimental observation. However, in practice, this effect appears to be difficult to observe because of the presence of the masking redistribution effect.[13,14]

We shall therefore also consider another surface thermoelectric effect, namely, branch imbalance in the quasiparticle spectrum under the influence of a temperature gradient, which is an extension of the effect examined in Refs. 9–12 to the case of a bulk superconductor. We shall show that, in this situation, the surface assumes a certain potential U_T relative to distant portions of the volume, which can be measured by a method analogous to that described in Refs. 9 and 11. The potential U_T is not very sensitive (in contrast to Φ_T) to the nature of surface scattering (although "glancing" particles again play the leading role in this case). As will be seen, the contribution of phonon drag to U_T can be at least comparable with the diffusion contribution. It is probable that these contributions can be separated because they have a different temperature dependence and because of a peculiar size effect (due to the dependence of τ_{ph} on the size of the specimen at low enough temperatures). The important point is that measurement of U_T will enable us to investigate the kinetics of surface quasiparticles in a superconductor placed in a magnetic field, thus avoiding the influence of the masking effect.[13,14]

It is well known (see, for example, Ref. 19) that, if a temperature difference is established between the ends of a metal specimen of macroscopic size, the quasiequilibrium distribution in the electron and phonon systems (corresponding to a temperature gradient ∇T_0 common to electrons and phonons) is established only at a sufficient distance from the thermal contact (exceeding the characteristic electron diffusion length l_d for scattering on phonons). In the contact region, on the other hand, this distribution is essentially of the nonequilibrium type. The point is that, since the electron thermal conductivity $\varkappa_e$ of a metal that is not too "dirty" is much greater than the phonon conductivity $\varkappa_{ph}$, electrons are largely responsible for the transfer of heat "within the body" of the specimen. However, the thermal flux due to electrons on the boundary of the specimen is zero, so that there is a contact region of finite size (of the order of the phonon mean free path l_{ph} in the metal) in which phonons are responsible for the transfer of heat. Since $\varkappa_e \gg \varkappa_{ph}$, the phonon flux in this region is much greater than in the quasiequilibrium region. On the other hand, the effective phonon temperature difference ΔT_{ph} over the length l_{ph} is of order

$$\nabla T_0 l_{ph} (l_d/l_{ph})^2 \sim \nabla T_0 l_{ph} (\varkappa_e/\varkappa_{ph})$$

and may be comparable with (or even greater than) the electron temperature difference across the entire specimen. These considerations are also valid for a superconductor when $T \sim T_c$. It is natural to assume that a high contact phonon flux will produce an appreciable drag effect. However, in the case of thermal contact with a dielectric, the condition that the normal current on the specimen boundary must be zero ensures that, as we shall see, the drag current can only reach values of the order of the "bulk" thermoelectric current of diffusion origin. However, a different situation obtains when the specimen is in contact with a superconductor with much higher $T_c = T_{c\mathrm{II}}$. On the one hand, the thermal conductivity in the second superconductor may be due to phonons if the quasiparticle excitations at the corresponding temperature are "frozen out" so that the above considerations become valid in this situation as well. On the other hand, since, for $T \ll \Delta_{\mathrm{II}}$, practically all the quasiparticles in the first superconductor undergo Andreev reflection by the separation boundary between the two superconductors, the boundary condition on this boundary demands that there be a branch imbalance in the excitation spectrum[1)] and not that the normal quasiparticle current must be zero. We shall show, on the basis of the above considerations, that the contact drag current of normal excitations can be quite large in this case, so that its contribution to the thermoelectric effect may appreciably exceed the contribution of the "bulk" thermoelectric current. Since, in practical thermoelectric experiments, the superconductors making up the thermoelectric circuit usually have very different values of T_c (the "active" pure superconductor has the lower T_c), it may well turn out that the situation that we are considering will be relevant to such experiments.

1. DRAG EFFECT IN A SUPERCONDUCTING FILM DEPOSITED ON A DIELECTRIC OR SUPERCONDUCTOR WITH HIGHER T_c

Let us suppose that a superconducting film of thickness $d \lesssim l_{ph} \sim \hbar v_F / T$ (l_{ph} is the phonon mean free path in the corresponding metal for $T \sim T_c$) is deposited on a substrate in the form of a crystalline dielectric. Suppose, further, that a temperature gradient[2)] ∇T is established in the substrate along the surface. The phonon distribution function in the dielectric then contains the following contribution, which is odd in the phonon momenta $\hbar q$:

$$N_{\mathbf{q}}^1 \sim (\mathbf{q}\nabla T)\frac{w^2}{\omega}\tau_{ph}\left(\frac{\omega}{T}\right)\frac{\partial N_0}{\partial\omega} \sim (-\mathbf{q}\nabla T)\frac{w^2\tau_{ph}}{\omega T}N_0, \qquad (1)$$

where w is the velocity of sound and N_0 is the equilibrium phonon distribution function. For simplicity, we shall neglect the difference between the elastic parameters of film and substrate, and will also assume that a perfect acoustic contact is established between the two, i.e., we shall suppose that the phonon transmission coefficient k_t between substrate and film is equal to unity (for $k_t < 1$, our subsequent estimates must be multiplied by k_t). Under these conditions, the order of magnitude of the phonon distribution function in the film is again given by (1). [Strictly speaking, on the boundary with vacuum, N_q differs from that given by (1) because the reflection of phonons by this boundary is not purely specular; however, this difference does not affect the order of magnitude of the quantity given by (1) nor the structure of this expression, and we shall neglect it.] Substituting (1) into the quasiparticle-phonon collision integral, we obtain the following expression for the addition n_1 to the quasiparticle distribution function in the film (see Appendix):

$$-\mathbf{v}\frac{\xi}{\varepsilon}\nabla T\frac{\varepsilon}{T}\frac{\partial n_0}{\partial\varepsilon} + I^-_{e-ph} = \frac{n_1}{\tau_{im}}\frac{|\xi|}{\varepsilon}, \qquad (2)$$

where $\varepsilon = (\xi^2 + \Delta^2)^{1/2}$, $\xi = E_p - \mu$, E_p is the electron dispersion law in the normal state, μ is the electron chemical potential, τ_{im} is the electron relaxation time on impurities, and, for $T \sim T_c$, we have

$$I^-_{e-ph} \sim \frac{-(\mathbf{p}\nabla T)}{p_F}\left(\frac{\tau_{ph}}{\tau_{e-ph}}\right)\frac{T}{\Theta_D}w\left[\frac{\partial n_0}{\partial\varepsilon} - mw^2\frac{\partial^2 n_0}{\partial\varepsilon^2}\operatorname{sign}\xi\right]. \quad (3)$$

Hence, it is clear that the ratio of the contribution of drag processes to the thermoelectric current j_n of normal excitations to the corresponding diffusion contribution [determined by the first term on the left of (2)] is measured by

$$\frac{\tau_{ph}}{\tau_{e-ph}}\frac{w}{v_F}\frac{T}{\Theta_D}\frac{\mu}{T} \sim \frac{\tau_{ph}}{\tau_{e-ph}}. \qquad (4)$$

In practice, (4) must be multiplied by the phonon transmission coefficient k_t. However, since $\tau_{ph}/\tau_{e-ph} \gg 1$ (for example, for $T \sim 4$ K, $\tau_{e-ph} \sim h\Theta_D^2/T^3 \sim 10^{-9}$ s, $\tau_{ph} \sim L/w \sim 10^{-5}$ s, $\tau_{ph}/\tau_{e-ph} \sim 10^4$), the phonons in the substrate may be expected to have an appreciable influence on the thermoelectric effect even when the acoustic contact is not perfect. In view of the foregoing, we find that, when $T \sim T_c$,

$$\mathbf{j}_n \sim \frac{L}{w\tau_{e-ph}}\eta\nabla T, \qquad (5)$$

where η is the thermoelectric coefficient of the metal in the normal state. Thus, when the dielectric substrate is not too "dirty," j_n exhibits a peculiar size effect.

As noted above, the substrate in the above situation can also be a superconductor with a high T_c. If we neglect the scattering of phonons by defects, we then have

$$\tau_{ph} \sim \min\left\{\frac{\hbar}{T}\frac{v_F}{w}\exp\left(\frac{\Delta_2}{T}\right), \frac{L}{w}\right\}.$$

It is clear that, when the thermoelectric magnetic flux is measured, the contacts with the second component of the thermoelectric ring must be placed on the free surface of the film.[3)] In accordance with the calculations reported in Ref. 14, the observed value of Φ_T is then exclusively determined by the surface current density of normal excitations (at depths of the order of the magnetic field penetration depth λ) and is therefore insensitive to the electrodynamic properties of the system due to the presence of the superconducting substrate.

2. THERMOELECTRIC EFFECTS IN A MASSIVE SUPERCONDUCTOR IN THE PRESENCE OF A SUPERCONDUCTING CURRENT

Consider a massive superconductor carrying a superconducting current on its surface (for example, due to Meissner screening) and let us suppose that

$$\Delta \gg pv_s \gg T. \qquad (6)$$

Since the total quasiparticle energy is $\tilde{\varepsilon} = \varepsilon + \mathbf{p}\cdot\mathbf{v}_s$, the surface layer (whose thickness is determined by the rate at which the magnetic field decreases) contains a localized group of quasiparticles which are reflected from the surface of the specimen in the usual way and from the v_s profile in the Andreev manner (see, for example, Refs. 20 and 14 and 21). Let us investigate the effect of phonon drag on this group of particles. In view of (6), the characteristic distance $\bar{z}$ of the trapped particles from the surface is given by $\bar{z} \sim (T/p_F v_s^0)$, where $v_s^0 = v_s$ $(z=0)$, z is measured from the surface along the inward normal to the superconductor, and it is clear that $\bar{z} \ll l_{ph}$. Hence, the phonon distribution function is given by (1), where

$$\tau_{ph} \sim \min\left\{\frac{\hbar}{T}\frac{v_F}{w}\exp\left(\frac{\Delta}{T}\right), \frac{L}{w}\right\},$$

and L is the size of the specimen. To calculate the thermoelectric current j_n and the thermoelectric potential U_t, we use the kinetic equation given in Ref. 15, i.e., we shall assume the validity of the classical description of the trapped particles, the condition for which can be written in the form (cf. Ref. 21)

$$\bar{z} \gg \hbar \bar{v}_z [\Delta p_F v_s^0 \bar{z}/\lambda]^{-1/2}, \tag{7}$$

where $\bar{v}_z$ is the characteristic value of the corresponding velocity component. In general, when (6) is satisfied, we obtain the requirement $T^3/\Delta(p_F v_s)^2 \gg (\xi/\lambda)^2$, where ξ is the coherence length. This condition is relatively rigid and corresponds to a pure type II superconductor. However, on the one hand, the approach based on the kinetic equation provides us with a very clear physical interpretation; on the other hand, we shall see that the resulting qualitative conclusions are valid irrespective of whether (7) is satisfied. This is probably also true for the estimated orders of magnitude of the various quantities in which we are interested. We note that (7) is much more readily satisfied for "glancing" particles with low v_z, which, as we shall see, provide the main contribution to the effects that we are considering. For example, when the "thermoelectric" potential U_T is calculated, the significant values are $v_z \sim v_F(T/p_F v_s)^2(\lambda/l_e)$, where l_e is the electron mean free path in the normal state, so that (7) reduces to $(p_F v_s)^2/T\Delta \gg (\xi/l_e)^2$. We can therefore write

$$\left(\frac{\xi}{\varepsilon}\mathbf{v}+\mathbf{v}_s\right)\nabla n_p^1 + \frac{\partial}{\partial z}(\mathbf{p}\mathbf{v}_s)\frac{\partial n_p^1}{\partial p_z} + \frac{n^1}{\tau_{im}^*}$$
$$= -I_{e-ph}^- + \frac{\xi}{\varepsilon}(\mathbf{v}\nabla T)\frac{\varepsilon}{T}\frac{\partial n_0}{\partial \varepsilon} \equiv \mathscr{F}, \tag{8}$$

where τ_{im}^* is the effective relaxation time of quasiparticles on impurities, which depends on $|\xi|$ and v_z. In the present case,

$$\frac{1}{\tau_{im}^*} \sim \frac{1}{\tau_{im}}\frac{|\xi|}{\varepsilon}\frac{T}{p_F v_s^0}.$$

The contribution of phonon drag in this case has the form (see Appendix)

$$I_{e-ph}^- \sim -\frac{(\mathbf{p}\nabla T)}{p_F}\frac{\tau_{ph}}{\tau_{e-ph}}\frac{T}{\Theta_D}\left(\left(\frac{T}{\Delta}\right)^{1/2}\frac{\partial n_0}{\partial \varepsilon} - mw^2\frac{\partial^2 n_0}{\partial \varepsilon^2}\frac{\xi}{(T\Delta)^{1/2}}\right). \tag{9}$$

The solution of the transport equation (8) can be expressed in terms of the integral over the classical trajectory of the particle:

$$n^1(t) = \int_C^t dt' \mathscr{F}(t') \exp\left[-\int_{t'}^t dt'' \frac{1}{\tau_{im}^*}\right]. \tag{10}$$

The constant of integration C depends on the boundary conditions on the surface. For specular reflection, $C = -\infty$. Motion over trajectories is then periodic, with each period containing two reflections from the v_s profile (which change the sign of ξ without changing the direction and magnitude of momentum to within T/μ) and two reflections from the surface (which involve a change in the sign of v_z). The period $\tilde{T}$ of the trajectory is given by

$$\int_0^{\tilde{T}/4} \frac{dt}{v_i \xi/\varepsilon} = z^*,$$

where z^* is the coordinate of the turning point on the v_s profile.

In the general case, the boundary condition is

$$n^1(v_z\xi/\varepsilon > 0) = \rho^* n^1(v_z\xi/\varepsilon < 0), \tag{11}$$

where the parameter ρ^* represents specular reflection of the quasiparticles and, generally speaking, may be different from the corresponding parameter ρ in the normal metal. It is clear, however, that $\rho^* \geqslant \rho$. When $1-\rho \ll 1$ (this is valid for a group of glancing particles which, as we shall see, provide the main contribution), we find using (11) and the periodic nature of the trajectories that

$$n^1(t) = \left\{1 - \rho^{*2}\exp\left(-\int_0^{\tilde{T}}\frac{dt}{\tau_{im}^*}\right)\right\}^{-1}\int_{t-\tilde{T}}^t dt' \mathscr{F}(t')$$
$$\times \exp\left\{-\int_{t'}^t \left[\frac{1}{\tau_{im}^*} + \ln\rho^* \sum_i \delta(t''-t_i)\right]dt''\right\}, \tag{12}$$

where the time t_i correspond to reflections from the surface.

Let us first calculate the thermoelectric current due to surface particles. It is readily seen that, if we use the estimate given by (4), the main contribution is due to drag effects and only the part I_{e-ph}^- which is even in ξ is important. As a result, and using the fact that $\mathscr{F}(t')$ has a constant sign, we find from (12) that

$$n^{1+} \sim \overline{I_{e-ph}^-}\min\left\{\frac{2}{1-\rho^*}\frac{z^*}{|v_z||\xi|/\varepsilon}, \tau_{im}^*\right\}$$
$$\sim \overline{I_{e-ph}^-}\left(\frac{\Delta}{T}\right)^{1/2}\frac{p_F v_s}{T}\min\left\{\frac{2}{1-\rho^*}\frac{\lambda}{|v_z|}\left(\frac{T}{p_F v_s}\right)^2, \tau_{im}\right\} \tag{13}$$

(the bar represents averaging over the period). The drag current can be expressed directly in terms of n^1:

$$\mathbf{j}_n = \sum_{\mathbf{p}} \mathbf{v} n^1.$$

It is clear from (13) that, for sufficiently pure specimens for which n^{1+} is controlled by the diffuseness of reflection from the surface, integration over p produces a divergence for small v_z, which emphasizes the contribution of glancing particles. It is clear from (12) that this divergence is cut off for

$$v_z \sim v_F(1-\rho^*)z^* \Big/ \left(v_F \frac{|\xi|}{\varepsilon} \tau_{im}^*\right).$$

We shall also take into account the fact that the specular parameter ρ^* will, in general, depend on the angle of incidence of a particle, so that the diffuseness measure for particles with small ϑ is smaller than the mean.[4)] We shall not examine in detail the characteristics of surface scattering (which depend on the particular properties of the surface) but, instead, will introduce the phenomenological relation $(1-\rho^*) = (1-\rho_0)(v_z/v_F)^\alpha$. In view of the foregoing, and using (9), we find that the current density is given by

$$j_n \sim \eta \nabla T \frac{\tau_{ph}}{\tau_{e-ph}} \exp\left(-\frac{\Delta}{T}+\frac{p_F v_s}{T}\right)\left(\frac{\Delta}{T}\right)^{1/2}\left[\cos(\widehat{\mathbf{v}_s \nabla T}) + \frac{T}{p_F v_s}(1-\cos\widehat{\mathbf{v}_s \nabla T})\right] \min[1, A\psi(A)]F(z),$$

$$A = \frac{2\lambda}{v_F\tau_{im}}\left(\frac{T}{p_F v_s}\right)^{1/2}\frac{1}{1-\rho_0}, \quad \psi(x) = \frac{2}{\pi}\begin{cases} x^{-\alpha/(\alpha+1)}, & \alpha\neq 0 \\ |\ln x|, & \alpha = 0 \end{cases}, \tag{14}$$

$$\eta \sim \frac{2\pi^2}{9}\frac{eT}{m}\tau_{im}\nu_e,$$

where ν_e is the density of states on the Fermi surface and the function $F(z)$ determines the reduction in the current j_n along the inward normal to the specimen over distances $\bar{z} \sim \lambda T/(p_F v_s)$. In accordance with (14), the surface current of normal excitations, j_n, in an ordinary thermoelectric circuit corresponds to the thermoelectric magnetic flux

$$\Phi_T \sim \left(\frac{j_n}{\nabla T}\right)\frac{4\pi}{c}\delta T\lambda\bar{z} = \Phi_0\left(\frac{4j_n}{\nabla T}\right)\frac{\lambda^2\delta T}{\hbar}\frac{T}{p_F v_s}, \tag{15}$$

where δT is the temperature difference. We note that, in this situation, it is easier to establish large temperature gradients because of the low thermal conductivity of the superconductor for $T \ll T_c$.

A more stringent limitation on possible measurements of Φ_T is related to the fact that such measurements reduce to the determination of the temperature dependence of the corresponding magnetic flux. At the same time, when an external magnetic field is present (due to the presence of v_s), there is also a temperature-dependent contribution $\Delta\Phi$ to the flux in the aperture of the thermoelectric ring, due to the trivial dependence $\lambda(T)$ (redistribution effect[13,14]). In our case, $T \ll p_V v_s \ll \Delta$ and, by analogy with Ref. 14, we have

$$\Delta\Phi \sim \frac{\Phi_0}{4\pi^2}\left(\frac{p_F v_s}{\Delta}\right)\frac{\delta T}{T}\frac{R}{\xi}\exp\left(-\frac{\Delta}{T}+\frac{p_F v_s}{T}\right)\frac{T^{1/2}\Delta^{1/2}}{(p_F v_s)^2}, \tag{16}$$

where R is a characteristic linear dimension of the circuit. If we substitute $T \sim 2$ K, $\tau_{e-ph} \sim 10^{-9}$ s, $T/T_c \sim 1/4$,

$$\frac{\tau_{ph}}{\tau_{e-ph}} \sim \frac{v_F}{w}\left(\frac{T}{\Theta_D}\right)^2 \exp\left(\frac{\Delta}{T}\right) \sim 10^4,$$

$p_F v_s \sim \Delta/2$, $1-\rho_0 \sim 1/2$, $\alpha = 2$ (as in the theory given in Ref. 22), and $\tau_{im} \sim 10^{-9}$ s, we find from (14)–(16) that $\Phi_T/\Delta\Phi \sim 0.01$ (whereas $\Phi_T \sim 0.1\Phi_0$ for $T \sim 1$). Thus, despite the fact that Φ_T is relatively large, the effect appears to be difficult to observe because of the considerable masking effect. It is, nevertheless, observable because of the different dependence of these effects on v_s, ∇T, and the geometry of the experiment. In principle, these differences enable us to compensate the redistribution effect in particular situations (see Ref. 14 for further details).

Comparison of the contribution due to the surface thermoelectric drag effect with the bulk phenomenon (due to the presence of quasiparticles within the bulk of the specimen, the number of which is exponentially small) shows that the surface effect predominates when

$$\frac{(T\Delta)^{1/2}}{(p_F v_s)}\left[\frac{2\lambda}{v_F\tau_{im}}\left(\frac{T}{p_F v_s}\right)^{1/2}\frac{1}{1-\rho_0}\right]^{1/(\alpha+1)}\exp\left(\frac{p_F v_s}{T}\right) \gtrsim 1. \tag{17}$$

We now turn to the analysis of the part of the function n^1 which is odd in ξ, which describes the branch imbalance in the quasiparticle spectrum and is due to the appearance of the gauge-invariant potential U:

$$U_T\nu_e = \sum_{\mathbf{p}}\frac{\xi}{\varepsilon}n^{1-}. \tag{18}$$

The determination of n^{1-} on the basis of (12) is complicated to some extent by the fact that the integrand has an alternating sign. However, simple rearrangement (see Appendix) leads to the following estimate for the part of the function n^{1-} which is even in v_z:

$$\frac{1}{2}[n^{1-}(v_z>0)+n^{1-}(v_z<0)]|_{z=0} \sim \frac{z^{*2}\,\mathrm{sign}\,\xi}{2(\xi^2/\varepsilon)v_z^2\tau_{im}^*} \times\left[-\frac{(\mathbf{p}\nabla T)}{p_F}\frac{\tau_{ph}}{\tau_{e-ph}}\frac{T}{\Theta_D}mw^3\frac{\partial^2 n_0}{\partial\varepsilon^2}+\frac{|\xi|}{\varepsilon}(\mathbf{v}\nabla T)\frac{\varepsilon}{T}\frac{\partial n_0}{\partial\varepsilon}\right]. \tag{19}$$

The function $[n^{1-}(v_z>0)+n^{1-}(v_z<0)]$ decreases monotonically with distance from the surface, and vanishes at the point z^*. It is clear that, as before, integration in (18) will lead to a divergence in the region of low v_z, which emphasizes the contribution of glancing particles. This divergence is cut off when

$$|v_z| \sim v_F z^*\left(v_F\frac{|\xi|}{\varepsilon}\tau_{im}^*\right)^{-1}.$$

Using this result and integrating (18), we finally obtain

$$U_T|_{z=0} \sim \frac{\lambda}{2\pi}\frac{(\mathbf{v}_s\nabla T)}{v_s}\frac{T}{p_F v_s}\left(\frac{\Delta}{p_F v_s}\right)^{1/2}\exp\left(-\frac{\Delta}{T}+\frac{p_F v_s}{T}\right) \times\left[\frac{\tau_{ph}}{\tau_{e-ph}}\frac{mw^2}{\Theta_D}\frac{w}{v_F}+C\left(\frac{T}{\Delta}\right)^{1/2}\right], \tag{20}$$

where $C \sim 1$. The second term in brackets is due to the diffusion thermoelectric effect, whereas the first is due to the drag effect. It is clear that the relative importance of the drag effect is determined by the parameter $(\tau_{ph}/\tau_{e-ph})(mw^2/\mu)(\Delta/T)^{1/2}$, which may be of order unity for low enough temperatures (or, more precisely, high enough T_c), for which the phonon mean free path is restricted by the dimensions of the specimen. Since

$$\tau_{ph} = \min\left(\frac{\hbar}{T}\frac{v_F}{w}e^{\Delta/T}, \frac{L}{w}\right),$$

the contribution of drag can be isolated from the diffusion background either by exploiting the difference in the exponential dependence on T or the size effect.

We now note one further important point. The potential U_T is measured with the aid of a tunnel S–N contact by the balance method, i.e., a determination is made of the external bias voltage V that must be applied to the contact to stop the current.[9,11] This balance voltage is related to U_T by $V = U_T/g$, where g is the normalized conductivity of the contact (i.e., the ratio of the conductivity of the S–N contact to the conductivity of the N–N contact). Since g decreases exponentially with decreasing T, this balances the corresponding exponential dependence of U_T (although the sensitivity of the null-detecting galvanometer must, of course, increase with decreasing T).

We now consider the above surface effect for low v_s for which $(p_F v_s) \ll T$.[5] The contribution of trapped particles is then small[6] and we obtain the following expression for the function n^{1-}:

$$\frac{1}{2}[n^{1-}(v_z>0)+n^{1-}(v_z<0)] = \int_0^\infty \frac{dz'}{(|\xi|/\varepsilon)v_z}\mathscr{F}\left[\exp\left\{-\frac{z+z'}{v_z\tau_{im}}\right\} + \exp\left\{-\frac{|z'-z|}{v_z\tau_{im}}\right\}\right], \tag{21}$$

where, in this case, the expression for $\mathscr{F}$ is the first-order term in the expansion of the right-hand side of (8) in powers of pv_s/T, since it is only in this order that we have a contribution that is even in the velocity. Finally, for $l_e = v_F\tau_{im} \gg \lambda$ we have

$$U_T \sim \mu \frac{2\lambda}{\pi^2 v_F}\frac{\mathbf{v}_s\cdot\nabla T}{T}e^{-\Delta/T}\left[\left(\frac{\Delta}{T}\right)^{1/2}\frac{\tau_{ph}}{\tau_{e-ph}}\frac{mw^2}{\Theta_D}\frac{w}{v_F}+C\right] \times \left|\ln\left(\frac{l_e}{\lambda}\right)\right| e^{-z/\lambda}. \tag{22}$$

It follows from this expression that the magnitude of the surface effect due to the diffusion contribution is of the same order as the bulk effect in the films examined in Refs. 9–12 if we put $\tau_{im} \sim \lambda/v_F$.

3. THERMOELECTRIC EFFECTS IN THE NEIGHBORHOOD OF THE THERMAL CONTACT WITH DIELECTRIC OR SUPERCONDUCTOR WITH HIGH T_c

Consider a high-purity superconductor at $T \sim T_c$. It occupies the half-space $x > 0$ and is in thermal contact on the $x = 0$ plane with a material whose thermal conductivity is due to phonons (we shall label it with subscript II), i.e., a dielectric or superconductor with $T_{c\text{II}} \gg T$. We shall suppose that heat is introduced into the superconductor from medium II so that, well away from the contact, in the region of quasilocal equilibrium between the electron and phonon systems, there is a temperature gradient $\nabla T_0 \to X$. This gradient will be assumed to be low enough so that, at any rate, $\nabla T_0 l_d \ll T$. The electron and phonon distribution functions in the neighborhood of the contact will be written in the form

$$n_\mathbf{p}(x)=n_0(T_0)+n_\mathbf{p}^1(x), \quad N_\mathbf{q}(x)=N_0(T_0)+N_\mathbf{q}^1(x),$$

where $T_0 = T_0^0 + (\nabla T_0)x$. To describe the nonequilibrium distribution in the neighborhood of the contact, we turn to the transport equations for the electrons and phonons, linearized in the nonequilibrium increments $n_\mathbf{p}^1$ and $N_\mathbf{q}^1$.

The transport equation for the phonons is

$$\mathbf{w}_\mathbf{q}\cdot\nabla N_\mathbf{q}^1 = -\frac{1}{\tau_{ph}(\omega)}N_\mathbf{q}^1+\hat{I}_{ph-e}(n_\mathbf{p}^1)-\mathbf{w}_\mathbf{q}\cdot\nabla T_0\frac{\partial N_0}{\partial T}. \tag{23}$$

The first term on the right-hand side describes the absorption of phonons by equilibrium quasiparticles, and the second, the contribution of the nonequilibrium addition $n_\mathbf{p}^1$. It will be clear later that $N^1 \sim (\alpha T_0/T)(l_d^2/l_{ph})N_0$, and $n^1 \lesssim (\nabla T_0/T)l_d n_0$[so that $\hat{I}_{ph-e}(n^1) \lesssim \tau_{ph}^{-1}(\nabla T_0/T)l_d N_0$]. Since $l_d \gg l_{ph}$, we shall confine our attention to the solution of the homogeneous equation (denoting it by $\tilde{N}^1$), which ensures that the boundary conditions are satisfied and describes the transfer of heat by phonons near the contact:

$$\tilde{N}_\mathbf{q}^1(x) \sim N_{\mathbf{q}0}^1 \exp\left(-\frac{x}{w_x\tau_{ph}(\omega)}\right)\theta(q_x>0), \tag{24}$$

where θ is the Heaviside function. In general, energy conservation yields

$$\int d^3q w_\mathbf{q}\hbar\omega N_{\mathbf{q}0}^1 \sim \varkappa_e \nabla T_0.$$

The detailed form of the boundary condition for $N_\mathbf{q}$ depends both on the phonon distribution function in medium II and on the frequency and angular dependence of the phonon transmission coefficient k_t of the boundary. We shall not analyze this here, and confine our attention to simple estimates. We shall suppose that the boundary condition for $N_\mathbf{q}^1$ corresponds to the frequency dependence

$$N_{\mathbf{q}0}^1(\omega)=C\frac{\partial N_0}{\partial t}\theta(q_x>0),$$

i.e., formally, it has the same form as when there is a discontinuity in the phonon temperature at the contact (which occurs for $k_t \to 0$):

$$N_{\mathbf{q}0}^1=[N_0(T_0+\Delta T_{ph})-N_0(T_0)]\theta(q_x>0) \approx \Delta T_{ph}\frac{\partial N_0}{\partial T}\theta(q_x>0). \tag{25}$$

This description is valid, in particular, if the phonon distribution in medium II may be looked upon as being in local equilibrium (with temperature depending on position), if the elastic scattering of phonons within it (for example, by the boundaries) predominates over inelastic scattering, and if the frequency dependence of the diffusion coefficient $D_{ph\,\text{II}}(\omega)$ and the angular and frequency dependence of k_t can be neglected. Condition (25) then corresponds to the "mating" of the phonon flux of given frequency in medium II, $j_\omega = D_{ph}\nabla N_{\text{II}}(\omega) = D_{ph}\nabla T_{ph}\partial N_0/\partial T$, with the flux in the metal at $x = 0$, $j_\omega \sim k_t N_{q_0}^1 w_q$, if we take $\Delta T_{ph} \simeq \nabla T_{ph} D_{ph}/wk_t$. We note that, since $\Delta T_{ph} \sim \nabla T_{ph} l_{ph\,\text{II}}$, there is no contradiction with the mating condition for the local-equilibrium parts of the distribution function at $x = 0$, since the variation in $N_{ph\,\text{II}}$ over distances $\sim l_{ph}$ cannot be allowed for within the framework of the hydrodynamic approximation.

Condition (25) is convenient because it substantially facilitates the analysis of the collision operator. It will be seen later that estimates obtained in this way are also valid for more general boundary conditions. We shall also use the fact that the main contribution is due to phonons with energies $\hbar\omega \sim T$, and that the contribution of low-energy phonons is small in comparison with the phase volume.[7] We shall thus introduce certain effective values of τ_{ph} and l_{ph}.

We now turn to the equation for the electrons. Substituting $\tilde{N}_q^1$ in the electron-phonon collision integral (see Appendix) and separating out the contributions which are even and odd in p_x, we obtain the following estimates:

$$\tilde{I}^+ \sim -\frac{n_0(T_0+\Delta T_{ph})-n_0(T_0)}{2\tau_{e-ph}} e^{-x/l_{ph}}$$

$$\approx \frac{1}{2\tau_{e-ph}} \Delta T_{ph} \frac{\varepsilon}{T} \frac{\partial n_0}{\partial \varepsilon} e^{-x/l_{ph}}, \tag{26a}$$

$$\tilde{I}^- \sim \frac{p_x}{p_F} \frac{1}{\tau_{e-ph}} \frac{T}{\Theta_D} \Delta T_{ph} \frac{\partial n_0}{\partial \varepsilon} e^{-x/l_{ph}}. \tag{26b}$$

We shall consider the case of sufficiently pure specimens, so that $l_{ph} \ll l_e \lesssim l_d$. Since the scale of variation in $\tilde{I}^+$, $\tilde{I}^-$ is much smaller than l_e, we shall reduce the action $\tilde{I}$ to some effective boundary conditions for the hydrodynamic equations. We shall define the $x = x_1$ plane so that $l_{ph} \ll x_1 \ll l_e$. For $x > x_1$, the transport equation, which does not contain the small-scale inhomogeneity, will reduce to the hydrodynamic equation for the parts of the distribution function which are even and odd in ξ:

$$\frac{1}{\nu_\varepsilon} \operatorname{div} \mathbf{j}_\varepsilon = \operatorname{div}(-D\nabla n_\varepsilon^{1+})$$

$$= -\sum_{\mathbf{p}} \delta(\varepsilon-\varepsilon_{\mathbf{p}}) \{\hat{I}_{e-ph}(n_\varepsilon^{1+}, N_0) + \hat{I}_{e-ph}(n_0, N_{\mathbf{q}}^1)\}, \tag{27a}$$

$$\operatorname{div} \mathbf{j}_n = \operatorname{div}(-\sigma \nabla U) = -U\nu_\varepsilon/\tau_b, \tag{27b}$$

where

$$n_\varepsilon^{1+} = \sum_{\mathbf{p}} n^1 \delta(\varepsilon-\varepsilon_{\mathbf{p}});$$

$$\mathbf{j}_\varepsilon = \sum_{\mathbf{p}} \mathbf{v} \frac{\xi}{\varepsilon} n^1 \delta(\varepsilon-\varepsilon_{\mathbf{p}}) - \nu_\varepsilon D \nabla T_0 \frac{\partial n_0}{\partial T},$$

$$U = \frac{1}{\nu_\varepsilon} \sum_{p} n^1 \frac{\xi}{\varepsilon},$$

and τ_b is the relaxation time of the branch imbalance in the quasiparticle spectrum. To derive the boundary conditions for (27), we solve the transport equation for $0 \leqslant x \leqslant x_1$[8]:

$$\mathbf{v} \cdot \frac{\xi}{\varepsilon} \nabla n^1 = -(\tilde{I}^+ + \tilde{I}^-) - \frac{1}{\tau_{im}}(n^1 - \langle n^1 \rangle)$$

$$- \frac{1}{\tau_b} \langle n^1 \rangle^- - \hat{I}_{e-ph}(\langle n^1 \rangle^+), \tag{28}$$

where angle brackets represent averaging over the surface with given ξ, and $\langle\rangle^+$ and $\langle\rangle^-$ represent, respectively, the ξ-even and ξ-odd parts. The boundary condition at $x = 0$ corresponds to the vanishing of the current j_ε. On the other hand, the conditions for j_n and U depend on the form of the contact. For the contact with the dielectric [case (a)], $j_{n|x=0} = 0$. For the contact with the superconductor with $T_{cII} \gg T$ [case (b)], the boundary condition corresponds to Andreev reflection. We then have $U_{|x=0} = 0$. Simple integration of (28) finally gives the following conditions for $x = x_1$:

$$n_\varepsilon^{1+}|_{x_1} = C_1(\varepsilon); \quad j_\varepsilon|_{x_1} = -\tilde{I}^+ l_{ph} \nu_\varepsilon,$$

$$U|_{x_1} = \begin{cases} C_2 & \text{(a)} \\ -\nu_\varepsilon^{-1} \sum_{\mathbf{p}} \tilde{I}^-|_{x=0} v_x^{-1} l_{ph} \operatorname{sign} \xi & \text{(b)} \end{cases};$$

$$j_n|_{x_1} = \begin{cases} -\sum_{\mathbf{p}} \frac{l_{ph}^2}{\tau_l v_x} \tilde{I}^-|_{x=0} & \text{(a)} \\ C_3 & \text{(b)} \end{cases}, \tag{29}$$

where C_1, C_2, and C_3 are certain constants. It is clear that, in this particular situation [in which (25) and (26) are valid], the boundary condition for j_ε is satisfied if

$$n_\varepsilon^{1+} = 0, \quad n_\varepsilon = n_0(T_0^0 + \nabla T_0 x), \tag{30}$$

or, in other words, we have the local-equilibrium particle distribution for the entire region $x > x_1$ in which thermal conductivity is of the electron type.[9] We then obtain

$$\frac{\Delta T_{ph}}{\tau_{e-ph}} \frac{l_{ph}}{D} \sim \nabla T_0 \quad \text{or} \quad \Delta T_{ph} \sim \nabla T_0 l_d \left(\frac{l_d}{l_{ph}}\right). \tag{31}$$

On the other hand, in the general case [when the expression for $\tilde{N}^1$ is different from that given by (25)], the energy dependence of the current j_ε at $x = x_1$ may differ from the corresponding quasiequilibrium distribution (30). Local equilibrium [corresponding to (30)] is then established over lengths $\sim l_d$ [the estimate given by (31), which follows from the continuity of the flux, will, of course, remain valid]. We note that, when the estimates given in the Appendix are taken into account, the function n_ε will decrease monotonically with increasing x even for $x_1 \leqslant x \lesssim l_d$.

Since we are assuming that $\nabla T_0 l_d \ll T_0$, the coordinate dependence of the parameters τ_{e-ph}, τ_{ph} can be neglected. Inclusion of the nonequilibrium addition may be important only for the evaluation of the superconducting parameters Δ and N_s. It is then clear that, if the condition

$$\nabla T_0 l_d \lessdot (T_c - T), \tag{32}$$

is also satisfied, the specific form of the distribution n^{1+} for $x \lesssim l_d$ can be ignored [having used (30) for all x]. If, on the other hand, $\nabla T_0 l_d \gtrsim T_c - T$, estimates based on the use of (30) are valid, at any rate, to within an order of magnitude.

Turning now to (27b), we note that τ_b may, in general, depend on x because of the dependence $\tau_b(\Delta)$. Thus, if the imbalance relaxes on phonons, then $\tau_b \sim \tau_{e-ph}(\Delta/T_c)^{-1}$, i.e.,

$$\tau_b \propto \left(\frac{T_c}{T_c-T}\right)^{1/2}, \quad l_b \equiv (\tau_{im}\tau_b v_F^2)^{1/2} \propto \left(\frac{T_c}{T_c-T}\right)^{1/4}.$$

This dependence is unimportant if $\nabla T_0 l_b \lesssim T_c - T$. However, since it is weak, we shall neglect it in approximate estimates when we solve (27b) in the general case, as well.

Using the boundary conditions (21) and the estimates given by (31), we find from the solution of (27b) that

$$j_n \approx -\sum_p \frac{l_{ph}^2}{\tau_b v_x} \tilde{I}^- |_{x=0} e^{-x/l_b} \sim \eta \nabla T_0 \frac{T^2}{\Theta_D^2} \frac{\mu}{\Theta_D} \frac{\tau_{e-ph}}{\tau_b} e^{-x/l_b} \quad \text{case (a)}, \qquad (33a)$$

$$j_n \approx -\sum_p \frac{l_{ph}}{v_x l_b} D\tilde{I}^- |_{x=0} e^{-x/l_b} \sim \eta \nabla T_0 \left(\frac{\tau_{im}}{\tau_b}\right)^{1/2} \frac{\mu}{\Theta_D} e^{-x/l_b} \quad \text{case (b)}, \qquad (33b)$$

These two solutions determine the contribution of the phonon drag in the contact region to the thermoelectric effect. It is clear that, in the case of thermal contact with a dielectric, this contribution can only be of the order of the contribution of the ordinary thermoelectric current. However, in the case of contact with a high-T_c superconductor [case (b)], the "contact" contribution may substantially exceed the "bulk" contribution. If we evaluate the thermoelectric addition to the magnetic flux by analogy with (2) for a ring consisting of superconductors, we find, using (33b) and (30), that the contribution due to the contact region is

$$\Phi_T \sim \frac{\Phi_0}{2\pi} \frac{2m}{v_e \mu e \hbar} \eta \left(\frac{\tau_{im}}{\tau_b}\right)^{1/2} \frac{\mu}{\Theta_D} \ln\left(\frac{T_c - T_0^0 - \nabla T_0 l_b}{T_c - T_0^0}\right). \qquad (34)$$

The ratio of this to the "bulk" contribution[1-4] is of order

$$\left(\frac{\tau_{im}}{\tau_b}\right)^{1/2} \cdot \frac{\mu}{\Theta_D} \ln\left(\frac{T_c - T_0^0 - \nabla T_0 l_b}{T_c - T_0^0}\right) \Big/ \ln\left(\frac{T_c - T_0^0 - \nabla T_0 L}{T_c - T_0^0}\right), \qquad (35)$$

where L is the characteristic length of the "active" superconductor. It is clear that, for sufficiently "clean" specimens ($\tau_{im} \sim \tau_b$), this ratio may be two or three orders of magnitude bigger than unity.

As far as the temperature dependence (34) is concerned, we find that, since $l_b \ll L$, it is stronger than that predicted in Ref. 2. The transition from the $(T_c - T_0)^{-1}$ law to the logarithmic law occurs not for $T_c - T_0 \sim \nabla T_0 L$ (as in the bulk case), but for $T_c - T_0 \sim \nabla T_0 l_b$.[10] We note that a stronger dependence of the effect was observed in some experiments[8] than that predicted in Ref. 2.

We now draw attention to an important point. If the temperatures of the "hot" and "cold" junctions are measured by special probes attached to the specimen, and the heat flux into these probes is negligible, they will record the local-equilibrium phonon temperature which, for $x > l_{ph}$, is equal to, or almost equal to, the electron temperature. This fixes the value of the gradient ∇T_0 (or, more precisely, the temperature difference $\nabla T_0 L$). In its turn, the instant of the superconducting transition determines the effective electron temperature in the region $x < l_{ph}$. Consequently, such measurements yield no information about the phonon temperature jump in the contact region $\Delta T_{ph} \sim \nabla T_0 l_{ph} \varkappa_e / \varkappa_{ph}$ which, in general, may exceed the measured temperature difference $\sim \nabla T_0 L$.

Our estimates thus show that analysis of the contact region is very important in experimental studies of the thermoelectric magnetic flux in a superconducting circuit. In particular, if the "active" superconductor (with $T_c \sim T$) is clean enough, whereas the "passive" superconductor has an appreciably greater T_c and is not too clean (which corresponds to the typical experimental situation), the contribution of contact effects may exceed the bulk contribution[2-4] by several orders of magnitude, and may lead to a different temperature dependence of the magnetic flux. It seems that these estimates may be relevant for measurements of the absolute thermo-emf in circuits consisting of a normal metal and a superconductor (with $T_c \gg T$), in which case the "contact" drag current j_n ensures that there is a corresponding contact contribution to the thermo-emf.

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APPENDIX

1. Transformation of the electron-phonon collision operator

Substituting the phonon distribution function (1), which is odd in **q**, into the standard expression for the electron-phonon collision operator of a superconductor (see, for example, Ref. 15), we obtain

$$I_{e-ph}^- = \frac{2\pi}{\hbar} \frac{(\mathbf{p}\nabla T)}{p_F} \int d^3q \, |C|^2 \frac{(\mathbf{qp})}{p_F} \frac{\hbar w^2}{T} \tau_{ph} \frac{N_0}{T} \times \left\{ \left(1 + \frac{\xi\xi' - \Delta^2}{\varepsilon\varepsilon'}\right)(n_{\tilde{\varepsilon}'} - n_{\tilde{\varepsilon}}) \times [\delta(\tilde{\varepsilon} - \tilde{\varepsilon}' + \hbar\omega) - \delta(\tilde{\varepsilon} - \tilde{\varepsilon}' - \hbar\omega)] - \left(1 - \frac{\xi\xi' - \Delta^2}{\varepsilon\varepsilon'}\right) \times (1 - n_{\tilde{\varepsilon}'} - n_{\tilde{\varepsilon}}) \delta(\tilde{\varepsilon} + \tilde{\varepsilon}' - \hbar\omega) \right\}, \qquad (A1)$$

where

$$\xi' = \xi_{\mathbf{p}+\hbar\mathbf{q}}, \quad \xi = \xi_{\mathbf{p}}, \quad \tilde{\varepsilon} = \varepsilon + \mathbf{p}\mathbf{v}_s, \quad \varepsilon = (\xi^2 + \Delta^2)^{1/2}.$$

Consider, to begin with, the case where $v_s = 0$, $T \sim T_c (\Delta \ll T)$.

A. Processes in which the number of quasiparticles is conserved

The coherence factor selects processes with sign $(\xi\xi') > 0$. Conservation laws give

$$(\mathbf{p} \cdot \mathbf{q}) = -\hbar q^2/2 \pm \omega m \operatorname{sign} \xi,$$

where the signs correspond to the sign of $\hbar\omega$ in the argument of the δ function. The difference $n_{\varepsilon'} - n_\varepsilon$ must now be expanded up to the second order in $(\varepsilon' - \varepsilon)$, and it is then clear that the first order provides the contribution to the part of I_{e-ph} which is even in ξ and the second order to the odd part. The result is given by (3).

B. Processes with the creation of pairs of excitations

The coherence factor selects processes with sign $(\xi\xi') < 0$. The conservation law gives

$$(\mathbf{p} \cdot \mathbf{q}) = -\hbar q^2/2 - \omega m \operatorname{sign} \xi;$$

both the even and odd contributions are proportional to $(l - n_\varepsilon - n_{\varepsilon'}) > 0$. It is clear that processes involving pair creation contribute to I^-_{e-ph} with the same sign and to the same order as processes with the conservation of particles. However, for simplicity, we do not write them out in (3) because they have no effect on the order of magnitude given by the final expressions.

We now turn to the case $T \ll T_c$, $v_s \neq 0$. Pair production can now be neglected and it is seen that, since $\hbar q \ll p_F$, the $\mathbf{p}\cdot\mathbf{v}_s$ terms in the arguments of the δ functions will vanish. The coherence factor $[1 + (\xi\xi' - \Delta^2)/\varepsilon\varepsilon']$ goes over to the form $\sim(\xi + \xi^1)^2/2\Delta^2$ and, if we recall that $\varepsilon' - \varepsilon \sim (\xi'^2 - \xi^2)/2\Delta$, we find that the δ functions transform to

$$\delta\left\{\frac{1}{\hbar}\frac{|\xi+\xi'|}{2\Delta}\left[(\mathbf{p}\cdot\mathbf{q})+\frac{\hbar q^2}{2}\mp\frac{2\Delta m\omega}{\xi'+\xi}\right]\right\}.$$

Bearing in mind the summation over sign ξ', we can readily see that the even and odd parts provide the following combinations, respectively:

$$\hbar\omega\frac{\partial n}{\partial\varepsilon}\frac{\hbar q^2}{2}\frac{|\xi'+\xi|}{\Delta},\qquad \frac{\xi}{|\xi+\xi'|}m\omega(\hbar\omega)^2\frac{\partial^2 n}{\partial\varepsilon^2}.$$

The final result is given by (9).

2. Expression for the function n^{1-}

Since we are only interested in the part which is even in v_z, we note that the replacement $v_z \to -v_z$ is equivalent to $dt \to -dt$ from the point of view of motion over trajectories. In view of this, we transform (12) as follows:

$$n^{1-}(v_z>0)+n^{1-}(v_z<0)=\left[1-\rho^{*2}\exp\left(-\int_0^{\tilde T}\frac{dt}{\tau^*_{im}}\right)\right]^{-1}$$
$$\times\int_{t-\tilde T}^{t+\tilde T}dt'\mathscr{F}^-(t')\exp\left\{-\left|\int_{t'}^{t}dt''\left[\frac{1}{\tau^*_{im}}+\ln\rho^*\sum_i\delta(t''-t_i)\right]\right|\right\},\qquad\text{(A2)}$$

where $\mathscr{F}^-$ is the odd (in ξ) part of $\mathscr{F}$. It is clear that

$$\int_{t-\tilde T}^{t+\tilde T}dt'\mathscr{F}^-(t')=0.$$

Using the periodicity properties of motion over trajectories, it may be shown that, on the one hand, the first-order term in the expansion of the exponential in the numerator again provides no contribution to the integral and, on the other hand, the integral is identically equal to zero as $\tau^*_{im} \to \infty$. The nonzero terms in the expansion of the numerator are thus found to be proportional to $(1/\tau_{im})^2$ and $(\ln\rho^*/\tau_{im})$. Next, if the point $z(t)$ corresponds to the turning point z^* on the v_s profile, the numerator is an integral of the product of an even and an odd function, and is therefore zero. The integral increases monotonically as z shifts toward the surface, and (19) is obtained when $z = 0$.

3. Analysis of I_{ph-e} in the contact region

Let us represent $\tilde N^1_q$ (defined for $q_x > 0$) as the sum of the even and odd (in $\mathbf{q}$) parts defined for all $\mathbf{q}$:

$$\tilde N^1=\tilde N^{1+}+\tilde N^{1-};\quad \tilde N^{1+}={}^1/_2\tilde N^1(|q_x|);\quad \tilde N^{1-}={}^1/_2[N^1(\mathbf{q},x=0)-N^1(-\mathbf{q},x=0)]e^{-x/|w_x|\tau_{ph}}.\qquad\text{(A3)}$$

The collision integral linearized in $\tilde N^1$ assumes the form

$$I(n_0,\tilde N^1)=-\frac{2\pi}{\hbar}\int d^3q|C|^2\left\{\left(1+\frac{\xi'\xi-\Delta^2}{\varepsilon\varepsilon'}\right)\times(n_{\mathbf{p}'}-n_{\mathbf{p}})([\delta(\varepsilon_{\mathbf{p}}-\varepsilon_{\mathbf{p}'}-\hbar\omega)\right.$$
$$-\delta(\varepsilon_{\mathbf{p}}-\varepsilon_{\mathbf{p}'}+\hbar\omega)]\tilde N^{1-}$$
$$+[\delta(\varepsilon_{\mathbf{p}}-\varepsilon_{\mathbf{p}'}+\hbar\omega)+\delta(\varepsilon_{\mathbf{p}}-\varepsilon_{\mathbf{p}'}-\hbar\omega)]\tilde N^{1+})$$
$$\left.+\delta(\varepsilon_{\mathbf{p}}+\varepsilon_{\mathbf{p}'}-\hbar\omega)\left(1+\frac{\xi'\xi-\Delta^2}{\varepsilon\varepsilon'}\right)(1-n_{\mathbf{p}}-n_{\mathbf{p}'})(\tilde N^{1+}+\tilde N^{1-})\right\}.\qquad\text{(A4)}$$

Analysis of the angular dependence shows that the contribution $\tilde N^{1+}$ appears in the zero order of the expansion of I_{e-ph} in terms of the parameter $\cos(\overline{pq}) \ll 1$ (see Appendix 1) and the contribution $\tilde N^{1-}$ appears in the first order, where the angular dependence of the decay law can be neglected in approximate estimates. When the conservation laws are taken into account, all this leads to (26). Moreover, we have neglected in (26b) the part which is odd in ξ and which describes the contribution of the drag to thermal conductivity for $mw^2/\Theta_D \ll 1$. We note that it is readily seen from (A4) that, whatever the particular form of N^{1+}, we have

$$\tilde I^+|_{\varepsilon=0,\,\Delta=0}=I(n_0,\tilde N^{1+}|_{\varepsilon=0,\,\Delta=0})=0,$$

and $\tilde I^+ \sim \varepsilon/T$ for $\varepsilon \ll T$. At the same time, $\tilde I^+ < 0$ for all $\varepsilon \gg \Delta$, which corresponds to an increase in the number of particles. On the other hand, $\tilde I^+$ is different from zero only in the first order in $\Delta/T \ll 1$ for $\varepsilon \sim \Delta$.

[1] In this case, the normal current j_n satisfies the boundary condition $j_n = -j_s$, where j_s is the current of the superconducting condensate. This follows from the electrodynamics of superconductors.

[2] We note that, if $\varkappa_{e1} d \ll \varkappa_{ph2} L$, where the subscripts 1 and 2 refer to the film and the substrate, respectively, and L is the thickness of the substrate, the temperature distribution is independent of the presence of the film. For $T \sim T_c$, this condition can be rewritten in the form $\pi^{-2}(\tau_{e1}/\tau_{ph2})(\Theta_{D2}/T)^2(v_F/w)(d/L) \ll 1$, where $\tau_e < d/v_F$ is the electron relaxation time.

[3] Measurement of the magnetic field localized in the region of the contact with the substrate, and analogous to that examined in Ref. 1 for a simply-connected inhomogeneous superconductor, appears to be difficult.

[4] Soffer[22] has analyzed the reflection of electrons from the surface (in the normal state) and has shown that, when there is no correlation between reflection from different points of the surface, $\rho = \exp[-\cos^2 h^{-2}(2p_F r)^2]$, where the scale r characterizes the roughness of the surface and ϑ is the angle of incidence of a particle on the surface, $\cos^2\vartheta = (v_z/v_F)^2$.

[5] We note that the condition for the validity of the classical description that we are using leads to the restriction $(\xi/\Delta)(v_F/v_z) \gg (\xi/\lambda)$.

[6] Although one would expect an enhanced contribution due to particles with low ξ, because of the presence of the singularity in the density of states, this singularity is compensated by the factor (ξ/ε) in the expression for U_T given by (18).

[7] We are assuming that $D_{ph_{\parallel}}$ does not have a strong divergence for $\omega \to 0$, i.e., that scatterring by impurities is not the principal mechanism for the relaxation of phonon momentum.

[8] In this region, we neglect terms due to the differentiation of the local-equilibrium function $n_0(T_0)$.

[9] Since $x > x_1$, the contribution of phonons to the thermal conductivity is negligible, the scale of variation in $N^1_{\mathbf{q}}$ is of the order of $l_{ph} \ll l_d$, and the establishment of quasilocal equilibrium in the electron-phonon system [described by (30)] is determined precisely by the electron kinetics; we shall therefore ignore the behavior of $N^1_{\mathbf{q}}$ for $x \gg x_1$.

[10)] We note that the estimate given by (34) is based on the use of (30) for n_ε^+. Since, for $x < l_d$, the distribution n_ε^+ may, in general, differ somewhat from (30) [independently of the assumption expressed by (25)], this, in turn, will lead to a modification of (34) for $T_c - T \lesssim \nabla T_0 l_d$. However, by virtue of the condition $\nabla n^+_{\varepsilon|x=x_1} < 0$ (and hence $\nabla N_{s|x=x_1} < 0$), this modification appears to be unimportant.

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