Electromagnetic effects in an intense single-crystal field

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It is shown that a situation exists that lends itself to modern experimentation from the viewpoint of research into quantum electrodynamics in an intense external field.

1. INTRODUCTION

A hypothesis was advanced relatively recently, on the basis of the Klein paradox, that it is impossible to produce a force field that can impart to an electron, over the Compton length, an energy exceeding m_0c^2, and that the Dirac theory is valid only for field weaker than $E_0 = m_0^2c^3/e\hbar$ (Ref. 1). The observation of quantum effects in an intense electric field is therefore of fundamental interest from the viewpoint of checking on the validity of quantum electrodynamics for fields of such an intensity. Unfortunately, the intensities of all the presently existing three-dimensional macroscopic fields are smaller than E_0 by many orders of magnitude, therefore (under conditions of significant momentum transfer to the field) the probabilities of many effects are exponentially suppressed and unobservable. It is nonetheless possible to observe certain quantum effects by using ultrarelativistic particles. It is known that there exist microscopic fields whose intensity exceeds significantly the macroscopic fields now obtainable under terrestrial conditions. It turns out that by creating special initial conditions it is possible to realize an intermediate experimental situation wherein the field is microscopic in one or two directions and macroscopic in the others (two or one, respectively). This situation is realized when light ultrarelativistic charged particles move along a crystallographic plane or axis, with their transverse motion finite and nonrelativistic and taking place in a sufficiently strong inhomogeneous field of these planes (axes). In certain single crystals (. . . W, Au . . .) the depth of the well produced by a crystallographic plane is $U_0 \approx 10^3$ eV (the plane-well depth is $U_0 \approx 10^2$ eV), and the effective width of the well is of the order of the screening radius. The electric field intensity of an axis is $E \approx 10^{12}$ V/cm ($E \approx 10^{11}$ V/cm for a plane). For electrons of energy $\varepsilon \approx 10$ GeV (in the frame where the longitudinal velocity is zero), the axis electric field intensity is comparable with the critical electric field $E_0 \approx 1.32\times10^{16}$ V/cm. This makes it possible to investigate experimentally, under terrestrial conditions and at now attainable electron energies, certain quantum-electrodynamics effects in intense fields, such as [2]: 1) the influence of external field on the electron emission intensity; 2) the influence of external fields on particle decay; 3) photo- and electroproduction of e^-e^+ pairs. The influence of nonlinear classical and quantum effects on relativistic-particle emission intensity in channeling was previously taken into account in some studies, e.g., in Ref. 3 (on this basis we can interpret certain fine details of the spectral density of the radiation in the corresponding experiments).

We investigate in this paper photo- and electroproduction of relativistic e^-e^+ pairs when each component of the produced pair executes a nonrelativistic finite motion in the transverse direction and a free relativistic motion along a crystallographic axis or plane.

2. PHOTOPRODUCTION OF e^-e^+ PAIRS

When relativistic particles move along selected crystallographic directions, their interaction with the crystal reduces to interaction of the particles with the averaged potential of the crystallographic axis or plane.[4] The averaged potential does not depend on the time. The problem is therefore stationary. To analyze this problem it is convenient to use the Dirac equation written in two-component form. This system of first order differential equations reduces to a single second-order differential equation for the function $\varphi(\mathbf{r}) = \chi(\mathbf{r})[\varepsilon - V(x) + m_0c^2]^{-1/2}$:

$$\{(\varepsilon - V(x))^2 - m_0^2c^4\}\varphi(\mathbf{r}) = \hat{\mathbf{p}}^2\varphi(\mathbf{r}) + U_1(x)\varphi(\mathbf{r}), \qquad (1)$$

$$U_1(x) = -\frac{1}{2}\left\{\frac{V''(x)}{\varepsilon - V(x) + m_0c^2} + \frac{3}{2}\left(\frac{V'(x)}{\varepsilon - V(x) + m_0c^2}\right)^2 - \frac{2V'(x)}{\varepsilon - V(x) + m_0c^2}\sigma_x(\sigma_\parallel \mathbf{p}_\parallel)\right\}.$$

The distinguishing feature of this problem is that $\varepsilon \gg V(x)$. It is therefore possible to neglect in Eq. (1) the effective potential $U_1(x)$. In this approximation, Eq. (1) is a Klein-Gordon equation.

On the other hand, the commutator is

$$[\hat{\mathscr{H}}_D, \hat{\mathscr{H}}_K]\mathscr{H}_K^{-1} = o[V(x)/\mathscr{H}_K],$$

where

$$\hat{\mathscr{H}}_D = (\sigma\hat{\mathbf{p}}) + \gamma_0 m_0c^2 + V(x), \quad \hat{\mathscr{H}}_K = (\hat{\mathbf{p}}^2c^2 + m_0^2c^4)^{1/2} + V(x).$$

Therefore the wave function of a particle in an external field can be represented ultimately in operator form

$$\psi(\mathbf{r}, t) = U_s(\hat{\mathbf{p}})\exp\{-i[\hat{\mathscr{H}}t/\hbar]\}\varphi(\mathbf{r}) + o[V(x)/\mathscr{H}]. \qquad (2)$$

Here $\varphi(\mathbf{r})$ is the wave function of the particle and is a solution of the Klein-Gordon equation in the field of the crystallographic plane $V(x)$ (or axis)

$$U_s(\hat{\mathbf{p}}) = [2\hat{\mathscr{H}}_1]^{-1/2}\begin{vmatrix} [\hat{\mathscr{H}}_1 + m_0c^2]^{1/2}\varphi_s \\ \dfrac{(\sigma\hat{p})}{[\hat{\mathscr{H}}_1 + m_0c^2]^{1/2}}\varphi_s \end{vmatrix}$$

are spinors that describe the spin properties of the particles, $\hat{p} = -i\hbar\nabla$,

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$$\hat{\mathscr{H}}=[\mathbf{p}^2c^2+m_0{}^2c^4]^{1/2}+V(x)\equiv\hat{\mathscr{H}}_1+V(x).$$

The wave function $\psi(\mathbf{r},t)$ was obtained accurate to $V(x)/\varepsilon\ll1$. Of the same order of smallness are the quantum effects connected with the mutual noncommutativity of the operators of the dynamic variables of the particles.

Indeed, the velocity operator is of the form

$$\mathbf{v}=\frac{1}{i\hbar}[\mathbf{r},\hat{\mathscr{H}}]_-=\frac{1}{i\hbar}[\mathbf{r},\hat{\mathscr{H}}_1]_-, \tag{3}$$

where $\hat{\mathscr{H}}\equiv\hat{\mathscr{H}}_1+V(x)$, $\hat{\mathbf{p}}=-i\hbar\nabla$. From (3) we obtain an operator equation for $\mathbf{v}$:

$$\mathbf{v}+[\hat{\mathscr{H}}_1,\mathbf{v}]_-\hat{\mathscr{H}}^{-1}/2=\hat{\mathbf{p}}\hat{\mathscr{H}}^{-1}.$$

Solving it by a variational method, we have

$$\mathbf{v}=\hat{\mathbf{p}}\hat{\mathscr{H}}^{-1}+\sum_{n=1}^{\infty}(-1)^n[\hat{\mathscr{H}}_1[\hat{\mathscr{H}}_1\ldots[\hat{\mathscr{H}}_1,\hat{\mathbf{p}}]_-\ldots]_-\hat{\mathscr{H}}^{-n-1}/2^n.$$

Since $[\hat{\mathscr{H}}_1\hat{p}]_-=0$, it follows that

$$\mathbf{v}=\hat{\mathbf{p}}\hat{\mathscr{H}}^{-1}. \tag{4}$$

The commutator is

$$[\hat{\mathscr{H}},\hat{\mathbf{p}}]_-\hat{\mathscr{H}}^{-1}=i\hbar V'(x)\hat{\mathscr{H}}^{-1}. \tag{5}$$

Using (5) we obtain from (4)

$$[\hat{\mathscr{H}},\mathbf{v}]_-=i\hbar V'(x)\hat{\mathscr{H}}^{-1}. \tag{6}$$

It follows from the foregoing analysis that the noncommunativity of the dynamic variables, which determines the magnitude of the quantum effects for particles moving in an arbitrary electric field, is of the order of $V'(x)/\varepsilon\ll1$.

Therefore the sequence of the operator factors in $\psi(\mathbf{r},t)$ is immaterial.

During the first stage we consider the probability of photoproduction of e^-e^+ pairs in accord with the theory of quantum transitions, on the basis of the operator method of Ref. 5. Next, using the intermediate result, we obtain by the Weizsäcker-Williams method the probability of electroproduction of e^-e^+ pairs by an ultrarelativistic electron in the presence of an inhomogeneous field of a crystallographic plane (axis), a field producing a potential well for the nonrelativistic transverse motion of the components of the produced pair. The matrix element of the transition, which describes the photoproduction of e^-e^+ pairs in first-order perturbation theory, is

$$\mathscr{H}_{i,f}=\frac{(-i)e(4\pi)^{1/2}}{(2\pi)^2\omega}\int dt\int\varphi_f{}^*(\mathbf{r})\exp\left\{i\frac{\hat{\mathscr{H}}t}{\hbar}\right\}U_s{}^+(\hat{\mathbf{p}})$$
$$\times e^{i\varkappa\mathbf{r}-i\omega t}(\mathbf{l}\alpha)U_{\bar{s}}(-\hat{\mathbf{p}})\exp\left\{i\frac{\hat{\mathscr{H}}}{\hbar}t\right\}\varphi_i(\mathbf{r})\,d\mathbf{r}. \tag{7}$$

Here $\varphi_{i,f}(\mathbf{r})$ are the coordinate functions of the antiparticles and particles, respectively. The factors $\exp\{i[\hat{\mathscr{H}}t/\hbar]\}$ transform the Schrödinger operators between them into Heisenberg-representation operators that depend explicitly on the time. The differential probability summed over the positron states is of the form

$$dW=\sum_i|\mathscr{H}_{i,f}|^2\rho. \tag{8}$$

Since the integration with respect to time has not yet been carried out in (7), so that the energy conservation has not been separated and the summation over the positron wave functions $\varphi_i(\mathbf{r})$ is not restricted by any conditions, the summation can be carried out with the aid of the equation

$$\sum_i\varphi_i{}^*(\mathbf{r})\varphi_i(\mathbf{r}')=\delta(\mathbf{r}-\mathbf{r}'),$$

which asserts the completeness of the system of functions $\varphi_i(\mathbf{r})$. As a result we obtain

$$dW=\frac{\alpha}{(2\pi)^2\omega}$$
$$\times\left\langle\varphi_f(\mathbf{r})\left|\iint dt_1dt_2e^{-i\omega(t_2-t_1)}R(t_2)e^{i\varkappa\mathbf{r}(t_2)}\exp\left\{2i\frac{\hat{\mathscr{H}}}{\hbar}(t_2-t_1)\right\}\right.\right.$$
$$\left.\times e^{-i\varkappa\mathbf{r}(t_1)}R^+(t_1)|\varphi_f(\mathbf{r})\right\rangle\rho, \tag{9}$$

where the time-dependent operator is

$$R(t)=U_s{}^+(\mathbf{p}(t))\mathbf{l}\alpha U_s(-\mathbf{p}'(t)),\quad \mathbf{p}'(t)=\hbar\varkappa-\mathbf{p}(t),$$

and ρ is the statistical weight of the electronic states. The calculation of (9) reduces to finding a combination of noncommuting operators:

$$L(t)=\exp[-i\omega(t_2-t_1)]e^{i\varkappa\mathbf{r}(t_2)}\exp\left\{2i\frac{\hat{\mathscr{H}}}{\hbar}(t_2-t_1)\right\}e^{-i\varkappa\mathbf{r}(t_1)}. \tag{10}$$

Recognizing that the longitudinal motion is free, commutation of this combination with the aid of the "untangling" operations yields

$$L(t)=\exp\{i[\varkappa_xx(t_2)-\varkappa_xx(t_1)$$
$$-\omega(t_2-t_1)(1-\beta_\parallel\cos\theta)+\omega\sin^2\theta(t_2-t_1)]\}. \tag{11}$$

This makes the quantities in the integrand of (9) commuting (at the required accuracy). Consequently the integration of the matrix element over the electronic functions $\varphi_f(\mathbf{r})$ reduces to replacing the operators with the classical values (functions of the time) of the corresponding quantities.

We now sum the quantity $R(t_2)R^+(t_1)$ over the polarizations of the fermion and boson fields:

$$\sum[U_s{}^+(\hat{\mathbf{p}}(t_2))(\mathbf{l}\alpha)U_{\bar{s}}(-\hat{\mathbf{p}}'(t_2))]$$
$$\times[U_{\bar{s}}{}^+(-\hat{\mathbf{p}}'(t_1))(\mathbf{l}\alpha)U_s(\mathbf{p}(t_1))]\approx\sin^2\theta, \tag{12}$$

where φ_s are two-component spinors that describe the spin properties of the particles (account is taken here of the fact that the longitudinal motion of the particles and antiparticles is free and that the interaction of the spin with the crystal field is weak), and θ is the angle between $\varkappa$ and $\mathbf{p}_-$ (of the electron). Under the conditions of the problem considered, a contribution to the e^-e^+ pair production is made by the entire trajectory of the components of the produced pair, so that the process is characterized by probability not per unit time, but by the probability "for all time." We ultimately obtain the equation

$$dW=\frac{\alpha}{(2\pi)^2\omega}\sin^2\theta|M|^2d^3p_-, \tag{13}$$

where

$$M=\int_{-\infty}^{\infty}dt\exp\{i[\omega t(1-\beta_\parallel\cos\theta)-t\omega\sin^2\theta-\varkappa x(t)\sin\theta]\}. \tag{14}$$

Expanding in (14) $x(t) = v_\perp t + \dot{v}_\perp t^2/2 + \ldots$ and retaining the first two terms, we obtain after integrating (13):

$$W = \frac{2\alpha}{3}\left(\frac{m_0c^2}{\hbar\omega}\right)\frac{E_0}{E}; \qquad (15)$$

where $E = U_0/e^-d$ is the crystal-field intensity, U_0 is the depth of the potential well, d is its width and is of the order of the screening radius, and e^- is the electron charge. The photoproduction onset is determined by the equation

$$\left(\frac{m_0c^2}{\hbar\omega}\right)\frac{E_0}{E} = 1. \qquad (16)$$

The condition (16) means that for the onset of the considered photoproduction process it is necessary that the field of intensity $\gamma E \equiv E(\hbar\omega/m_0c^2)$ perform over the Compton length work equal to m_0c^2.

3. ELECTROPRODUCTION OF e^-e^+ PAIRS

In the second stage we consider, by the Weizsäcker-Williams method, the electroproduction proper of e^-e^+ pairs by an ultrarelativistic electron moving along a crystallographic plane (axis). The electromagnetic field of an ultrarelativist charged particle is close to the field of the light wave. The action of a fast electron is equivalent to the action of the spectrum of real photons. Therefore the electroproduction of e^-e^+ pairs by a fast electron in a crystal field can be regarded as production of e^-e^+ pairs under the action of the photons of this spectrum. The distribution of the photons that represent the field of the relativistic particle is given by the expression[6]

$$h(\omega)\,d\omega = \frac{2\alpha}{\pi}\ln\left[\frac{\gamma m_0c^2}{\hbar\omega}\right]\frac{d\omega}{\omega}. \qquad (17)$$

Consequently, the electroproduction probability is

$$P = \frac{4}{3}\frac{\alpha^2}{\pi}\frac{E_0}{\gamma E}\left\{1 - \frac{\gamma E}{E_0}\left[\ln\left(\frac{E_0}{\gamma E}\right) + 1\right]\right\}. \qquad (18)$$

Electroproduction of e^-e^+ pairs likewise has a threshold. The threshold condition has the same meaning and form as in the case of photoproduction:

$$E_0/\gamma E = 1.$$

Other conditions being equal, the threshold energy of a photon moving in the axial-channeling regime is lower than in the case of planar channeling. In addition, the threshold energy is lower the higher the atomic number of the material. In this sense, the most convenient is the regime of axial channeling in tungsten or gold single crystals. The threshold energy in these single crystal is, according to (20), $\varepsilon_{\min} \approx 10$ GeV. Near the threshold the photoproduction probability is of the order of the fine-structure constant $\alpha \approx 1/137$, and the electroproduction probability is $\sim\alpha^2$, which makes this effect observable.

We note that although the distance between the electron and the positron is of the order of 1 Å, and the relative velocity of the components of the produced pair is $v_\perp/c < \alpha$, their mutual Coulomb interaction can be neglected compared with the interaction of the particles with the crystal field. On the other hand, after the emission of the particle from the crystal this Coulomb interaction of e^- and e^+ becomes substantial ($v_\perp/c < \alpha$, $d \approx 1$ Å), so that the pairs are in the upshot relativistic positrons.

4. INTERPRETATION OF RESULT

1) It is clear that the described pair-electroproduction process can be used as a monochromatic source of relativistic pairs of positronium type, besides its use for the investigation of nonlinear quantum effects on the basis of this process.

2) In our case the e^-e^+ pairs are produced by equivalent photons of the primary electrons. Nonetheless, it appears that the considered pair-production process can serve as a model of a supercritical atom at rest,[7] whose nuclear field is capable of extracting electrons from a Dirac background (tunneling of an electron with negative energy though a barrier). To verify this it suffices to transform to a reference frame in which the longitudinal velocity of the primary particle is equal to zero. In such a reference frame the potential of the crystallographic plane (axis) is multiplied, as a result of the Lorentz transformation, by a γ factor that plays the role of a continuously varying coupling constant. For an arbitrary potential well of finite depth, with increasing well depth (due to the growth of the γ factor) the lowest state of the electron drops lower and at a certain critical depth well reaches a value $-m_0c^2$. Pair production from vacuum then becomes possible. Obviously, the condition of spontaneous pair production from vacuum coincides with the threshold condition of the pair electroproduction process considered above. In addition, the pair components execute fictitious motion in a transverse direction, imitating thereby the long-lived "quasistationary" states that appear in the case of a supercritical atom at rest. This permits this problem to be studied with modern accelerators.

3) It is known that in an intense external field the vacuum can become polarized and its refractive index becomes different from unity, so that a "vacuum" Čerenkov effect can be observed.[8] Replacing the magnetic field intensity by that of the electric field acting on a particle moving along a crystallographic plane (axis), we can analyze qualitatively this process in analogy with the investigation of the "vacuum" Čerenkov effect in a magnetic field. The condition for the onset of such radiation then takes the form

$$(\varepsilon_{min}/m_0c^2)(E/E_0) \approx [2\pi/\alpha J(\chi)]^{1/2} \approx 59,$$

where $\varepsilon_{\min}$ is the total threshold energy of the particle. In a field $E \approx 10^{12}$ V/cm we have $\varepsilon_{\min}/m_0c^2 \approx 10^5$; for electrons this corresponds to an energy $\varepsilon_{\min} \approx 10^2$ GeV. This can be used to produce a "vacuum" Čerenkov counter based on a single crystals (for cosmic particles whose energy is sufficient for the onset of the effect).

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