Photodetachment of an electron bound to a small-radius center in a magnetic field

Yu. A. Gurvich and A. S. Zil'bermints

Moscow State Pedagogical Institute

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We obtain the system of wave functions of the continuous spectrum of an electron in the field of a small-radius potential (SRP) in a magnetic field **H**. The interaction with the center is taken rigorously into account in the SRP approximation. We show that bound and quasibound states with angular momentum projection m appear at an attraction center in the case of positive amplitude $f_{|m|}$ of scattering of an electron with angular momentum $l = |m|$ and with low energy. Expressions are obtained for the energies of these states. The cross section $\sigma^H(\omega)$ for photodetachment in a magnetic field is obtained (ω is the frequency of the external radiation). The cross section executes oscillations due to successive participation of new Landau bands in the absorption as ω is increased. The structure of these oscillations depends on the sign of $f_{|m|}$. Some opinions are expressed concerning the possibility of analyzing these oscillations in experiment. It is shown that the character of the oscillations must be taken into account when determining in experiment the binding energy of the electron in **H**. An attempt is made at a qualitative analysis of the available experimental data in light of the results.

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1. The properties of D^- and A^+ centers in semiconductors have lately been the object of intensive study.[1] One of the main sources of information on these centers is measurement of the photodecay cross section $\sigma(\omega)$ as a function of the frequency ω of the external radiation. Great interest attaches to measurement of the cross section $\sigma^H(\omega)$ in a magnetic field **H**. Such measurements were made on D^- centers in CdS, Ge, and Si (Refs. 2, 3, and 4, respectively). A number of peaks were observed on the $\sigma^H(\omega)$ plots located at a distance close to $\omega_c = |e|H/m_e c$ (e and m_e are the charge and effective mass of the electrons) and due to successive inclusion in the absorption of new Landau bands as ω increases. (In Refs. 3 and 4 was measured the photoconductivity, whose frequency dependence coincides with the $\sigma(\omega)$ dependence.[5]) The experiments raise the question of calculating $\sigma^H(\omega)$.

The D^- (A^+) centers are customarily regarded as analogs of the H^- ion. The simplest model of the H^- ion is based on the small-radius-potential (SRP) approximation. The cross section $\sigma(\omega)$ calculated in the SRP approximation agrees very well with experiment.[6] The same calculation, when performed for semiconductors, yields perfectly satisfactory results for D^- centers in silicon.[5] It is natural to attempt to calculate $\sigma^H(\omega)$ in the SRP approximation.

We note that this problem is in fact outside the scope of the theory of D^- (A^+) centers. It is of interest for all cases dealing with photodetachment of a charge bound to a small-radius center in the presence of **H** such as for deep impurities in superconductors, whose optical properties can frequently be described by the SRP model,[7] for H^- ions, and for a number of other objects. (Oscillations of $\sigma^H(\omega)$ were observed in Ref. 8 for negative sulfur ions S^-.) Semiconductors, on the other hand, are of particular interest for similar problems, for it is relatively easy to realize in them the strong-field conditions $\hbar\omega_c \gg kT$ and $a_H \lesssim a$, where $a_H = (\hbar c/|e|H)^{1/2}$ and a is the decay length of the wave function of the bound state. One can therefore hope to study the effect of **H** on the "intrinsic" (existing at $\mathbf{H} = 0$) bound state, and to observe "magnetic" (which appear at $H \neq 0$) bound and quasibound states.

To calculate $\sigma^H(\omega)$ it is necessary to know the wave functions of the final states—the states Ψ_ν of the continuous spectrum. The simplest approach is to take the "free" functions ψ_ν, i.e., to disregard the influence of the SRP on the final states, as in Refs. 8 and 9. This approach is perfectly satisfactory at $H = 0$ or at $H \neq 0$ but with $\mathbf{E} \parallel \mathbf{H}$ (**E** is the electric field of the external radiation). At $\mathbf{E} \perp \mathbf{H}$, however (this is implied hereafter) this approach leads to a cross section that deviates when the final-state energy coincides with the bottom of some Landau band, owing to the state density becoming infinite at the bottom of the band. Thus, the most interesting part of the spectrum, the oscillation peaks, cannot be described in this approach. To be able to do so we must take into account the influence of various perturbations on the final states. It is natural to start with allowance for the effect of the SRP itself, which is in fact the purpose of the present article.

Optical transitions in an SRP field in $H \neq 0$ were investigated also in Ref. 10. The interaction of the electron with the SRP was taken into account in the Born approximation. The initial states for the transitions from bound states to the continuum were taken to be only magnetic. The transitions considered were: 1) only for the Landau band that is the highest possible for the given frequency, into states located in the immediate vicinity of the bottom of this band, and 2) into quasibound states. The equations of Ref. 10 for the absorption coefficient are valid in narrow frequency regions corresponding to the indicated final states. It is impossible to track with these equations the dependence of the absorption on ω in an interval comparable with the period ω_c of the oscillations, nor can the bottom frequency of any Landau band be approached from below.

We obtain below, in a representation with quantum

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numbers $\nu = \{k, n_\rho, m\}$ (see Ref. 11. p. 525 of Russian original), the system of the functions Ψ_ν of an electron in a field **H** in the form of an incident + diverging wave in the presence of an SRP $V(r)$ of arbitrary form. To describe Ψ_ν it suffices to know the coefficient $c_{|m|}$ in the relation

$$f_{|m|}(k_0) = c_{|m|} k_0^{2|m|},$$

where $f_{|m|}(k_0)$ is the scattering amplitude of an electron with angular momentum $l = |m|$ and wave number $k_0 \to 0$ at $H = 0$. It is assumed that

$$r_0 \ll \lambda \equiv \hbar(2m_e E_\nu)^{-1/2}, \quad \beta_{|m|} = 2^{-1/2}(2|m|+1)!!\, c_{|m|} a_H^{-2|m|-1} \ll 1; \tag{1}$$

here r_0 is the radius of the potential and E_ν is the electron energy. In the approximation (1), the influence of the SRP on Ψ_ν is taken accurately into account. It is shown that in an attraction field, under the condition $c_{|m|} > 0$, there arise at $H \neq 0$ ($\mathbf{H} \| z$) bound and quasibound states with angular-momentum projection $l_z = |m|$ (in the Born approximation of the interaction with the center[10] such states always exist for arbitrary m), and their energies were obtained.[1)]

We calculate $\sigma^H(\omega)$ in a weak **E** for transitions from arbitrary (both intrinsic and magnetic) bound states with $m = 0$ to higher continuum states with $m = \pm 1$. The equations derived allow us to track the $\sigma^H(\omega)$ dependence over the first few oscillations. We have observed that $\sigma^H(\omega)$ vanishes (rather than going to infinity as when the functions ψ_ν are used) when the final-state energy coincides with the bottom of some Landau band. The behavior of $\sigma^H(\omega)$ in the vicinity of the bottom depends on the sign of c_1: at $c_1 < 0$ there is a peak above the bottom of the band, and at $c_1 > 0$ there are two peaks on the two sides of the band. Questions connected with measurement of $\sigma^H(\omega)$ are discussed. An attempt is made to compare qualitatively the theory with the experimental results obtained in Ref. 4.

2. We determine the wave functions Ψ_ν of the continuum. They satisfy the equation

$$\Psi_\nu(\mathbf{r}) = \psi_\nu(\mathbf{r}) + \int G(\mathbf{r}, \mathbf{r}'; E_\nu) V(r') \Psi_\nu(\mathbf{r}') d^3\mathbf{r}'. \tag{2}$$

Here $V(r)$ is a potential with radius r_0; ψ_ν are the functions in the absence of $V(r)$;

$$G(\mathbf{r}, \mathbf{r}'; E) = \sum_{\nu'} \psi_{\nu'}(\mathbf{r}) \psi_{\nu'}^*(\mathbf{r}') (E - E_{\nu'} + i\delta)^{-1} \tag{3}$$

is the Green function in a magnetic field. Equation (2) was solved in Ref. 4 for $r_0 \ll \lambda$ in connection with the scattering problem. Let us recall the solution procedure. At short distances $r \ll \lambda$ we can write

$$G(\mathbf{r}, \mathbf{r}'; E) = -(m_e/2\pi\hbar^2)|\mathbf{r} - \mathbf{r}'|^{-1} + G'(\mathbf{r}, \mathbf{r}'; E). \tag{4}$$

The first term is the Green function of a free electron with zero energy, and G' is that part of the Green function which describes the influence of **H**; it remains finite as $\mathbf{r}' \to \mathbf{r}$. At small r we assume that

$$\Psi_\nu = M_\nu \Phi, \tag{5}$$

where Φ satisfies the equation

$$\Phi(\mathbf{r}) = \varphi(\mathbf{r}) - \frac{m_e}{2\pi\hbar^2} \int V(r') |\mathbf{r} - \mathbf{r}'|^{-1} \Phi(\mathbf{r}') d^3\mathbf{r}', \tag{6}$$

and φ is a function of the free zero-energy electron and has the same asymptotic form as ψ_ν at small r. Substituting (4) and (5) in (2) and eliminating the term containing $|\mathbf{r} - \mathbf{r}'|^{-1}$, we obtain with the aid of (6), in the zeroth approximation in r/λ, an equation that does not contain a dependence on r and can yield M_ν. Again substituting (5) with the obtained value of M_ν in (2), we obtain Ψ_ν for arbitrary r. We note that in Ref. 14 φ was taken in the form $\varphi = \exp(ikz) \approx 1$. In this case the scattered wave [the second term on the right-hand side of (2)] turns out in the lowest approximation in r/λ to be independent of the angle α in the xy plane. By the same token, the effect of the center only on the state with $m = 0$ is taken into account.

We shall need the functions Ψ_ν for $m \neq 0$. For ψ_ν we choose functions in a magnetic field in the representation $\nu = \{n_\rho, m, k\}$. Then

$$E_\nu = E_{n_\rho m, k} = \hbar\omega_c[n_\rho + {}^1/_2(m + |m| + 1)] + \hbar^2 k^2/2m_e. \tag{7}$$

For φ we choose a spherical wave with angular momentum $l = m$ and with a projection of m that has at small r the same asymptotic form as ψ_ν:

$$\varphi \sim \psi_\nu \sim \rho^{|m|} e^{im\alpha},$$

where ρ is the radius vector in the xy plane. Repeating now the procedure described above, we obtain an equation in which, in the lowest approximation in r/λ, all the terms depend on the coordinates like $\rho^{|m|}\exp(im\alpha)$. Mutual cancellation of this factor yields M_ν. The final form of Ψ_ν is

$$\begin{aligned}\Psi_\nu = \Psi_\nu^{(+)}(\mathbf{r}) = \psi_\nu(\mathbf{r}) + i\beta_{|m|} \frac{1}{|m|!} \left[\frac{(n_\rho + |m|)!}{n_\rho!}\right]^{1/2} K_{N_m}^{|m|} \\ \times \left\{ \left[\frac{(N_m + |m|)!}{N_m!\,\varepsilon}\right]^{1/2} \psi_{N_m,m}^{\perp}(\rho) \exp\left[i\frac{|z|}{a_H}(2\varepsilon)^{1/2}\right] \right. \\ \left. - i\left[\frac{(N_m + |m| + 1)!}{(N_m + 1)!(1 - \varepsilon)}\right]^{1/2} \psi_{N_m+1,m}^{\perp}(\rho) \exp\left[-\frac{|z|}{a_H}(2(1-\varepsilon))^{1/2}\right] \right\}.\end{aligned} \tag{8}$$

Here $\psi_\nu^\perp(\rho)$ are the functions of the transverse motion in a magnetic field,

$$K_{N_m}^{|m|}(\varepsilon) = \left[1 - i\beta_{|m|} \sum_{n=0}^{N_m+1} \frac{(n + |m|)!}{|m|!\,n!} (N_m + \varepsilon - n)^{-1/2}\right]^{-1}, \tag{9}$$

($x^{1/2} = +i(-x)^{1/2}$ at $x < 0$), and the coefficient $\beta_{|m|}$ is defined in (1). The number N_m is determined in the following manner. We write E in the form

$$E_\nu = \hbar\omega_c(N + {}^1/_2 + \varepsilon), \tag{10}$$

where N is an integer and $0 \leqslant \varepsilon < 1$. Then $N_m = N - (|m| + m)/2$ is the number of the highest Landau level having an angular momentum m and energy that does not exceed E_ν. The superscript (+) indicates that the function (8) consists of an incident wave (ψ_ν) plus a diverging wave.

3. Equation (8) is valid at $\varepsilon^{1/2} \ll 1$ or $(1 - \varepsilon)^{1/2} \ll 1$. (The second or first term in the curly bracket should be respectively omitted.) If $|\beta_{|m|} \ll 1$, however (as is assumed below), it is precisely in this region that the influence of the SRP comes into play. Outside these regions, the diverging wave is

$\propto |\beta_{|m|}|$ and can be neglected. We have discarded in (8) terms proportional to $|\beta_{|m|}|$ at arbitrary ε. It is easy to verify that the functions Ψ_ν are orthogonal and normalized to

$$\delta(k-k')\delta_{m,m'}\delta_{n_\rho,n'_\rho}.$$

If the potential has no shallow real or virtual level, the estimate $|c_{|m|}| \sim r_0^{2|m|+1}$ is valid; we then have at $|m| \sim 1$

$$|\beta_{|m|}| \sim (r_0/a_H)^{2|m|+1} \ll 1$$

(we note that $\lambda \lesssim a_H$). If, however, a level ε_0 such that $|\varepsilon_0| = \hbar^2\varkappa^2/2m_e$ does exist, then $|c_0|$ increases by $(r_0\varkappa)^{-1}$ times, and $|c_m|$ with $m \neq 0$ increases by $(r_0\varkappa)^{-2}$ times (Ref. 11, § 133), and the condition $|\beta_{|m|}| \ll 1$ becomes stronger than $r_0 \ll \lambda$. If, however, at $H = 0$ there exists a bound state with angular momentum $l = |m|$ and zero energy, then $f_{|m|} \to \infty$ and the theory developed here does not hold for any H.

4. If $N_m \neq 0$, the amplitude of the scattered wave in (8) has at $\beta_{|m|} > 0$ poles at

$$\varepsilon = \varepsilon(N_m, |m|) = \varepsilon'(N_m, |m|) - i\varepsilon''(N_m, |m|)$$

$$= 1 - \beta_{|m|}^2 \left[\frac{(N_m+|m|)!}{N_m!|m|!}\right]^2 \times \left[1 + 2i\beta_{|m|} \sum_{n=0}^{N_m-1} \frac{(n+|m|)!}{n!|m|!}(N_m-n)^{-1/2}\right]. \quad (11)$$

The quantity $\varepsilon_{qu}(N_m,|m|) = 1 - \varepsilon'(N_m,|m|)$ is the dimensionless (in units of $\hbar\omega_c$) binding energy of the quasibound state with angular momentum below the bottom of the N_m th band; the presence of $\varepsilon''(N_m, |m|)$ is due to the possibility of decay of this state via transition of an electron into lower bands with the same m. If, however, $N_m = 0$ there is also a pole at

$$\varepsilon = -\varepsilon_b(|m|) = -\beta_{|m|}^2, \quad (12)$$

where $\varepsilon_b(|m|)$ is the binding energy of the bound state with angular momentum m. Thus, for those m for which $\beta_{|m|} > 0$ bound states appear under the bottom of the band $N_m = 0$ and quasibound states appear under the bottoms of the remaining bands. The latter are well defined at not very large N_m if $\beta_{|m|} \ll 1$, namely $\varepsilon''/\varepsilon_{qu} \ll 1$. Bound states with $m = 0$ were obtained in Ref. 15. Bound and quasibound states with arbitrary m were investigated in Ref. 10 in the Born approximation. We note that in the Born approximation $c_{|m|} > 0$ in an attracting field, and bound (and quasibound) states with arbitrary m are always present. The value (1) of ε_b was obtained in Ref. 16 without the use of the Born approximation.

We emphasize that the results obtained at $|\beta_{|m|}| \ll 1$ are valid for arbitrary potentials and arbitrary m. At $m = 0$ and $m = \pm 1$ these results coincide formally with the results of Ref. 12 obtained in the Born approximation $|\beta_{|m|}| \ll 1$ for a rectangular square well.

It follows from (8) that $\Psi_{N_m}, m,k(\mathbf{r}) \to 0$ as $\varepsilon \to 0$. The reason is that in the state $\{N_m, m, k\}$ the motion is effectively one-dimensional, and in the two-dimensional case a particle with energy close to zero is completely reflected from the well. Therefore, as will be shown below, the absorption cross section is zero when the final-state energy coincides with the bottom of any Landau band.

5. We proceed now to calculate the photodetachment cross section in a weak electric field. We shall assume that the initial (bound) state Ψ_i has $m = 0$. The function Ψ_i can be easily determined from the expressions given in Ref. 17, Chap. 7. The final functions must be taken to be of the form of incident + converging waves $\Psi_\nu^{(-)}$. They are obtained from $\Psi_\nu^{(+)}$ in accordance with the rule

$$\Psi^{(-)}_{n_\rho,m,k} = (\Psi^{(+)}_{n_\rho,-m,-k})^*.$$

Transitions are possible into the states $m = \pm 1$ (hereafter we assume $m = \pm 1$ throughout). For the cross section for the transition into a state with definite m (circular polarization) we obtain

$$\sigma_{i;m}^H(\omega) = \sum_{n_\rho=0}^{N_m} \sigma_{i;n_\rho,m}(\omega) = \sigma_0 \zeta^{-1}(3/2, I_i)(I_i+N-\varepsilon)^{-1}$$

$$\times (I_i+N-m+\varepsilon)^{-2} |K^1_{N_m}(\varepsilon)|^2 \sum_{n_\rho=0}^{N_m} (n_\rho+1)(N_m+\varepsilon-n_\rho)^{-1/2}; \quad (13)$$

$$\sigma_0 = 8\pi^2 e^2/m_e c\omega_c, \qquad \varepsilon = (\omega/\omega_c) - I_i - N.$$

Here $\zeta(x,y)$ is the generalized Riemann zeta function (it stems from the renormalization of Ψ_i); the parameter $I_i = 1/2 - E_i/\hbar\omega_c$ is the dimensionless binding energy; $E_i = E_i(H)$ is the bound-state energy; N is the integer part of $(\omega/\omega_c) - I_i$. The separate term in the sum over n_ρ is the cross section $\sigma^H_{i;n\rho,m}(\omega)$ for the transition to a band with given n_ρ. The factor $|K|^2$ describes the influence of the SRP on Ψ_ν. At $\beta_1 = 0$ this influence is absent:

$$\Psi_\nu = \psi_\nu, \qquad K_{N_m}{}^1(\varepsilon) = 1.$$

The $E_i(H)$ dependence was investigated in Ref. 17, Chap. 7. For the intrinsic bound state in a weak field we have $|E_i(H)| \approx |E_i(0)| \gg \hbar\omega_c$. In this case

$$\Psi_i = (\varkappa/2\pi)^{1/2} e^{-\varkappa r}/r, \qquad \varkappa = \hbar^{-1}(2m_e|E_i|)^{1/2}.$$

Then $I_i \gg 1, \zeta(\frac{3}{2},I_i) = 2I_i^{-1/2}$, and the arithmetic mean $\sigma^H_{i;1}$ or $\sigma^H_{i;-1}$ (the cross section for linear polarization) coincides at $\beta_1 = 0$ with that obtained in Ref. 9, where deep centers in the Lucovsky model[7] were considered in the approximation $\Psi_\nu = \psi_\nu$. For a magnetic initial state we have $I_i \ll 1$. Then

$$\zeta(3/2, I_i) \approx I_i^{-3/2} = \beta_1^{-3}.$$

The cross section $\sigma^H_{i;m}(\omega)$ for $I_i \ll 1$ yields at $\varepsilon \ll 1$, after replacing the coefficient c_1 by its value in obtained in the Born approximation, the absorption coefficient obtained in Ref. 10. Accordingly, at

$$|\varepsilon - \varepsilon'(N_m, |m|)| \lesssim \varepsilon''(N_m, |m|)$$

we obtain the absorption coefficient obtained in Ref. 10 for the transition into quasibound states.

6. Let us investigate in general outline the $\sigma^H_{i;m}(\omega)$ dependence. In the background region, i.e., outside the regions $\varepsilon \ll 1$ and $1 - \varepsilon \ll 1$, the factor $K^1_{N_m}(\varepsilon) \simeq 1$ and the $\sigma^H_{i;m}(\omega)$ dependence does not differ from the one obtained in Ref. 9, namely, $\sigma^H(\omega)$ decreases smoothly with increasing ω. We shall not discuss this region. The role of $K^{1}_{N_m}(\varepsilon)$ is significant when E_v is close to the bottom of some Landau band and lies above ($\varepsilon \ll 1$) it or below ($1-\varepsilon \ll 1$). We shall refer to the strong changes of $\sigma^{H}_{i;m}(\varepsilon)$ near the bottom of the band N_m as "oscillation," to which we assign a number N_m (0_m, 1_m, etc.) Let us consider the oscillation numbered N_m. At $\varepsilon = 0$ the cross section $\sigma^{H}_{i;m}(\omega) = 0$. Near and above the bottom of the band N_m we have

$$\sigma^{H}_{i;m} = (N_m+1)\,\varepsilon^{1/2}/[\varepsilon+\beta_1^2(N_m+1)^2]. \tag{14}$$

This cross section has the form of a peak whose maximum is located at $\varepsilon_u(N_m) = (N_m+1)^2\beta_1^2$ above the bottom of the band N_m ("upper" peak). The relative value of the maximum is $\sim\beta_1^{-1}$ and does not depend on N_m. The peak is asymmetric with $\sigma^{H}_{m;i} \propto \varepsilon^{1/2}$ on the left and $\sigma^{H}_{i;m} \propto \varepsilon^{-1/2}$ on the right; its half-width is $\sim\beta_1^2$.

The behavior of the cross section near and below the bottom of the band N_m depends on the sign of β_1. We note that it is assumed that an initial bound state exists, we are dealing with an attracting potential. Nonetheless, the amplitude f_1 and with it β_1 can have, in principle, either sign. Let $\beta_1 < 0$. We then have at $1-\varepsilon \ll 1$

$$\sigma^{H}_{i;m} \propto |K^{1}_{N_m}(\varepsilon)|^2 \approx [1+|\beta_1|(N_m+1)(1-\varepsilon)^{-1/2}]^{-2}. \tag{15}$$

The cross section in the narrow region $0 < 1-\varepsilon < \beta_1^2(N_m+1)^2$ decreases as $\varepsilon \to 1$ from values corresponding to the background to be zero. At $\beta_1 > 0$,

$$\sigma^{H}_{i;m} \propto |K^{1}_{N_m}(\varepsilon)|^2 \approx \left[\left(1-\beta_1\frac{N_m-1}{(1-\varepsilon)^{1/2}}\right)^2 + \beta_1^2 b_{N_m}^2\right]^{-1},$$

$$b_{N_m} = \sum_{n=0}^{N_m-1} \frac{n+1}{(N_m-n)^{1/2}}. \tag{16}$$

The plot of (16) is a peak whose maximum lies $\varepsilon_1(N_m)$ below the bottom of band N_m ("lower" peak). Its position coincides with the energy $\varepsilon_1(N_m, |m|)$ of the quasibound state under the bottom of the N_m band. The relative value of the maximum is $\sim\beta_1^{-2}b_{N_m}^{-2}$. Near the maximum we have

$$\sigma^{H}_{i;m} \propto \{[\varepsilon-\varepsilon'(N_m,|m|)]^2 + \varepsilon''^2(N_m,|m|)\}^{-1}. \tag{17}$$

The width $\sim\beta_1^3$ of the peak is determined by the lifetime of the quasi-stationary state.

Thus, depending on the sign of β_1, two spectrum types are possible. The schematic form of $\sigma^{H}_{i;-1}$ is shown in Fig. 1 for $|\beta_1| = 0.1$. In those regions where the cross sections for $\beta_1 > 0$ and $\beta_1 < 0$ are different, the curve for $\beta_1 < 0$ is shown dashed. It is assumed that $I_i \gg 1$ (intrinsic bound state undistorted by a magnetic field). In this case, at not too large N, the dependences of the third and fourth factors in the right-hand side of (13) on ω can be neglected.

For the 0_m oscillation we have $\varepsilon'' = 0$ and $\sigma^H \to \infty$ at $\varepsilon = 1-\beta_1^2$—a transition to a bound state below the bottom of the lower band, with angular momentum m. This lower peak takes in our approximation the form of a delta-function. To obtain a finite peak it is necessary to take into account the influence of some external perturbations (phonons, impurities). This problem is outside the scope of the present paper. We note that external perturbations not only make the delta-function peak (unbounded line to the left of zero in Fig. 1) finite, but lead also to smearing of the oscillation picture. In particular, the cross section on the bottom of the Landau band no longer vanishes, and at $\beta_1 > 0$ the upper and lower peaks merge into a single one.

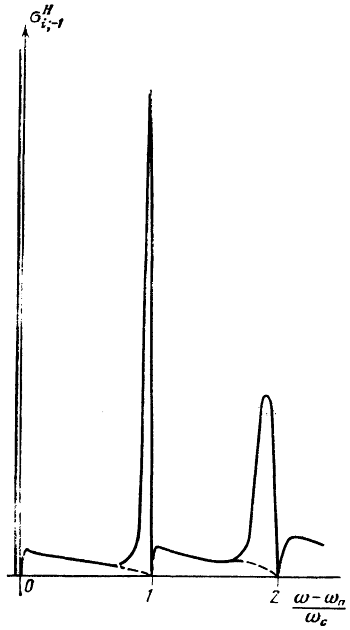

FIG. 1. Dependence of $\sigma^{H}_{i;-1}$ on $(\omega-\omega_{\text{thr}})/\omega_c$ ($\omega_{\text{thr}} = \omega_c I_i$ is the threshold frequency) for $\beta_1 = 0.1$. The dashed lines show $\sigma^{H}_{i;-1}$ at $\beta_1 < 0$ in those regions where the cross sections for $\beta_1 > 0$ and $\beta_1 < 0$ are different. The vertical straight line to the left of the origin is the delta-function lower peak 0_{-1}.

7. We discuss now the possibility of using the developed theory for an analysis of experimental results. We focus our attention on experiments with D^- centers. By virtue of a number of factors that complicate the experimental situation, the effects considered by us could hardly play an important role in Ref. 8. Of course, for some detailed analysis we must take into account the influence of the external perturbations on the spectrum and, primarily on the delta-function peak. We can therefore make here only some general remarks. It follows from the foregoing that the distance $\Delta\omega$ between neighboring peaks does not equal ω_c and can be either smaller (at $\beta_1 > 0$) or larger than ω_c. Furthermore $\Delta\omega$ must depend in a definite manner on the number of the oscillation and on the value of H. The character of these dependences can yield information on the magnitude and sign of β_1 and on the (possible) dependence of β_1 on H_1, as well as on the influence of external perturbations.

Next, much attention is paid of late to the binding energy $E_i(H)$ for SRP, and in particular for D^- centers. This question is discussed in a large number of papers, including Refs. 2–4, 17, and 18. In principle, a direct method of measuring $E_i(H)$ is to determine the threshold frequency $\omega_{\text{thr}} = \omega_c I_i$ for the $\sigma^H(\omega)$ dependence. This, however, is not as simple a problem as might seem at first glance. Attempts were made in Refs. 2–4 to determine the position of ω_{thr} on the spectra. It was assumed in Ref. 2 that ω_{thr} lies below the first maximum, while in Refs. 3 and 4 the two were assumed to coincide. From the results above it follows that ω_{thr} may be located (and is most likely to be located, see below) also above the first maximum. Therefore the determination of ω_{thr} calls for a special experimental investigation (possibly, measurement of the absorption at $\mathbf{E}\|\mathbf{H}$). We note that to analyze the oscillations it is desirable to perform the measurements with circular polarization. Linear polarization, as in Refs. 2–4, greatly complicates the problem because of superposition of the peaks with $m = \pm 1$.

We calculated the values of $\Delta\omega$ for the spectra obtained in Ref. 4. (The measurements in Ref. 3 were made in the Voigt configuration and the condition $\mathbf{E}\perp\mathbf{H}$ was not satisfied; two peaks, probably of different origin,[18] were observed in Ref. 2.) Almost none of the $\Delta\omega$ agree with ω_c and in the overwhelming majority of cases $\Delta\omega < \omega_c$. If this is so, then $\beta_1 > 0$, as is to be expected on the basis of the analogy with the H^- center (there is no intrinsic bound state with angular momentum $l = 1$). The relative deviation of $\Delta\omega$ from ω_c reaches 0.1, i.e., patently in excess of the limit of the measurement error. This means that in Ref. 4 as well as in Refs. 2 and 4, there are lower peaks and ω_{thr} should be located above the first maximum.

We discuss now qualitatively the relation between the intensities of the photoconductivity peaks obtained in Ref. 4. We recognize first that in Ref. 4 the radiation was not polarized (although the condition $\mathbf{E}\perp\mathbf{H}$ was satisfied). The photoconductivity is therefore proportional to the sum of the cross sections for $m = \pm 1$. The cross section $\sigma^H_{i;-1}(\omega)$ is shown in Fig. 1. Since $N_1 = N_{-1} + 1$, the cross section $\sigma^H_{i;+1}(\omega)$ is obtained by shifting the curve in the figure to the right by unity. Then the peak with the lowest frequency will be peak 0_{-1}, the next one the sume of the peaks $1_{-1} + 01$, next $2_{-1} + 1_1$, etc. Since the summed peaks have the same number $N_m - (|m| + m)/2 \equiv N$, it is convenient to use the latter to number the peaks in the case of unpolarized or linearly polarized radiation: $N = 0, 1, 2, \ldots$.

In Ref. 4 are given sets of photoconductivity curves for As and Li impurities in Si. For both As and Li there are five peaks in a field $H = 2$T, with the peak $N = 1$ the highest. With increasing H ($H = 4$T and 6T), the paks $N = 0$ and $N = 1$ become equalized (all other peaks are already outside the frequency range).

We have assumed that $\beta_1 > 0$ and that the delta-function peaks have in fact a certain finite height comparable with that of the peak 1_{-1} in Fig. 1. Then, if the prescription given above is used to obtain the cross section for linear polarization, then the highest peak turns out to be $N = 1$, as is the case in the experiment in a field $H = 2$T.

It must now be recalled that the curve in Fig. 1 corresponds to the case $I_i \gg 1$, when the third and fourth factors on the right side of (8) can be regarded as constant (and equal for $\sigma^H_{i;-1}$ and $\sigma^H_{i;+1}$). In Ref. 4 the inequality $I_i \gg 1$ does not take place ($I_i \lesssim 1$ at $H = 6$T). It must therefore be recognized that these factors are different for $\sigma^H_{i;-1}$ and $\sigma^H_{i;+1}$, and decrease noticeably with increasing ω at a rate that is faster the larger H. It is found then that the ratio of the heights of the peaks $N = 1$ and $N = 0$ should decrease with increasing H from values larger than unity, as is indeed observed in Ref. 4.

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[1)]After this paper went to press, two papers[12,13] were published with methods of determining the spectrum of weakly bound states of a small-radius well at $H \neq 0$ for abritrary $\beta_{|m|}$. The method of Ref. 12, however, calls for knowledge of the wave functions for $H = 0$ at $r \lesssim r_0$. Specific results were obtained for a rectangular potential well. To calculate the spectrum by the method of Ref. 13 it suffices to know the scattering length and the effective radius at $H = 0$. In particular, Eq. (12) of the present paper was obtained in Ref. 13.

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