Undamped oscillations of NLC director in the field of an ordinary light wave

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We have investigated experimentally some characteristic features of orientational aberration self-focusing of an ordinary light wave in a homotropically oriented NLC. A qualitative explanation of these features is offered.

It was noted in the first experiments aimed at observing the optically induced Fréedericksz effect[1] that the interactions of the fields of extraordinary and ordinary light waves with a homotropically oriented octyl-cyano-biphenyl (OCB) nematic liquid crystal (NLC) differ greatly. At oblique incidence of the light wave on an NLC, the characteristic pattern of the aberrational self-focusing that accompanies the reorientation of the NLC molecules in narrow light beams[2] is stable in the first case and unstable in the second, where the number of observed aberration rings changes periodically (oscillates) with time. The oscillations were observed at incidence angles $\alpha \lesssim 20°$. At larger angles α there was no aberration pattern at all, even at a laser radiation power $P \sim 300$ mW. The period of the oscillations and their amplitude depend on the incidence angle α and on the laser radiation power P. The nonstationary character of the self-focusing indicates that the NLC director also oscillates in the field of an ordinary light wave.

The purpose of the present study was a detailed investigation of the distinctive features of director reorientation in the field of an ordinary light wave, as well as an explanation of these features using simple models.

1. ORGANIZATION AND RESULTS OF EXPERIMENT

The object of the investigation was a homotropically oriented DCB crystal with $L = 150\ \mu$m at a temperature 37 °C.

The cell with the crystal was placed in the focal spot of the beam of a cw argon laser (ILA-120 by Carl Zeiss, Jena or model 171 by Spectra Physics, USA) focused by a lens of focal length $f = 270$ mm; the spot radius was $w_0 \approx 44\ \mu$m. The cell plane was vertical and perpendicular to the plane containing the unperturbed director $\mathbf{n}_0$ and the wave vector $\mathbf{k}$ of the light beam incident on the crystal. The incidence angle (the angle between $\mathbf{n}_0$ and $\mathbf{k}$, α_0, inside the crystal) was varied by rotating the cell around the vertical axis. The laser beam polarized in the vertical plane (ordinary wave) was displayed, after passing through the NLC, on a screen perpendicular to $\mathbf{k}$.

The picture on the screen has the following dependence on the incidence angle.

At large incidence angles $\alpha > 20°$ no changes take place in the laser beam passing through the crystal up to a laser-beam power $P \sim 300$ mW (no investigations were made at $P > 300$ mW to prevent damage to the crystal), i.e., no orientational self-focusing takes place.

At small incidence angles $1\text{–}2° < \alpha < 20°$ orientational self-focusing of the light beam sets in and the characteristic aberration pattern is observed on the screen. This pattern, however, is pulsating: the rings appear, "collapse" after some time, reappear, collapse again, etc. The effect has a threshold, viz., self-focusing occurs only if the light-beam power P exceeds a certain threshold $P_{\rm thr}$. The polarization of the aberration rings is likewise not constant in time.

Detailed investigations reported in Ref. 3 have shown the following:

1) The threshold power $P_{\rm thr}$ depends on the angle α. Figure 1 shows the experimental plot of $P_{\rm thr}(\alpha)/P_{\rm thr}(0)$ (curve 1). It can be seen from Fig. 1 that the threshold power $P_{\rm thr}(\alpha)$ increases monotonically with increasing α. As $\alpha \to 0$ the threshold power $P_{\rm thr}(\alpha) \to P_{\rm thr}(0)$.

2) The character of the oscillation of the number $N(t)$ of the aberration rings and of the beam divergence $\theta(t)$ depends on the angle α and on the laser radiation power P. Figure 2 shows by way of illustration the experimental dependences of N on the time t at different values of α and P.

Analysis of all the experimental $N(t)$ plots for $\alpha > 3°$ shows (as illustrated in Fig. 2) that the value $N_1^{\max}$ of the first

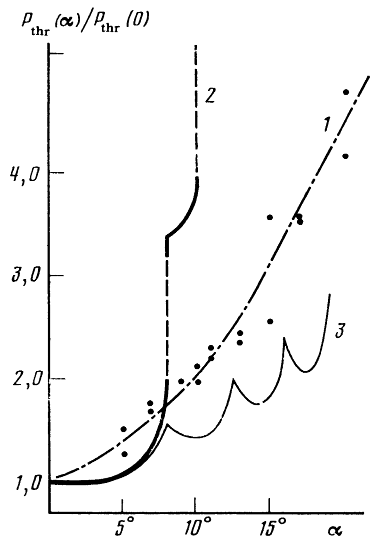

FIG. 1. Ratio of threshold power $P_{\rm thr}(\alpha)$ of Fréedericksz transition in the field of an ordinary light wave obliquely incident on a crystal at an angle α to the threshold power $P_{\rm thr}(0)$ at normal incidence: 1—experimental plot; 2—theoretical plot in the plane-wave approximation; 3—theoretical plot calculated by a variational method; $P_{\rm thr}(0) = 50$ mW.

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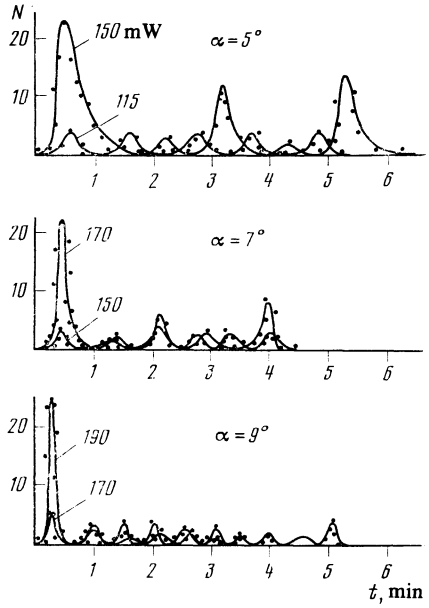

FIG. 2. Experimental time dependence of the number N of aberration rings at different values of α and P.

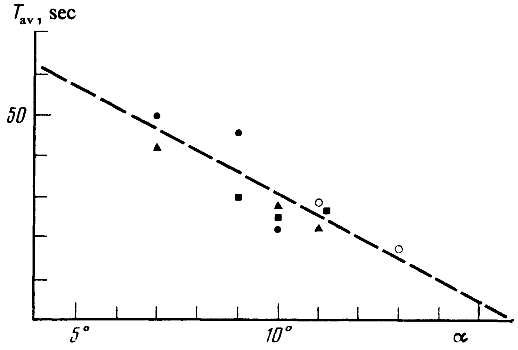

FIG. 3. Experimental plots of the average oscillation period T_{av} vs the crystal rotation angle: ●—$P = 135$ mW, ▲—170 mW, ■—190 mW, ○—210 mW.

maximum of the function $N(t)$ at $P(\alpha) \sim (2\text{–}3)P(0)$ is several (2–10) times larger than $N_n^{\max}$ of all the succeeding maxima ($n > 1$). No regularity in the variation of the maxima $N_n^{\max}$, starting with the second ($n \geqslant 2$) could be observed. In some cases the values of $N_n^{\max}(n \geqslant 2)$ are practically equal for different n, and in others they differ from one another, sometimes considerably (by 2–3 times), but usually they do not exceed $N_1^{\max}$. With decreasing power $P(\alpha) N_1^{\max}$ ceases to be distinguishable from $N_n^{\max}$ of the remaining maxima for each of the α.

An aperiodic nonmonotonic time variation of N is observed at $\alpha \sim 1°$. N reaches a maximum 2–3 min after the start of the illumination, decreases to zero slowly, within 6–8 min, and thereafter no more aberration rings appear. A stationary aberrational pattern of self-focusing orientation is always observed at $\alpha = 0$ regardless of the position of the incident-light polarization plane. The number N of the aberration rings is then determined primarily by the light-beam power.

The measured temporal characteristics of the oscillatory regime of orientational aberrational self-focusing were the time $t_1^{\min}$ from the start of the crystal illumination to the instant of first collapse of the aberration pattern, and the oscillation period of the aberration-ring oscillations.

It was established that starting with the second ($n = 2$) maximum all the succeeding maxima ($n > 2$) follow one another at almost equal time intervals $T_n(\alpha, P) = t_{n+1}^{\max} - t_n^{\max}$($t_n^{\max}$ corresponds to $N_n^{\max}$). This allows us to introduce an oscillation period defined here as

$$T_{av}(\alpha, P) = \frac{1}{n-1} \sum_{i=2}^{n} T_i,$$

where $n - 1$ is the number of averaged periods.

Figure 3 shows the experimentally obtained dependence of $T_{av}(\alpha, P)$ on α (The period T_{av} for each angle α at constant P was obtained by averaging the time intervals between 5 to 10 maxima). The period T_{av} is practically independent of the beam power P.

As for the time $t_1^{\min}$, it was found to decrease with increasing α [at constant $P(\alpha)$]. The explicit dependence of $t_1^{\min}$ on P in the investigated P interval could not be found.

3) The polarization of the beams that form the aberration pattern in orientational self-focusing in an ordinary light wave differs from that of the incident radiation[1)] and varies strongly with time. This can be deduced from observations of the aberration rings. Recall that the aberration rings are not quite regular in shape. They are elongated in a direction perpendicular to the polarization of the incident radiation, and the "elongation" of the rings allows us to track the polarization of the incident light beam and all its variations as the beam passes through the NLC.

Observations of the aberration pattern in our case show that after a time $t_1^{\min}$ the polarization in the transmitted beam changes from its vertical position in the incident beam and at the instant $t = 0$, to horizontal at the instant $t = t_1^{\min}$. At the instant $t_1^{\max}$ the polarization plane of the beam makes an angle $\sim 45°$ with the vertical. It remains practically horizontal for all the succeeding maxima ($n \geqslant 2$).

2. DISCUSSION OF EXPERIMENTAL RESULTS

The observed features of the orientational self-focusing of an ordinary light wave can be first interpreted qualitatively as follows.

If an ordinary light wave with $\mathbf{E} \perp \mathbf{n}_0$ is obliquely incident on an NLC, the vectors $\mathbf{k}$ and $\mathbf{n}_0$ are not collinear. When the director is tilted away from the horizontal plane, two waves will therefore propagate in the interior of the crystal, ordinary $\mathbf{E}_o$ and extraordinary $\mathbf{E}_e$. This alters greatly the polarization of the incident wave and leads to a substantial change of the interaction between the field and the director. A consequence of these complicated processes is an oscillatory regime of the self-focusing and an increase of the self-focusing threshold with increasing incidence angle. Let us explain this in somewhat greater detail.

The external bulk force $\mathbf{G}$ exerted on the director by the electric field $\mathbf{E} = \mathbf{E}_o + \mathbf{E}_e$ is[4]

$$\mathbf{G} = \frac{\Delta\varepsilon}{4\pi} (\mathbf{n}\mathbf{E}_e)\mathbf{E}_e + \frac{\Delta\varepsilon}{4\pi} (\mathbf{n}\mathbf{E}_e)\mathbf{E}_o. \tag{1}$$

With increasing angle between the director **n** and the vector **k** in the course of the change of director orientation, the anisotropy increases and the energy of the ordinary wave incident on the crystal is effectively transferred to the extraordinary one. When the anisotropy becomes very large, the phase difference between the ordinary and extraordinary waves changes greatly and the second term of Eq. (1) oscillates rapidly over the length of the crystal. Clearly, the force corresponding to this term cannot cause a significant reorientation of the director. That is to say, when considering the action of a strong optical electric field on the director, only the field of the extraordinary wave need be considered. Since the director is forced to become parallel to the field of the extraordinary electric wave, the moment of the forces exerted on the director by the electric field is perpendicular to the plane containing $\mathbf{E}_e$ and **n**, i.e., to the plane containing **k** and **n**. But the director is acted upon also by elastic forces that tend to return it to the initial position $\mathbf{n}_0$. The moment of these forces is thus perpendicular to the plane containing **n** and $\mathbf{n}_0$. Since the vectors **k** and $\mathbf{n}_0$ are not collinear in our case, the moments of the electric and elastic forces are also noncollinear, and cannot therefore balance each other.

The resultant torque causes the director to precess towards the horizontal plane. This decreases the fraction of the light-field energy going into the extraordinary wave, making the angle between **n** and **k** smaller. This manifests itself in experiment as an approach of the aberration-ring polarization to horizontal and a decrease in the number of the aberration rings. The director approaches gradually the unperturbed state. The entire process is then repeated. Since it starts out then from a somewhat different position, the amplitude and period of this oscillation differ from those of the preceding one.

With increasing angle α_0, the increase of the anisotropy leads to a more substantial change of the polarization of the incident radiation, and this decreases the amplitude and period of the oscillations.

A quantitative description of oscillating self-focusing of an oblique beam in an NLC encounters the following difficulties. In our experiments the laser beam had a focal spot (transverse radius) comparable with the thickness L of the NLC sample (the characteristic parameter ${}^2g = \sqrt{2}L/\pi w_0$ is equal to 1.5). The limited transverse dimension of the beam plays therefore a rather important role in the director reorientation. Thus, even at normal incidence ($\alpha = 0$) the field-intensity threshold is 6.3 times the corresponding value for a plane wave. This is why the theory developed by a number of workers for plane unbounded waves[5,6] cannot be applied directly to our experimental results. An attempt can be made to take the influence of the limited transverse dimension into account either by introducing a phenomenological factor g in the exact plane-wave solution for the threshold[2,6] (curve 2 of Fig. 1) or by finding a correction by the Ritz method[7] (curve 3 of Fig. 1). It can be seen that both approaches lead to a qualitatively correct description of the threshold curve. For a better quantitative agreement, numerical methods must apparently be invoked.

A theoretically even more complicated problem is that

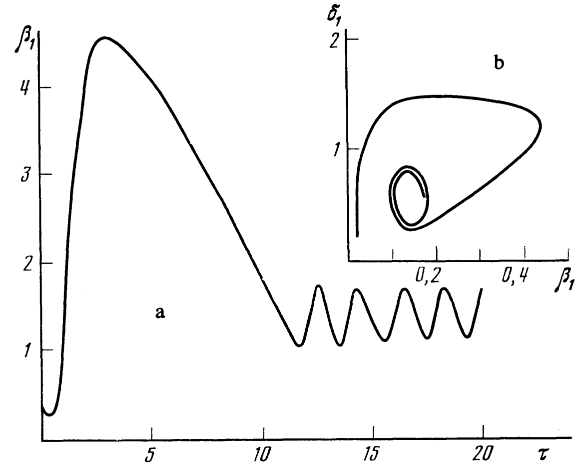

FIG. 4. Oscillations of director **n**, plotted with β_1 and τ (a) and with β_1 and δ_1 (b) as coordinates, in the center of the crystal at $\alpha_0 \approx 1°$ and at a power 5 times the threshold; β_1 is the change of the angle between the director **n** and the wave vector **k**; δ_{12} is the angle between the horizontal plane and the director projection on the plane perpendicular to $\mathbf{n}_0$; τ is the dimensionless time.

of the oscillating self-focusing observed in our experiment. We have taken here the first step towards the development of a mathematical model of self-focusing. A numerical analysis of the system of Maxwell's equations that describe the reorientation of the director in the electric field of a light wave, carried out in the plane-wave approximation with allowance for only the lowest spatial harmonics of the director-deflection angles ($\beta(t)$ and $\delta(t)$ in Fig. 4), shows a limit cycle, i.e., the presence of undamped oscillations. Their calculated period, $T \sim 60$ sec for $\alpha_0 \approx 1°$ and for a beam power 5 times the threshold, is in satisfactory agreement with experiment. The model of director reorientation is made additionally complicated by the need for taking into account higher longitudinal modes and by the complex transverse structure that differs, generally speaking, from Gaussian. In our opinion, however, the limit cycle is preserved also in the complicated model. Since different limit cycles (different oscillation periods) are to be expected for different spatial modes, the director oscillations and the self-focusing periods can become quasiperiodic.

[1)]In contrast to the polarization effects described in Ref. 2, we are dealing here with the change of the polarization of the aberration pattern as a whole.

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