Self-trapping in crystals and nonlinear wave processes: self-trapping barrier for plasma cavitons

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An analogy is drawn between some nonlinear wave phenomena (plasma solitons, their instability, plasma collapse) and related objects and phenomena in the physics of self-trapping in condensed media. The analogy is found to be far-reaching; the main difference is that for ordinary self-trapping the rate constant is determined only by the parameters of the material, while for plasma formations it is also proportional to the number of involved plasmons. It is shown that for plasma cavitons there exists a self-trapping barrier W_N corresponding to the energy of formation of a critical caviton, which subsequently is spontaneously compressed via plasma collapse or by dissipation (N is the number of plasmons trapped by the caviton). The height of the barrier is $W_N \propto N^{-1}$, i.e., it decreases with increasing N, while the critical-caviton radius $R_N \propto N$. It is shown from symmetry considerations that the critical caviton is asymmetric: it has a flattened (disk-like) shape in its central part. In the conclusion, a brief comparison is presented with some numerical calculations and with the available experimental data on spike turbulence and caviton formation.

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A definite analogy exists between a purely quantum phenomenon, namely self-trapping (after Landau) in condensed phases and certain wave phenomena that evolve in the region of a strong nonlinearity. To be specific, we shall consider predominantly one of them, the formation of plasma cavitons, which can occur in the course of the plasma collapse first considered by Zakharov.[1] This analogy manifests itself mathematically most fully within the framework of the continual theory. The continual self-trapping theory is valid if the scale of the produced formations is large compared with the lattice constant d. It is convenient to compare it with hydrodynamic theory of a quasineutral plasma. Physically, the analogy between these two greatly different groups of phenomena is best traced under conditions when they can be described in the adiabatic approximation. The rapid subsystem that is described by the Schrödinger equation of the stationary states is formed in this case by electrons (holes, excitons) in solids and by plasmons in a plasma. The slow subsystem constitutes respectivley phonons and ions.

In Sec. 1 we shall establish the similarity of the Lagrangians for the systems indicated above and elucidate both the analogies and the substantial differences between them. Thus, a correspondence is established between the Langmuir solitons,[2] their instability,[3] and plasma collapse[1] on the one hand, and the analogous objects and processes in self-trapping physics, on the other. The ensuing differences are due mainly to two causes. First, the wave function in the self-trapping theory is normalized, whereas its analog in the nonlinear plasma theory—the envelope of the high-frequency field-spans over an extensive region of space and the number of plasmons N stored in this space begins in fact to play the role of the effective coupling constant. Second, the collapse theory is usually constructed as a dynamic theory of the type of self-focusing theory, in which the dissipative processes remain usually in the background and as a rule weaken the self-focusing.[4] In the self-trapping theory, on the contrary, the dissipative processes are assumed to be strong enough to ensure the self-trapping process, which evolves so as to decrease the system energy, and the principle calculated quantities are as a rule the stationary values of the energy. As applied to wave-collapse theory, a similar approach ensures dissipative evolution of the system towards formation of cavitons after the system passes through the self-trapping barrier (see below).

In Sec. 2 we construct the theory of the self-trapping barrier for the formation of plasma cavitons, i.e., we find the lowest saddle that separates, spatially, on the surface of the adiabatic potential of the ions, the homogeneous state of the plasma and the state that evolves towards caviton formation. It is shown that the critical caviton corresponding to the barrier has a reduced symmetry—it is not spherical but has an oblate (lentil-shaped) form; the nonsphericity of the caviton follows completely from the arguments based on symmetry considerations.

Also formulated are certain results that are valid also outside the region of applicability of the adiabatic approximation.

1. SELF-TRAPPING IN CERTAIN NONLINEAR PHENOMENA IN A PLASMA

We start with writing down the Lagrangians of two different systems. One system is a quasiparticle (for the sake of argument, a hole) in a crystal, belongs to a triply degenerate band, and interacts with longitudinal acoustic oscillations with the aid of one (hydrostatic) deformation potential.[5] These conditions were chosen to obtain a maximum analogy with plasma; for the same purpose we retain in the deformation only the contribution connected with the bulk compression modulus (this limitation, however, is not particularly

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essential). The other system is a quasineutral plasma with account taken of the electric field of the plasma oscillations and of the field of the electromagnetic waves.

The Lagrangian of the first system is of the form

$$L=\frac{\rho}{2}\left(\frac{\partial \mathbf{Q}}{\partial t}\right)^2-\frac{\rho s^2}{2}(\operatorname{div}\mathbf{Q})^2+i\psi^*\frac{\partial\psi}{\partial t}$$
$$-\frac{1}{2m_l}|\operatorname{div}\psi|^2-\frac{1}{2m_t}|\operatorname{rot}\psi|^2-C\operatorname{div}\mathbf{Q}(\mathbf{r})|\psi(\mathbf{r})|^2. \quad (1)$$

Here $\mathbf{Q}(\mathbf{r})$ is the deformation vector, ρ is the density, s is the speed of the longitudinal sound, ψ is a three-component vector wave function that describes a hole belonging to the triply degenerate band,[5] C is the deformation potential, and m_l and m_t are the effective masses. The notation used in (1) for the kinetic energy of the hole (the fourth and fifth terms) is particularly convenient,[6] inasmuch as the masses corresponding to holes with different helicities λ are explicitly separated in it. For a plasma, however $\mu = m_l/m_t \gg 1$, and we shall therefore consider this case below. The last term describes the interaction of the hole with the lattice deformation.

It is convenient to transform to dimensionless variables, leaving unchanged the normalization integral

$$\int|\psi(\mathbf{r})|^2 d^n r=1; \quad (2)$$

The form in which the volume $d^n r$ is written presupposes that dimensionalities $n \leqslant 3$ of space can be considered. Of course the dimensionality of the density ρ depends here on n. The conversion to dimensionless units is effected by the transformation

$$\mathbf{r}\to r_0\mathbf{r},\quad t\to t_0 t,\quad \mathbf{Q}\to Q_0\mathbf{Q},\quad L\to L_0 L, \quad (3)$$

and the energy scale is determined by the quantity $g = L_0 r_0^3$. The quantities are dimensional in the left-hand side of (3) and dimensionless in the right. If we define

$$r_0=\left(\frac{m_l C^2}{\rho s^2}\right)^{1/(n-2)},\quad t_0=\frac{r_0}{s},\quad Q_0=\frac{1}{m_l C r_0},\quad g=\frac{1}{m_l r_0^2}, \quad (4)$$

then the dimensionless Lagrangian

$$L=\frac{1}{2}\left(\frac{\partial \mathbf{Q}}{\partial t}\right)^2-\frac{1}{2}(\operatorname{div}\mathbf{Q})^2+ia\psi^*\frac{\partial\psi}{\partial t}$$
$$-\frac{1}{2}|\operatorname{div}\psi|^2-\frac{\mu}{2}|\operatorname{rot}\psi|^2-\operatorname{div}\mathbf{Q}|\psi(\mathbf{r})|^2, \quad (5)$$

where $a = m_l r_0 s$. Formulas (4) determine the characteristic scales of all the physical quantities.

Writing down in standard fashion[7] the Lagrangian equations for ψ:

$$\frac{\partial L}{\partial\psi^*}-\sum_s\frac{\partial}{\partial x_s}\frac{\partial L}{\partial(\partial\psi^*/\partial x_s)}-\frac{\partial}{\partial t}\frac{\partial L}{\partial\dot\psi^*}=0 \quad (6)$$

and similarly for ψ^* and $\mathbf{Q}$, we obtain the equations of motion

$$ia\frac{\partial\psi}{\partial t}+\frac{1}{2}\nabla(\nabla\psi)-\frac{\mu}{2}[\nabla\times[\nabla\times\psi]]-\psi\operatorname{div}\mathbf{Q}=0, \quad (7)$$
$$\partial^2\mathbf{Q}/\partial t^2-\Delta\mathbf{Q}-\nabla|\psi(\mathbf{r})|^2=0; \quad (8)$$

we assume here that curl $\mathbf{Q} = 0$, since the last term in (8) does not excite vortical motions. Introducing the generalized momenta

$$\mathbf{P}(\mathbf{r})=\delta L/\delta\dot{\mathbf{Q}}(\mathbf{r})=\dot{\mathbf{Q}}(\mathbf{r}),$$
$$\Pi=\delta L/\delta\dot\psi=ia\psi^*,\quad \Pi^*=\delta L/\delta\dot\psi^*=0, \quad (9)$$

we obtain the Hamiltonian

$$H(\mathbf{Q},\psi)=\frac{1}{2}\mathbf{P}^2+\frac{1}{2}(\operatorname{div}\mathbf{Q})^2$$
$$+\frac{1}{2}|\operatorname{div}\psi|^2+\frac{\mu}{2}|\operatorname{rot}\psi|^2+|\psi(\mathbf{r})|^2\operatorname{div}\mathbf{Q}. \quad (10)$$

As seen from (4), the formulas obtained here become meaningless at $n = 2$. This case calls for a special treatment. The dimensionless Lagrangian has here, in contrast to (5), the form

$$L=\frac{1}{2}\left(\frac{\partial \mathbf{Q}}{\partial t}\right)^2-\frac{1}{2}(\operatorname{div}\mathbf{Q})^2+ia\psi^*\frac{\partial\psi}{\partial t}-\frac{1}{2}|\operatorname{div}\psi|^2$$
$$-\frac{\mu}{2}|\operatorname{rot}\psi|^2-\left(\frac{mC^2}{\rho s^2}\right)^{1/2}|\psi(\mathbf{r})|^2\operatorname{div}\mathbf{Q}, \quad (11)$$

i.e., it contains one extra dimensionless coeffiicent compared with (5). The corresponding Hamiltonian is

$$H=\frac{1}{2}\left(\frac{\partial \mathbf{Q}}{\partial t}\right)^2+\frac{1}{2}(\operatorname{div}\mathbf{Q})^2+\frac{1}{2}|\operatorname{div}\psi|^2$$
$$+\frac{\mu}{2}|\operatorname{rot}\psi|^2+\left(\frac{mC^2}{\rho s^2}\right)^{1/2}|\psi(\mathbf{r})|^2\operatorname{div}\mathbf{Q}, \quad (12)$$

with the dimensional and dimensionless variables connected by formulas similar to (3), except that now

$$r_0=\frac{1}{(m_l g)^{1/2}},\quad t_0=\frac{1}{s(m_l g)^{1/2}},\quad Q_0=\frac{g^{1/2}}{s},\quad a=s\left(\frac{m_l}{g}\right)^{1/2}, \quad (13)$$

and the energy scale g remains arbitrary.

We write down now the analogous equations for a quasineutral plasma. We assume that in the high-frequency electric field the dominant frequencies are close to the plasma frequency $\omega_p = (4\pi e^2 n_0/m)^{1/2}$, where m is the electron mass and n_0 is the equilibrum density of the homogeneous plasma. We introduce, as usual, the smooth envelope $\tilde{\mathbf{E}}$ of the electric field $\mathbf{E}_0$:

$$\mathbf{E}_0(\mathbf{r},t)=\frac{1}{2}[\tilde{\mathbf{E}}(\mathbf{r},t)\exp(-i\omega_p t)+\text{c.c.}]. \quad (14)$$

The plasma Lagrangian can then be written in the form

$$L=\frac{\rho}{2}\left(\frac{\partial \mathbf{Q}}{\partial t}\right)^2-\frac{\rho s^2}{2}(\operatorname{div}\mathbf{Q})^2+\frac{i}{8\pi\omega_p}\tilde{\mathbf{E}}^*\frac{\partial\tilde{\mathbf{E}}}{\partial t}$$
$$-\frac{3T_e}{16\pi m\omega_p^2}(\operatorname{div}\tilde{\mathbf{E}})^2-\frac{c^2}{16\pi\omega_p^2}|\operatorname{rot}\tilde{\mathbf{E}}|^2-\frac{e^2 n_0}{4m\omega_p^2}|\tilde{\mathbf{E}}|^2\operatorname{div}\mathbf{Q}. \quad (15)$$

Here ρ is the plasma density, s is the speed of ion sound, c is the speed of light, T_e is the electron temperature; usually

$T_e \gg T_i$ (T_e is the ion temperature). The quantity $\mathbf{Q}(\mathbf{r})$ has the meaning of the hydrodynamic Lagrangian variable with that peculiarity that the differentiation is with respect to the coordinate of the point $\mathbf{r}$; the first term in (15) is then the kinetic energy of the plasma and the second is the work done against the gaskinetic pressure forces. The remaining terms are connected with the field $\tilde{\mathbf{E}}$, which plays the role of the wave function. Thus, the fourth term is the energy of the longitudinal field (reckoned from $N\omega_p$, where N is the total number of plasmons), i.e., the kientic energy of the plasmons. It can be easily obtained if it is recognized that the effective mass m_l of the plasmon determined from the known plasmon dispersion law[8]

$$\omega(k) \approx \omega_p + 3T_e k^2/2m\omega_p,$$

is equal to $m_l = m\omega_p/3T_e$, and the plasmon density n_p and the energy density of the longitudinal field $\rho(\omega)$ (Ref. 9), in terms of which n_p is directly expressed, are equal to

$$\rho(\omega) = \frac{1}{8\pi}\frac{d}{d\omega}(\omega\varepsilon(\omega))\overline{|\mathbf{E}_0(\mathbf{r},t)|^2} = \frac{1}{8\pi}|\tilde{\mathbf{E}}|^2, \qquad n_p = \frac{|\tilde{\mathbf{E}}|^2}{8\pi\omega_p}. \tag{16}$$

The superior bar in (16) denotes averaging with respect to time; in the calculation of $\rho(\omega)$ we used the formula for the dielectric constant $\varepsilon(\omega) = 1 - \omega_p^2/\omega^2$ and, after differentiating, we put $\omega \approx \omega_p$; for a plane wave the fourth term in (15) reduces to $n_p k^2/2m_l$. The third term considered jointly with teh fourth ensures a correct plasmon dispersion law. The fifth term is the magnetic-field energy $\mathbf{H} = -ic\,\mathrm{curl}\tilde{\mathbf{E}}/\omega_p$; the transverse-electric-field energy is small compared with it in terms of the parameter $T_e/mc^2 \ll 1$. Also negligibly small is the energy of the quasistatic ambipolar-diffusion field (see Sec. 2). Secondly, the last term in (15) is the energy of electrons with density $\delta n = n_0\,\mathrm{div}\mathbf{Q}$ in the field of the pondermotive forces with potential $e^2|\tilde{\mathbf{E}}|^2/4m\omega_p^2$ (which is the same for the longitudinal and transverse field).[10,11] From the Lagrangian (15) follow Zakharov's equations[1] for $\mathbf{E}$ and δn in the form in which they were expressed by Kuznetsov.[12]

It is convenient to change the normalization of the field $\tilde{\mathbf{E}}$ in such a way that its square yields n_p directly. We introduce

$$\mathbf{E}(\mathbf{r}) = (8\pi\omega_p)^{-1/2}\tilde{\mathbf{E}}(\mathbf{r}), \qquad n_p = |\mathbf{E}(\mathbf{r})|^2 \tag{17}$$

and

$$I_1 \equiv \int |\mathbf{E}(\mathbf{r})|^2 d^3r = \int n_p(\mathbf{r})\,d^3r = N, \tag{18}$$

where N is the total number of plasmons. Then (15) takes the form

$$L = \frac{\rho}{2}\left(\frac{\partial \mathbf{Q}}{\partial t}\right)^2 - \frac{\rho s^2}{2}(\mathrm{div}\,\mathbf{Q})^2 + i\mathbf{E}^*\frac{\partial \mathbf{E}}{\partial t} - \frac{1}{2m_l}|\mathrm{div}\,\mathbf{E}|^2 - \frac{1}{2m_t}|\mathrm{rot}\,\mathbf{E}|^2 - \frac{\omega_p}{2}|\mathbf{E}(\mathbf{r})|^2\mathrm{div}\,\mathbf{Q}, \tag{19}$$

where $m_t = \omega_p/c^2$; usually $\mu = mc^2/3T_e \gg 1$. In this form the Lagrangian (19) conicides with (1) apart from the substitution $C \to \omega_p/2$. Therefore a transformation similar to (4) can reduce it to the dimensionless form (5) so that the Hamiltonian (10) is again obtained.

It is obvious that I_1 is an integral of the motion[1] and both a renormalization integral for the Schrödinger equation (17) and an adiabatic invariant corresponding to conservation of the number N of the plasmons.

Thus, the Lagrangians in the self-trapping theory and in the nonlinear plasma theory practically coincide. However, a substantial difference exists in the normalized conditions: for a plasma, in constast to (2), the condition (18) is valid, since it contains the number of plasmons N, a quantity determined by the degree of pumping of the plasma, by the geometric dimensions of the pumped region, by the sizes of the fluctuation, etc. We shall show that the value of the integral of motion N exerts a substantial influence on the behavior of the system.

The second difference lies in the very approach of the treatment of the problem. In plasma theory one usually solves dynamic equations of the type (7) and (8) with definite initial conditions. The underlying idea is in many respects close to self-focusing theory[13]; e.g., plasma collapse is considered predominantly as the result of plasmon focusing (since the three-dimensional time-dependent Schrödinger equation is equivalent to a four-dimensional parabolic equation), which is self-consistent with the plasma motion, and not as the result of dissipation. (By dissipation we understand here viscuous hydrdynamic friction with the number of plasmons conserved.) Therefore most papers deal with the study of he nondissipative equations (7) and (8), which conserve the energy integral

$$I_2 \equiv \mathscr{H} = \int H d^3r \tag{20}$$

[H is defined by Eq. (10)] and a rigid phase relation between the density field and the velocity; in some cases one investigates also the perturbing action of the dissipation (see, e.g., Refs. 14–16). Unfortunately, the problem for a plasma is substantially more complicated then in self-focusing theory,[4,13] since in place of the known initial condition—the profile of the incident beam— it is necessary to choose arbitrary initial conditions corresponding to turbulent-plasma configurations whose statistical weight is difficult to estimate. The main advantage of the dynamic approach is the possibility of calculating, within the framework of the assumed model, the growth rates, the characteristic times, and the detailed course of the process.

On the contrary, in self-trapping theory the basic concept is the adiabatic potential of the system, the particular attention is being paid to finding stationary values of the adiabatic potential: the stable state—the absolute minimum, metastable local minima, unstable states—saddle points. It is assumed here that the direction of the processes is determined not by the dynamic processes,[1)] but by the dissipation: the system tends to go over into a state with lower energy. This approach is not claimed to describe completely the course of the process; nonetheless, as will be shown below, it provides a new view of certain processes in a plasma and makes it possible to include in natural manner in plasma theory symmetry arguments that have apparently not been used heretofore in this field. A total synthesis of these approaches, which has not been realized so far, consists of a consistent allowance of dissipation in the theory of collapse and induced emission of phonons—in self-trapping theory.

We consider now several examples. We apply to them the standard methods of Pekar's adiabatic self-trapping theory.[18] We change first from the density of the Hamiltonian H to the total Hamiltonian $\mathscr{H}$ in acccordance with (20). Then the condition $\delta\mathscr{H}/\delta Q(\mathbf{r}) = 0$ yields then

$$\operatorname{div}\mathbf{Q}=-\mathbf{E}^2(\mathbf{r}), \tag{21}$$

and after eliminating $\mathbf{Q}(\mathbf{r})$ we obtain for the field $\mathbf{E}(\mathbf{r})$

$$\mathscr{J}[\mathbf{E}]=\mathscr{H}[\mathbf{Q}[\mathbf{E}],\mathbf{X}\mathbf{E}]=\int\left\{\frac{1}{2}|\operatorname{div}\mathbf{E}|^2+\frac{\mu}{2}|\operatorname{rot}\mathbf{E}|^2-\frac{1}{2}|\mathbf{E}|^4\right\}d^nr. \tag{22}$$

Further minimizatin of $\mathscr{J}$ with respect to E should be carried out under the additional condition (18). It is convenient to introduce first a new function

$$\mathbf{\Psi}(\mathbf{r},t)=\frac{1}{N^{1/2}}\mathbf{E}(r,\mathbf{t}), \tag{23}$$

which satisfies the standard normalization condition (2), and the functional

$$\tilde{\mathscr{J}}_N[\mathbf{\Psi}]=\frac{1}{N}\mathscr{J}_N[\mathbf{E}]=\int\left\{\frac{1}{2}|\operatorname{div}\mathbf{\Psi}|^2+\frac{\mu}{2}|\operatorname{rot}\mathbf{\Psi}|^2-\frac{N}{2}|\mathbf{\Psi}|^4\right\}d^nr. \tag{24}$$

Varying $\tilde{\mathscr{J}}_N$ with respect to ψ with the normaliztion preserved, we obtain

$$-\frac{1}{2}\nabla(\nabla\mathbf{\Psi})+\frac{\mu}{2}[\nabla[\nabla\mathbf{\Psi}]]-N|\mathbf{\Psi}|^2\mathbf{\Psi}=\Omega_N\mathbf{\Psi}. \tag{25}$$

It is important to note that since the functional $\tilde{\mathscr{J}}[\mathbf{\Psi}]$ is obtained from $\mathscr{H}$ by variation with respect to $\mathbf{Q}(\mathbf{r})$, it has by itself no direct physical meaning in the adiabatic theory: only its stationary values (extrema, saddles) have a clear meaning.

For the extremals of the functionals $\mathscr{J}_N$ there exist simple relations between the individual terms in the right-hand side of (24). They can be obtained if, following the usual method of deriving the quantum-mechanical viral theorem,[19] one carries out in (24) a scale transformation that preserves the normalization:

$$\mathbf{\Phi}(\mathbf{r})=\lambda^{n/2}\mathbf{\Psi}(\lambda\mathbf{r}). \tag{26}$$

Then

$$\tilde{\mathscr{J}}_N[\mathbf{\Phi}]=\lambda^2\int\left\{\frac{1}{2}|\operatorname{div}\mathbf{\Psi}|^2+\frac{\mu}{2}|\operatorname{rot}\mathbf{\Psi}|^2\right\}d^nr-\lambda^n\frac{N}{2}\int|\mathbf{\Psi}|^4d^nr. \tag{27}$$

Since $\mathbf{\Psi}$ is an extremal of the function $\tilde{\mathscr{J}}_N[\mathbf{\Psi}]$, the equality $(d\tilde{\mathscr{J}}_N/d\lambda)_{\lambda=1}=0$, should hold, from which it follows that

$$\int\left\{|\operatorname{div}\mathbf{\Psi}|^2+\mu|\operatorname{rot}\mathbf{\Psi}|^2\right\}d^nr=\frac{n}{2}N\int|\mathbf{\Psi}|^4d^nr. \tag{28}$$

We denote the left-hand side of (28) by $2K$, where K is the kinetic energy; according to (10) and (21)–(24), the integral

$$U=\frac{N}{2}\int|\mathbf{\Psi}|^4d^nr$$

has the meaning of potential energy, and $V=-2U$ the meaning of interaction energy. Then we have as a result of (28) and (24)

$$\tilde{\mathscr{J}}_N=\frac{n-2}{n}K,\quad \Omega_N=\frac{n-4}{n}K,\quad 2K=nU=-\frac{n}{2}V, \tag{29}$$

and from (18), (20), and (24) it follows that

$$I_2=I_1\tilde{\mathscr{J}}_N. \tag{30}$$

Relations similar to (29) are known in self-trapping theory[18,20] and Eq. (30) with $\mu\to\infty$ was obtained by another method in Ref. 21.

By way of the first example we consider Rudakov's planar plasma soliton.[2] To this end we assume that $\mathbf{\Psi}=\mathbf{\Psi}(x)$ and has only one nonzero projection, along x. In this case curl $\mathbf{\Psi}=0$ and Eq. (25) reduces to

$$-\frac{1}{2}\frac{d^2\Psi}{dx^2}-N_s|\Psi|^2\Psi=\Omega_{N_s}\Psi, \tag{31}$$

where N_s is the number of plasmons per (dimensionless) unit area of the soliton. The only solution of (31) that decreases at infinity is well known: taking (29) at $n=1$ into account we obtain

$$\Psi_{N_s}(x)=\frac{N_s^{1/2}/2}{\operatorname{ch}(N_sx/2)},\quad \Omega_{N_s}=-\frac{N_s^2}{8},\quad \tilde{\mathscr{J}}_{N_s}[\Psi]=\frac{1}{3}\Omega_{N_s}. \tag{32}$$

To change over to the solution of the problem of self-trapping in a one-dimensional chain, we need only put $N_s=1$ in (31); we obtain then the known result[22,23]:

$$\psi(x)=1/2\operatorname{ch}(x/2),\quad \Omega=-1/8,\quad \mathscr{J}=\Omega/3. \tag{33}$$

In both cases $\mathscr{J}<0$ and is the minimum of the total energy (including the lattice-deformation energy).

Comparison of (32) and (33) shows the whereas in the self-trapping problem all the quantities are uniquely determined by the parameters of the material [cf. (4)], in the theory of plasma solitons there remains a free parameter N_s. If, using (4) (with $n=1$), we define Ω that enters in (33) in dimensional units, then it turns ouf that $\Omega\propto C^4$; it is then natural to regard the deformation potential C as a dimensional coupling constant. Comparison with (32) shows in this case that the number N_s of the plasmons enters in the dimensional energy in the combination $(N_sC^2)^2$, i.e., it increases the coupling constant by a factor $N_s^{1/2}$. We shall verify below that the conclusion that an increase of N is equivalent to strengthening of the coupling is of general character.

We note also that both plasma solitons and self-trapped one-dimensional particles can move; in particular, one can calculate their effective mass and other parameters.[2,24]

To consider self-trapping in a two-dimensional system it is necessary, starting from (12), to eliminate div $\mathbf{Q}$ in analogy with (21) and write the total Hamiltonian

$$\mathscr{J}[\psi]=\mathscr{H}[\mathbf{Q}[\psi],\mathbf{X}\psi]=\int\left\{\frac{1}{2}|\operatorname{div}\psi|^2+\frac{\mu}{2}|\operatorname{rot}\psi|^2\right\}d^2r-\frac{1}{2}\left(\frac{mC^2}{\mu s^2}\right)^{1/2}\int|\psi(\mathbf{r})|^4d^2r. \tag{34}$$

If we carry out a scale transformatin of the type (26), we see that both terms vary in proportion to λ^2. This means that self-trapping sets in[22,25] at a critical coupling-constant value $mC^2/\rho s^2 \sim 1$ (it is assumed that $\mu \gtrsim 1$); a similar result, namely the presence of a critical power, is known also in the theory of self-focusing of a cylindrical beam.[26] When the coupling constant reaches the critical value, the self-trapping takes place without a threshold, i.e., by continuously varying ψ it is possible to achieve a monotonic lowering of $\mathscr{J}[\psi]$, starting with $\mathscr{J}=0$; the absence of a threshold follows from (29) with $n=2$. In theory of a plane soliton, the effective coupling consant is arbitrarily large, to the extent that the ratio N/N_s, i.e., the area of the soliton, is large. Therefore the soliton is unstable and breaks up into bunches when $\tilde{\mathscr{J}}[\Psi]$ is continuously lowered—this instability is observed also in the kinetic analysis.[3]

In the three-dimensional case ($n=3$) the functional $\tilde{\mathscr{J}}[\Psi]$ is not bounded from below,[2)] a fact that can be seen from (27), where the last term decreases without limit with increasing. This corresponds to "'collapsing" the self-trapped state[27] or the plasma caviton[1]; of course, the employed description is valid only over scales $r \gg d$ for self-traping and at $r \gg l_D$ for plasma cavitons (l_D is the Debye length). It must be emphasized that in contrast to the theory of plasma collapse[1] we arrive at the formation of plasma cavitons from energy rather than dynamic considerations. The energy arguments indicate that the caviton can certainly be produced if $\tilde{\mathscr{J}}_N[\Psi]$ becomes negative already on functions Ψ with space scale $R > l_D$, i.e., within the framework of applicability of the quasineutral theory.[3] The condition for this, as follows from (24), is of the form $N > T_e^2 l_d/e^2\omega_p$, where N is the number of plasmons in the cluster; consequently, only sufficiently large clusters can produce cavitons. After contraction to a scale $\sim l_D$, the electric field $\tilde{\mathbf{E}}_f$ in such a cluster should satisfy the condition $|\tilde{\mathbf{E}}_f|^2/n_0T_e > 1$; it follows from the constancy of N upon compression of the cluster. We note, incidentally, that in plasma cavitons produced by pumping with an electron beam, a ratio $\mathbf{E}^2_{\max}/4\pi n_0 T_e \approx 1$ was recorded.[28] It appears that a similar criterion (or an even more stringent one, since it must include the hydrodynamic energy of the plasma) should exist also in plasma-collapse theory. Indeed, if it is assumed that the collapse is due to strong outflow of plasmons at large momenta and is accompanied by a loss of energy, the energy deficit can be covered only on account of the thermal energy of the electrons and ions; such a mechanism seems artifical and to our knowledge has not been discussed.

We approach now the nub of our problem. In the theory of self-tapping in three-dimensional systems, the transition from the homogeneous state $|\psi(\mathbf{r})|^2 = 1/\mathscr{V}$ is the normalization volume, corresponding to $\mathscr{J}[\psi]=0$, to the self-trappng state with $\mathscr{J}[\psi] \to -\infty$ cannot be effected by decreasing $\mathscr{J}$ continuously: it is necessary to surmount a self-trapping barrier[20,22,29] corresponding to the lower saddle point on the surface of the adiabatic potential. The height of this barrier is $W = \mathscr{J}_N^{(s)}[\mathbf{E}] > 0$, where the superscript s indicates that the value of the functional $\mathscr{J}$ is taken at the saddle point, and the subscript N, as usual, is the number of plasmons. The Ψ function on the barrier is defined by an equation of type (25). This equation was previously analyzed in its scalar variant,[30,31] and it wa shown that there exists an infinite number of eigenvalues with $\Omega_N < 0$ which decrease at infinity. Since according to (29) we have at $n=3$

$$W_N = \mathscr{J}_N^{(s)}[\mathbf{E}] = -N\Omega_N,$$

we are are interested in the solution with a maximum Ω_N, which ensures a minimum barrier W_N; we take into account consistently in this case the vector structure of Eq. (25).

In pure thermodynamic language, W_N is the minimum isothermal work necessary to produce the critical caviton containing N plasmons; this caviton will hereafter be contracted alredy spontaneously, i.e., without performance of outside work, in other words, the barrier caviton is the critical one.

2. HEIGHT OF SELF-TRAPPING BARRIER AND SHAPE OF BARRIER CAVITON

In the Schrödinger equation (25) and in the function $\mathscr{J}_N$ [Eq. (24)] the two terms that make up the kinetic energy are of different order of magnitude, since $\mu = mc^3/3T_e \gg 1$ is large compared with the relativistic parameter. Strictly speaking, allowance for the curl team exaggerates the accuracy somewhat, since the relativistic corrections to the plasmon dispersion law[8] were not taken into account. We have retained above the curl term in order to preserve the general vector structure of the equations. It describes the transverse field produced as a result of the spatial inhomogeneity of the longitudinal field; in this sense it is analogous to the weak longitudinal field in self-focusing theory.[32,33] It may turn out physically to be responsible for the emission of electromagnetic waves from plasma cavitons. Its influence on $\mathscr{J}$ at $\mu \gg 1$, however, is small; it can be shown[4)] that the corrections to $\mathscr{J}$ are proportional to μ^{-2}. We can therefore put curl $\Psi = 0$ (in analogy with the theory of acceptor centers[6]). In the functional (24) this can be done by choosing $\Psi = \nabla\chi$. In Eq. (25), so as not to increase the order of equation, it is convenient to proceed in a different manner, separating in the linear term its potential part:

$$-\frac{1}{2}\Delta\Psi(\mathbf{r}) + \frac{N}{4\pi}\,\mathrm{grad\,div}\int\frac{(\Psi^*,\Psi)\Psi(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}d^3r' = \Omega_N\Psi(\mathbf{r}). \tag{35}$$

This equation will subsequently be convenient for establishment of the asymptotic behavior of $\Psi(\mathbf{r})$ as $\mathbf{r}\to\infty$. In essence, in both cases we have performed one and the smae operation—we retained only the longitudinal states with $\lambda = 0$ in the wave function Ψ of the electromagnetic field, which has a spin $S=1$.

To choose correctly the class of functions that describe the lower saddle point, we disregard for the time being the violation of the spherical symmetry (which will inevitably set in later) and consider the solutions of a linear Schrödinger equation of the type (35), in which the second term is replaced by an attracting spherically symmetrical potential. In this case the solutions of (35) should be spherical vectors[35] made up of spherical functions and bases spinors with spin $S=1$. Their set, corresponding to the minimal $\mathscr{J}_N$, should belong to the vector representation of the rotation group, i.e., to the same representation as the initial electric field. The reasoning is that this choice ensures the minimum of zeros of the wave function (in analogy with the choice of the ground state of the acceptor centers[36,5]). Then in the bases chosen as X, Y, and Z the spehrical vector ψ_0 corresponding to a projection $M = 0$ of the total angular momentum $J = 1$ is of the form

$$\Psi_0=\varphi_1(r)\begin{vmatrix}0\\0\\1\end{vmatrix}+\varphi_2(\mathbf{r})\begin{vmatrix}xz\\yz\\z^2-r^2/3\end{vmatrix}, \tag{36}$$

where φ_1 and φ_2 are spherically symmetrical functions (with $\varphi_1(0)$, $\varphi_2(0)\neq 0$). However, the scalar square

$$(\Psi_0^*(\mathbf{r}),\Psi_0(\mathbf{r}))$$

$$=\left\{\varphi_1^2-\frac{2}{3}r^2\varphi_1\varphi_2+\frac{1}{9}r^4\varphi_2^2\right\}+z^2\left\{2\varphi_1+\frac{1}{3}r^2\varphi_2\right\}\varphi_2, \tag{37}$$

which corresponds according to (21) to the field of the inhomogeneous deformation because of the factor z^2 in the second term, has only axial but not spherical symmetry. Therefore in the problem considered the vector character of the field $\tilde{\mathbf{E}}(\mathbf{r})$ leads to spontaneous (essentially, Jahn-Teller) breaking of the symmetry of the self-trapping barrier. Assuming that minimum symmetry breaking takes place, compatible with the lifting of the degeneracy, we shall assume that the functions $\varphi_1(\mathbf{r})$ and $\varphi_2(\mathbf{r})$ preserve central symmetry and axial symmetry with respect to the z axis.

Before we proceed to the numerical calculation, we shall dwell on certain simple physical considerations that permit a better understanding of the structure of the states corresponding to the self-trapping barrier. In the barrier states, the densities $n(\mathbf{r})$ of the electrons and $p(\mathbf{r})$ of the positive ions, as well as the electric field $\mathfrak{G}(\mathbf{r})$ of the ambipolar diffusion are constant in time, while the field $\tilde{\mathbf{E}}(r,t)$ in (14) can vary only in phase, i.e.,

$$\tilde{\mathbf{E}}(\mathbf{r},t)=\tilde{\mathbf{E}}(\mathbf{r})e^{-i\Omega t}.$$

The condition that there be no constant electron and hole fluxes due to the pondermotive forces, diffusion, and drift in the field $\mathfrak{G}$, are of the form

$$-\frac{e^2}{4m\omega_p^2}\nabla|\tilde{\mathbf{E}}|^2-\frac{\nabla\{(n_0+\tilde{n})T_e\}}{n_0+\tilde{n}}-e\mathfrak{G}=0,$$
$$-\frac{\nabla\{(n_0+\tilde{p})T_i\}}{n_0+\tilde{p}}+e\mathfrak{G}=0. \tag{38}$$

Here $\tilde{n}$ and $\tilde{p}$ are nonequilibrium increments to the densities. Assuming that the quasineutrality ($\tilde{n}\approx\tilde{p}$) and linearity ($|\tilde{n}|\ll n_0$) conditions are satisfied, as well as that the potential of the pondermotive forces is small, $e^2|\tilde{\mathbf{E}}|^2/4m\omega_p^2\ll T_e$, and neglecting the spatial dependence of T_e and T_i, we obtain[5)]

$$\frac{\tilde{n}(\mathbf{r})}{n_0}\approx-\frac{e^2|\mathbf{E}|^2}{4m\omega_p^2(T_i+T_e)}\qquad \mathfrak{G}=-\frac{T_i}{T_e+T_i}\frac{e\nabla|\tilde{\mathbf{E}}|^2}{4m\omega_p^2}. \tag{39}$$

At the center of the caviton the field $|\tilde{\mathbf{E}}|$ of the envelope has a maximum, the density n_0+n has a minimum, and the static field $\mathfrak{G}(0)=0$. This distribution corresponds fully to the physical fixture of caviton formation (incidentally, it answers in natural fashion the question of the possible existence of a burst of $n(\mathbf{r})$ near the center of the caviton, a burst that occurred in some earlier studies[1,37]; in the second of them it was demonstrated that spherical supersonic collapse is impossible).

Writing the equations for the vibrational motions against the background of the static solution (39), we easily arrive at Eq. (25). In this equation Ω_N is the frequency shift: the frequency of the local plasma mode produced near the barrier caviton is $\omega_p+\Omega_N$ (with, of course $\Omega_N<0$). The first term in (25) appears in natural fashion as a result of the spatial dispersion of the conductivity[8]

$$m\frac{\partial\mathbf{v}}{\partial t}=-e\left\{\mathbf{E}_0-\frac{3T_e}{m\omega_p^2}\nabla(\nabla\mathbf{E}_0)\right\}. \tag{40}$$

This relation together with the Maxwellian equation

$$\frac{\partial^2\mathbf{E}_0}{\partial t^2}+c^2\,\mathrm{rot\,rot}\,\mathbf{E}_0=4\pi e(n_0+n)\frac{\partial\mathbf{v}}{\partial t} \tag{41}$$

with account taken of $|\Omega_N|\ll\omega_p$ and of (39) leads right away, after changing to dimensionless units, to (25).

We proceed now to a more detailed analysis of the behavior of the function $\Psi(\mathbf{r})$. From a comparison of (36) and (35) it follows that the dominant term in the expansion of the second term of (35) as $r\to\infty$ will be the one connected with the deviation from zero of the z-projection of the vector

$$\int|\Psi(\mathbf{r}')|^2\Psi(\mathbf{r}')d^3\mathbf{r}'.$$

This in turn means that $\Psi(\mathbf{r})$ has the following asymptotic behavior (cf. Ref. 38)

$$\Psi(\mathbf{r})\propto\nabla\partial(r^{-1})/\partial z, \tag{42}$$

i.e., Ψ is anisotropic and decreases like r^{-3}. The behavior as $r\to 0$ is clear from (36), while $\varphi_1(\mathbf{r})$ and $\varphi_2(\mathbf{r})$ are smooth functions.

The numerical calculation was carried out by a variational method by minimizing the functional (24) with $\Psi=\nabla\chi$ and the factor N, previously excluded with the aid of a scale transformation. For χ we choose a linear combination of the functions

$$B_{ln}z^l(r^2+b^2)^{-(n+1/2)}$$

with different l and n satisfying the correct behavior as $\mathbf{r}\to 0$, and ensuring satisfactory approximation also at $r\gtrsim 1$. We note that as $\mu\to\infty$ the leading term in the asymptotic form of χ as $r\to\infty$ is $z/(r^2+b^2)^{3/2}$, and the correct behavior at $r\lesssim b$ is ensured by other terms. At finite μ, the asymptotic variation of χ is exponential, but the necessry quality of the approximation of $\tilde{\Psi}$ with the aid of functions with power-law decrease was monitored both by the convergence of the process with expansion of the bases, and by the use of a similar approximation to the exact solution of Eq. (31). The parameter b was initially maintained fixed, and the function $\tilde{\mathscr{J}}_N(b)$ was calculated by minimization with respect to all B_{ln}. The change of b corresponds to a transition from the free states $b\to\infty$) to the self-trapped ones ($b\to 0$). The maximum $\tilde{\mathscr{J}}_N^{(s)}=\max_b\tilde{\mathscr{J}}_N(b)$ gave the lowest barrier, and from the width of the maximum, which constituted a rather broad plateau, it was possible to deduce a sufficient flexibility of the approximating functions. The final result, in dimenensional units, for the barrier height $W_N = \mathcal{J}_N^{(s)}$ and for the frequency shift $\Omega_N = -\mathcal{J}_N^{(s)}$ is of the form

$$W_N \approx 1.25\cdot 10^3 \frac{g}{N}, \quad \Omega_N = -\frac{W_N}{N}, \quad g = \frac{27 T_e^4 l_D^2}{\pi^2 e^4 \omega_p}. \tag{43}$$

Here $l_p^2 = T_e/m\omega_p^2$ and it is assumed that $T_e \gg T_i$. Attention is called to the large numerical coefficient in W_N. The spatial scale of the barrier caviton is

$$R_N \sim 10^{-2}\, r_0, \qquad r_0 = \pi e^2 \omega_p N/3T_e^2; \tag{44}$$

and the numerical small coefficient of r_0 is the result of the large coefficient in W_N.

It is of interest to note that the quality g, as follows from its initial definition (4), contains C^4 in the denominator at $n = 3$. Therefore, according to (43) we have $\Omega_N \propto (NC^2)^{-2}$. This agrees with fully with the statement that an increase of N is equivalent to an increase of the effective coupling constant, a statement made above when discussing Eq. (32).

A very important fact is that the barrier caviton is anisotropic: it has in its inner part a disk-like shape of a strongly flattened ellipsoid of revolution.[6)] The isolines for the central section of the caviton are shown in Fig. 1. The predominant contribution to the normalization integral for the wave function $\Psi(\mathbf{r})$ of the local plasmon mode is made by the cental part of the caviton (thus, inside the surface $q = 0.225$ is concentrated approximately 80% of the normalization integral, and consequently an equal part of the deficit of the plasma density and of the energy of the high-frequency field ($\tilde{\mathbf{E}}$). The spatial distribution of the field $\tilde{\mathbf{E}}(\mathbf{r})$ is illustrated by Fig. 2. The decisive role for all the physical processes should be played by the central part of the caviton. In particular, lowering the plasma density in it $\bar{n}(\mathbf{r}) \propto -|\Psi(\mathbf{r})|^2$ [cf (39)] produces a "resonator" [1] that forms the considered local mode; the field in the resonator is practically parallel to the symmetry axis z.

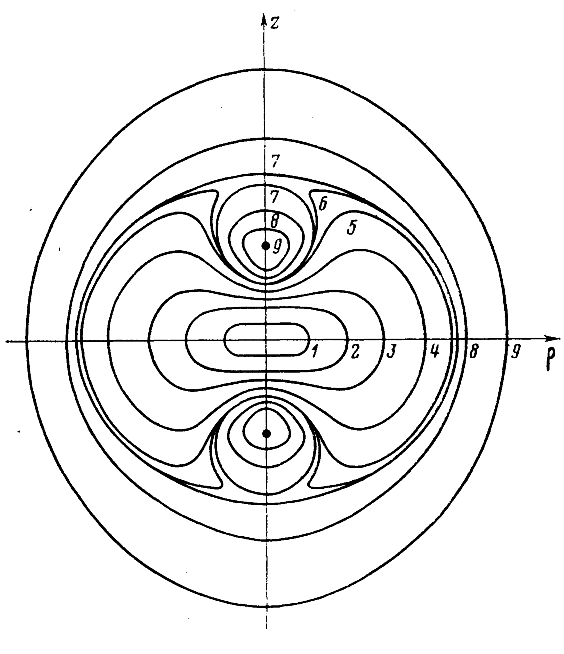

FIG. 1. Isolines in a critical caviton. The central section $z\rho$ is shown (z is the symmetry axis and ρ is the polar radius) the parameter is $q = |\Psi(\mathbf{r})|^2/|\Psi(0)|^2$. Curves: *1*–$q = 0.63$; *2*–$q = 0.225$; *3*–$q = 0.075$; *4*–$q = 0.027$; *5*–$q = 0.019$; *6*-$q = 0.0175$; *7*–$q = 0.0173$; *8*–$q = 0.013$; *9*–$q = 0.006$. The distance between eyelets is $\approx 4.5\times 10^{-2} r_0$ [see (44)].

The course of the isolines in the peripheral region is much more complicated; the complete picture is illustrated in Fig. 1. It can be seen from it that with increasing distance from the internal region the disks "shrink" in their central part and the isolines acquire the shape of a butterfly wing. On the symmetry axis z there are two deep dips of the function $|\Psi(\mathbf{r})|^2$ (the centers of the "eyelets" in Fig. 1). The isolines go around these points in a complicted manner. At the value of the parameter $q_{\rm cr} \approx 0.0174$ (see the caption of Fig. 1), bifurcation takes lace: the two "claws" that surround each "eyelet" (curves 6) are joined. In this case, instead of one isoline at $q > q_{\rm cr}$ there appear three isolines: two of them enclose the eyelets, and the third the entire figure as a whole. At first this isoline is slightly flattened, but with decreasing q it becomes elongated. From (42) it follows that the asymptotic form of these isolines as $q \to 0$ is

$$r^6 \approx (1+3\cos^2\vartheta)/0.6q, \quad \cos\vartheta = z/r. \tag{44a}$$

Therefore at $q \to 0$ the ratio of the axes of the isolines is $d(z)/d(\rho) = 2^{1/3}$. The isolines that enclose the eyelets compress into points at $q \approx 0.001$. It is interesting to note that the eyelets are points where the plasma density has local maxima with $n \approx n_0$. In contrast to the spherically symmetrical barrier caviton with $J = 0$ (see the end of this section), in the present case these maxima are located on the periphery of the caviton (and not at its center!). Although there is also numerical calculations in regions where $|\Psi(\mathbf{r})|^2$ is small are the least reliable, the picture shown in Fig.s 1 and 2 does not change qualitatively when the number of parameters B_{ln} is increased from 5 to 16; Figs. 1 and 2 correspond to a calculation with 16 parameters.

It is natural to expect the shape of the central part of the barrier caviton (Figs. 1 and 2) to remain flattened when the caviton is compressed. Flattened cavitons were predicted earlier by certain numerical calculations for two -dimensional models (see, e.g., Ref. 39). It follows from the foregoing that there were due to the vector character of the electric field and should therefore not appear in scalar models of the collapse (see, e.g., Ref. 40). One more difference from the scalar model is the abrupt increase of the numerical coefficient in (43): in the isotropic model with mass m_l it would be equal to ≈ 44 (as against $\approx 1.25\times 10^3$).

Finally, we emphasize that $W_N \propto N^{-1}$, i.e., it decreases with increasing number N of the trapped plasmons.

We have considered above a state corresponding to the minimum value of W. Besides the function Ψ_0 (Eq. (36)) it is necessary to consider two other spherical vectors $\Psi_{\pm 1}$ corresponding to the projections $M = \pm 1$ of the same total angular momentum $J = 1$. If W is formally calculated for the functions $\Psi_{\pm 1}$, the numerical coefficient of W_N in the equation similar to (43) increases to ≈ 1600, and the caviton becomes elongated in this case. But since the states $\Psi_{\pm}$ do not

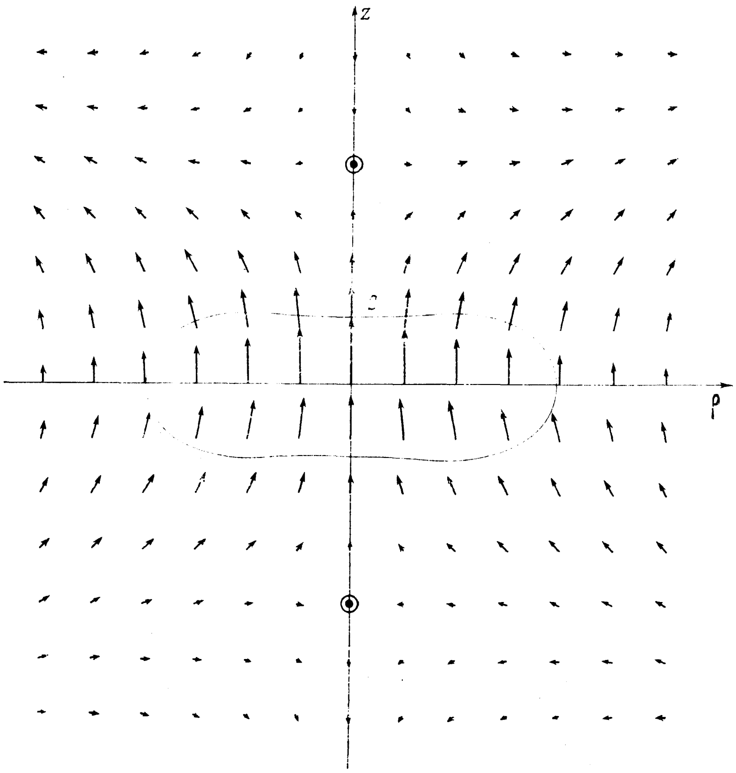

FIG. 2. Distribution of high-frequency electric field $\widetilde{\mathbf{E}}(\mathbf{r})$ in the $z\rho$ palne for the cirtical caviton. The lengths of the arrows are proportional to $\widetilde{\mathbf{E}}$ at the corresponding points (the scale is arbitrary). Curve 2 and the eylets are the same as in Fig. 1.

form a Kramers doublet, this pair of states is unstable and the symmetry should be further lowered. Although in principle it can lead to states that are not axisymmetric (this question was not investigated by us numerically), it seems to us more probable that, in final analysis, after passing through the axially-asymmetric configurations the axial symmetry is restored and states of the type $M = 0$ are produces, but with a symmetry axis other than the z axis.

In an external constant uniform magnetic field $\mathfrak{G}$ oriented along the quantization axis, the degeneracy of the states $\Psi_{\pm 1}$ is lifted. Therefore the instability referred to above in the preceding paragraph vanishes. Then, in principle, it can be assumed that owing to the Zeeman energy the value of W for one of the states $M = \pm 1$ turns out to be lower than for $M = 0$. In an approximation linear in $\mathfrak{G}$, there is added to the left-hand side of (25) the term

$$\tfrac{1}{2} i [\Psi \times \omega_c],$$

where $\omega_c = e\mathfrak{G}/mc$ (Ref. 12), and to the functional (24) is added the term

$$\tfrac{1}{2} i\omega_c \int [\Psi^* \Psi] d^3 r.$$

Since the vector $\Psi_{\pm 1}$ is potential, we write it in the form

$$\Psi_{\pm 1} = \nabla \{(x \pm iy)\varphi(\rho, z)\}.$$

Substitution in the preceding formula leads to the integral

$$\mp \pi\omega_c \int_0^\infty \int_{-\infty}^{+\infty} \frac{\partial(\rho^2|\varphi|^2)}{\partial\rho} d\rho dz = 0,$$

since $\rho^2|\varphi|^2 = 0$ at $\varphi = 0$ and at $\rho = \infty$. Thus, in the linear approximation the magnetic field does not influence the height of the barrier and the shape of the caviton. Among the higher barriers there is also a barrier corresponding to a function Ψ with angular momentum $J = 0$ and a spherically symmetrical caviton (see also Refs. 1 and 37). This Ψ is of the form

$$\Psi_{J=0}(\mathbf{r}) = \begin{vmatrix} x \\ y \\ z \end{vmatrix} \varphi(r), \tag{45}$$

where $\varphi(r)$ is a spherically symmetrical function that decreases exponentially as $r \to \infty$. The corresponding barrier is $W_N(J=0) \approx 1600 g/N$. Incidentally, it is precisely such a function which leads to an unphysical burst of the density at $r = 0$. Indeed, from

$$\Psi_{J=0}(r=0) = \Psi_{J=0}(r=\infty) = 0$$

and Eqs. (38) it follows that $n(r=0) = n(r=\infty) = n_0$.

A comparison of the barriers obtained for cavitons of different shape shows that the lowest is the barrier (43), which corresponds to flattened cavitons.

3. DISCUSSION OF RESULTS

Let us draw some conclusions from Eqs. (43) and (44). We take it into account here that the total energy I_2 [Eq. (20)] is conserved or decreases (when dissipation is taken into account). Therefore, if in a certain regin of space, with radius R, the plasma density is lowered and a cluster of N coherent plasmons with energy I_2 is present, then at $I_2 < W_N$ the cluster will spread out if $R > R_N$, and contrast with formation of a caviton if $R < R_N$. Thus, the barrier separates plasma states with different type of evolution. If $I_2 > W_N$, both types of behavior are possible. At $I_2 < 0$, the plasma bunch will always contract; this last fact is well known.[1]

It is appropriate to note here that the calculated quantity W_N permits definite conclusions to be drawn concerning the behavior of the system not only in the hydrodynamic situation considered above, but also in the static approximation,[1] when the first term of th Lagrangian (15) is left out and it is assumed that the density distribution $\bar{n}(\mathbf{r})$ is determined by the instantantous value of $|\mathbf{E}(\mathbf{r},t)|^2$ in accordance with (39). In this case the evolution of the system is determined by the time-dependent Schrödinger equation for $\widetilde{\mathbf{E}}(\mathbf{r},t)$, and the conserved quantity, having the meaning of the total energy, is the functional $\mathscr{I}_N[E]$, which is determined by Eq. (24) [since the first term in (10) is omitted in the static approximation]. It is very important that in the static (in contrast to the adiabatic!) approximation the functional $\widetilde{\mathscr{I}}_N[\Psi]$, defined by Eq. (24), means something over the entire space of the functions $\Psi(\mathbf{r})$, and its numerical value can be specified arbitrarily. What is critical is the value of W_N/N, where W_N is the lower saddle. The equation $\widetilde{\mathscr{I}}[\Psi] = \text{const} \leqslant W_N/N$ defines two hypersurfaces in functional space, and the functions Ψ_N, which are solutions of (25) and differ from one another only by a phase factor $e^{i\alpha}$, belong to both hypersurfaces and form a saddle-point set on which they are joined together. These are unstable equilibrium points, and small perturbations cause the system to go over either towards a homogeneous state, or towards a deepening of the caviton. Although both dynamic and dissipative processes can contribute to the caviton development, it can be stated that in the case of dynamic evolution solutions with self-similar behavior in all of space are impossible (in the static approximation considered here), provided only that $I_1 = N < \infty$. Indeed, it can be seen from (27) that $\widetilde{\mathscr{I}}_N$ cannot remain constant under the scale transformation (26), regardless of the time dependence of the transformation constant $\lambda(t)$. At $\widetilde{\mathscr{I}} < W_N/N$ the direction of the evolution depends on the ratio of the respective characteristic radii R and R_N of the initial state Ψ and of the functions Ψ_N (see the preceding paragraph). At $\widetilde{\mathscr{I}} > W_N/N$, evolution is also possible in both directions, but it becomes more and more difficult to predict the direction of the evolution with increasing distance of $\widetilde{\mathscr{I}}_N$ from W_N/N, since the succeeding saddles come into play.

We turn now to the adiabatic regime. We can raise the question of the probability of fluctuation formation, in the plasma, of a barrier caviton, i.e., of a state from which the system can become self-compressed (by a dissipative or nondissipative mechanism). Calculation of the probability, strictly speaking, calls for knowledge of the correlation functions of a strongly turbulent plasma. But the simplest estimate can be obtained from thermodynamic considerations, if one assumes the existence of a plasmon temperature T_p and of their average density that fixes the chemical potential of the plasmons ξ. Using a Gibbs distribution with variable number of particles,[41] we can then write down the probability of formation of a barrier state:

$$w_N \propto \exp\left(-\frac{2W_N}{T_i}\right)\exp\left(\frac{\zeta-\Omega_N}{T_p}N\right). \qquad (46)$$

Here $2W_N$, according to (29), is the energy of formation of the fluctuation of the density forms the barrier W_N, in the absence of plasmons; therefore, the first factor is the usual Boltzmann multiplier. The second factor reflects both the entropy loss and the energy gain connected with the density N of the plasmons on one local mode. Recognizing that $W_N = -N\Omega_N$, and putting $(T^*)^{-1} = 2T_i^{-1} - T_p^{-1}$, we obtain

$$w_N \propto \exp\left\{-\frac{W_N}{T^*} + \frac{\zeta N}{T_p}\right\}.$$

The optimal fluctuation occurs at

$$N_{\text{opt}} \approx (W_1 T_p/|\zeta| T^*)^{1/2}. \qquad (47)$$

It can be seen that this estimate can be meaningful only at $T_p > T_i/2$ (when $T^* > 0$). At lower T_p, the formation of the fluctuation already ceases to be a limiting factor: since the change of the sign of T^* takes place when T^* pases through the value $\pm\infty$, according to (47) there are produced small cavitons whose energy can subsequently increase by trapping new plasmons. Actually one deals here already with a cold plasmon gas, whose instability is well known.[42]

The creation of cavitons was discussed throughout above under conditions when there is no external source of plasmon excitation. Yet in many recent experiments there was successfully observed the onset of a spiking turbulence that develops in cavitons under conditions of external excitation (longitudinal high-frequency field or electron beam, see, e.g., Refs. 43 and 44 and the literature cited therein). For a one-dimensional model, an approximate theory of caviton formation with allowance for external pumping and disipative processes was developed in Ref. 15. The formation of cavitons under conditions of external pumping can be naturally understood from the viewpoint developed above it it is recognized that the pumping should lead to an increase of N, i.e., to a continuous accumulation of plasmons on the mode induced by fluctuations; therefore the decrease of $W_N \propto N^{-1}$ with increase N should lead to shutoff at a definite instant of time: once the barrier is surmounted, the process begins to develop spontaneously in a direction towards compression of the plasma bunch and formation of a caviton.

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[1)] The only exception is seemingly Ref. 17, where dynamic equations in imaginary time were considered for tunnel self-trapping. In plasma theory, however, such exponentially slow processes have to our knowledge not been investigated.

[2)] We recall that we have confined ourselves (Sec. 1) to interaction via a deformation potential and therefore excluded polarons from consideration from the very outset.[18]

[3)] The very same criterion is the condition for the absence of strong Landau damping.

[4)] It must, however, be noted that the entire theory (see also below) is burdened with large numerical coefficients. In particular, it turns out that $(W(\infty) - W(\mu)/W(\infty) \approx 50/\mu^2$, therefore the relativistic effects can manifest themselves relatively early. The $W = W(\mu)$ dependence that follows from the functional (2) is given in Ref. 34.

[5)] Starting from Eq. (39) we can easily verify the correctness of the statement made in Sec. 1, that the contribution of the energy of the ambipolar-diffusion field $\mathfrak{G}$ to the total energy is small. Thus, using the equality $\delta n = n_0 \operatorname{div} \mathbf{Q}$, as well as the fact that the field $\mathfrak{G}$ varies over a scale of the dimension of the barrier R_N (see (44) below), it is easy to verify that the ratio of the energy of the field $\mathfrak{G}$ to the second term in (15) is of the order of $(l_D/R_N)^2$; the smallness of this factor coincides with the quasineutrality condition.

[6)] The decrease of the parameter μ weakens the degree of flattening of the caviton and increases the contribution of the transverse electromagnetic fields to its formation; these two tendencies are closely related.

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