Excitation dynamics of a band of levels in a resonant external field

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The dynamics of a system consisting of a ground state and a band of levels in a resonant external field is analyzed over the entire range of intensities and rise times of this field. The spectrum of quasienergy states of the system is analyzed in detail. All of these states, except those with the highest and lowest quasienergies, are determined primarily by no more than two levels of the band, so that the number of levels which are populated is essentially equal to the number of quasienergy states involved in the resonance. The number of such states is determined as a function of the rise time and intensity of the field. Conditions are derived for the two cases of adiabatic imposition and instantaneous imposition of the field. At a resonance at the band edge, the condition for an adiabatic imposition becomes more stringent. The behavior of the intensities of all the levels of the system over the entire time interval is studied. In particular, when the field is imposed instantaneously the recursive behavior of the populations is eventually replaced by a quasirandom behavior for the ground state and by quasiperiodic behavior for the levels in the band. Consequences of the dynamic Stark effect at the band edges are analyzed. Even in a weak field the system can be completely excited into the band, while in a strong field levels can be excited immediately in several localized regions of the band. A differential-difference equation is constructed for the population amplitude of the ground state in an arbitrary field for the case of a band of equidistant levels. The range of applicability of this equation is discussed. A weak anharmonicity in the spectrum of band levels may intensify the recursive effects for the population of the ground state.

1. INTRODUCTION

Theoretical interest in the behavior of many-level systems in intense resonant fields has increased sharply in recent years, primarily because of experimental results on many-photon dissociation of polyatomic molecules. Since a quasicontinuum of states is excited in this process, interest has been attracted to the problem of a transition which is induced by resonant radiation from a ground state to a band of levels. Transitions between levels within the band have not been considered. Various aspects of the excitation of such a system were studied in Refs. 1–8.

The general dynamics of the filling of a band of levels during the instantaneous imposition of a field was analyzed in Refs. 1 and 2 without consideration of the band structure. Numerical calculations were carried out in Refs. 1–3, and analytic calculations in Ref. 4, on the dynamics of the population of the ground level during the instantaneous imposition of a field which excites the system into a band of equidistant levels with equal transition dipole moments (for brevity we will say simply an "equidistant band"). The analysis was carried out over the time interval $0\leqslant t\leqslant 2T_0$, where $T_0 = 2\pi/\Delta\nu$, and $\Delta\nu$ is the distance between the levels in the band. The effect of the adiabatic (infinitely slow) imposition of a field on the emptying of the ground levels in an ultrastrong field $|f|\gg\Delta\nu N_0^{1/2}$ was studied in Refs. 5 and 6; here f is the field-induced broadening of one level of the band, and N_0 is the effective number of levels. The behavior of the population of the ground level over a long time after the instantaneous imposition of a field, for a band with a random distribution of dipole moments, was studied in Ref. 7. Finally, analytic results were derived in Ref. 8 to describe the population of the ground level during the instantaneous imposition of a monochromatic field in the case of an infinitely wide equidistant band.

In all these studies attention has been focused on the dynamics of the population of the ground level, rather than on the details of the filling of the band itself. Furthermore, where the structure of the band has been taken into account the analysis has been restricted by rather stringent limitations. As a result, we do not yet have an overall picture of the excitation process.

In the present paper we analyze the dynamics of a system consisting of a ground level and a band over the entire time interval. We impose no restrictions on the field intensity or rise time. Our approach will be to analyze the spectrum of quasienergy states of the system[1)] (§3). We show that even in a strong field $\Delta\nu < |f|\ll\Delta\nu N_0^{1/2}$ nearly all of these states are determined primarily by only one or two levels of the band. Consequently, the width of the "excitation region" of the band, i.e., the number of levels in the band which are filled as the ground level is emptied, depends primarily on how many quasienergy states are involved in the process. This number in turn depends directly on the way in which the field is imposed. We will therefore examine the two limiting cases of an instantaneous imposition of a field (§5) and an adiabatic imposition (§6) as well as the intermediate case in which the field is imposed over a finite time to (§7).

We will also examine some specific consequences of the dynamic Stark effect which stem from the important changes in the indices of the band levels which are in resonance near the band edge.

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In §4 we derive a compact differential-difference equation describing the dynamics of the population of the ground level in an arbitrary field for a system with an equidistant band. This is a generalization of an equation derived in Refs. 8. We determine the range of applicability of this equation. We show that a weak anharmonicity in the spectrum of the band may, under certain conditions, intensify the recursive effects which are characteristics of an equidistant band when a field is imposed instantaneously.

We will ignore relaxation effects, since in the collisionless regime they should have little influence on the excitation of the system.

2. DYNAMIC EQUATIONS

We assume that the band consists of N levels with energies $\hbar\omega_n$ $(n = 1, \ldots, N)$. The levels of the band are coupled to the ground level by a resonant external field

$$\tilde{E}=\tilde{E}_0(t)e^{-i\omega t}+\text{c.c.},$$

where $\tilde{E}_0(t)$ is the slowly varying field amplitude, and the condition $|\omega_n - \omega| \ll \omega$ holds for all n. In the absence of relaxation, and in the resonant approximation, the following equations[1] then hold for this system:

$$i\dot{a}=\sum_{n=1}^{N} f_n^*(t)b_n, \quad i\dot{b}_n=(\Delta_n-\Delta)b_n+f_n(t)a, \tag{1}$$

where $a(t)$ and $b_n(t)$ are the amplitudes the populations of the ground level and of level n, $\Delta = \omega - \omega_c$ is the deviation of the carrier frequency of the field from the center of band,[2)] $\Delta_n = \omega_n - \omega_c$, $f_n(t) = \tilde{E}_0 d_n/\hbar$ is the field-induced broadening of level n, and d_n is the dipole moment of the transition to level n.

We are referring to the lower singlet level as the "ground level," although, strictly speaking, this may not be the case. We will simply assume that the complete system is initially in this level:

$$a(-\infty)=1, \quad b_n(-\infty)=0.$$

From system (1) we find

$$b_n=-i\int_{-\infty}^{t} f_n(t_1)a(t_1)\exp\{i(\Delta-\Delta_n)(t-t_1)\}dt_1,$$

$$\dot{a}=-\int_{-\infty}^{t} a(t_1)\exp\{i\Delta(t-t_1)\}\sum_{n=1}^{N} f_n^*(t)f_n(t_1) \times \exp\{-i\Delta_n(t-t_1)\}dt_1. \tag{2}$$

If the external field is monochromatic, i.e., if $\tilde{E}_0(t) = \text{const}$, then we can analyze system (1) by a quasienergy approach.[9,10,1,5] The solution of (1) can then we written in the form

$$A=\sum_{k=1}^{N+1} c_k A_k \exp\{-iE_k t\},$$

$$A=\{a, b_1, \ldots, b_N\}, \quad A_k=\{a_k, b_{1k}, \ldots, b_{Nk}\},$$

where the term $A_k\exp\{-iE_k t\}$ respresents a quasienergy state with a quasienergy E_k. Here we have

$$E_k=\sum_{n=1}^{N}|f_n|^2(E_k-\Delta_n+\Delta)^{-1},$$

$$b_{nk}=f_n a_k(E_k-\Delta_n+\Delta)^{-1}, \tag{3}$$

$$a_k=\left[1+\sum_{n=1}^{N}|f_n|^2(E_k-\Delta_n+\Delta)^{-2}\right]^{-1/2}.$$

Let us assume that at $T \geqslant 0$ the external field is monochromatic and that the initial conditions are $a(t=0) = a(0)$, $b(t=0) = b_n(0)$. Using the orthogonality relations

$$\sum_{k=1}^{N+1} b_{nk}^* b_{mk}=\delta_{nm}, \quad \sum_{k=1}^{N+1} a_k^* b_{nk}=0, \quad \sum_{k=1}^{N+1}|a_k|^2=1,$$

we then find

$$c_k=a_k^* a(0)+\sum_{n=1}^{N} b_{nk}^* b_n(0). \tag{4}$$

3. QUASIENERGY STATES OF THE SYSTEM

Equations (3) have been written down in several places (e.g., Refs. 1, 2, 5, and 6), but there has been no detailed analysis of the quasienergy spectrum, even though it is obvious that it is the structure of this spectrum which determines all the particular features of the behavior of the system. We intend to repair this omission.

The system is described by $N + 1$ quasienergy states. Of these, $N - 1$ are "interior" states (Fig. 1), by which we mean that their quasienergies are restricted to definite intervals:

$$\Delta_k > E_k+\Delta > \Delta_{k-1}, \quad k=2, \ldots, N. \tag{5}$$

Two of the states are "edge" states, whose quasienergies are not limited to a definite interval:

$$E_1 < \Delta_1-\Delta, \quad E_{N+1} > \Delta_N-\Delta.$$

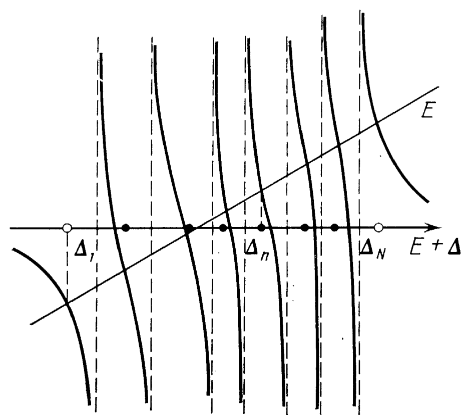

FIG. 1. Determination of the quasienergy spectrum for an arbitrary band of N levels. ●—Interior quasienergies; ○—edge quasienergies.

It follows from (3) that in the absence of a field we would have

$$E_k=\begin{cases}\Delta_{k-1}-\Delta, & \Delta_{k-1}-\Delta>0\\ \Delta_k-\Delta, & \Delta_k-\Delta<0\\ 0, & \Delta_{k-1}<\Delta<\Delta_k\end{cases}$$

(by definition, $\Delta_0 = -\infty$, $\Delta_{N+1} = +\infty$); i.e., under these conditions the quasienergies are identical to the ordinary level energies of the system (shifted by $\hbar\omega$ for the case of the band). As the field is strengthened, the quasienergies shift monotonically to the right if $\Delta_{k-1} - \Delta > 0$ or to the left if $\Delta_k - \Delta < 0$. For the interior states, condition (5) of course holds at all times, and the limiting value for $E_k (k = 2, \ldots, N)$ is the corresponding root of the equation

$$\sum_{n=1}^{N} |d_n|^2 (E_k-\Delta_n+\Delta)^{-1}=0,$$

for which condition (5) also holds. The quasienergies of the edge states increase in modulus without bound as the field is strengthened: $E_1 \to -\infty$, $E_{N+1} \to +\infty$.

Let us examine the interior states. We assume that the band has no distinct levels, i.e., that $|d_n|^2$ and $(\Delta_n - \Delta_{n-1})$ are comparable in magnitude for all n. We write E_k in the form

$$E_k=\varepsilon_k-\Delta=\varepsilon_k^0-\tilde{\varepsilon}_k-\Delta,$$

where

$$\varepsilon_k^0=\frac{|d_k|^2\Delta_{k-1}+|d_{k-1}|^2\Delta_k}{|d_{k-1}|^2+|d_k|^2},$$
$$|d_{k-1}|^2>\tilde{\varepsilon}_k\frac{|d_{k-1}|^2+|d_k|^2}{\Delta_k-\Delta_{k-1}}>-|d_k|^2. \qquad (6)$$

Let us transform the sum on the right side of Eq. (3) for E_k. In this sum, we leave unmodified only the two terms with $n = k$ and $k - 1$, which are the most important terms. Since for $n \neq k,\ k-1$ we have

$$|\tilde{\varepsilon}_k|/|\varepsilon_k^0-\Delta|\lesssim 1/3,$$

where we are using (6), we use the approximation

$$(E_k+\Delta-\Delta_n)^{-1}\approx(\varepsilon_k^0-\Delta_n)^{-1}+\tilde{\varepsilon}_k(\varepsilon_k^0-\Delta_n)^{-2}$$

for the other terms in this sum. We then find from (3) an equation for $\tilde{\varepsilon}_k$, which gives us quite accurate values of E_k for the interior states for an arbitrary external field strength:

$$\varepsilon_k^0-S(k)-\Delta-\tilde{\varepsilon}_k[1+Z(k)]=\frac{|f_k|^2}{\varepsilon_k^0-\tilde{\varepsilon}_k-\Delta_k}+\frac{|f_{k-1}|^2}{\varepsilon_k^0-\tilde{\varepsilon}_k-\Delta_{k-1}}, \qquad (7)$$

where

$$S(k)=\sum_{n\neq k,k-1}\frac{|f_n|^2}{\varepsilon_k^0-\Delta_n},\quad Z(k)=\sum_{n\neq k,k-1}\frac{|f_n|^2}{(\varepsilon_k^0-\Delta_n)^2},$$
$$S(k)=\frac{1}{4}(|f_k|^2+|f_{k-1}|^2)S^0(k).$$

The quantity $S(k)$ is an increasing function of k, with $S(1) < 0$ and $S(N) > 0$. Within the band there is accordingly a certain k_c for which the condition $S^0(k_c) \approx 0$ holds. We call this point in the band the "center of the band," and we will refer to S below as the "S shift." Equation (7) is cubic in $\tilde{\varepsilon}_k$. We wish to emphasize that we are interested in only one of the roots of Eq. (7), namely, that which satisfies condition (6). We will not use Cardan's formula to solve for this root; instead we proceed immediately to the most important cases.

A. Weak field: $|f_n| \ll (\Delta_n - \Delta_{n-1})/2$

In this case, with $\Delta_{k-1} - \Delta - S > 0$ or $\Delta_k - \Delta - S < 0$, noting that we have $Z \ll 1$ in a weak field, and retaining on the right side of (7) only the term with the smaller denominator, we find

$$\varepsilon_k=\Delta_{\bar{k}}+1/2(\Delta+S-\Delta_{\bar{k}})+[1/4(\Delta+S-\Delta_{\bar{k}})^2+|f_{\bar{k}}|^2]^{1/2}(2k-2\bar{k}-1), \qquad (8)$$

$$a_k=\left[1+\frac{|f_{\bar{k}}|^2}{(\varepsilon_k-\Delta_{\bar{k}})^2}\right]^{-1/2},\quad b_{nk}=\delta_{n\bar{k}}\left[1+\frac{(\varepsilon_k-\Delta_{\bar{k}})^2}{|f_{\bar{k}}|^2}\right]^{-1/2}.$$

Here $\bar{k} = k - 1$ when the first inequality holds, and $\bar{k} = k$ when the second holds. For $\Delta_k > \Delta + S > \Delta_{k-1}$ if $|\Delta + S - \Delta_{\bar{k}}| \gg |f_{\bar{k}}|$ for $\bar{k} = k, k - 1$, we have, in a first approximation

$$E_k=S+\frac{|f_{k-1}|^2}{\Delta+S-\Delta_{k-1}}+\frac{|f_k|^2}{\Delta+S-\Delta_k},\quad a_k\approx 1.$$

When $|\Delta + S - \Delta_{\bar{k}}| \leqslant |f_{\bar{k}}|$, for $\bar{k} = k$ or $k - 1$, on the other hand, we have, by analogy with (8),

$$E_k=1/2(S+\Delta_{\bar{k}}-\Delta)+\text{sign}\,(k-\bar{k}+1/2)\,[1/4(\Delta+S-\Delta_{\bar{k}})^2+|f_{\bar{k}}|^2]^{1/2}, \qquad (9)$$

$$a_k=\left[1+\frac{|f_{\bar{k}}|^2}{(E_k-S)^2}\right]^{-1/2},\quad b_{\bar{k}k}=-\left[1+\frac{(E_k-S)^2}{|f_{\bar{k}}|^2}\right]^{-1/2}.$$

In a weak field, one of two situations is possible: (1) The quasienergy state nearly coincides with some stationary state; (2) the approximation of a two-level system is valid [see (9)]. The effect of the band is manifested globally in the existence of an S shift of the resonant level of the band due to the dynamic Stark effect. This shift may be quite large even in a weak field, since at the band edge, $k \sim 1, N$ for $N \gg 1$, we have

$$|S|\sim[|f|^2/(\Delta_n-\Delta_{n-1})]_{\text{av}}\ \ln N.$$

In other words, in the case of a wide band, S may become greater than the distance between adjacent levels in the band.

B. Strong and ultrastrong fields: $|f_k| \gg \frac{1}{2}(\Delta_k - \Delta_{k-1})$

$$|\varepsilon_k^0-\Delta-S|\ll(|f_k|^2+|f_{k-1}|^2)/(\Delta_k-\Delta_{k-1}), \qquad (10)$$

then

$$|\tilde{\varepsilon}_k|\ll|\varepsilon_k^0-\Delta_{k-1}|,\quad \tilde{\varepsilon}_k=(\varepsilon_k^0-\Delta-S)/Z_0(k),$$

where

$$Z_0(k)=\sum_{n=1}^{N}\frac{|f_n|^2}{(\varepsilon_k^0-\Delta_n)^2}\gg 1.$$

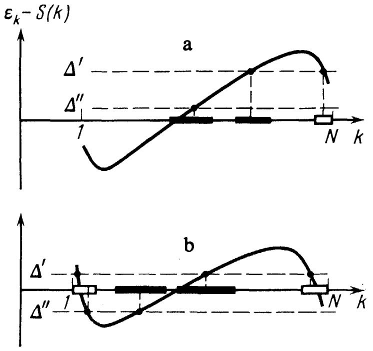

FIG. 2. Regions in the band in which condition (10) holds at a relatively low field strength (a) and at a relatively high field strength (b), for various frequency deviations of the field. In the case of an instantaneous field imposition, these are the regions in the band in which the levels are excited most efficiently the black rectangles along the abscissa show the central excitation region, while the open rectangles show the edge excitation region.

Then

$$a_k \approx Z_0(k)^{-1/2} \ll 1, \quad b_{nk} \approx \frac{d_n}{\varepsilon_k{}^0 - \Delta_n} \Big[\sum_{p=1}^{N} \frac{|d_p|^2}{(\varepsilon_k{}^0 - \Delta_p)^2} \Big]^{-1/2}. \tag{11}$$

In a strong, $(\Delta_N - \Delta_1) \gg |f_k| \gg (\Delta_k - \Delta_{k-1})/2$, condition (10) may hold in several regions in the band because of the large S shift at the band edges. Let us consider, for example, an equidistant band. In this case (10) take the form

$$\Big| 2\pi k - \Delta T_0 - \frac{\beta}{\pi} \ln \frac{k + N/2}{1 - k + N/2} \Big| \ll \frac{2\beta}{\pi}, \tag{12}$$

where $\beta = |f|^2 T_0^2/2$. For $\beta < 2\pi^2 N/\ln N$, there may be one or two regions in the band in which condition (10) holds (Fig. 2a). For $2\pi N > \beta > 2\pi^2 N/\ln N$, there may be two or three such regions (Fig. 2b). In the central region the number of levels is on the order of β, while in the edge regions there are $1 + N \exp(-2\pi^2 N/\ln N)$ levels, i.e., significantly fewer. Further, estimates show that several clearly localized regions—defined by inequality (10)—can exist only in a very broad band, with $N > 10^4$. When

$$|\Delta_{\bar{k}} - \Delta - S| \gg \frac{|f_k|^2 + |f_{k-1}|^2}{\Delta_k - \Delta_{k-1}},$$

where $\bar{k} = k - \theta(\Delta_{\bar{k}} - \Delta - S)$ [$\theta(x)$ is the unit step function], we have

$$\varepsilon_k = \Delta_{\bar{k}} + \frac{(|f_k|^2 + |f_{k-1}|^2)|d_{\bar{k}}|^2}{(\Delta_{\bar{k}} - \Delta - S)(|d_k|^2 + |d_{k-1}|^2)},$$
$$a_k = |\varepsilon_k - \Delta_{\bar{k}}|/|f_{\bar{k}}|, \quad b_{nk} \approx \delta_{n\bar{k}}. \tag{13}$$

The contributions of the main level to these states are much smaller than those in (11).

Since $Z/(Z_0 - Z) \lesssim 1/4$, a good approximation of $\tilde{\varepsilon}_k$ in a strong field, for any value of the parameter

$$|\varepsilon_k{}^0 - \Delta - S|\,(\Delta_k - \Delta_{k-1})/(|f_k|^2 + |f_{k-1}|^2),$$

is the expression

$$\tilde{\varepsilon}_k = \frac{(\Delta_k - \Delta_{k-1})(|d_{k-1}|^2 - |d_k|^2)}{2(|d_k|^2 + |d_{k-1}|^2)} - \frac{|f_k|^2 + |f_{k-1}|^2}{2(\varepsilon_k{}^0 - \Delta - S)}$$
$$\pm \frac{1}{2} \Big[(\Delta_k - \Delta_{k-1})^2 + \Big(\frac{|f_k|^2 + |f_{k-1}|^2}{\varepsilon_k{}^0 - \Delta - S} \Big)^2 - 2 \frac{(\Delta_k - \Delta_{k-1})(|f_{k-1}|^2 - |f_k|^2)}{\varepsilon_k{}^0 - \Delta - S} \Big]^{1/2}.$$

This expression is found from (7) by ignoring $\tilde{\varepsilon}_k$ on the left side. The sign is chosen to satisfy the equalities (5).

In strong and ultrastrong fields, the contributions b_{nk} to essentially all of the interior states are independent of the field intensity. In region (10), where the levels of the band are mixed to the greatest extent, just two levels with $n = k$ and $n = k - 1$ dominate in the quasienergy state, while in regions (13) a single level dominates. Consequently, if the interior states play a leading role in filling a band, the width of the excitation region will be essentially equal to the number of states involved the process. In turn, this number depends strongly on the way in which the field is imposed, as we will see in the subsequent sections of this paper.

The structure of the edge states in a strong field differs substantially from the structure of the interior states. As the field is intensified, progressively more levels at the corresponding band edge become involved in these edge states. When the field becomes ultrastrong,

$$|\Delta|, \quad (\Delta_N - \Delta_1) \ll \Big[\sum_{n=1}^{N} |f_n|^2 \Big]^{1/2},$$

on the other hand, the entire band becomes involved in the edge state:

$$E_{1,N} \approx \pm \Big[\sum_{n=1}^{N} |f_n|^2 \Big]^{1/2}. \tag{14}$$

The edge states describe a generalized two-level system, and the band acts as an upper level with a substructure.

4. BAND OF EQUIDISTANT LEVELS

We now consider a band of equidistant levels $(\Delta_{n+1} - \Delta_n = \Delta\nu, n = 0, \mp 1, ...)$ in an arbitrary field; the levels are counted from the center of the band. We assume that the number of levels is infinite and that the n dependence of $|d_n|^2$ is smooth; we are thereby ignoring effects due to sharp edges on the band. We transform to the dimensionless quantities

$$\tau = t/T_0, \quad \delta = \Delta T_0, \quad s = ST_0 = \beta S^0/T_0 = \beta s^0.$$

We assume that the number of band levels which are actively involved in the resonant excitation in N_{res} and that these levels lie in a single, well-localized, excitation region. We assume that this region is displaced by an amount $\varphi/2\pi$ from the resonance, where $|\varphi| \ll |\delta|$.

The kernel of integral equation (2),

$$\Phi(\tau - \tau_1) = \sum_n |d_n|^2 \exp\{-2\pi i n(\tau - \tau_1)\},$$

is periodic in $\tau - \tau_1$ with a period of 1. If the band is sufficiently wide,

$$N_0=\sum_n \frac{|d_n|^2}{|d_n|^2_{max}} \gg 1, \quad N_0 \gg N_{res},$$

where N_0 is the effective number of levels in the band, the kernel $\Phi(\tau-\tau_1)$ will be nonzero only in small neighborhoods $\Delta\tau = 1/N_0$ around the times $\tau-\tau_1 = m$. We use the replacement

$$a(t)=\tilde{a}(t)\exp\left\{-i\int_{-\infty}^{\tau}\varphi(\tau_1)\,d\tau_1\right\}.$$

We assume that $\tilde{a}(t)$ and also the intensity and frequency of the external field vary slowly over a time $\Delta\tau$ and that the dipole moments vary quite smoothly:

$$\begin{gathered}\frac{1}{N_0}\left|\frac{1}{\tilde{a}}\frac{d\tilde{a}}{d\tau}\right|, \quad \frac{1}{N_0}\left||f|^{-1}\frac{d|f|}{d\tau}\right| \ll 1, \\ \frac{1}{N_0^2}\left|\frac{d(\delta_1+\varphi)}{d\tau}\right| \ll 1, \quad |d_n|^{-2}\left|\frac{d|d_n|^2}{dn}\right| N_{res} \ll 1,\end{gathered} \tag{15}$$

where

$$f=|f|\exp\left\{i\int_{-\infty}^{\tau}\delta_1\,d\tau_1\right\}.$$

By virtue of condition (15), we can limit the integration in Eq. (2) for the amplitude a to small intervals $\tau_{\rm in}$ near the times $\tau_1 = \tau - m$. In other words, we can restrict the integration to $|\tau - m - \tau_1| \lesssim \tau_{\rm in}$, where

$$m=0,1,\ldots;$$

$$\frac{1}{N_0} < \tau_{in} < \left|\tilde{a}^{-1}\frac{d\tilde{a}}{d\tau}\right|, \quad \left||f|^{-1}\frac{d|f|}{d\tau}\right|, \quad \frac{1}{N_{res}}.$$

The results of the integration are essentially independent of $\tau_{\rm in}$, and in establishing the integration limits we can set $\tau_{\rm in} = \infty$ everywhere we find $\tau_{\rm in}$. The equation for the amplitude a then becomes

$$\begin{aligned}da/d\tau=&-{}^1/_2\,T_0^2|f_{n_0}(\tau)|^2(1+is_{n_0}{}^0(\tau))\,a(\tau)\\ &-T_0^2\sum_{k=1}^{\infty}\exp\{ik\delta\}f_{n_k}{}^*(\tau)f_{n_k}(\tau-k)\,a(\tau-k), \quad a(-\infty)=1,\end{aligned}$$

where

$$n_m=[\delta+\varphi(\tau-m)-\delta_1(\tau-m)]/2\pi,$$

$$s_{n_m}{}^0(\tau)=2\int_0^{\infty}d\tau_1\int_{-\infty}^{+\infty}\frac{|d_n|^2}{|d_{n_m}|^2}\sin[(\delta+\varphi(\tau)-\delta_1(\tau)-2\pi n)\tau_1]\,dn.$$

This equation can be transformed into a differential-difference equation of the neutral type:

$$\begin{aligned}&\tilde{E}_0^*(\tau-1)\,[da/d\tau+\beta_{n_0}(\tau)\,(1+is_{n_0}{}^0(\tau))\,a(\tau)]=\tilde{E}_0^*(\tau)\,e^{\{i\delta\}}\\ &\times[da(\tau-1)/d\tau-\beta_{n_1}(\tau-1)\,(1-is_{n_1}{}^0(\tau-1))\,a(\tau-1)],\end{aligned} \tag{16}$$

where $\beta_n = \frac{1}{2}T_0^2|f_n(\tau)|^2$. Equation (16), combined with the equations

$$b_n=-i\int_{-\infty}^{\tau}T_0f_n(\tau_1)\,a(\tau_1)\exp\{i(\delta-2\pi n)(\tau-\tau_1)\}\,d\tau_1,$$

gives a complete description of the dynamics of the system under restrictions (15). Galbraith *et al.*[8] have derived a particular case of Eq. (16) for an infinitely wide equidistant band ($s^0 = 0$, $\varphi = 0$) and for a monochromatic field $E_0 = \text{const}$. For this band, Eq. (16) is exact.

It can be seen from (16) that the dynamics of the population of the ground level is determined primarily by the local characteristics of the band in excitation region. The quantity $\beta_{nm}s^0_{nm}$ is the sum as the S shift introduced in the preceding section. The shift φ of this region with respect to the resonance stems from the dynamic Stark effect, but the value of φ is determined not only by βs^0 but also by the way in which the field is imposed. It can be shown that we have

$$0\leqslant|\varphi|\leqslant\beta|s^0|. \tag{18}$$

In particular, we have $\varphi = 0$ when the field is applied adiabatically or $\varphi = \beta s^0$ when it is applied instantaneously. For an unbounded equidistant band, φ is always zero, since any point of the band is the center of the band, and we have $s^0 \equiv 0$.

If Eq. (16) is to be correct, the excitation region must be unique. For an adiabatic field imposition this condition holds in any case at arbitrary intensities, under the sole restriction $\Delta_N > \Delta > \Delta_1$. For instantaneous field imposition, for bands characterized by a unique width parameter N_0, Eq. (16) holds if

$$\begin{aligned}&\beta_c\ll\pi^2N_0/2, \quad \text{for } |\delta|<\pi N_0,\\ &|\delta|\gg\beta_c \quad \text{for } |\delta|>\pi N_0,\end{aligned} \tag{19}$$

where the subscript c means the center of the band. If, however, the band edge is characterized by a width $N_1 \ll N_0$, then at $\beta_c < \pi N_1$ no additional restrictions on (19) arise. If $\beta_c > \pi N_1$, on the other hand, a truncated wedge is cut out of region (19). The shape of this wedge varies slightly with the particular form of the band, but it can be estimated from (Fig. 3).

$$\left|\pi N_0-\frac{\beta_c}{\pi}-\delta-\frac{\beta_c}{\pi}\ln\frac{2N_0\pi^2}{\beta_c}\right|<\beta_c, \quad \beta_c>\pi N_1,$$

which, as it turns out, corresponds to the existence of a unique region in the band in which condition (10) holds.

As we mentioned in §3, the width of the edge excitation regions is significantly smaller than that of the central region. For this reason, Eq. (16) always gives a good description of the latter in a strong field or when the central region

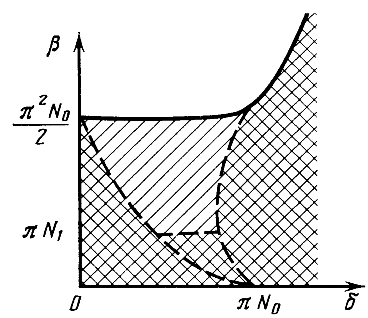

FIG. 3. Region of applicability of Eq. (16) for instantaneous imposition of a field. Hatched region—Band with smooth edges; double-hatched region—band with sharp edges.

in narrower than N_0. In this case the second inequality in (19) can be replaced by the less stringent inequality $|\delta|\gg 8\beta_c N_0$ at $|\delta|>\pi N_0$. Equation (16) is thus applicable in all cases except that in which there is a resonance with the entire band as a whole; i.e., the limiting case of an ultrastrong field goes beyond the applicability of Eq. (16). Although the S shift may be significant, it is always less than the deviation from the center of the band, $|s|\ll|\delta|$, in the range of applicability of Eq. (16). Consequently, using inequality (18), we can ignore the dependence of the S shift on the shift φ, which we can determine by solving Eq. (17), at an accuracy to within the half-width of the excitation region, of course.

Let us examine the quasienergy states of Eq. (16). In terms of dimensionless variables we have

$$b_{nk}=(2\beta)^{1/2}a_k(\varepsilon_k-2\pi n+\delta)^{-1},$$

$$a_k=\left[1+\frac{1}{2}\beta+\frac{1}{2\beta}(\varepsilon_k-\beta s^0)^2\right]^{-1/2},\quad \varepsilon_k=\beta s^0+\beta\operatorname{ctg}\frac{(\varepsilon_k+\delta)}{2}. \tag{20}$$

In a weak field, the equidistant nature of the band gives rise to no effects beyond those in (8) and (9). In a strong field, $\beta>\pi^2/2$, we find from (20)

$$\varepsilon=\pi(2k+1)-\delta-2\operatorname{arctg}Y,$$

$$b_{nk}=\frac{2}{\pi}\left[1+2(k-n)-\frac{2}{\pi}\operatorname{arctg}Y\right]^{-1}(1+Y^2)^{-1/2},$$

$$a_k=\{2/[\beta(1+Y^2)]\}^{1/2},$$

$$Y(k)=(2\pi k+\pi-\delta_0)/\beta,\quad \delta_0=\delta+\beta s^0. \tag{21}$$

It is not difficult to see that the approximate general expression derived for the quasienergies in the preceding section agrees very accurately with (21) in the case of an equidistant band. It follows from (21) that the ground level enters about β quasienergy states with an identical weight. In these states we have $Y^2\lesssim 1$ [cf. condition (10)], and 80% of the contribution comes from the $n=k$ and $n=k-1$ levels. Because of the last term in Eq. (21) for the quasienergy, their spectrum is nonlinear, but at $Y^2\ll 1$ this nonlinearity is weak. This circumstance is a distinctive feature of the system consisting of a ground level and an equidistant band.

5. INSTANTANEOUS IMPOSITION OF A FIELD

For the instantaneous imposition of a monochromatic field we find from (4)

$$a(t)=\sum_{n=1}^{N+1}|a_k|^2\exp\{-iE_kt\}, \tag{22}$$

$$b_n(t)=\sum_{n=1}^{N+1}a_k^*b_{nk}\exp\{-iE_kt\}. \tag{23}$$

In a weak field, if $\Delta+S=\Delta_n$, the system behaves as if it were a two-level system. At a resonance at the band edge, because of the large S shift the level which is captured in the resonance may be entirely different from the level "which was aimed at" ($\Delta=\Delta_n$), and be shifted a distance S from this target level toward the edge. The time average of the population, $|a(t)|^2$, is no less than 0.5.

In a strong field, expression (22) is dominated by states which satisfy condition (10). Their number is, in order of magnitude,

$$\beta_{av}=2\pi^2[|f_n|^2(\Delta_n-\Delta_{n-1})^{-2}]_{av}\gg 1,$$

where the average is over the excitation region. If the band is not equidistant, the different quasienergies are essentially random numbers with respect to each other from the very outset. Accordingly, the quasienergy states completely lose their phase coherence in a time of only $2\pi[\beta_{av}\Delta\nu_{av}]^{-1}$ after the field is imposed, and the population amplitude $a(t)$ behaves as if it were a complex, "nearly Gaussian" noise:

$$\langle a(t)\rangle=0,\quad \langle|a(t)|^2\rangle=\sum_{k=1}^{N+1}|a_k|^4\sim\beta_{av}^{-1}, \tag{24}$$

where the angle brackets mean an average over a time on the order of $2\pi/\Delta\nu_{av}$.

If, on the other hand, the band is symmetric ($\Delta_n=-\Delta_{N-n+1}$), the noise will be not complex but real exactly at a resonance at the center of the band, because of the relation $E_k=-E_{N-k+2}$.

During the emptying of the ground level in a band in which condition (10) holds, regions appear in which the time-average populations are also of the same order of magnitude as (24). In contrast with the ground level, however, these populations are more reminiscent of periodic functions of the time. As was shown in §3, two levels of the band dominate the resonant quasienergy state. Corresponding, in (23) two states, $k=n$ and $k=n+1$, play a leading role for levels from the excitation region. We can therefore write

$$|b_n(t)|^2=|a_n|^2|b_{nn}|^2+|a_{n+1}|^2|b_{nn+1}|^2+a_n^*b_{nn}a_{n+1}b_{nn+1}^*\times\exp\{-i(E_n-E_{n+1})t\}+\text{c.c.}+\Phi_0(t),$$

where $\Phi_0(t)$ is a small increment, $a_{\bar n}$ and $b_{n\bar n}$ are given by (11), and $\bar n=n,n+1$.

Outside the excitation region, expression (23) is dominated by a single state, and $|b_n|^2$ remains constant over time, much lower than the population of the resonant levels. Because of the large S shifts, the excitation regions may be shifted toward the band edges by a distance greater than their width by a factor of $\Delta\nu_{av}S^0$.

In an extremely strong field, two edge states become dominant, and we arrive at the approximation of a generalized two-level system, which has been discussed in many papers (see, e.g., Refs. 1, 2, and 5).

If the excitation occurs into an equidistant band, recursive phenomena will arise during the instantaneous imposition of a strong monochromatic field.[1,4,8] In this case Eq. (16) becomes

$$\dot a(\tau)+\beta(1+is^0)a(\tau)=e^{i\delta}[\dot a(\tau-1)-\beta(1-is^0)a(\tau-1)]\theta(\tau-1),$$

$$a(0)=1. \tag{25}$$

Its solution is

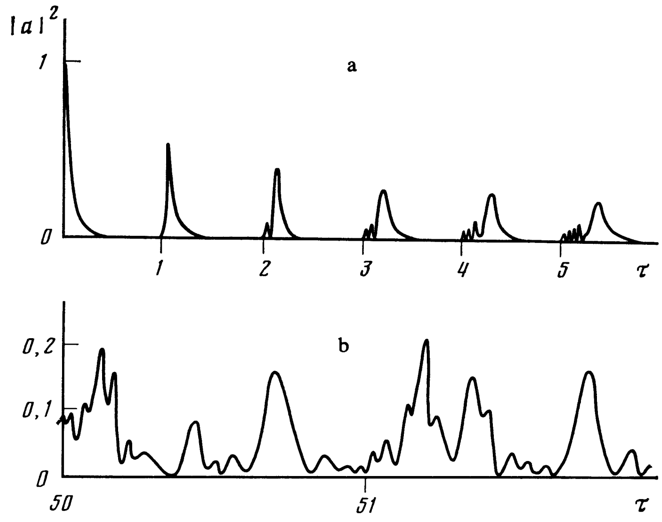

FIG. 4. Time evolution of the population of the ground level in the case of equidistant band (numerical calculations). $N = 60, \beta = 20, \delta = 0$. a—Recursive phenomena ("bursts") at $\tau < \beta$; b—quasirandom behavior at $\tau > \beta$.

$$a(\tau) = \exp\{-\beta(1+is^0)\tau\} + \sum_{n=1}^{\infty}\theta(\tau-n)\exp\{-\beta(1+is^0)(\tau-n)+in\delta\} \times L_n^{-1}(2\beta(\tau-n)), \quad (26)$$

where $L_n^{-1}(x)$ are Laguerre polynomials. Solution (26) was derived in Ref. 8 with $s^0 = 0$ and without a discusssion of its range of applicability.[3)]

A characteristic feature of (26) in a strong field is the recursive behavior of the population $|a|^2$ at $\tau < \beta$. In the interval $n \leqslant \tau \leqslant n+1$ the function $|a(\tau)|^2$ is a series of n "bursts" of increasing amplitude; the amplitude does not depend on the field intensity (the maximum amplitude is on the order of $n^{-2/3}$, where $n = 2, 3, \ldots$; see Fig. 4a). The recursive behavior of this population is a manifestation of the linearity of spectrum (21) at $Y^2 \ll 1$. Over a time of order β, however, there is a complete loss of phase coherence, and at $\tau > \beta$ the population $|a|^2$ starts to behave in complete accordance with the general rules as a random function which is, because of the equidistant nature of the band, quasiperiodic with a period of $1 + \beta^{-1}$ (Fig. 4b).

In addition to (15), we have some other restrictions on the applicability of solution (26):

$$e^{\beta}\frac{\beta}{\pi N^2} < 1, \quad \beta^2\left|\frac{d^2\alpha(n)}{dn^2}\right|_{n=\beta+(\delta_0/2\pi)} \ll 1,$$

where $\alpha(n)$ is a nonlinear increment in the spectrum of the band, and $\Delta_n = \Delta\nu(n+\alpha)$. If the first of these conditions does not hold, a transition will take place from an exponential emptying of the ground level to a power-law emptying[14] at the ends of the intervals $(n, n+1)$, $\beta \gg n$:

$$|a|^2 \sim \beta^2/[N^4(\tau-n)^2].$$

The reason for this result is that the contribution of $|a_k|^2$, as a function of the continuous parameter k, has discontinuities $\Delta|a(k)|^2 \sim \beta/N^2$ at $k = 0$ and $N+2$. Since $\beta \ll N$, however, the changes in (26) can be regarded as unimportant to the present analysis. If there is a pronounced anharmonicity in the spectrum of the band, i.e., if the second of the conditions is violated, recurrence phenomena will not be observed because of the important nonlinearity of the quasienergy spectrum. If, on the other hand, the anharmonicity in the band is slight, coherent effects may in fact be amplified in a certain intensity range. The explanation is as follows: The quasienergy spectrum of an equidistant band is nonlinear. For the most important states this nonlinearity is on the order of k^3/β^3, where $k \lesssim \beta$. When the band is anharmonic, the nonlinearity of the spectrum is on the order of $k^3/\beta^3 + \alpha(k)$. If $\alpha(k)$ and k^3/β^3 differ in sign, the spectrum becomes linearized in the region important for coherent effects under the condition $|\alpha(\beta)| \sim 1$. For $\alpha(k) = \alpha_0 k^2$ with $\alpha_0 k < 0$, for example, the spectrum becomes linearized, but if $\alpha_0 k > 0$ it does not. The linearization of the spectrum, however, even for half of the quasienergies can contribute an amplifying effect, as is seen in the results of the numerical calculations (Fig. 5). This compensatory effect is seen most vividly in the case of a symmetric band with a negative anharmonicity, with an exact resonance at the center of band. In this case the entire spectrum becomes linearized, and the condition for maximum amplification of the bursts is $\alpha_0\beta^2 \approx 1$. With a further increase in the field intensity, of course, the nonlinearity of the band spectrum by itself begins to disrupt the quasienergy spectrum, and the coherent effects gradually disapear.

We now consider the population of the levels in the band. From (21) we find

$$b_n = \frac{2}{\pi}\left(\frac{2}{\beta}\right)^{1/2}\sum_{-\infty}^{+\infty}\frac{\exp\{-i(\pi(2k+1) - 2\,\mathrm{arctg}\,Y)\tau\}}{(1+Y^2)[1+2(k-n)-(2/\pi)\,\mathrm{arctg}\,Y]}. \quad (27)$$

Knowing $a(\tau)$, we find $b_n(\tau)$ for any time from (17). We will not write out these solutions in their general form; instead we consider the most important cases.

For the resonant levels in the band, i.e., for $|2\pi n - \delta_0| \ll \beta$, and in a strong field, we have

$$b_n = \left(\frac{2}{\beta}\right)^{1/2}\sum_{k=-\infty}^{+\infty}\frac{2}{\pi(1+2(k-n))}\exp\{-iE_k\tau\} \approx -i(2\beta)^{1/2}\int_0^{\tau} a(\tau_1)\,d\tau_1.$$

If $\tau < \beta$, the change in the population $|b_n|^2$ is also recursive. For example, for resonant levels with $0 \leqslant \tau \leqslant 3$ we have

$$|b_n|^2 = \frac{2}{\beta}[1-e^{-\beta\tau}]^2 \quad \text{for} \quad 0 \leqslant \tau \leqslant 1,$$

$$|b_n|^2 = \frac{2}{\beta}[1-2(1+\beta(\tau-1))e^{-\beta(\tau-1)}]^2 \quad \text{for} \quad 1 \leqslant \tau \leqslant 2,$$

$$|b_n|^2 = \frac{2}{\beta}[1-2(1+\beta(\tau-2)+\beta^2(\tau-2)^2)e^{-\beta(\tau-2)}]^2 \quad \text{for} \quad 2 \leqslant \tau \leqslant 3.$$

To the extent that the phase coherence is lost at $\tau \geqslant \beta$, a be-

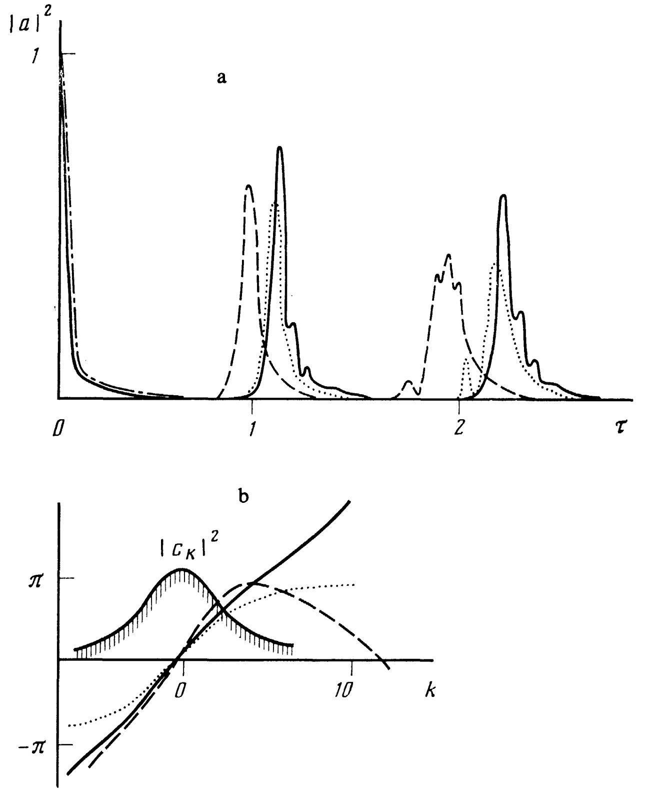

FIG. 5. Effect of weak anharmonicity on the height of the coherent "bursts" $\beta = 14$, $N = 160$. a: Solid Line—$\alpha_0 = -0.005$, $\delta = 0$; dashed line—$\alpha_0 = 0.005$, $\delta = 20$; dotted line—$\alpha_0 = 0$, $\delta = 0$. b: Nonlinear part of the quasienergies and of the $|c_k|^2$ contribution versus the index of the quasienergy state. A weak anharmonicity has an obvious linearizing effect. The curves are drawn for common values of k for clarity.

havior independent of the particular form of the band becomes dominant, and the population of the resonant level becomes reminiscent of a periodic function (Fig. 6). From (27) we find

$$|b_n(\tau)|^2 \approx \frac{2}{\beta}\left[0{,}2+1{,}6\sin^2\pi\left(1-\frac{2}{\beta}\right)\tau\right].$$

For nonresonant levels ($|2\pi n - \delta_0| \gg \beta$) we can distinguish in (27) two ensemble of states which play a leading role in the dynamics of the populations: $k = n$ and $|2\pi k - \delta_0| \leqslant \beta/2$. We thus have

$$b_n \approx \frac{(2\beta)^{1/2}}{2\pi n-\delta_0}\left[\exp(-iE_n\tau)-\sum_k \frac{2}{\beta}\frac{\exp(-iE_k\tau)}{1+Y^2(k)}\right]$$
$$\approx \frac{(2\beta)^{1/2}}{2\pi n-\delta_0}[\exp(-iE_n\tau)-a(\tau)].$$

The population of a nonresonant level also initially changes in a recursive manner, and later, as $a(\tau)$ converts into noise, the second ensemble becomes unimportant, and $|b_n(\tau)|^2$ approaches a constant value $|b_n(\tau)|^2 = 2\beta(2\pi n - \delta_0)^{-2}$. In the general case, over a long time, the time average of the population is

$$\langle|b_n(\tau)|^2\rangle = 2\beta/[\beta^2+(2\pi n-\delta_0)^2],$$

in agreement with the result derived in Refs. 1 and 2 for a band with a continuum of levels. Over a long time the populations in the band thus change in a quite regular way: The populations of the resonant levels vary periodically, and those of the nonresonant levels remain constant. The quasirandom nature of the change in the population of the ground level, in contrast, results from a drift of the phase of the population amplitudes of the resonant levels of the band with respect to each other.

6. ADIABATIC IMPOSITION OF A FIELD

In the case of an infinitely slow imposition of the field, the system will of course be in a definite quasienergy state with a quasienergy which satisfies the condition $E_k(-\infty) = 0$, $\Delta_k > \Delta > \Delta_{k-1}$, in this case. This quasienergy and all the other characterstics of this state must vary continuously with a change in the field.

The condition for adiabatic behavior can be found by the general procedure for finding the probability for a transition induced by an adiabatic perturbation (§53 in Ref. 15).

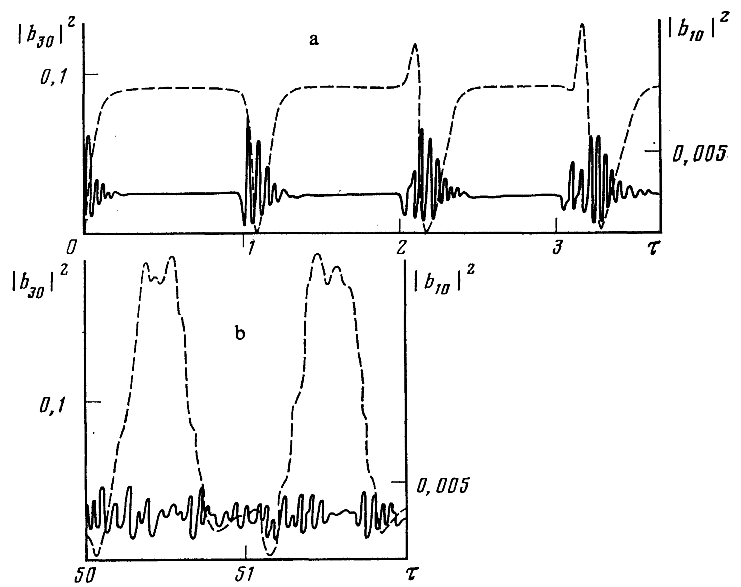

FIG. 6. Time evolution of the population of a level in an equidistant band (numerical calculations). a—Recursive behavior at $\tau < \beta$; b—periodic behavior of the population for resonant levels and a constant value with superimposed noise for nonresonant levels at $\tau > \beta$. $\beta = 20$, $\delta = 0$, $N = 60$. Solid curve) $n = 10$ (nonresonant levels); dash curve) $n = 30$ (resonant levels). The level index is measured from the lower edge of the band.

For the probability $\omega_{kk\pm1}$ for a transition from the initial state to a neighboring state

$$\omega_{k\,k\pm1}=\exp\left\{-2\left|\operatorname{Im}\int_{t_1}^{t_\pm}(E_{k\pm1}-E_k)\,dt\right|\right\},$$

where $E_k(t_\pm)=E_{k\pm1}(t_\pm)$, the time $t_\pm$ has a positive imaginary part, and t_1 is any point on the real axis. For definiteness we consider the upper edge of the band ($S^0>0$). We assume that a field $\tilde{E}_0(t)=\tilde{E}_0(0)\exp(t/t_0)$ at $t<0$ and a field $\tilde{E}_0(t)=\tilde{E}_0(0)$ at $t\geqslant0$. For $\frac{1}{2}S^0\Delta_\pm\ll1$ we find

$$\omega_{k\,k\pm1}=\exp\{-\pi t_0\Delta_\pm\}\ll1,$$

where $\Delta_+=\Delta_k-\Delta$ and $\Delta_-=\Delta-\Delta_{k-1}$. This is the well-known criterion for the adiabatic imposition of a field. For $(1/2)S^0\Delta_\pm\gg1$, i.e., at the band edge, we again find a probability $\omega_{kk-1}=\exp\{-\pi t_0\Delta_-\}$ for a transition to the state $(k-1)$. For a transition to the $(k+1)$ state, however, we now have $\omega_{kk+1}=\exp\{-2\pi t_0/S^0\}$. This is an analog of the Landau-Zenner effect, which occurs because the distance between neighboring quasienergy terms initially decreases and then increases. For the general case the condition for adiabatic behavior is

$$\pi t_0\min(2/S^0,\ \Delta_\pm)\gg1. \tag{28}$$

The usual condition for adiabatic behavior thus becomes more stringent at the band edge.[4] In a strong field the system is nearly entirely excited into the band, and the "superselectivity" studied in Ref. 5 for the case of an extremely strong field occurs. At the center of the band, condition (10) holds, and the level populations are determined by (11). Essentially two levels, with $n=k$ and $k-1$, are populated in the band. At a resonance at the band edge, expressions (13) apply. Only a single level is populated, and we have

$$|a|^2=\left\{\frac{4|d_k|^2}{|f_k|S^0(|d_k|^2+|d_{k-1}|^2)}\right\}^2\approx\frac{|a|_c^2}{(S^0)^2}\frac{32}{(\Delta_k-\Delta_{k-1})^2},$$

where $|a|_c^2$ is the residual population of the ground level for the case of a resonance at the center of the band. When there is a resonance at the band edge, the ground level is thus emptied to an even greater extent than it would be in the case of a resonance at the center of the band, as was pointed out previously by Peterson.[6] It can be seen from these results that the rate at which the field is imposed is not as important for the total population of the band as in the case of an extremely strong field (in which case the integrated populations of the band differ by a factor of two). In a strong field, on the other hand, regardless of the rate at which the field is applied, the system goes nearly entirely into the band. In this case, however, there are important differences in the distribution of the system among levels. While in the case of an instantaneous field imposition there are equal probabilities for the filling of something on the order of $\beta\gg1$ levels, in the case of an adiabatic imposition only one or two levels are filled.

Because of the large S shift at the band edge even in a weak field, there can be an essentially complete excitation of the system. If $\beta s^0\ll\pi$, i.e., at the center of the band, the time average of the population of the ground level cannot be less than 0.5. For this population to decrease even to 0.5, we would need a very accurate resonance with some level in the band. For a resonance at the band edge, on the other hand, a situation with $\beta s^0\gg\pi$, but $\beta<\pi^2/2$ would be completely possible. It follows from (8) that we have $|a|^2=4(|f|S^0)^{-2}\ll1$ in this case, and the system is excited almost entirely into a definite level. In order to achieve this efficient excitation it is sufficient to simply "aim" in the vicinity of the band edge; it is not necessary to seek any narrow resonances.

If condition (28) does not hold, but the condition $t_0(\Delta_k-\Delta_{k-1})\gg1$ nevertheless does, the dynamics of the system is described by a combination of two states. For clarity we consider an equidistant band with an exact resonance at the center ($\delta=0$). We then have

$$a(\tau)=2\left(2+\beta+\frac{\varepsilon^2}{\beta}\right)^{-1/2}\cos\int_{-\infty}^{\tau}\varepsilon\,d\tau_1,$$

$$\varepsilon=\beta(\tau)\operatorname{ctg}\frac{\varepsilon}{2},\qquad \pi>\varepsilon>0, \tag{29}$$

$$b_n(\tau)=\left[\beta\left(1+\frac{\beta}{2}+\frac{\varepsilon^2}{\beta}\right)^{-1}\right]^{1/2}$$

$$\times\left[\frac{4\pi n}{4\pi^2n^2-\varepsilon^2}\cos\int_{-\infty}^{\tau}\varepsilon\,d\tau_1+\frac{2i\varepsilon}{4\pi^2n^2-\varepsilon^2}\sin\int_{-\infty}^{\tau}\varepsilon\,d\tau_1\right].$$

In a strong field, at $\tau>0$, we have

$$|a(\tau)|^2=\frac{2}{\beta}\left[1+\cos\left(\psi_0+2\pi\left(1-\frac{2}{\beta}\right)\tau\right)\right],$$

$$\psi_0=2\int_{-\infty}^{0}\varepsilon\,d\tau_1,$$

$$|b_n(\tau)|^2=\frac{4}{\pi^2(1-4n^2)^2}\left[(1+4n^2)+(4n^2-1)\times\cos\left(\psi_0+2\pi\left(1-\frac{2}{\beta}\right)\tau\right)\right] \tag{30}$$

Expressions (29) and (30) generalize the result derived in Ref. 5 for a three-level system to the ground level. Specifically, in the case of a resonance with a slow imposition of the field the amplitude of the Rabi oscillations of the population of this level decreases to a small value as the field intensity increases. In a sense, the Rabi oscillations are carried into the band, and their frequency is nearly equal to the distance between the levels in the band in the case of a strong field. The time averages of the populations in the band in this case are slightly different from those in the adiabatic case, but we see from (30) that, as in the adiabatic case, the populations in the band do not depend on the field intensity. With a probability of 0.85 the system populates one of the three levels $n=0,\pm1$.

7. INTERMEDIATE CASE OF FIELD IMPOSITION

We now consider the real case in which the field is applied over a finite time. For clarity we assume an equidistant band. This assumption does not restrict the generality of the analysis, since the specific features of such a band are seen only in the recursive phenomena resulting for instantaneous field imposition, and we will not be focusing on those phenomena in this section of the paper. Furthermore, we assume $\mathrm{mod}_{2\pi}\delta \approx \pi$, since the subtle effects which arise in a weak field in the case $|\pi - \mathrm{mod}_{2\pi}\delta| \approx \pi$ were discussed at the end of the preceding section. We assume $\beta(\tau) = \beta \exp(2\tau/\tau_0)$ at $\tau \leqslant 0$ and $\beta(\tau) = \beta$ at $\tau \geqslant 0$. We first consider a resonance at the center of the band, with $|s^0| \ll 1$. At $\tau_0 > 1$, we have adiabatic behavior. At $\beta < 1$, regardless of the way in which the field is imposed, no more than one state will be at resonance, and there is no essential difference between the cases in which the field is imposed in different way. We therefore assume $\tau_0 \ll 1$ and $\beta \gg 1$. At $\tau < 0$, even in the case of equidistant levels, the phase memory of the band plays no role [the right side of (16) is exponentially small]. Working directly from (2), assuming

$$|d_n|_{max}^{-2} \sum_l |d_n|^2 \exp\{-i\delta_n(\tau-\tau_1)\} = N_0 \exp\{-2N_0|\tau-\tau_1|\}$$

for clarity, we find

$$\tilde{\tau}\ddot{a}(\tilde{\tau}) + {}^1/_2(1+2N_0\tau_0)\dot{a}(\tilde{\tau}) + {}^1/_2\beta N_0\tau_0^2 a(\tilde{\tau}) = 0, \quad \tilde{\tau} = e^{2\tau/\tau_0}. \tag{31}$$

A solution of Eq. (31) can be expressed in terms of a Bessel function:

$$a(\tau) = \Gamma\left(N_0\tau_0 + \frac{1}{2}\right)\left(\frac{x}{2}\right)^{(1-2N_0\tau_0)/2} J_{N_0\tau_0-1/2}(x),$$
$$x(\tau) = (2\beta N_0)^{1/2}\tau_0 e^{\tau/\tau_0}. \tag{32}$$

Hence

$$b_n(0) = -\frac{i}{N_0^{1/2}}\Gamma\left(N_0\tau_0 + \frac{1}{2}\right) \times \exp\left\{-i(2\pi n-\delta)\tau_0 \ln\left[\left(\frac{\beta N_0}{2}\right)^{1/2}\tau_0\right]\right\} \times \int_0^{x(0)} \left(\frac{x_1}{2}\right)^{(1-2N_0\tau_0)/2+i(2\pi n-\delta)\tau_0} J_{N_0\tau_0-1/2}(x_1)\,dx_1. \tag{33}$$

If

$$\tau_0 \min\left(\beta, \frac{\pi^2}{2}N_0\right) \gg 1, \tag{34}$$

then $|a(0)|^2 \ll 1$, and we can set $x(0) = \infty$ in (33). We then find

$$|b_n(0)|^2 = \pi\tau_0\left[\mathrm{ch}\frac{\pi}{2}(2\pi n-\delta)\tau_0\right]^{-1}$$

It is not difficult to see that the edge states are of minor importance in this case, and we have $|c_k|^2 \approx |b_k(0)|^2$. Under condition (34), the dynamics of the system is thus dominated by something on the order of τ_0^{-1} states. The width of the excitation region is also of order τ_0^{-1}, and the average population in it is of order τ_0. We then find

$$\langle|a(\tau)|^2\rangle = 2/\beta, \quad |a(\tau)|_{max}^2 \leqslant \left[\tau_0 \min\left(\beta, \frac{\pi^2 N_0}{2}\right)\right]^{-1} \ll 1.$$

The time average of the population, $|a|^2$, is the same as in the adiabatic case, while the recursive effects are less pronounced. We will call the situation discussed above, in which the parameters of the excitation region depend strongly on the rate of the field imposition, the "intermediate case" of field imposition. If, on the other hand,

$$\tau_0 \min\left(\beta, \frac{\pi^2}{2}N_0\right) \ll 1,$$

then $a(0) = \cos[(\beta\tau_0 \min(2N_0\tau_0, 1))^{1/2}]$,

and we are essentially dealing with the instantaneous field imposition approximation.

We turn now to the case of a resonance at the band edge. For definiteness, we consider the upper edge: $s^0 \gg 11$. If $\beta < \pi/s^0$, we have the same situation as in the case $\beta < 1$ at the center of the band. If $\beta(\tau) > \pi/s^0$, but $\beta(\tau) \lesssim 1$ and $2\pi\tau_0 < s^0$, we can apply the Landau-Zenner analysis to a transition between quasienergy states, by a procedure analogous to that of the preceding section. It is not difficult to see that as the field is increased these states are populated with a probability

$$|c_{k+k_1}|^2 = \frac{2\pi\tau_0}{s^0}\exp\left\{-\frac{2\pi\tau_0 k_1}{s^0}\right\}, \quad k_1 = 0, 1, \ldots, \tag{35}$$

where $(k + k_1)$ is the index of the state, and $2\pi k > \delta > 2\pi(k-1)$. The states are filled up to the state of index

$$k_0 = \min[\beta(\tau)s^0, \pi N_0 - \delta]/2\pi;$$

here

$$\omega_{k+k_0} = |c_{k+k_0}|^2 = \exp\{-2\pi k_0\tau_0/s^0\}.$$

If $s^0 < \tau_0(\pi N_0 - \delta)$ and $\tau_0\beta(\tau) \gg 1$ (i.e., if $\beta < 1$, $\tau_0 > \beta^{-1}$ or if $\beta > 1$, $\tau_0 > 1$), the distribution of the system among states can be assumed already formed, and expression (35) gives us the contributions of states which the description of the dynamics of the system. Something on the order of $s^0/2\pi\tau_0 \gg 1$ levels in the band are filled, and we have $|b_n|^2 \approx |c_n|^2$. The population of the ground level is the same as in the adiabatic case: $|a|^2 = 1/\beta(s^0)^2$. If, on the other hand,

$$\tau_0 \min(\beta, (\pi N_0 - \delta)/s^0) < 1,$$

then we have $|c_{k+k_0}| \approx 1$, we can use the instantaneous field imposition approximation. The population $|a|^2$ is approximately unity, and the system is not excited into the band.

If $1 < \beta < (\pi N_0 - \delta)/s^0$ at $\tau_0 < 1$, then we find the following result from (16) for $\tau \leqslant 0$:

$$a(\tau) = \exp\left\{-\frac{\beta\tau_0}{2}(1+is^0)\exp\left(\frac{2\tau}{\tau_0}\right)\right\}.$$

From (17) we then have

$$b_n(0) = \left(\frac{\tau_0}{2(1+is^0)}\right)^{1/2}\gamma\left[\frac{1}{2}(1+(2\pi n-\delta)\tau_0), \frac{\beta\tau_0}{2}(1+is^0)\right] \times \exp\left\{-\frac{i(2\pi n-\delta)\tau_0}{2}\ln\left[\frac{\beta\tau_0}{2}(1+(s^0)^2)^{1/2}\right] + \frac{1}{2}(2\pi n-\delta)\tau_0 \,\mathrm{arctg}\, s^0\right\}, \tag{36}$$

where $\gamma(x, y)$ is the incomplete gamma function. For $\beta\tau_0/2 \ll 1$ we have $|a(0)| \approx 1$, corresponding to instantaneous field imposition. For $\beta\tau_0/2 \gg 1$ we find from (36)

$$|a(0)|\ll 1, \quad |b_n(0)|^2 = \frac{\pi\tau_0 \exp\{(2\pi n-\delta)\tau_0 \operatorname{arctg} s^0\}}{(1+(s^0)^2)^{1/2} \operatorname{ch}[{}^1/_2\pi(2\pi n-\delta)\tau_0]}. \tag{37}$$

This is the intermediate case of field imposition. The contributions of the quasienergy states are given by (4) and (37), and we have $|c_k|^2 \approx |b_k(0)|^2 = \langle |b_k|^2 \rangle$. The width of the excitation region, $s^0/2\pi\tau_0$, can become greater than the width (β) in the case of instantaneous field imposition. When the resonance occurs too close to the band edge, and the conditions β, $1/\tau_0 \gg (\pi N_0 - \delta)/s^0$ hold, the system is basically in an edge state. If $\beta < N_0$, we have the nonresonant case of an instantaneous field imposition, and the population $|a|^2$ is close to unity. As the field is strengthened, first the edge of the band and then the entire band begin to be captured into this state. At $\beta \gg N_0$, the population $|a|^2$ reaches 0.5, and the band becomes uniformly populated: $|b_n|^2 = (2N_0)^{-1}$ [see (14) for an arbitrary band]. This picture is analogous to the adiabatic imposition of an ultrastrong field with $\Delta > \Delta_N$.

8. CONCLUSION

From this study of the dynamics of a system consisting of a ground level and a band, carried out for various rates of the field imposition and for various field strengths, we can draw several conclusions:

1) In a strong field the contributions of the levels of the band to the quasienergy states are essentially independent of the field strength, and one or two levels are dominant in the interior states. The number of states participating in the excitation is determined by both the field imposition rate and the frequency deviation from the center of the band. The width of the excitation region in the band is equal to the number of such states.

2) For a resonance at the band edge, the customary condition for adiabatic field imposition becomes more stringent ($2\pi\tau_0/s^0 \gg 1$), and even in a weak field ($\beta \ll \pi^2/2$, $\beta s^0 > \pi$) there can be an essentially complete excitation of the system into the band.

3) During the instantaneous imposition of the field, something on the order of β levels are excited in the band, and the populations of these levels are nearly periodic functions of time.

4) For instantaneous field imposition in a very broad band ($N_0 \geq 10^4$), several excitation regions may arise: In addition to the central region, levels at the band edges may be drawn into a resonance because of the large S shift.

5) For a band of equidistant levels, with smoothly varying dipole moments, we have derived a differential-difference equation for the population amplitude of the ground level for a field which varies arbitrarily with time. It has been shown that this equation is applicable except in the approximation of a generalized two-level system.

6) Recursive effects for the population of the ground level may be enhanced by a slight anharmonicity in the spectrum of the band.

7) In a strong field, the time average of the population of the ground level is nearly independent of the field imposition rate and is on the order of β^{-1}. For the cases of instantaneous and intermediate field imposition, this population behaves like a random function of the time.

8) For the intermediate case of the field imposition, the width of the excitation region is on the order of $(1 + (s^0)^2)^{1/2}/2\pi\tau_0$, and the width at the band edge may be greater than that in the case of instantaneous field imposition.

The results of this study show that an analysis of actual experiments on the excitation of many-level systems absolutely must incorporate the effects of all the parameters of the external field (the intensity, the imposition rate, and the frequency deviation) on the nature of the excitation of the system. These effects may be varied and quite complicated.

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[1)]For brevity below, we will call the quasienergy states simply "states," while the stationary states of the system in the absence of a field are "levels."

[2)]The center of the band is defined in §3.

[3)]Mathematically, solution (26) is analogous to the solution derived in Refs. 11 and 12 in an analysis of intramolecular transitions and in Ref. 13 for the density matrix of a two-level system in a multifrequency field with an equidistant mode spectrum in the splitting approximation.

[4)]This point was not mentioned in Ref. 6, where the case of an adiabatic field imposition with a large S shift was analyzed.

[1]A. A. Makarov, V. T. Platonenko, and V. V. Tyakht, Zh. Eksp. Teor. Fiz. **75**, 2075 (1978) [Sov. Phys. JETP **48**, 1044 (1978)].

[2]V. M. Akulin and A. M. Dykhne, Zh. Eksp. Teor. Fiz. **73**, 2098 (1977) [Sov. Phys. JETP **46**, 1099 (1977)].

[3]M. Quack, J. Chem. Phys. **69**, 1282 (1978).

[4]J. H. Eberly, J. J. Yeh, and C. M. Bowden, Chem. Phys. Lett. **86**, 76 (1982); J. Chem. Phys. **76**, 5936 (1982).

[5]M. V. Kuz'min and V. N. Sagonov, Zh. Eksp. Teor. Fiz. **83**, 50 (1982) [Sov. Phys. JETP **56**, 27 (1982)].

[6]C. L. Peterson, C. D. Cantreel, and R. S. Burkey, Opt. Commun. **43**, 123 (1982).

[7]V. M. Akulin, Zh. Eksp. Teor. Fiz. **83**, 984 (1982) [Sov. Phys. JETP **56**, 533 (1982)].

[8]H. W. Galbraith, J. R. Acherhalt, and P. W. Milonni, J. Chem. Phys. **78**, 790 (1983); P. W. Milonni, J. R. Acherhalt, H. W. Galbraith, and Mei-Li Shin, Phys. Rev. **A28**, 32 (1983).

[9]Ya. B. Zel'dovich, Zh. Eksp. Teor. Fiz. **51**, 1492 (1966) [Sov. Phys. JETP **24**, 1006 (1966)]; Usp. Fiz. Nauk **110**, 139 (1973) [Sov. Phys. Usp. **16**, 427 (1973)].

[10]V. I. Ritus, Zh. Eksp. Teor. Fiz. **51**, 1544 (1966). [Sov. Phys. JETP **24**, 1041 (1966)].

[11]G. C. Stey and R. W. Gilerd, Physica (Utrecht) **60**, 1 (1972).

[12]R. Lefebure and J. Savolainen, J. Chem. Phys. **60**, 2509 (1974).

[13]V. Yu. Finkel'shteĭn, Zh. Eksp. Teor. Fiz. **78**, 2138 (1980) [Sov. Phys. JETP **51**, 1072 (1980)].

[14]A. I. Andryushin, A. E. Kazakov, and M. V. Fedorov, Zh. Eksp. Teor. Fiz. **76**, 1907 (1979) [Sov. Phys. JETP **49**, 966 (1979)].

[15]L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika, Nauka, Moscow, 1974 (Quantum Mechanics, Pergamon Press, Oxford).

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