Stimulated emission during axial channeling

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A quantum-mechanical analysis shows that it would be possible to achieve stimulated emission in an axisymmetric focusing electric field by a mechanism based on the nonuniform spacing of levels in the electron energy spectrum and on recoil effects during the emission and absorption of photons by the electrons.

1. INTRODUCTION

We consider a quantum-mechanical system consisting of an electron beam moving in, and interacting with an axisymmetric electromagnetic field. Motion of this type can be arranged, for example, in an axial magnetic undulator or in a crystal during axial channeling of charged particles, with the longitudinal electron velocity $v_{\parallel}$ (the component along the axis of the undulator or the crystal) being much larger than the transverse velocity $v_{\perp}$ (th component in the plane perpendicular to the axis).

Stimulated emission in a system of this type can be described as a change in the number of photons, $N(\varkappa)$, by means of the equation

$$\frac{dN(\varkappa)}{dt}=\frac{1}{2}N(\varkappa)\int\{[f(\mathbf{p}+\hbar\varkappa)-f(\mathbf{p})]w^{+}+[f(\mathbf{p})-f(\mathbf{p}-\hbar\varkappa)]w^{-}\}\frac{d\mathbf{p}}{(2\pi)^3},\qquad(1)$$

where $f(\mathbf{p})$ is the electron distribution function in momentum space, $w^{-}=w^{(\lambda)}_{\mathbf{p}\to\mathbf{p}-\hbar\varkappa}$ is the probability for the stimulated emission by an electron with a momentum $\mathbf{p}$ of a photon with a momentum $\hbar\varkappa$ and a polarization $\mathbf{e}_{\lambda}$ ($\lambda=1,2$) and $w^{+}=w^{(\lambda)}_{\mathbf{p}\to\mathbf{p}+\hbar\varkappa}$ is the probability for stimulated absorption. The factor 1/2 arises because the stimulated emission has been separated from the absorption in (1).

We consider a quasi classical approximation, with $\hbar\varkappa$ small in comparison with $\mathbf{p}$. Expanding the integrand in (1), we can then put Eq. (1) in the following form,[1] with an accuracy to $o(|\hbar\varkappa/\mathbf{p}|)$:

$$\frac{dN(\varkappa)}{dt}=N(\varkappa)\int w(\varkappa,\mathbf{p})\hbar\varkappa\frac{\partial f(\mathbf{p})}{\partial\mathbf{p}}\frac{d\mathbf{p}}{(2\pi)^3},\qquad(2)$$

where $w(\varkappa,\mathbf{p})$ is the probability for spontaneous emission in the classical approximation.

The state of a relativistic particle being channeled in a crystal or in a magnetic undulator is characterized by "transverse-energy" levels of high indices, which are closely spaced. Most of the emission, however, comes from dipole transitions between these levels, since the condition $v_{\perp}/v_{\parallel}\ll 1$ holds. We will be analyzing Eq. (2) for the case of monoenergetic beams, so we can rewrite (2) as

$$\frac{dN(\varkappa)}{dt}=N(\varkappa)\int\left[\hbar\varkappa\frac{\partial f(\mathbf{p})w(\varkappa,\mathbf{p})}{\partial\mathbf{p}}-f(\mathbf{p})\hbar\varkappa\frac{\partial w(\varkappa,\mathbf{p})}{\partial\mathbf{p}}\right]\frac{d\mathbf{p}}{(2\pi)^3}.\qquad(3)$$

The first term in (3) represents a stimulated electron-emission mechanism that involves a level population inversion and was studied in Refs. 2–5. For monoenergetic beams this term vanishes after an integration over the momentum, because the distribution function vanishes at the integration limits.

The second term in (3) can be written in the quasi classical approximation as the difference between the probability for stimulated emission and that for absorption of a photon of momentum $\hbar\varkappa$ by an electron of momentum $\mathbf{p}$ during a quantum transition of the electron to the state $\mathbf{p}-\hbar\varkappa$ (emission) or $\mathbf{p}+\hbar\varkappa$ (absorption):

$$w^{+}-w^{-}=2\hbar\varkappa\partial w(\varkappa,\mathbf{p})/\partial\mathbf{p}.$$

This quantum-mechanical system is therefore described by three effective levels. The mechanism for the induced emission in this system is determined by the nonuniform spacing of levels in the electron energy spectrum and by recoil effects during the emission and absorption of photons of the radiation field by the electron.[1] The radiation field can evidently be intensified under the condition $\hbar\varkappa\partial w(\varkappa,\mathbf{p})/\partial\mathbf{p}<0$; i.e., a necessary condition for intensification is that the probability for the spontaneous emission by an electron at a given frequency and at a definite $\varkappa$ decrease with increasing electron energy (or momentum).

Below we analyze the possibility of achieving stimulated emission by specifically this induced-emission mechanism. We use the quantum-mechanical approach of Ref. 1.

2. CHOICE OF A MODEL FOR THE EXTERNAL FIELD AND CALCULATION OF TRANSITION MATRIX ELEMENTS

We consider a relativistic charged particle moving through the potential field produced by a string of atoms (axial channeling). In this motion regime the total energy of the particle is much larger than the energy of the interaction of the charge of the particle with the field produced by the atomic string.

The motion of the charge along the z axis during axial

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channeling is described by the Dirac equation, whose wave functions take the following form[6] in the cylindrical coordinate system r, φ, z in an axisymmetric focusing electric field $E_r = -A_0'(r)$:

$$\Psi(\mathbf{r}, t) = \frac{N}{\sqrt{r}} e^{-icKt+ik_3z} \begin{vmatrix} (N_3+\zeta k_0)^{1/2} g_1 e^{i(l-1)\varphi} \\ i\varepsilon_0 (N_3-\zeta k_0)^{1/2} g_2 e^{il\varphi} \\ \zeta\varepsilon_0 (N_3-\zeta k_0)^{1/2} g_1 e^{i(l-1)\varphi} \\ i\zeta (N_3+\zeta k_0)^{1/2} g_2 e^{il\varphi} \end{vmatrix}, \quad (4)$$

where $E = c\hbar K$ is the energy, N is a normalization factor, $N_3 = (k_0^2 + k_3^2)^{1/2}$, $\varepsilon_0 = \operatorname{sign} k_3, \hbar k_0 = m_0 c$, and the functions $g_1(r)$ and $g_2(r)$ satisfy the system of equations

$$g_1' - \frac{l_0}{r} g_1 + (N_3 + \zeta \bar{K}) g_2 = 0, \quad l_0 = l - \frac{1}{2},$$
$$g_2' + \frac{l_0}{r} g_2 + (N_3 - \zeta \bar{K}) g_1 = 0, \quad \bar{K} = K + \frac{e_0 A_0(r)}{c\hbar}. \quad (5)$$

Wave functions (4) are eigenfunctions of the spin operator S_3, which determines the projection of the spin of the electron on the z axis ($\zeta = 1$ means that the projection is along z, while $\zeta = -1$ means that it is opposite to z; Ref. 6) in the proper frame of reference of the electron:

$$S_3\Psi = \zeta\hbar N_3 \Psi, \quad \mathbf{S} = m_0 c \rho_3 \boldsymbol{\sigma} + \rho_1 \hat{\mathbf{P}}, \quad (6)$$

where ρ_1, ρ_3, and $\boldsymbol{\sigma}$ are the Dirac matrices.

We consider stimulated transitions of the electron caused by a plane, monochromatic, circularly polarized electromagnetic wave. We consider the case which is the most interesting from the physical standpoint, that in which the wave photons are propagating along the direction of the electron's motion: $\varkappa = \{0,0,\varkappa\}$. For relativistic electrons in undulator motion we would have

$$\beta_\perp^2/(1-\beta_3) \ll 1, \quad (7)$$

and the radiation would reach a maximum at the angle $\theta = 0$. Here $\beta_\perp K = \varepsilon$ and $\beta_3 K = k_3$ are respectively the transverse and longitudinal energies of the electron.

Working by the standard methods of quantum electrodynamics,[7] we find the probabilities for stimulated transitions with the wave functions in (4) at $\theta = 0$:

$$w^{\mp} = N_0 \frac{(1+\eta\chi)}{2} J_{\mp}^2 g(\varkappa^{\mp}) \left\{ \frac{(1+\zeta\zeta')}{2} (N_3 N_3^{\mp} + k_3 k_3^{\mp} + k_0^2) + \frac{(1-\zeta\zeta')}{2} (N_3 N_3^{\mp} - k_3 k_3^{\mp} - k_0^2) \right\},$$

$$N_0 = \frac{8\pi e^2 c (2\pi)^4}{\hbar\varkappa L^3}, \quad J_- = \left(\int_0^\infty g_1 g_2^- dr \right)^2,$$

$$J_+ = \left(\int_0^\infty g_2 g_1^+ dr \right)^2, \quad \eta = \operatorname{sign} l. \quad (8)$$

The minus sign represents a quantum transition of an electron with emission of a photon, while the plus sign means a transition accompanied by an absorption; L is the normalization distance; and $\chi = \pm 1$ corresponds to right-hand and left-hand polarizations of the wave.

The function $g(\varkappa^{\mp})$ determine the dependence of this process on the interaction time. If the "lifetime" τ of the initial state of the electron is significantly longer than the scale times of the spontaneous quantum transitions, but shorter than the "time-of-flight" $t = d/c\beta_3$, where d is the length of the system, then we have[1]

$$g(\varkappa^{\mp}) = 4\tau[1 + (2\tau c)^2 (K - K^{\pm} \pm \varkappa)^2]^{-1}. \quad (9)$$

If $\tau \gg t$, then

$$g(\varkappa^{\mp}) = [4/\pi t c^2 (K - K^{\pm} \pm \varkappa)^2] \sin^2[(K - K^{\pm} \pm \varkappa) ct/2]. \quad (10)$$

We emphasize that t in (10) is not the instantaneous time but the time of flight of the electron through the interaction region. It is related in an unambiguous way to the linear dimensions of the interaction region along the direction of the channeling axis in the crystal or along the undulator axis. The probability for a transition per unit time in the case $\tau \gg t$ can therefore be determined in the usual way (see Ref. 7, for example), by dividing by t the expression for the probability for the transition of an electron from its initial state to final states in some given interval dp.

A characteristic feature of the probabilities for stimulated transitions in (8) is the factor $(1 + \eta\chi)$, which describes the dependence on the polarization of the electromagnetic wave (at $\theta = 0$). An electron interacts with an electromagnetic wave of only right-hand or only left-hand circular polarization, depending on the direction of its own angular momentum, i.e., the transition probability is nonzero if $\eta\chi = 1$.

If the total energy of the electron is much larger than its transverse energy, system (2) has the asymptotic solution

$$g_1 \approx (1+\zeta)\Phi(r), \quad g_2 \approx (\zeta-1)\Phi(r), \quad (11)$$

where $\Phi(r)$ is an unknown function. Using (11), we then find from (8) that the probabilities for spin-flip transitions depend strongly on the initial orientation of the electron spin:

$$w^- \approx D \frac{(1+\eta\chi)}{2} \frac{(1-\zeta\zeta')}{2} (1+\zeta) g(\varkappa^-),$$
$$w^+ \approx D \frac{(1+\eta\chi)}{2} \frac{(1-\zeta\zeta')}{2} (1-\zeta) g(\varkappa^+), \quad D = \frac{\pi e^2 c}{2\hbar\varkappa L^3} \frac{k_0^2 \varkappa^2}{N_3^4}. \quad (12)$$

It follows from (12) that electrons with spin $\zeta = 1$ emit with spin flip, losing energy. The electrons with spin $\zeta = -1$, in contrast, undergo a stimulated absorption and acquire energy. The stimulated emission during axial channeling thus affects the dynamics of the electron.[8]

To pursue the analysis of the stimulated emission we need to specify the potential $A_0(r)$ which performs the axial focusing of the charged particle in the axial-channeling regime. It is an extremely complicated problem[9] to determine the potential $A_0(r)$ explicitly. It is no simpler matter to integrate system (5), even if the potential is known. The problem does simplify substantially, however, if it is assumed that the axially channeled electron moves along a helix, undergoing small oscillations about some equilibrium radius. In this case the motion of the electron can be described by the example of the exactly solvable model of a "rigid" cylindrical rotator.[8] Analysis of the motion of charged particles in axial channeling in more realistic potentials, "Coulomb" and "parabolic,"[10] leads to expressions for the rotation energy (this energy is determined in our case for an analysis of the stimulated emission) which are the same as the corresponding expressions derived from the model of motion along a helix near an equilibrium radius.

3. THE ROTATOR MODEL

We consider a rigid cyclindrical rotator. In this model[8,11] it is assumed that the motion occurs along the surface of a fixed rigid cylinder of radius $r = R$. According to this assumption, the derivatives of the functions g_1 and g_2 with respect to r in (5) vanish; the integrals in (8) drop out; and we set $r = R$ in all the expressions. In this case the functions g_1 and g_2 are given by

$$\left|\begin{matrix} g_1 \\ g_2 \end{matrix}\right| = \frac{\eta}{\sqrt{2}} \left|\begin{matrix} \left(1+\zeta\dfrac{N_3}{K}\right)^{1/2} \\ \\ \left(1-\zeta\dfrac{N_3}{K}\right)^{1/2} \end{matrix}\right|,$$

$$K^2=k_0^2+\varepsilon^2+k_3^2, \quad \varepsilon^2=\left(\frac{l_0}{R}\right)^2, \quad N=\frac{1}{2\pi}\frac{1}{(2N_3)^{1/2}}.$$

The difference between the power levels of the stimulated emission and absorption in the classical approximation for the rotator model is

$$W^- - W^+ = \frac{(e c\mathscr{E})^2}{4\varkappa E}\frac{(1+\eta\chi)}{2}\frac{(1+\zeta\zeta')}{2} \times\{[g(\varkappa^-)-g(\varkappa^+)]K\beta_\perp^2+[g(\varkappa^-)+g(\varkappa^+)] \times[\beta_3\beta_\perp^2\varkappa-(1-\beta_\perp^2)(1-\beta_3)\varkappa_0]\}, \quad (13)$$

where $c\varkappa_0$ is the classical frequency of spontaneous emission, and $\mathscr{E}$ is the field of the electromagnetic wave.

Expression (13) is the same as that for stimulated emission in a uniform magnetic field. Furthermore, it can be shown (by the method of kinetic relations,[1] for example) that expression (13) holds for the motion of an electron in an arbitrary axisymmetric electromagnetic field (for $\theta = 0$) if the kinetic energy of the electron is given approximately by $E = c\hbar(k_0^2 + \varepsilon^2 + k_3^2)^{1/2}$, where $c\hbar\varepsilon$ is the energy of the revolution at the equilibrium radius. It thus becomes possible to generalize the results of this analysis to the case of an asymmetric magnetic field.

To analyze expression (13) by the quantum-mechanical approach we need to know how the electron revolution energy $c\hbar\varepsilon$ in the given field depends on the quantum numbers characterizing the state of the electron. This dependence can be found in the semiclassical approximation from the classical relativistic equations of motion.

4. EQUILIBRIUM RADIUS OF THE ORBIT OF REVOLUTION

In an asymmetric focusing electric field $E_r = -A_0'(r)$ the equilibrium radius of the orbit of revolution is determined by the equation

$$e_0A_0'(r)(E+e_0A_0)+(Mc)^2/r^3=0, \quad (14)$$

where M is the classical angular momentum. In the quasi-classical approximation we would have $M = \hbar l$.

If the potential of the electric field is

$$e_0A_0(r)=-u_0r^\alpha, \quad (15)$$

then the equilibrium orbital radius in field (15) in the paraxial approximation, $|e_0A_0(r)/E| \ll 1$, is given approximately by

$$R=[(\hbar lc)^2/\alpha u_0E]^{1/(2+\alpha)}.$$

In the cases $\alpha = -1$ (a Coulomb potential) and $\alpha = 2$ (a parabolic potential), potential (15) gives a good approximation of the motion of electrons during axial channeling for various regimes of the motion in the crystal.[10]

In an axisymmetric magnetic field the equilibrium orbital radius is determined by the equation

$$e_0A'(r)-Mc/r^2=0. \quad (16)$$

For the magnetic vector potential $A(r) = br^{1-q}/(2-q)$, where we would have a uniform magnetic field in the case $q = 0$ or a focusing magnetic field in the case $0 < q < 1$,[12] we find from (16)

$$R=[l/\gamma]^{1/(2-q)}, \quad \gamma=e_0b(1-q)/c\hbar(2-q). \quad (17)$$

For these cases, the dependence of the revolution energy on the quantum numbers can therefore be determined from

$$\varepsilon^2=\bar{\varepsilon}l^\mu K^\rho. \quad (18)$$

Expression (18) describes various models, depending on the parameters μ and ρ:

1) motion in electric field (15),

$\mu=2\alpha/(2+\alpha), \quad \rho=2/(2+\alpha), \quad \bar{\varepsilon}=(\alpha u_0/\hbar c)^{2/(2+\alpha)};$

2) a rigid cylindrical rotator,

$\mu=2, \quad \rho=0, \quad \bar{\varepsilon}=R^{-2};$

3) the uniform magnetic field (17),

$q=0, \quad \mu=1, \quad \rho=0, \quad \bar{\varepsilon}=\gamma^{1/2};$

4) a focusing magnetic field,

$\mu=2(1-q)/(2-q), \quad \rho=0, \quad \bar{\varepsilon}=\gamma^{1/(2-q)}.$

5. STIMULATED EMISSION

If the stimulated emission is determined by the lifetime τ [see (9)], then we find the following expression for the power of the stimulated emission in undulator regime (7) from (13), using (18):

$$W^- - W^+ = \frac{8M_0\tau}{1+\lambda^2}\left\{\frac{2\lambda^2\beta_\perp^2}{x(1+\lambda^2)}\left[\frac{(\mu-1)}{\mu}\frac{(1-\beta_3)}{\beta_\perp^2}\varkappa_0^2 +\rho\varkappa_0^2-x\varkappa\right]-\varkappa_0(1-\beta_3)\right\},$$

$$M_0=\frac{(e c\mathscr{E})^2}{4\varkappa E}\frac{(1+\eta\chi)}{2}\frac{(1+\zeta\zeta')}{2}, \quad (19)$$

where the frequency difference $x = \varkappa_0 - \varkappa$ must be larger than the corresponding quantum corrections to the frequency of the spontaneous emission, and $\lambda = 2\tau c x(1-\beta_3)$.

At $\mu = 1$ and $\rho = 0$ we find from (19) an expression derived previously[1] for stimulated emission. In this case the total power determined by (19) is negative; i.e., the system will absorb energy.

For a parabolic electric potential (15) with $\alpha = 2$ ($\mu = 1, \rho = 1/2$), the emission field can be "amplified" if the frequency difference satisfies $x > 0$:

$$W^- - W^+ = \frac{8M_0\tau}{(1+\lambda^2)}\left\{\frac{2(\lambda\beta_\perp\varkappa_0)^2\rho}{x(1+\lambda^2)} - \varkappa_0(1-\beta_3)\right\}.$$

If $\mu \neq 1$, we find the following from (19) in the undulator regime:

$$W^- - W^+ = \frac{8M_0\tau\varkappa_0(1-\beta_3)}{(1+\lambda^2)}\left\{\frac{2\lambda^2\varkappa_0}{(1+\lambda^2)}\frac{(\mu-1)}{x\mu} - 1\right\}. \quad (20)$$

It follows from (20) that an amplification of the emission is possible if $(\mu - 1)/x\mu > 0$, in which case we would have

$$|x| > \left|\frac{(\mu-1)}{\mu}\frac{(1-\beta_3)}{\beta_\perp^2}\frac{\varkappa_0^2}{K}\right|.$$

If the stimulated emission is determined by the electron time of flight t in (10), the total radiation-field energy (ΔE) emitted or absorbed over this time has a physical meaning. In particular, in the undulator regime we find from(13), using (18),

$$\Delta E = \frac{4}{\pi}M_0 t^3(1-\beta_3)c\frac{\sin z}{z^3}\left\{\left(z - 2\,\mathrm{tg}\frac{z}{2}\right)\right.$$
$$\left.\times\left[\varkappa x - \frac{(\mu-1)}{\mu}\frac{(1-\beta_3)}{\beta_\perp^2}\varkappa_0^2 - \rho\varkappa_0^2\right]\beta_\perp^2 - x(1-\beta_3)\varkappa_0\,\mathrm{tg}\frac{z}{2}\right\}, \quad (21)$$

where $z = x(1-\beta_3)ct$.

For a uniform magnetic field ($\mu = 1, \rho = 0$) we find from (21) that the electrons absorb energy from the radiation field.

For a parabolic electric potential ($\alpha = 2$) we find from (21) that the radiation field can be amplified it $x < 0$:

$$\Delta E = \frac{4}{\pi}M_0 t^3(1-\beta_3)c\varkappa_0\frac{\sin z}{z^3}\left\{\left(2\,\mathrm{tg}\frac{z}{2} - z\right)\right.$$
$$\left.\times\ \rho\beta_\perp^2 - x(1-\beta_3)\,\mathrm{tg}\frac{z}{2}\right\}.$$

If $\mu \neq 1$, we find from (21) for the undulator regime that the total energy absorbed or emitted over the time t is

$$\Delta E = -\frac{8M_0 d^3(1-\beta_3)^2\varkappa_0^2}{c^2\pi}\frac{(\mu-1)}{\mu}\frac{d}{dz}\left[\frac{1}{z^2}\sin^2\left(\frac{z}{2}\right)\right]. \quad (22)$$

This result—that the total energy emitted or absorbed is proportional to the cube of the length of the system and to the derivative with respect to z written in (22)—is well known for several specific magnetic undulators.[13]

Expression (22), derived in this paper, describes the stimulated emission of electrons whose energy of revolution is given by (18). As was shown above, this dependence of the revolution energy is characteristic of the motion of electrons during axial channeling and in an axial magnetic undulator with "soft" field focusing.

It follows from (22) that in focusing magnetic field (17) an electron will absorb energy from the radiation field: $(\mu - 1)/\mu < 0$. For the model of a rigid rotator ($\mu = 2$) and for electric field (15), in contrast, the interaction of the electron with the radiation field will result in an increase in the energy of the radiation field if $(\mu - 1)/\mu > 0$ ($\alpha < 0; \alpha > 2$).

CONCLUSION

The stimulated emission is described in the quasiclassical approximation by Eq. (3). It is determined by two possible mechanisms (a population inversion and a nonuniform spacing of energy levels). The contributions of the different mechanisms for stimulated emission, which are described by the first and second terms in(3), depend strongly on the properties of the electron beam (the width of the electron energy distribution, the length of the system, etc.).

The possibility of emission in a crystal as a result of an ordinary level population inversion was demonstrated in Refs. 2–4. In the present paper it has been shown that an amplification of an external radiation field can be achieved in a crystal by using monoenergetic electron beams. The mechanism is based on the nonuniform spacing of electron energy levels and recoil effects during the emission or absorption of a photon by an electron. A similar mechanism for stimulated emission is the governing mechanism in free-electron lasers.[13]

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