Transfer anisotropy in a turbulent plasma

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We formulate a theory for transfer phenomena in a plasma with developed ion-sound turbulence. A transfer anisotropy effect caused by a temperature gradient is revealed. The corresponding fluxes transverse to the effective force vector (1) which generates the turbulence turn out to be considerably larger than the longitudinal fluxes in a plasma with a relatively small degree of nonisothermality. For a strongly nonisothermal plasma a suppression of the transverse fluxes takes place and corresponds to a growth of the thermal insulation of the current-carrying plasma filaments.

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It has been established experimentally[1] that under conditions when ion-sound turbulence (IST) occurs the relationships characterizing transfer proceses in the plasma differ qualitatively from those in a laminar plasma. As a result of the development during the last twenty years of a theory of IST[2–8] it has become possible to formulate a model[9] which eliminates the qualitative contradictions between theory and experiment. At the same time the description of transfer in a turbulent plasma in Ref. 9 was limited to the simplest situations when the directions of the electrical field in the plasma and of the density and electron temperature gradients were the same. This was primarily due to the necessity to compare the consequences of the new theory[9] with the results of preceding papers in which a similar limitation was used.

In the present communication we free ourselves from such a limitation and, using the IST model of Ref. 9, we predict the effect of the anisotropy of transport phenomena in a turbulent plasma caused by the inhomogeneous distribution of the electron temperature $I_e(\mathbf{r})$. The physical cause of the transfer anisotropy predicted by us is the anisotropy of the IST itself, which is determined by the direction of the effective force density vector:

$$en_e\mathbf{E}-\nabla(n_e\varkappa T_e), \qquad (1)$$

where $\mathbf{E}$ is the electrical field strength and e and n_e are the electron charge and number density.

To reveal the transfer anisotropy effect we propound here the basic propositions of plasma kinetics at a high level of IST, when the electron mean free path caused by the scattering by turbulent pulsations is small compared to the characteristic spatial size of the inhomogeneity of the particle distribution, and hence when the energy acquired by an electron along a mean free path length turns out to be much smaller than its thermal energy. Under those conditions one can use the kinetic equation for the electrons in the following approximate form:

$$f_0\left(\nabla\ln(n_eT_e)-\frac{e\mathbf{E}}{\varkappa T_e}+\frac{\partial\ln T_e}{\partial\mathbf{r}}\left[\frac{v^2}{2v_{Te}^2}-\frac{5}{2}\right],\mathbf{v}\right)=\frac{e^2}{2\pi m_e^2}\frac{\partial}{\partial v_i}\int d\mathbf{k}\frac{k_ik_j}{k^2}\frac{\omega_s^3}{\omega_{Li}^2}N(\mathbf{k})\delta(\omega_s-\mathbf{kv})\frac{\partial f}{\partial v_j}, \qquad (2)$$

which is the same as the first approximation of the Chapman-Enskog method.[10] Here f and f_0 are the nonequilibrium and the equilibriumelectron Maxwell distributions, and their difference is small. In Eq. (2) we also used the following notation: m_e is the electron mas, $v_{Te}=(\kappa T_e/m_e)^{1/2}$ is the thermal velocity, ω_{Li} and $\omega_s(k)$ are the ion Langmuir frequency and the ion-sound frequency, $N(\mathbf{k})$ is the number of ion-sound waves with wavevector $\mathbf{k}$.

In our further considerations we take the z axis along the direction of the vector (1) and we choose the zx plane to coincide with the plane in which lie the vectors ∇T_e and (1). Further we shall use spherical coordinates $\mathbf{v}=\{v,\theta_v,\varphi_v\}$ $\mathbf{k}=\{k,\theta_k,\varphi_k\}$ taking the z axis as the polar axis. In correspondence with that

$$N(\mathbf{k})=N(k,\theta_k,\varphi_k),\qquad f(v)=f_0(v)[1+\Psi(v,\theta_v,\varphi_v)],$$

while $|\psi|\ll 1$. Finally, we use the fact that the main contribution to the IST energy, and also to the corresponding transfer coefficients, comes from ion-sound waves with $kr_{De}<1$ when $\omega_s=kv_s$ [$r_{De(i)}$ is the electron (ion) Debye radius, and v_s the ion sound speed]. The turbulent pulsation distribution for such a long-wavelength IST can according to Ref. 9 be written in the form

$$N(k,\theta_k,\varphi_k)=N(k)\Phi(\theta_k,\varphi_k),$$

$$N(k)=4\pi n_e\varkappa T_e v_{Ti}^{-2}\gamma_s(k)k^{-5}\ln(kr_{De})^{-1},$$

where $v_{Ti}=(\kappa T_i/m_i)^{1/2}$ is the ion thermal velocity and $\gamma_s(k)=(\pi/8)^{1/2}(\omega_{Li}/\omega_{Le})\omega_s(k)$ the ion-sound damping rate due to the electrons. As a result the kinetic Eq. (1) for the electrons can be written in the following form:

$$\cos\theta\left[\frac{\partial}{\partial z}\ln(n_eT_e)-\frac{eE_z}{\varkappa T_e}+\left(\frac{v^2}{2v_{Te}^2}-\frac{5}{2}\right)\frac{\partial\ln T_e}{\partial z}\right]+\sin\theta\cos\varphi\left(\frac{v^2}{2v_{Te}^2}-\frac{5}{2}\right)\frac{\partial\ln T_e}{\partial x}$$
$$=\frac{\nu_N v_{Te}^3}{v^4}\left\{\frac{\partial}{\partial\cos\theta}\left[\sin^2\theta\chi_2(\sin\theta,\varphi)\frac{\partial\Psi}{\partial\cos\theta}-\frac{v_s v}{v_{Te}^2}\sin\theta\chi_1(\sin\theta,\varphi)\right]+\frac{\partial}{\partial\varphi}\left[\frac{\xi(\sin\theta,\varphi)}{\sin^2\theta}\frac{\partial\Psi}{\partial\varphi}\right]\right\}, \qquad (3)$$

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where $\nu_N = \omega_{Li} r_{De}^2/(8\pi)^{1/2} r_{Di}^2$ while the functions χ_n and ξ are determined by the following formulae ($n = 1, 2$):

$$\chi_n(\sin\theta,\varphi)=\frac{1}{2}\int d\Omega_k\left(\frac{\cos\theta_k}{\sin\theta}\right)^n\Phi(\theta_k,\varphi_k)\delta\left(\frac{\mathbf{kv}}{kv}\right),$$
$$\xi(\sin\theta,\varphi)=\frac{1}{2}\int d\Omega_k\sin^2\theta_k\sin^2(\varphi-\varphi_k)\Phi(\theta_k,\varphi_k)\delta\left(\frac{\mathbf{kv}}{kv}\right). \quad (4)$$

Similarly the kinetic equation for the ion-sound waves gives (cf. Ref. 9)

$$-\gamma_i-\gamma_e+\frac{\pi v_{Te}^2\omega_s}{n_e}\int_0^\infty f_0\,dv\int d\Omega_v\,\delta\left(\frac{\mathbf{kv}}{kv}\right)\left[\cos\theta_k\frac{\partial}{\partial\cos\theta_v}+\frac{\sin\theta_k}{\sin\theta_v}\sin(\varphi_k-\varphi_v)\frac{\partial}{\partial\varphi_v}\right]\Psi(v,\theta_v,\varphi_v)$$
$$-\gamma_s\int\frac{d\Omega_{k'}}{2\pi}\frac{[\mathbf{kk'}]^2(\mathbf{kk'})^2}{(kk')^4}\Phi(\theta_{k'},\varphi_{k'})=0. \quad (5)$$

This equation corresponds to equating to zero the nonlinear growth rate of the ion-sound waves[9] which is the sum of the damping rates of the waves due to the ions γ_i and due to the equilibrium electrons γ_e, the quasilinear growth rate for the instability by the nonequilibrium electrons (third term), and the nonlinear damping rate caused by the induced scattering of the waves also by the ions (last term).

Bearing in mind the fact that Eq. (3) is linear we can write the function ψ in the form $\psi = \psi_1 + \psi_2$, where ψ_1 is determined by Eq. (3) when $\partial T_e/\partial x = 0$, while ψ_2, on the other hand, corresponds to Eq. (3), if in its inhomogeneous part we retain solely the term $\propto \partial \ln T_e/\partial x$. It is then clear that the v dependence of ψ_2 has the form $v^4[(v^2/2v_{Te}^2) - 5/2]$. This leads to the fact that ψ_2 does not contribute at all to the quasilinear damping rate of Eq. (5). This means that the x dependence of the electron temperature does not affect the IST at all. Hence, the theory of the spectral distribution of the IST does not differ under our conditions from the one expounded in Ref. 9 and this, in particular, enables us to assume that the function ϕ is independent of $\varphi_k(\phi(\theta_k,\varphi_k) = \phi(\cos\theta_k))$. This leads to the fact that the functions χ_n and ξ are functions only of the angle θ. Using this we get from Eq. (3)

$$\frac{\partial\Psi_1(v,\theta)}{\partial\cos\theta}=\frac{v^4}{2\nu_N v_{Te}^3}\frac{1}{\chi_2(\sin\theta)}\left\{\frac{eE_z}{\varkappa T_e}-\frac{\partial}{\partial z}\ln(n_eT_e)-\left[\frac{v^2}{2v_{Te}^2}-\frac{5}{2}\right]\frac{\partial\ln T_e}{\partial z}\right\}+\frac{v_sv}{v_{Te}^2}\frac{\chi_1(\sin\theta)}{\sin\theta\cdot\chi_2(\sin\theta)}, \quad (6)$$

$$\Psi_2(v,\theta,\varphi)=\frac{v^4}{\nu_N v_{Te}^3}\left(\frac{v^2}{2v_{Te}^2}-\frac{5}{2}\right)\frac{\partial\ln T_e}{\partial x}\cos\varphi\cdot y(\theta), \quad (7)$$

where the function $y(\theta)$ is given by the equation

$$\frac{\partial}{\partial\cos\theta}\left[\sin^2\theta\cdot\chi_2(\sin\theta)\frac{\partial y}{\partial\cos\theta}\right]-\frac{\xi(\sin\theta)}{\sin^2\theta}y=\sin\theta. \quad (8)$$

We further use the results of Ref. 9 for the IST spectrum, obtained in the region of not too strong forces and not too strong gradients, when

$$K_N=3\pi\frac{\omega_{Le}^2}{\omega_{Li}^2}\frac{r_{Di}^2}{r_{De}}\left|\frac{e\mathbf{E}}{\varkappa T_e}-\frac{\partial\ln(n_eT_e)}{\partial\mathbf{r}}\right|\ll1. \quad (9)$$

We can then use for the function $\phi(\cos\theta_k)$ the following expression

$$\Phi(x)=\frac{4K_N}{3\pi x}\frac{d}{dx}\frac{x^4}{1-x+\Delta},\quad \Delta=\varepsilon+\delta, \quad (10)$$

which describes the developed IST.[9] In this formula ε is a small parameter:

$$\varepsilon\cong(8K_N/3\pi)\ln K_N^{-1}\ll1.$$

The quantity δ is determined by the damping rate γ_i of the Cherenkov damping of ion-sound waves by ions:

$$\delta=\frac{\omega_{Le}}{\omega_{Li}}\left(\frac{r_{De}}{r_{Di}}\right)^3\exp\left(-\frac{r_{De}^2}{2r_{Di}^2}-\frac{3}{2}\right) \quad (11)$$

and may not be small for not very large values of ZT_e/T_i where Z is the multiplicity of the ionization of the ions. Using the angular distribution (10) we can write Eq. (4) in the following form:

$$\chi_n(\sin\theta)=K_NX_n(\sin\theta),\quad \xi(\sin\theta)=K_N\Xi(\sin\theta), \quad (12)$$

where

$$X_1(\Delta,x)=\frac{1+\Delta}{x}X_2(\Delta,x)-x, \quad (13)$$

$$X_2(\Delta,x)=\frac{4}{3\pi}\left\{-\frac{\pi}{2}(1+\Delta)+\frac{x[2x^2-(1+\Delta)^2]}{(1+\Delta)^2-x^2}+\frac{(1+\Delta)^4\left[\frac{\pi}{2}+\arcsin\frac{x}{1+\Delta}\right]}{[(1+\Delta)^2-x^2]^{3/2}}\right\}, \quad (14)$$

$$\Xi(\Delta,x)=\frac{4}{3\pi}\left\{-x-\frac{\pi}{2}(1+\Delta)+\frac{(1+\Delta)^2}{[(1+\Delta)^2-x^2]^{1/2}}\left[\frac{\pi}{2}+\arcsin\frac{x}{1+\Delta}\right]\right\}. \quad (15)$$

It follows from these formulae that under our conditions one may say that the effective collision frequency is inversely proportional to K_N, i.e., inversely proportional to the modulus of the vector (1). Furthermore, Eqs. (12) to (15) show that it is feasible to use the function $z(\theta) = K_N^{-1}y(\theta)$ which is determined by the differential equation

$$\frac{d}{d\cos\theta}\left[\sin^2\theta\cdot X_2(\Delta,\sin\theta)\frac{dz}{d\cos\theta}\right]-\frac{\Xi(\Delta,\sin\theta)}{\sin^2\theta}z=\sin\theta. \quad (16)$$

It makes sense to establish the boundary conditions which the solution of Eq. (16) satisfies. To do this we note that in the limit $|\sin\theta|\ll 1+\Delta$ it follows from (12) to (15) that

$$X_1(x)=\frac{32}{9\pi}\frac{x^2}{1+\Delta},\quad X_2(x)=\frac{x^2}{1+\Delta},\quad \Xi(x)=\frac{x^2}{3(1+\Delta)}. \quad (17)$$

In the limit $\sin^2\theta\ll1$ Eq. (16) therefore takes the form

$$\sin^2\theta\frac{d^2z}{d(\sin\theta)^2}+3\sin\theta\frac{dz}{d\sin\theta}-\frac{1}{3}z=(1+\Delta)\sin\theta. \quad (18)$$

The solution of this equation has the form ($\sin^2\theta\ll1$)

$$z(\theta)=\frac{3}{8}(1+\Delta)\sin\theta+C_1(\sin\theta)^{2/\sqrt{3}-1}+C_2(\sin\theta)^{-2/\sqrt{3}-1}. \quad (19)$$

By virtue of the condition that $z(\theta)$ be bounded Eq. (19) first gives $C_2 = 0$ and second, shows that Eq. (16) must be solved with the boundary conditions

$$z(0)=z(\pi)=0. \tag{20}$$

To establish the anisotropy of transfer phenomena caused in a turbulent plasma by the nonuniform electron temperature distribution, we use Eqs. (6) and (7) to find expressions for the electrical current density $\mathbf{j} = ed\mathbf{v}\cdot\mathbf{v}f$ and the thermal flux density $\mathbf{q} = (m_e/2)d\mathbf{v}\cdot v^2\mathbf{v}f$:

$$j_z=en_ev_s\{\alpha_j(\Delta)-(24/\pi)\beta_\parallel(\Delta)(\zeta)_z\}, \tag{21}$$

$$j_x=-en_ev_s(24/\pi)\beta_\perp(\Delta)(\zeta)_x, \tag{22}$$

$$q_z=n_e\varkappa T_ev_s\{\alpha_q(\Delta)-(160/\pi)\beta_\parallel(\Delta)(\zeta)_z\}, \tag{23}$$

$$q_x=-n_e\varkappa T_ev_s\beta_\perp(\Delta)(160/\pi)(\zeta)_x, \tag{24}$$

where we have introduced the notation

$$\zeta=|(e\mathbf{E}/\varkappa T_e)-\partial\ln(n_e\varkappa T_e)/\partial\mathbf{r}|^{-1}\nabla\ln T_e. \tag{25}$$

Now $\beta_\perp(\Delta)$ is determined by the solution of Eq. (16):

$$\beta_\perp(\Delta)=-\int_0^\pi d\theta\sin^2\theta\cdot z(\theta). \tag{26}$$

For the remaining coefficients in Eqs. (21) and (23) we have

$$\alpha_j(\Delta)=\frac{3}{2}(1+\Delta)+\left(\frac{16}{\pi}-\frac{3}{2}\right)\beta_\parallel(\Delta), \tag{27}$$

$$\alpha_q(\Delta)=\frac{15}{4}(1+\Delta)+\left(\frac{64}{\pi}-\frac{15}{4}\right)\beta_\parallel(\Delta),$$

$$\beta_\parallel(\Delta)=\frac{1}{2}\int_0^\pi d\theta\sin^3\theta\cdot[X_2(\sin\theta)]^{-1}. \tag{28}$$

In the limiting case of small departures from isothermal behavior

$$\Delta\approx\delta\gg1, \tag{29}$$

when we can use the approximate Eqs. (17) we have

$$\beta_\parallel(\delta)=\delta,\quad \alpha_j=\frac{16}{\pi}\delta,\quad \alpha_q=\frac{64}{\pi}\delta. \tag{30}$$

According to this the function $z(\theta)$ is given by the equation

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\sin^3\theta\frac{dz}{d\theta}-\frac{1}{3}z(\theta)=\delta\sin\theta. \tag{31}$$

Curve 4 of Fig. 1 depicts the solution of this equation which satisfies the boundary conditions (20). To this solution corresponds the value

$$\beta_\perp(\delta)=4.55\delta. \tag{32}$$

Comparison of the values of the coefficients $\beta_\parallel$ and $\beta_\perp$ obtained in the limit (29) allows us to confirm the anisotropy of the transfer. The fluxes transverse to the vector (1) and caused by the temperature gradient are characterized by appreciably larger transfer coefficients than the fluxes along that vector.

In the opposite limit of large departures from isothermal behavior when $\Delta\ll1$ the fluxes along the vector (1) were considered in Ref. 9. We then got

$$\beta_\parallel=0.18,\quad \alpha_j=2.15,\quad \alpha_q=6.74. \tag{33}$$

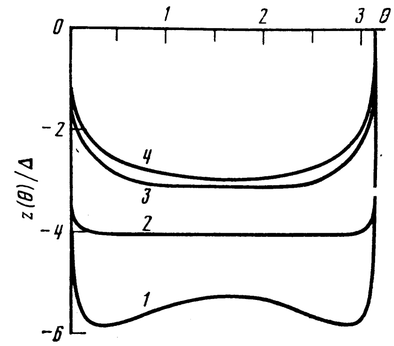

FIG. 1. Angular dependence of the anisotropic part of the electron distribution function $z(\theta)$ for various values of the parameter Δ (1: $\Delta=0.1$; 2: $\Delta=0.3$; 3: $\Delta=3$; 4: $\Delta\gg1$).

The solution of Eq. (13) necessary to find the transverse fluxes is given by Fig. 2 when $\Delta=0$. To this solution corresponds

$$\beta_\perp(0)=0.02. \tag{34}$$

Comparison of $\beta_\parallel(0)$ and $\beta_\perp(0)$ shows that in the limit where there are large departures from isothermal behavior there also occurs transfer anisotropy. However, in the opposite case where there is nearly isothermal behavior the fluxes caused by the temperature gradient in the direction across the vector (1) are now characterized by smaller transfer coefficients than along the vector (1). We given in Fig. 1 (curves 1, 2, and 3) the function $z(\theta)$ for several values of the parameter Δ demonstrating the change in the electron distribution anisotropy when the turbulence anisotropy (10) changes.

Finally, Fig. 3 demonstrates the functions $\beta_\parallel(\Delta)$ and $\beta_\perp(\Delta)$ obtained from numerical calculations. In that case for values $\Delta>0.3$ we have to a good approximation:

$$\beta_\parallel\approx0.14+\Delta,\quad \beta_\perp\approx0.86+4.55\Delta, \tag{35}$$

and correspondingly

$$\alpha_j(\Delta)\approx1.3+5.1\Delta,\quad \alpha_q\approx6.1+20.4\Delta. \tag{36}$$

One should stress that the fluxes in the limit of small departures from isothermal behavior are appreciably larger than those for the case when the departures are large in a plasma with the same value of the electron temperature. This is due to the fact that in a plasma with a smaller departure

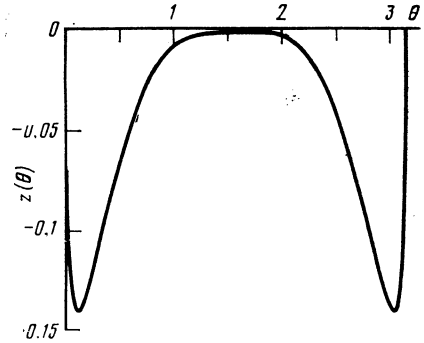

FIG. 2. Angular dependence of the function $z(\theta)$ in the case of weak damping of the ion-sound oscillations due to Cherenkov scattering with ions ($\Delta\ll1$).

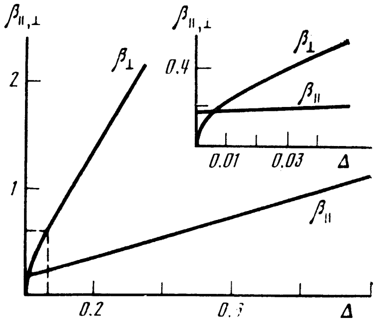

FIg. 3. The transfer coefficients $\beta_\parallel$ and $\beta_\perp$ as functions of the parameter Δ.

from isothermal behavior the level of the IST pulsations turns out to be correspondingly lower and the turbulence limitation of the fluxes therefore turns out to be less. Finally, the phenomenon of a qualitative change in the transfer anisotropy found by us when we change from the case (29) to the opposite one is determined by the change in the anisotropy of the angular distribution (10) of the IST. For the case of axial symmetry (no φ dependence) of the part of the electron distribution a suppression of the nonequilibrium number of electrons across the vector (1) occurs when the departure from isothermal behavior grows. For the axially nonsymmetric part of the distribution, comparison of Figs. 1 and 2 likewise demonstrates the steep suppression of the number of nonequilibrium electrons in the direction across the vector 1. Such a suppression is particularly important for transverse fluxes and this leads to the case when there are large departures from isothermal behavior in the plasma to the effect of turbulent thermal isolation of current carrying plasma filaments.

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