

Translational disequilibrium induced by resonance radiation pressure in a mixture of gases

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The effect of resonance radiation pressure (RRP) in the field of a traveling electromagnetic wave on a mixture of resonance and buffer gases is examined. It is shown that RRP can produce a strong perturbation of the velocity distribution of impurity particles in a narrow region near the resonance velocity. The shape of the nonequilibrium structures on the distribution function is found, and it is shown that an external nonselective force will enhance the stability of the process responsible for the evolution of the local (in velocity space) translational disequilibrium relative to the effect due to collisions with the buffer-gas particles.

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INTRODUCTION

There are at present two known selective mechanisms whereby resonance radiation affects translational degrees of freedom in a tenuous atomic gas. One of them relies on resonance radiation pressure (RRP)¹ and the other on the difference between the elastic scattering cross sections of excited and unexcited atoms that are responsible for light-induced diffusion (LID) of gases.² Such processes are interesting, above all, because they can be used to influence selectively and in a desired manner the translational state of microparticles, which is valuable in scientific and practical applications.¹⁻³

Monochromatization of atomic velocities in a collisionless gas by RRP in the field of a traveling magnetic wave, which relies on the highly nonlinear dependence of the RRP force on the velocity of the atoms, was predicted in Refs. 4–6. (The designation “monochromatization” seems to be more apt than the original “phasing” used in Ref. 5.) The theory has been extended to the case of atomic beams,⁷ and the first direct experimental studies of velocity monochromatization induced by a traveling-wave field in a sodium atomic beam have been reported.⁸ We also note that the author of Ref. 1 drew attention to the possibility of velocity monochromatization by the RRP force of the so-called mixed type.

In a sufficiently dense gas, collisions will scatter active particles out of the region of resonance with the electromagnetic field and will therefore, clearly, effectively prevent RRP-induced velocity monochromatization. The difference between the elastic-collision frequencies for excited Γ_{2s} and unexcited Γ_{1s} atoms will then also produce a translational disequilibrium, which will be reflected in the appearance of sharp structures on the velocity distribution of the atoms⁹ that will be entirely due to collisions. RRP in a traveling wave will continue to play a dominant role in stimulating the translational disequilibrium if the characteristic time¹ for velocity monochromatization by RRP, $\tau_R = (\hbar k^2/2m)^{-1}$ is comparable with or less than the resonance-particle mean free time:

$$\hbar k^2/2m = \omega_R \geq \Gamma_s. \quad (1)$$

This condition is readily satisfied for buffer-gas pressures $p \sim 1$ Torr and sufficiently short-wavelength transitions. For

example, in magnesium vapor, if we consider the $3^1S_0 - 3^1P_1$ transition with $\lambda = 2852 \text{ \AA}$ for a mean free path $l = 0.2 \text{ cm}$ at a vapor temperature $T = 649 \text{ K}$, we have $\omega_R = 1/\tau_R \sim 6 \cdot 10^5 \text{ s}^{-1}$, $\Gamma_s = 3 \cdot 10^5 \text{ s}^{-1}$, whereas, for the B-transition with $\lambda = 2348 \text{ \AA}$, we have $\omega_R \sim 2.5 \cdot 10^6 \text{ s}^{-1}$ and $\Gamma_s \approx 9.5 \cdot 10^5 \text{ s}^{-1}$.

When (1) is satisfied, RRP will induce a strong translational disequilibrium in a resonance gas, despite the fact that the frictional force $mu_0 \Gamma_s \sim F_{fr}$ may be much greater than the RRP force, i.e., $\hbar k \gamma / 2 = F_r$, $F_r / F_{fr} = \xi \ll 1$ (in LID theory,² RRP is neglected because ξ is small). This is connected with the local nature of the translational disequilibrium produced by RRP, which is reflected in the appreciable distortion of the velocity distribution function in a velocity interval of width γ/k that is much smaller than the width u_0 of the equilibrium distribution.

In this paper, we derive and analyze the shape of the nonequilibrium structures produced in the field of a traveling electromagnetic wave on the distribution function of a resonance gas when condition (1) is satisfied and spectral-line broadening is of the Doppler type. It is shown that a nonselective thermal force accelerating the particles (for example, ions) can enhance the stability of the process of velocity monochromatization as compared with the broadening effect of collisions with the buffer-gas particles.

1. BASIC MODEL. ASYMPTOTIC SHAPE OF THE NONEQUILIBRIUM COMPONENT OF THE DISTRIBUTION FUNCTION

Consider a mixture of a two-level resonance gas and a buffer gas in the field of a traveling electromagnetic wave of amplitude E_0 . Suppose that the gas pressure is low enough so that the elastic collision frequencies Γ_{si} between the ground-state and excited resonance particles, on the one hand, and the buffer-gas particles, on the other, are much smaller than the rates of longitudinal and transverse relaxation γ , γ_{\perp} ; the concentration n of the resonance impurity is much smaller than the concentration N of the buffer gas; and the absorption line is inhomogeneously broadened:

$$\Gamma_{si} \ll \gamma_{\perp}, \quad \gamma, \quad (2a)$$

$$n \ll N, \quad (2b)$$

$$\gamma_{\perp} \ll k u_0, \quad (2c)$$

where u_0 is the most probable velocity of random motion of the resonance particles in the absence of external field.

Let us use the quasiclassical approach to examine the equations for the density matrix $\hat{\rho}$ with allowance for collisions,¹⁰⁻¹² the recoil effect,^{1,10} and the constant external force $\mathbf{F} = F_0 \mathbf{e}_x$. We assume that the conditions given by (2) and $\omega_R \ll \gamma$, γ_{\perp} are valid, and eliminate the off-diagonal elements. In addition, we neglect quantum fluctuations in the RRP force. We then obtain, as in Ref. 9, the kinetic equation for the resonance-particle distribution function $f = \text{Sp} \hat{\rho}$:

$$\begin{aligned} & \theta^{-1} \nabla \partial f / \partial \mathbf{R} + \omega_R \mu \partial \tilde{\beta} L((v_x - \Delta') / \mu) f / \partial v_x \\ & + \mu \Omega_0 \partial f / \partial v_x = \hat{\text{St}}\{f\} = \hat{\text{St}}_1(\rho_{11}) + \hat{\text{St}}_2(\rho_{22}), \end{aligned} \quad (3)$$

where \mathbf{v} is the dimensionless velocity expressed in units of u_0 , v_x is the component of the velocity in the direction \mathbf{e}_x of propagation of the radiation, $\Delta' = \Delta / k u_0$ is the dimensionless detuning from resonance, \mathbf{k} is the wave vector,

$$\begin{aligned} \mu &= \frac{G}{k u_0}, \quad G = \left[\gamma_{\perp}^2 + \frac{4\eta d^2 E_0^2}{\hbar^2} \right]^{1/2} \\ \eta &= \frac{\gamma_{\perp}}{\gamma}, \quad \beta = \frac{4\eta d^2 |E_0|^2}{\hbar^2 G^2}, \quad \tilde{\beta} = \frac{\gamma \beta}{G}, \end{aligned}$$

d is the dipole moment matrix element, $\Omega_0 = F_0 |k| / m G$, m is the mass of the resonance particles, R is the dimensionless space variable, Θ is the characteristic macroscopic time ($\Theta = r_0 / u_0$, and r_0 is the characteristic spatial scale),

$$L(v_x - \Delta' / \mu) = \{ [(v_x - \Delta' / \mu)^2 / \mu^2] + 1 \}^{-1};$$

$\hat{\text{St}}\{f\}$ is the linear collision integral which, when the difference between the elastic scattering cross sections in the upper and lower levels is taken into account, has the form

$$\hat{\text{St}}\{f\} = -[\Gamma_{s1} + \Gamma_{s0} \beta L(v_x - \Delta' / \mu)] f(\mathbf{v})$$

$$+ \int \{ K_i(\mathbf{v}, \mathbf{v}') + K_0(\mathbf{v}, \mathbf{v}') \beta L(v_x' - \Delta' / \mu) \} f(\mathbf{v}') d^3 v', \quad (4)$$

and K_i are the collision kernels in states i ($i = 1, 2$), given by

$$K_0 = [K_2(\mathbf{v}, \mathbf{v}') - K_1(\mathbf{v}, \mathbf{v}')]/2,$$

$$\Gamma_{si} = \int K_i(\mathbf{v}', \mathbf{v}) d^3 v', \quad \Gamma_{s0} = (\Gamma_{2s} - \Gamma_{1s})/2.$$

If we use the order-of-magnitude formula $D \sim (\hbar k / m)^2 \gamma$ for the diffusion coefficient D in velocity space, and if we assume that the characteristic velocity scale δv of the region in which the departure from equilibrium takes place in saturating fields ($4\eta d^2 |E_0|^2 / \hbar^2 \sim \gamma_{\perp}^2$) $\delta v \sim \gamma_{\perp} / k$, we obtain the following condition for fluctuations in the RRP force to be negligible in comparison with collisions:

$$(\omega_R^2 \gamma / \gamma_{\perp}^2) \ll \Gamma_{s1}, \quad (5)$$

which, for $\omega_R \sim \Gamma_{s1}$, is essentially equivalent to the inequality $\omega_R \ll \gamma$, which is practically always satisfied for optical transitions. We note that the effect of acceleration by the constant external force F_0 on the spectral-line profile was not taken into account¹⁰ in the derivation of (3), so that we shall restrict it by the inequality

$$\Omega_0 \ll \gamma, \quad \gamma_{\perp}. \quad (6)$$

We also draw attention to the fact that, when (2a) is satisfied,

negative light pressure¹³ is also negligible, and was not taken into account in the kinetic equation given by (3).

Thus, under the above conditions, the original mathematical model represented by the kinetic equation given by (3) describes the following three basic processes: spontaneous RRP in the field of the traveling electromagnetic wave, particle (ion) acceleration by the constant external force, and collisions of resonance particles with buffer-gas particles in each of the possible quantum states.

Collisional processes will be simulated by specifying the collision kernels in the phenomenological form of Keilson and Storer:^{14,10,12}

$$\begin{aligned} K_i(\mathbf{v}, \mathbf{v}') &= \Gamma_{si} (\pi^{1/2} \Delta u)^{-3} \exp[-(v - \alpha v')^2 / \Delta u^2], \\ \Delta u &= (1 - \alpha^2)^{1/2}, \quad \alpha < 1. \end{aligned} \quad (7)$$

Moreover, we shall consider the case of sufficiently strong collisions for which the kernel width (in units of u_0) is

$$\Delta u \sim 1, \quad (8)$$

which is actually reached, for example, for $\alpha < 0.86$.²⁾ It seems quite obvious that such collisions have the greatest disturbing effect on the translational equilibrium induced by RRP.

In the most interesting situation, the absorption line is inhomogeneously broadened for $4\eta d^2 E_0^2 / \hbar^2 \sim \gamma_{\perp}^2$ and the parameter μ , introduced above, is small: $\mu \ll 1$. However, this does not allow us, in the first approximation, to neglect the terms on the left-hand side of (3) and to write down the distribution function in the form of the sum of the Maxwellian distribution and a small addition to it [when (1) is satisfied], since the derivatives with respect to velocity are large ($\sim 1/\mu$) in the region near resonance because the function $L((v_x - \Delta')/\mu)$ has a singularity in the small parameter. Physically, this is connected with the sharp selectivity of the RRP force which acts effectively only on a group of atoms with a particular velocity projection, which are concentrated in a very narrow (as compared with the width of the resonance distribution) region $\sim \mu$ in velocity space. This also leads to the well-defined local character of the induced translational disequilibrium, which must be taken into account in the asymptotic expansion of the solution of the kinetic equation given by (3). In view of the foregoing, we introduce the "fast" (local) variable $s = (v_x - \Delta')/\mu$, and seek the asymptotic solution of (3) in the form of a composite expansion, similarly to what is done in the theory of the boundary layer in problems with singular perturbations:¹⁵

$$f(\mathbf{v}) = j_0(\mathbf{v}) \{ \Pi(s, \mu) Q(\mathbf{v}, \mu) + h(\mathbf{v}, \mu) \}, \quad (9)$$

$$\Pi(s, \mu) = \Pi_0(s) + \mu \Pi_1(s) + \dots, \quad \Pi(\pm\infty) = 0, \quad (10)$$

$$Q(\mathbf{v}, \mu) = Q_0(\mathbf{v}) + \mu Q_1(\mathbf{v}) + \dots, \quad (11)$$

$$h(\mathbf{v}, \mu) = h_0(\mathbf{v}) + \mu h_1(\mathbf{v}) + \dots, \quad (12)$$

where $j_0 = \pi^{-3/2} e^{-v^2}$ is the Maxwellian distribution.

Consider, to begin with, the case where we can neglect the difference between the collision frequencies in the upper and lower states, $\Gamma_{s1} = \Gamma_{s2} = \Gamma_s$, $K_i = K$, and where the medium is spatially homogeneous, so that we can isolate the role of RRP in the evolution of the departure from equilibri-

um in pure form. We now introduce the function

$$\tilde{L}(s) = [\omega_R \beta L(s) + \Omega_0] \quad (13)$$

and substitute (9) in (3), so that, using the principle of detailed balancing $K(\mathbf{v} \mathbf{v}') f_0(\mathbf{v}') = K(\mathbf{v}', \mathbf{v}) f_0(\mathbf{v})$, we have

$$\begin{aligned} & \frac{\partial}{\partial s} (\tilde{L}(s) \Pi(s, \mu) Q(\mathbf{v}, \mu)) + \mu \tilde{L}(s) \Pi(s, \mu) \frac{\partial Q}{\partial v_x} \\ & - 2(\Delta' + \mu s) Q(\mathbf{v}, \mu) \Pi(s, \mu) \tilde{L}(s) \mu - \mu \cdot 2v_x \Omega_0 h(\mathbf{v}, \mu) \\ & + \mu \Omega_0 \frac{\partial h}{\partial v_x} - \mu \cdot 2\omega_R \beta (\Delta' + \mu s) L(s) h \\ & + \mu \cdot 2\omega_R \beta L(s) \frac{\partial h(\mathbf{v}, \mu)}{\partial v_x} + \tilde{L}'(s) h(\mathbf{v}, \mu) \\ & = -\Gamma_s (\Pi(s, \mu) Q(\mathbf{v}, \mu) + h(\mathbf{v}, \mu)) + \bar{K} h + \mu \bar{K}_1 (\Pi Q), \quad (14) \end{aligned}$$

where the integral operators are given by

$$\begin{aligned} \bar{K} h &= \int K(\mathbf{v}', \mathbf{v}) h(\mathbf{v}') d^3 v', \\ \bar{K}_1 (\Pi(s, \mu) Q(\mathbf{v}, \mu)) & \quad (15) \\ &= \int K(\mathbf{v}'(\mathbf{v}_t, \Delta' + \mu s), \mathbf{v}) \Pi(s, \mu) Q(\mathbf{v}'(\mathbf{v}_t, \Delta' + \mu s)) d^2 v_t ds, \\ & \mathbf{v}'(\mathbf{v}_t, \Delta' + \mu s) = (\Delta' + \mu s, v'_y, v'_z), \end{aligned}$$

and $\mathbf{v}_t = (v'_y, v'_z)$ is the transverse velocity component.

Let us substitute (10)–(12) in (14) and arrange the functions $Q_i(\mathbf{v})$ so that we can separate the fast and slow distribution functions in each of the orders under consideration. We then equate terms corresponding to equal powers of μ that depend separately on s and on v . This successively gives us

$$-\Gamma_s h_0 + \bar{K} h_0 = 0, \quad h_0(\mathbf{v}) = \bar{h}_0 = \text{const}; \quad (16)$$

$$h_0(\mathbf{v})/Q_0(\mathbf{v}) = C_1 = \text{const}, \quad Q_0 = \bar{h}_0/C_1 = \text{const}; \quad (17)$$

$$d(\tilde{L}(s) \Pi_0(s))/ds + \Gamma_s \Pi_0(s) = -\tilde{L}'(s) C_1, \quad \Pi_0(\pm\infty) = 0; \quad (18)$$

$$\begin{aligned} Q_1(\mathbf{v}) &= -h_1(\mathbf{v})/C_1, \quad -2\Omega_0 v_x C_1 = -\Gamma_s h_1 + \bar{K} h_1 + \bar{K}_1 \Pi_0(s) Q_0, \\ \bar{K}_1 \Pi_0(s) &= \int K(\mathbf{v}'(\mathbf{v}_t, \Delta'), \mathbf{v}) \Pi_0(s) ds d^2 \mathbf{v}_t, \quad (19) \end{aligned}$$

$$\begin{aligned} d(\tilde{L}(s) \Pi_1(s))/ds + \Gamma_s \Pi_1(s) &= \Delta' (2\omega_R \beta L(s) C_1 + 2\tilde{L}'(s) \Pi_0(s)), \\ \Pi_1(\pm\infty) &= 0. \quad (20) \end{aligned}$$

In deriving these expressions, we have used the following expansion for the integral term, which has a "smoothing" effect on the functions $\Pi_i(s)$ if the condition given by (8) is satisfied:

$$\mu \bar{K}_1 [(Q_0 + \mu Q_1) (\Pi_0 + \mu \Pi_1(s))] = \mu \bar{K}_1 (\Pi_0 Q_0) + O(\mu^2).$$

When (16)–(20) are satisfied, the original equation is satisfied to within $\sim \mu^2$. Moreover, if the distribution function is normalized to unity, we have $\bar{h}_0 = 1$, and the local translational disequilibrium³⁾ (LTD) induced by the RRP is determined by the principal part of the asymptotic expansion

$$f(\mathbf{v}) = f_0(\mathbf{v}) [\bar{\Pi}_0(v_x - \Delta'/\mu) + 1] + \dots, \quad (21)$$

where $\bar{\Pi}_0(s) = \Pi_0 Q_0$ and satisfies the ordinary first-order differential equation

$$\begin{aligned} d\bar{\Pi}_0/ds + (\Gamma_s + \tilde{L}'(s)) \bar{\Pi}_0^{-1}(s) \bar{\Pi}_0(s) &= -\tilde{L}'(s) \bar{h}_0, \\ \bar{\Pi}_0(\pm\infty) &= 0. \quad (22) \end{aligned}$$

We note that the local property of the translational disequilibrium [equivalent to the validity of the asymptotic behavior of the form of (21)] is essentially determined by the decrease in $L(s)$ and $L'(s)$ at infinity. On the other hand, because of the spreading effect of collisions, the influence of LTD on the non-resonance part of the velocity distribution is reflected only in the second order in the small parameter μ , since $\int \bar{\Pi}_0(s) ds = 0$ and the third term on the right-hand side of (19) is strictly equal to zero.

To conclude this section, we shall show how one can take into account the inhomogeneity and the difference between the elastic collision frequencies. In the weakly inhomogeneous case, the asymptotic expansion must also be sought in the form of (9), retaining the derivatives with respect to the spatial variables and assuming that $1/\Theta$ is a small parameter in (3). This produces a modification of only the first-approximation equations, but the zeroth-approximation equations remain unaltered. The quantity \bar{h}_0 is then an unknown function of the spatial variables, which can be found from the macroscopic transport equation under the corresponding boundary conditions:

$$-\frac{1}{3} \langle v^2 \rangle \frac{1}{\Gamma_s (1-\alpha) \theta} \frac{\partial \bar{h}_0}{\partial \mathbf{R}} + \frac{\mu \langle \tilde{L}' \rangle \bar{h}_0 \mathbf{e}_x}{\Gamma_s (1-\alpha)} = \mathbf{J}, \quad \text{div } \mathbf{J} = 0,$$

where we have neglected the higher-order terms and the averages are taken over the Maxwellian function.

The difference between the collision frequencies [when the relative difference $(\Gamma_{2s} - \Gamma_{1s})/\Gamma_{1s} = \nu$ is not very large] is most simply taken into account (although this can also be done directly when the asymptotic expansion is constructed) by reformulating the original problem. This is done by introducing a replacement that is possible for collision kernels of the form of (7) and which, in the absence of the nonselective force, has the form

$$\begin{aligned} \varphi &= (1 + \beta \nu L(s)) f, \quad \mu \rightarrow \mu_1 = \mu (1 + \beta \nu)^{1/2}, \\ \beta &\rightarrow \beta_1 = \beta / (1 + \beta \nu)^{3/2}. \quad (23) \end{aligned}$$

The problem of finding the shape of the nonequilibrium structures is then precisely reduced to that just considered. Henceforth, we shall confine our attention to the situation where the inhomogeneity is small (i.e., the tube of length L is closed):

$$\bar{L} \ll \frac{ku_0}{\gamma} \frac{mu_0^2}{\hbar k \gamma}, \quad \bar{L} \ll \frac{u_0}{\Gamma_{s1}} \frac{ku_0}{\beta \nu \gamma}.$$

2. RRP-INDUCED LOCAL TRANSLATIONAL DISEQUILIBRIUM

The RRP-induced local departure from translational equilibrium is described by the function $\bar{\Pi}_0((v_x - \Delta')/\mu)$, which differs appreciably from zero only in a narrow interval of width μ near resonance ($v_x = \Delta'$). It is clear from (22) that the source of the LTD is the dependence of the RRP force on velocity ($L'(s) \neq 0$) and that collisions appear as a factor that destroys the disequilibrium. In the absence of the nonselective external force, Eq. (22) yields

$$\bar{\Pi}_0(s) = 2(s^2 + 1) \exp(-p\Psi(s)) \cdot \int_{-\infty}^s \frac{t}{(t^2 + 1)^2} e^{p\Psi(t)} dt, \quad (24)$$

where $p = \Gamma_s / \omega_R \bar{\beta}$, $\Psi(s) = s^2/3 + s$.

The behavior of the function $\bar{\Pi}_0(s)$ at infinity is determined by the asymptotic behavior of the improper integral in (24):

$$\bar{\Pi}_0(s) \sim 2/s^3 p, \quad |s| \rightarrow \infty.$$

Figure 1 shows the shape of the nonequilibrium structure on the distribution function for three cases corresponding to different ratios of the mean free time to the monochromatization time ($p = \tau_R / \tau = \Gamma_s \tau_R$). On the right of resonance ($s > 0$) there is a peak, whereas, on the left of resonance, we have a valley of characteristic width $\delta s \sim 1$ (in dimensional units, $\sim G/k$). The spreading of the nonequilibrium structure with increasing collision frequency is quite clear and is due to the rapid scattering of particles that resonate with the electromagnetic field. The fact that there is no appreciable narrowing of the peak with increasing ratio $\omega_R \bar{\beta} / \Gamma_s = p^{-1}$ is not trivial. Thus, when the ratio of the mean free time to the monochromatization time is of the order of five, the width of the peak at the 0.97 level is $\delta s \sim 1$ and its center is shifted into the region of positive velocities by the amount $s_0 \sim 3$. Moreover, when $p \ll 1$, the following expression that follows from (24) is valid near the maximum of $\bar{\Pi}_0(s)$:

$$\bar{\Pi}_0(s) \approx 2.1 p^{1/3} \exp\{-3.8 p^{2/3} (s - s_0)^2\}, \quad s_0 = (2/p)^{1/3}, \quad (25)$$

and, consequently, the peak width increases with increasing parameter $1/p = \omega_R \bar{\beta} / \Gamma$ ($\delta s \sim 0.51/p^{1/3}$), whereas the height decreases. The spreading effect of collisions then suppresses monochromatization because the particles are pushed into the velocity interval near⁵ $(G/k) (\omega_R / \Gamma_s)^{1/3}$, in which the velocity gradient $F'(v_x)$ of the RRP force, which is responsible for the narrowing, is very small and the rate of

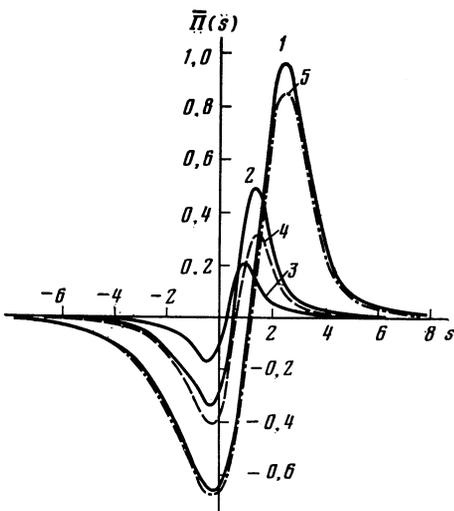


FIG. 1. Shape of the nonequilibrium structure on the distribution function of resonance particles in the field of a traveling electromagnetic wave for $p = 0.2$, $(\Gamma_{2s} - \Gamma_{1s})\bar{\beta} / \Gamma_{1s} = d = 0$ (1); $p = 1$, $d = 0$ (2); $p = 3$, $d = 0$ (3); $p = 1$, $d = 0.2$ (4); $p = 0.2$, $d = 0.2$ (5)

monochromatization is comparable with the rate of scattering.

The difference between the collision frequencies for excited and unexcited atoms, on the one hand, and the buffer gas, on the other (which can be taken into account in accordance with the previous section), is found to have a small effect on the shape of the RRP-induced nonequilibrium structure if the relative frequency difference is small: $\beta(\Gamma_{2s} - \Gamma_{1s}) / \Gamma_{1s} \ll 1$. Numerical calculations show that, as the relative frequency difference increases, the nonequilibrium structure is deformed to a greater extent for larger values of the parameter p . At the same time, the depth of the valley increases and the height of the peak decreases, since the resonance region is depleted of particles as a result of the rapid scattering of excited atoms.⁹ When $p \ll 1$, the difference between the collision frequencies has little effect on the RRP-induced nonequilibrium structure because the particles are expelled by the RRP force from the region of strict resonance in the time interval corresponding to the mean path, but the direction of qualitative changes remains the same as for $p > 1$.

The dashed curves in Fig. 1 show the shape of the nonequilibrium structures when both destabilizing factors are operating, namely, RRP and the difference between the elastic scattering of resonance particles in the upper and lower states. We note that the area under the structure is then always $\int \bar{\Pi}_0(s) ds \neq 0$, so that the macroscopic particle fluxes are also functions of the detuning.

3. LOCAL TRANSLATIONAL DISEQUILIBRIUM UNDER THE COMBINED ACTION OF RRP AND NONSELECTIVE EXTERNAL FORCE VELOCITY MONOCHROMATIZATION

Suppose the nonselective external force acts against the RRP force and completely cancels it for resonance velocities (see Fig. 3 below), i.e., $\Omega_0 = -\omega_R \bar{\beta}$. The corresponding experimental situation can be established, for example, for ions in the weakly ionized plasma of a gas discharge.

The function $\bar{\Pi}_0(s)$ can be expressed in terms of improper integrals, as follows:

$$\bar{\Pi}_0(s) = -(1 + 1/s^2) \exp\{-p\Psi_1(s)\} \int_{-\infty}^s \frac{2t}{(t^2 + 1)^2} \times \exp\{p\Psi_1(t)\} dt, \quad s > 0; \quad (26)$$

$$\bar{\Pi}_0(s) = -(1 + 1/s^2) \exp\{-p\Psi_1(s)\} \int_0^s \frac{2t}{(t^2 + 1)^2} \times \exp\{p\Psi_1(t)\} dt, \quad s \leq 0,$$

where $\Psi_1(s) = (1/s - s)$.

The following asymptotic representations of $\bar{\Pi}_0(s)$ ensue from (26):

$$\lim_{s \rightarrow +0} \bar{\Pi}_0(s) = \lim_{s \rightarrow -0} \bar{\Pi}_0(s) = 0, \quad \bar{\Pi}_0(s) \sim 2s/p, \quad s \rightarrow 0,$$

$$\bar{\Pi}_0(s) \sim 2/s^3 p, \quad |s| \rightarrow \infty. \quad (27)$$

Figure 2 shows the shape of the nonequilibrium structure on the distribution function for resonance ions in the

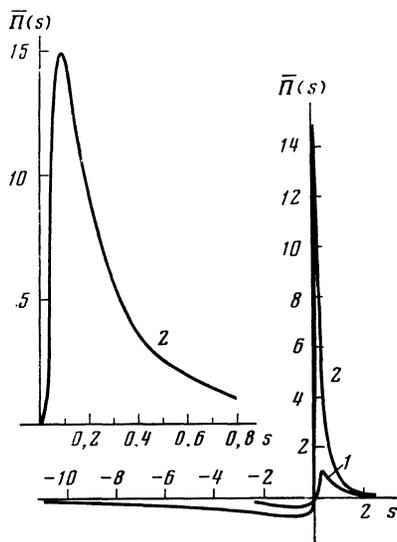


FIG. 2. Shape of the nonequilibrium structure on the distribution function of resonance particles subjected to the simultaneous application of the RRP and nonselective external force: $p = 1$ (1), $p = 0.2$ (1).

case of two different values of the parameter $p \sim \Gamma_s \tau_R$.

The most remarkable effect as compared with the case examined in the last section is the considerable narrowing of the peak as the parameter p decreases. In dimensional units, the width of the peak for $G \gtrsim \gamma_1$ is much smaller than the ratio γ_1/k , and its height for the situation shown in Fig. 2 ($p = 0.2$) is 14.9. To the left of the resonance, ($s < 0$), we have, as before, a valley although, in this case, it is less pronounced. On the right of Fig. 2, we show separately the nonequilibrium peak on a larger scale for $p = 0.2$. It is quite clear that the peak is asymmetric: its left-hand wing falls off more rapidly than the right-hand wing. In the immediate neighborhood of the maximum of the function $\bar{\Pi}_0(s)$, we have the following expression, which ensues from (26) and is valid⁴⁾ for $p \ll 1$:

$$\bar{\Pi}_0(s) \approx \frac{0.51}{p^2} \exp \left\{ -\frac{4}{p^2} \left(s - \frac{p}{2} \right)^2 \right\}. \quad (28)$$

From this, we have the following simple estimate for the width δs and height d_0 of the peak when $p \ll 1$:

$$\delta s \sim p/2, \quad d_0 \approx 0.51/p^2,$$

from which it is clear that, as p decreases, the number of

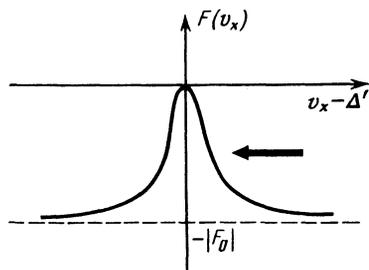


FIG. 3. Resultant force as a function of velocity. Arrow shows the direction of the nonselective part of the resultant force.

particles localized within the peak tends to increase. Thus, in this case, we are entitled to speak of velocity monochromatization of resonance ions. The physical interpretation of the increased stability of the monochromatization process under the action of the nonselective force is as follows.

In the absence of collisions, the resultant force (its direction is shown by the thick arrow in Fig. 3) impels the particle into the resonance region. Moreover, since $F(\Delta') = 0$, $F_{v_x} < 0$, for $v_x > \Delta'$, they accumulate in the region immediately adjacent to the point ($v_{x0} = \Delta'$) on the right, and this leads to effective ion-velocity monochromatization. We note that practically all ions with initial velocity components $t_M \sim mu_0/\hbar k \gamma$ ($t_M \sim 10^{-4}$ s for $\hbar k/mu_0 = 10^{-4}$, $\gamma = 10^8$ s) are localized within the region of the peak after a time $v_x > \Delta'$. Collisions should result in the broadening of the peak. However, in contrast to the case considered in the last section, the resultant force for $p \ll 1$ effectively assures that the collisions are compensated by the flux of scattered particles that enters the resonance region in which the velocity gradient of the force is high and velocity monochromatization takes place. In other words, the point $v_x \sim \Delta'$ is stable with respect to collisions that increase the x component of velocity (Fig. 3). On the other hand, in the absence of the nonselective force component, the scattered particles practically fall out of the region in which the RRP force that is selective in velocity operates and, moreover, the RRP force itself ejects particles from the resonance region in which the rate of monochromatization is a maximum.

The width of the nonequilibrium peak for $p \ll 1$ is sensitive to the difference between the elastic collision frequencies for excited and unexcited ions (which can be taken into account by renormalizing the parameters so that $\tilde{\beta} \rightarrow \tilde{\beta}_1 = \tilde{\beta} / (1 + \beta v)^{1/2}$, $f \rightarrow \varphi$, $\mu \rightarrow \mu_1$), and increases with increasing relative difference between the frequencies in proportion to

$$p_1 = (1 + \beta v).$$

We also note that fluctuations in the RRP force have a greater influence when the width of the peak is substantially reduced. Estimates of the role of diffusion with allowance for the narrowing of the peak lead to the following condition for an appreciable effect of fluctuations on the monochromatization process in the presence of collisions: $\omega_R^2 p_1^2 > \Gamma_{1s}$.

CONCLUSIONS

Thus, when (1) is satisfied, RRP may give rise to a considerable change in the distribution function of the resonance particles (present in the buffer gas) in a narrow interval near the resonance velocity $v_x = \Delta'$ (LTD). When the size of this interval is much smaller than the width of the undisturbed velocity distribution (~ 1), it is possible to establish the asymptotic (in the parameter $\mu \sim \gamma_1/ku_0 \ll 1$) separation of the nonequilibrium component of the distribution function, which is described by a function of the form $\bar{\Pi}((v_x - \Delta')/\mu)$. It is readily seen that this asymptotic procedure is possible for any collision kernel of width $\Delta u \gg \mu$, i.e., for any kernel describing large-angle scattering. Generally speaking, collisions broaden the nonequilibrium peak on the distribution

function even when the mean free time is very much greater than the velocity monochromatization time ($\tau > \tau_R$), and prevents effective monochromatization in velocity space. An external nonselective force of particular magnitude and direction has a stabilizing effect on velocity monochromatization, and can be used to obtain a tall narrow peak of width $\ll \gamma_1/k$ on the distribution function.

We note that the establishment of the LDE in a mixture of gases may be of considerable importance in those cases where a stationary departure from translational equilibrium must be set up.

¹⁾This is the time necessary for resonance particles in a saturating field to leave the region of exact resonance $\tau_R \sim (\gamma/k)(\hbar k \gamma / 2m)^{-1} = (\hbar k^2 / 2m)^{-1}$.

²⁾The case of weak collisions was partially analyzed in Ref. 9.

³⁾Local in the sense that the distribution function is appreciably perturbed in a narrow interval of width $\sim \mu$ near resonance.

⁴⁾The force given by (28) does not describe the wings of the peak.

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