Determination of baryon and baryon-resonance masses from quantum-chromodynamics sum rules. Nonstrange baryons

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The masses of the nucleon, the Δ isobar, and the N * \( J^P = \frac{1}{2}^+ \), \( T = 1 \) resonance as well as the amplitudes of virtual transitions of these states into quark currents are determined on the basis of the QCD sum rules with allowance for the higher-power corrections. The consistency of results obtained from the analysis of different sum rules is demonstrated.

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I. INTRODUCTION

Quantum chromodynamics (QCD) is now becoming a universally accepted strong-interaction theory. Therefore one of the most interesting and most important problems of QCD is a model-free description of hadron mass spectra. The QCD sum-rule method, first proposed in Ref. 1, has made possible much progress in the description of meson properties. This method was extended in Ref. 2 to include baryons, and the first results were obtained there on the masses of both strange and ordinary baryons (see also Ref. 3).

The present paper continues the investigation of the sum rules for baryons. We pursue simultaneously several goals. First, refinement of the sum rules obtained in Ref. 2, by taking higher-power corrections into account, and demonstration, by the same token, of the reliability of the values calculated in Ref. 2 for the nucleon mass and the isobar mass \( m_\Delta \), as well as of residues in the quark currents \( \beta_\alpha \) and \( \gamma_\alpha \). Second, formulation of a new independent set of sum rules from which \( m_\Delta \) and \( m_\Sigma \) can also be determined, and to check the consistency of these two methods of determining the isobar mass \( m_\Delta \) and \( m_\Sigma \). Third, calculations more definite than in Ref. 2, on the basis of the new sum rules, of the mass of the baryon resonance \( N^* \left( J^P = \frac{1}{2}^+ \right) \). Fourth, demonstration of the self-consistency of the results, on the basis of a large overdefined system of sum rules. Fifth, refinement of the amplitudes of the transition of a nucleon into quark currents, which determine the matrix element of the proton decay in the grand-unification theory and the asymptotic form of the nucleon electromagnetic form factor. Sixth and last, finding the magnitude of the quark-gluon condensate \( \langle 0 | \bar{q}q | a(l^2/2)G_\mu \nu u \rangle \) (0) from the conditions of best agreement of the sum rules.

2. THE METHOD

We describe briefly in this section the sum-rule method for baryons, define the notation, consider the properties of the baryon currents, and describe the sum-rule saturation procedure to be used subsequently.

We consider the polarization operator

\[
\Pi_{\alpha\beta}(q) = \frac{1}{\pi} \int e^{imQ} \langle 0 | T (\bar{q}_\alpha(x)q_\beta(x)) | 0 \rangle d^4x,
\]

where \( \eta_\alpha \) is a colorless spin vector made up of quark operators and having the quantum numbers of the baryons (baryon charge, isotopic spin, strangeness); \( \mu \) and \( v \) are the Lorentz indices; \( a \) and \( \beta \) are the spinor indices and will be omitted hereafter; \( m = m_q \).

The polarization operator \( \Pi_{\alpha\beta}(q) \) is calculated in the region \( q^2 = -1 \) GeV\(^2\) where, on the one hand, perturbation theory is still applicable, and on the other the contribution of the nonperturbative effects connected with the structure of the QCD vacuum is no longer small. Nonperturbative corrections will be taken into account when the operator expansion is used. The operator \( \Pi_{\alpha\beta}(q) \) contains various structures \( \langle \eta_\alpha, \eta_\beta, T, \gamma, \gamma, q, q, \ldots \rangle \). For the function at each structure in the polarization operator we write a dispersion relation

\[
\frac{1}{\pi} \int e^{imQ} \frac{\text{Im} f(s) ds}{s - Q^2} = \frac{1}{\pi} \int e^{imQ} \frac{\text{Im} f(s) ds}{s - Q_i^2}, \quad Q^2 = -q^2.
\]

Expression (2) is written in simplified form, since it would actually contain a sufficient number of the subtraction terms necessitated by the divergence of the dispersion integral. If however, we apply to (2) the Borel transformation proposed in Ref. 1,

\[
I(Q^2) = \frac{1}{\pi} \int e^{imQ} \frac{\text{Im} f(s) ds}{s - Q^2} = \frac{1}{\pi} \int e^{imQ} \frac{\text{Im} f(s) ds}{s - Q_i^2},
\]

then all the subtraction terms vanish and we get the equality

\[
I(M_i^2) = \frac{1}{\pi} \int e^{imQ} \frac{\text{Im} f(s) ds}{s - M_i^2}.
\]

Relations (4), written for each of the structure functions of the polarization operator, are sum rules whose left-hand sides are calculated on the basis of quantum chromodynamics, and the right-hand sides phenomenologically. Expressions (4) offer the following advantages over (2): 1) absence of subtraction terms; 2) stronger suppression, due to the factor \( \exp(-m/M_i^2) \), of the contribution of the heavy intermediate states, so that (4) can be used to study the lightest resonances. We consider the following currents \( \eta_\alpha \) with baryon quantum numbers:

\[\text{(continued on the next page)}\]
where \(a, b, c = 1, 2, 3\) are the color indices, \(C = -C^T\) is the charge-conjugation matrix, and \(\sigma_{a\beta} \equiv 2\delta_{\alpha\beta} - \gamma_5\gamma_\alpha \gamma_\beta\). The currents \(q\) and \(qt\) coincide with the currents used in Ref. 2 to obtain sum rules for the case of the nucleon and the \(A\) isobar. The current \(q_{\alpha\beta}\) corresponds to isotopic spin \(T = 3/2\) and to total angular momenta \(J = 3/2\) and \(1/2\) (nucleon, \(N^*\) resonance with \(\lambda = +\), \(\lambda = -\)) etc. We consider first the calculation of the left-hand sides of the sum rules. As already mentioned, the strong-interaction constant is quite small in the region \(|q| < 1\) GeV, and, neglecting \(a_{\alpha\beta}\), the principal diagram corresponding to the zeroth term of the operator expansion is the simple quark loop of Fig. 1a. The first term of the operator expansion (dimensionality \(d = 3\)) is proportional to the quark condensate \((\bar{q}q)(0)\) with \(\bar{q} = u\) or \(d\) (diagram e of Fig. 1). The case of baryon sum rules, however, is the most significant (see Ref. 2), since it causes the appearance of structures that violate the chiral invariance and, in particular, determines the values of the baryon masses. From among the operators with larger dimensionality, next in significance are the four-quark operators \(\bar{q}q\bar{q}q\bar{q}(d = 6)\), whose contribution to the sum rules is described by diagram c of Fig. 1 and, as discussed in Ref. 2, its importance is due to the presence of the large numerical coefficient \(-2\pi^2\) compared with diagram a of Fig. 1. In the vacuum mean values of the four-quark operators we shall use the factorization hypothesis (see Ref. 2), i.e., in the sum over the intermediate states, in the right-hand side of (4),

\[
\text{Im} \Pi_{\alpha\beta}(s) = n \langle 0|\bar{q}\gamma_\lambda q(0)\bar{q}\gamma_\mu q(0)|0\rangle \delta(\delta - M_{\alpha\beta,\mu\nu}),
\]

we shall retain only the contribution of the vacuum state, so that all the four-quark operators averaged over the vacuum reduce effectively to \(\langle 0|\bar{q}\gamma_\lambda q(0)\bar{q}\gamma_\mu q(0)|0\rangle\). Besides the operators indicated above, there appear in the sum rules also two operators with dimensionality \(d = 6\), namely \(\bar{q}q\gamma_\lambda G_{\alpha\beta} G_{\alpha'\beta'}(d = 5)\) and \(G_{\alpha\beta} G_{\alpha'\beta'}(d = 4)\). The most important among them are the contributions of the vacuum mean value of the operator \(\bar{q}q\gamma_\lambda G_{\alpha\beta} G_{\alpha'\beta'}\phi\) (diagrams f of Fig. 1), which are the corrections of first order in \(1/M^2\) to the structures that violate chiral invariance. In the calculation of the polarization operator \(P_{\alpha\beta}(q)\) (the left-hand side of the sum rules) we shall, using the factorization hypothesis, take into account also the vacuum mean values of the operators with dimensionality \(d > 6\):

\[
\langle 0|\bar{q}\gamma_\lambda q(0)\bar{q}\gamma_\mu q(0)|0\rangle \text{Fig. 1c), and } \langle 0|\bar{q}\gamma_\lambda q(0)\bar{q}\gamma_\mu q(0)|0\rangle \text{Fig. 1h). The contribution of the gluon condensate } \langle 0|\bar{q}\gamma_\lambda q(0)\bar{q}\gamma_\mu q(0)|0\rangle \text{ to the baryon sum rules was considered in Ref. 4 and, as already shown there, turned out to be relatively small. In our calculations we shall use the method developed in Ref. 4. The contribution of the operators with dimensionality } d > 6 \text{ was not taken into account previously. All the calculations will be carried out in the zero-mass quark approximation. It must be noted (see Erratum to Ref. 2) that consideration of operators with } d > 5 \text{ in the operator expansions calls also for allowance for the } x \text{ dependence of the vacuum mean values of the fermion operators } \langle 0|\bar{u}\gamma_\lambda u(0)|0\rangle. \text{ In our approxima}-
\]

\[
\langle 0|\bar{u}\gamma_\lambda u(0)|0\rangle \approx \sum_{x} \langle 0|\bar{u}(x)\gamma_\lambda u(0)|0\rangle = \delta_{\lambda 0} \delta_{x 0} \langle 0|\bar{u}\gamma_\lambda u(0)|0\rangle - \delta_{\lambda 0} \delta_{x 0} \langle 0|\bar{u}\gamma_\lambda u(0)|0\rangle - \frac{2}{3} \langle 0|\bar{u}\gamma_\lambda u(0)|0\rangle \delta_{x 0} \delta_{\lambda 0} - \frac{2}{3} \langle 0|\bar{u}\gamma_\lambda u(0)|0\rangle \delta_{x 0} \delta_{\lambda 0} - \frac{2}{3} \langle 0|\bar{u}\gamma_\lambda u(0)|0\rangle \delta_{x 0} \delta_{\lambda 0} - \frac{2}{3} \langle 0|\bar{u}\gamma_\lambda u(0)|0\rangle \delta_{x 0} \delta_{\lambda 0} - \frac{2}{3} \langle 0|\bar{u}\gamma_\lambda u(0)|0\rangle \delta_{x 0} \delta_{\lambda 0} - \frac{2}{3} \langle 0|\bar{u}\gamma_\lambda u(0)|0\rangle \delta_{x 0} \delta_{\lambda 0} - \frac{2}{3} \langle 0|\bar{u}\gamma_\lambda u(0)|0\rangle \delta_{x 0} \delta_{\lambda 0} - \frac{2}{3} \langle 0|\bar{u}\gamma_\lambda u(0)|0\rangle \delta_{x 0} \delta_{\lambda 0} - \frac{2}{3} \langle 0|\bar{u}\gamma_\lambda u(0)|0\rangle \delta_{x 0} \delta_{\lambda 0}).
\]

where : denotes subtraction of the perturbative part of the average over the vacuum. The last two terms of (7) were obtained using the factorization hypothesis. The \(x\) dependence of the vacuum mean value is represented in the diagrams as emission of a soft gluon by a quark with a small momentum.

We use in this paper the following notation for the correlators:

\[
\gamma_{\alpha\beta}^{(n)}(x) = \langle 0|\bar{u}(x)\gamma_\alpha u(0)|0\rangle \delta_{\lambda 0} \delta_{x 0} \langle 0|\bar{u}\gamma_\lambda u(0)|0\rangle - \delta_{\lambda 0} \delta_{x 0} \langle 0|\bar{u}\gamma_\lambda u(0)|0\rangle - \frac{2}{3} \langle 0|\bar{u}\gamma_\lambda u(0)|0\rangle \delta_{x 0} \delta_{\lambda 0} - \frac{2}{3} \langle 0|\bar{u}\gamma_\lambda u(0)|0\rangle \delta_{x 0} \delta_{\lambda 0} - \frac{2}{3} \langle 0|\bar{u}\gamma_\lambda u(0)|0\rangle \delta_{x 0} \delta_{\lambda 0} - \frac{2}{3} \langle 0|\bar{u}\gamma_\lambda u(0)|0\rangle \delta_{x 0} \delta_{\lambda 0} - \frac{2}{3} \langle 0|\bar{u}\gamma_\lambda u(0)|0\rangle \delta_{x 0} \delta_{\lambda 0} - \frac{2}{3} \langle 0|\bar{u}\gamma_\lambda u(0)|0\rangle \delta_{x 0} \delta_{\lambda 0} - \frac{2}{3} \langle 0|\bar{u}\gamma_\lambda u(0)|0\rangle \delta_{x 0} \delta_{\lambda 0} - \frac{2}{3} \langle 0|\bar{u}\gamma_\lambda u(0)|0\rangle \delta_{x 0} \delta_{\lambda 0} - \frac{2}{3} \langle 0|\bar{u}\gamma_\lambda u(0)|0\rangle \delta_{x 0} \delta_{\lambda 0} - \frac{2}{3} \langle 0|\bar{u}\gamma_\lambda u(0)|0\rangle \delta_{x 0} \delta_{\lambda 0}).
\]
g, and normalized into itself in transformations of the re-
normalization group, and their anomalous dimensionalities
are renormalization-covariant, i.e., each of them is transformed
into itself in transformations of the re-
normalization group, and their anomalous dimensionalities
are known. This circumstance allows us to calculate the
principal logarithmic corrections in the usual manner.

We denote by $\gamma$, the anomalous dimensionality of the
current $\eta$, and by $\gamma$, the anomalous dimensionality of the
operator $\Omega$, in the operator expansion of the $T$ product:

$$T(\eta, \gamma) = \sum \mathcal{C}(x) \eta(0).$$

The coefficients of the functions will then contain in the
momentum representation the factor

$$n(x^2) a(x^2) \gamma + n + m_n,$$  

where $n(x^2)$ is the effective QCD charge at the point $x^2$, $\gamma$ is
the normalization point of the operator expansion. For the
momentum representation $\gamma > 0$, the anomalous dimensionality
does not violate the factorization hypothesis to within
10%, and for the operators $\Omega G G G$, $\gamma = \gamma < 0$. The anomalous dimensionality of the operator
$g, G G G, 1/2 g G G \gamma$ is small (see Ref. 6), and we shall there-
fore set it equal to zero hereafter. The anomalous dimensionality
of the operator $\alpha, G^2$ is equal to zero, since this operator is
equal, accurate to terms $a^3$, to the trace of the energy-
momentum tensor of the gluon field. In the numerical calcu-
lations we shall assume $\alpha, G^2 = 4\pi/91 m_0 = 150$
MeV, $\mu = 0.5$ GeV.

We proceed now to consider the right-hand (phenomen-
ological) part of the sum rules. By virtue of the exponential
suppression of the large masses, the dominant contribution
in the right-hand side of $\delta$ is that of the lowest hadron states
in the channel with the given quantum numbers. The right-
hand side of (4) will therefore take accurate account of the
lowest baryon states (in the approximation of infinitely nar-
row resonances), and all the remaining heavier states will be
taken into account via a model. Namely, following Refs. 1
and 2, we approximate the entire remainder by the contribu-
tion of a continuum that starts at a certain threshold $q_0 = M^2$
and is determined by the simplest quark loop corresponding
to the given structure function (i.e., by the diagrams $a$, $b$, and
e, of Fig. 1 for structures with odd and even numbers of $\gamma$
matrixes, respectively).

The amplitude product $\langle \eta \gamma | R \rangle \langle R \eta | 0 \rangle$ in (6) takes
in the case of resonances with spin and parity $J^P = \frac{1}{2}^-$ and
$\frac{3}{2}^-$ the form

$$(0 \gamma_0 \eta^\prime) \langle \gamma_0 | R \rangle \langle R \eta | 0 \rangle = -\frac{\mu^2}{9} \langle \eta_0 \gamma_0 | R \rangle \langle R \eta | 0 \rangle = \frac{m_0^2}{9} \langle \eta_0 \gamma_0 | R \rangle \langle R \eta | 0 \rangle$$

where $M_0$ is the resonance mass; $\delta$, $\gamma$, and $\beta$ are constants
defined by the formulas

$$X(q_0 - q) = \frac{\mu^2}{9} \langle \eta_0 \gamma_0 | R \rangle \langle R \eta | 0 \rangle = \delta \langle \eta_0 \gamma_0 | R \rangle \langle R \eta | 0 \rangle$$

and

$$\delta = \frac{\mu^2}{9} \langle \eta_0 \gamma_0 | R \rangle \langle R \eta | 0 \rangle.$$
We minimize with respect to the parameter $\delta$ the disparities of the sum rules, and thus obtain the masses and residues of the baryons. The fact that several sum rules must be reconciled allows us to improve the accuracy with which the masses and residues of the resonances are determined.

The threshold $W$ of the continuum will not be specified beforehand, but will be determined from the condition of the best agreement of the sum rule in the region $R$. For each polarization operator we can, generally speaking, introduce two different continuums at structures with even and odd numbers of $\gamma$ matrices. The reason is that the contributions of resonance with different parities to a sum rule with an odd number of $\gamma$ matrices are of the same sign, whereas the sign of their contributions to sum rules with even numbers of $\gamma$ matrices are different. By the same token, the threshold of the model continuum can generally speaking be different in these two cases. Since we are restricting the size of the contribution of the continuum to the sum rules, it exerts no significant influence on the calculated masses and residues of the baryons. Even though we require that the sum rules be in agreement only in the region $\Omega$, the fact that the sum rules are satisfied also in the region where the continuum dominates indicates that, despite the crude model of the higher states, the continuum approximates them well enough, meaning that the correction to the baryon mass to account for the continuum is of the correct sign and order. The accuracy of our results is thereby improved.

3. THE NUCLEON

The matrix elements between the states of a nucleon and of the vacuum differ from zero for both currents $q,\bar{q}$, and $q_w$. Therefore the nucleon mass and its residues in the quark currents can be determined by considering the polarization operators $\Pi_{\mu_1}(\gamma_\mu,1,\bar{q})$, $\Pi_{\nu_1}(\gamma_\nu,2,\bar{q})$, and $\Pi_{\mu_1}(\gamma_\mu,1,\bar{q})$. It is clear that the agreement between results obtained from independent sum rules for these polarization operators can serve as an important check on the validity of the entire approach.

a) Sum rules for $\Pi_{\mu_1}(\gamma_\mu,1,\bar{q})$. The calculation of $\Pi_{\mu_1}(\gamma_\mu,1,\bar{q})$ with allowance for the diagrams shown in Fig. 1 leads to the following equations:

$$\Pi_{\mu_1}=\frac{g^2\ln(e^{g'-g})}{16(2\pi)^3}ax^3, \quad \Pi_{\nu_1}=\frac{6g^2\ln(e^{g'-g})}{32(2\pi)^3}ax^3, \quad \Pi_{\nu_1}=\frac{6g^2\ln(e^{g'-g})}{24(2\pi)^3}ax^3, \quad \Pi_{\nu_1}=0,$$

where on the left of the sum rules are indicated the structures for which they have been written, $m$ is the nucleon mass, $\beta_1 = [2\nu^2]\beta_1$, $W_1$ and $W_2$ are the thresholds of the continuum.

The reconciliation of sum rules in the region $\Omega$ yields the following values for the mass and the residue of the nucleon:

$$m=1.02\pm 0.12 \text{GeV}, \quad \beta_1=0.45\pm 0.15 \text{GeV},$$

The scatter of the values of the mass and of the residue are due to the uncertainty of the continuum threshold and to the fact that at a fixed value of the threshold it is possible to choose different $m$ and $\beta_1$ without substantially increasing disparity of the sum rules. Figure 2 shows the $M$ dependence of the left and right parts of these sum rules. The values of $m$ and $\beta_1$ [Eq. (19)] agree well with the corresponding value obtained in Ref. 2. The residue $\beta_1$ determines the amplitude of the proton decay in the SU(5) grand unification theory. Its value (17) confirms the proton lifetime calculations of Ref. 7.

![FIG. 2. Plots of the sum rules of the polarization operator $\Pi_{\mu_1}(\gamma_\mu,1,\bar{q})$.](image-url)
b) Sum rules for $\Pi_{\mu\nu}(q_{\mu},q_{\nu})$. A contribution to $\text{Im} \Pi_{\mu\nu}(q_{\mu},q_{\nu})$ is made only by states with spin $\frac{1}{2}$ (nucleon), with $a_{i} = 0$. The values of the residues $a_{i}$ and $\beta_{j}$ can also be related to each other. We note to this end that the current $\eta_{\mu}$ has the following property:

$$\eta^{\mu}_{\nu} = 0.$$  (20)

Using (13), (20), on the Dirac equation, we obtain

$$a_{0} = \frac{4b_{0}}{m}.$$  (21)

In the case of resonance with negative parity, we must replace $m$ by $-m$ in (21).

Calculation of the polarization operator leads to the equations

$$\Pi_{\mu\nu}^{\prime} = \Pi_{\mu\nu}^{\prime\prime} = 0,$$

$$\Pi_{\mu\nu}^{\prime'} = \frac{1}{2} \frac{1}{(2m)^{2}} \frac{1}{q^{2}} \left\{ \frac{1}{2} \alpha_{0} \left( \frac{a_{0}}{m} \right) \left( -\frac{q^{2}}{m^{2}} + 1 \right) + \frac{2}{9} \frac{q^{2}}{M^{2}} \right\} \eta_{\mu} \eta_{\nu} + \frac{1}{4} \frac{q^{2}}{m^{2}} \eta_{\mu} \eta_{\nu} \frac{a_{0}}{m},$$

$$\frac{1}{2} \frac{1}{a_{0}} \eta_{\mu} \eta_{\nu}.$$  (22)

These lead to three independent sum rules:

$$\langle \eta_{\mu} \eta_{\nu} \rangle = a_{0} \frac{1}{2} \frac{1}{(2m)^{2}} \frac{1}{q^{2}} \left\{ \frac{1}{2} \alpha_{0} \left( \frac{a_{0}}{m} \right) \left( -\frac{q^{2}}{m^{2}} + 1 \right) + \frac{2}{9} \frac{q^{2}}{M^{2}} \right\},$$

$$\langle \eta_{\mu} \eta_{\nu} \rangle = \frac{2}{9} \frac{a_{0}}{m} \frac{q^{2}}{M^{2}}.$$  (23)

From the sum rules (23) we obtain for the mass and residue of the nucleon

$$m = 1.05 \pm 0.15 \text{ GeV}, \quad \beta_{0} = 0.05 \pm 0.15 \text{ GeV}.$$  (24)

Figure 3 shows the behavior of the sum rules as a function of $M$ at the chosen values of the mass, residue, and continuum.

The residue $\beta_{0}$, which can be determined from (19) and (24),

$$\beta_{0} = (2.5 \pm 0.8) \times 10^{-4} \text{ GeV},$$

is of interest because it is connected with the constant $f_{0}$, in the asymptotic expression for the neutron electric form factor. Namely (see Ref. 8), as $Q^{2} 
\rightarrow \infty$ the electric form factor of the neutron is given by

$$F_{e}(Q^{2}) = \frac{2}{9} \frac{1}{q^{2}} \left\{ \frac{1}{2} \alpha_{0} \left( \frac{a_{0}}{m} \right) \left( -\frac{q^{2}}{m^{2}} + 1 \right) + \frac{2}{9} \frac{q^{2}}{M^{2}} \right\} \frac{1}{Q^{2}}.$$  (26)

where the constant $f_{0}$ is determined by the part of the matrix element

$$e^{\mu}(-q) \langle \nu C_{\mu}^{\nu} \rangle \delta^{\nu} \langle \nu | N \rangle = \frac{1}{24} \frac{q^{2}}{M^{2}} \frac{1}{q^{2}} \left\{ \frac{1}{2} \alpha_{0} \left( \frac{a_{0}}{m} \right) \left( -\frac{q^{2}}{m^{2}} + 1 \right) + \frac{2}{9} \frac{q^{2}}{M^{2}} \right\} \frac{1}{Q^{2}}.$$  (27)

that is proportional to $\eta_{\mu}$. Separating from the current in the matrix element (27) the current corresponding to the isospin $\lambda$, we can easily transform the left-hand side of (27) into

$$e^{\mu}(-q) \langle \nu C_{\mu}^{\nu} \rangle \delta^{\nu} \langle \nu | N \rangle = \frac{1}{24} \frac{q^{2}}{M^{2}} \frac{1}{q^{2}} \left\{ \frac{1}{2} \alpha_{0} \left( \frac{a_{0}}{m} \right) \left( -\frac{q^{2}}{m^{2}} + 1 \right) + \frac{2}{9} \frac{q^{2}}{M^{2}} \right\} \frac{1}{Q^{2}}.$$  (28)

where $\eta_{\mu}$ is defined in (5) and we have used Eq. (21) (the terms proportional to $\eta_{\mu}$ are left out). Hence

$$f_{0} = 1.5 \pm 0.8 \times 10^{-4} \text{ GeV}.$$  (29)

Substituting this value in (25) we obtain, say at $Q^{2} = 25 \text{ GeV}^{2}$

$$F_{e}(Q^{2}) = \frac{2}{9} \frac{1}{q^{2}} \left\{ \frac{1}{2} \alpha_{0} \left( \frac{a_{0}}{m} \right) \left( -\frac{q^{2}}{m^{2}} + 1 \right) + \frac{2}{9} \frac{q^{2}}{M^{2}} \right\} \frac{1}{Q^{2}}.$$  (30)

as against the experimental value

$$F_{e}(Q^{2})_{\text{exp}} = 4 \times 10^{-4} \text{ GeV}^{2},$$

i.e., besides the difference in sign, the form factor calculated from the asymptotic formula is smaller than the experimental value by almost two orders of magnitude even at $Q^{2} = 25 \text{ GeV}^{2}$.

c) Sum rules for $\Pi_{\mu\nu}(q_{\mu},q_{\nu})$. In contrast to the already considered sum rules, contributions to $\text{Im} \Pi_{\mu\nu}(q_{\mu},q_{\nu})$ are made not only by particles with spin $\frac{1}{2}$, but also by particles with spin $\frac{3}{2}$. It is therefore necessary to choose structures to which only fermions with spin $\frac{3}{2}$ contribute. Such structures can be chosen by using (11) and (12). They take the form

$$\frac{1}{2} \frac{1}{a_{0}} \frac{q^{2}}{m^{2}} \frac{1}{M^{2}} \frac{1}{q^{2}} \left\{ \frac{1}{2} \alpha_{0} \left( \frac{a_{0}}{m} \right) \left( -\frac{q^{2}}{m^{2}} + 1 \right) + \frac{2}{9} \frac{q^{2}}{M^{2}} \right\} \frac{1}{Q^{2}}.$$  (31)

The sum rules for these structures will be used in the calculation of the nucleon mass.

The calculated polarization operator $\Pi_{\mu\nu}(q_{\mu},q_{\nu})$ is
Comparing the relations between the different structure functions in the right-hand side with Eq. (12), we can readily see that these relations are very close to those obtainable, for example, if the right-hand sides of the sum rules were to contain only a resonance with spin $\frac{1}{2}$. This means that the contribution of spin-$\frac{1}{2}$ states to the sum rules for the polarization operator $\Pi_{\alpha\beta}(q_2^2)$ is strongly suppressed (by an order of magnitude or even more). Therefore the different corrections not taken into account by us, particularly the corrections $\sim \alpha_0$, may turn out to be important for the calculation of this contribution.

For the structures (25) we obtain from (26) the following sum rules:

$$\Pi_{\alpha\beta} = \frac{12}{3} \frac{q^2}{(2\pi)^2} \left[ \frac{1}{M^2 - m_1^2 - \frac{q^2}{2}} \right] \frac{1}{2} \left[ \frac{1}{M^2 - m_1^2} \right]$$

$$\Pi_{\alpha\beta} = \frac{2}{3} \frac{q^2}{(2\pi)^2} \left[ \frac{1}{M^2 - m_1^2 - \frac{q^2}{2}} \right]$$

$$\Pi_{\alpha\beta} = \frac{10}{16} \frac{q^2}{(2\pi)^2} \left[ \frac{1}{M^2 - m_1^2 - \frac{q^2}{2}} \right]$$

$$\Pi_{\alpha\beta} = \frac{2}{3} \frac{q^2}{(2\pi)^2} \left[ \frac{1}{M^2 - m_1^2 - \frac{q^2}{2}} \right]$$

$$\Pi_{\alpha\beta} = \frac{10}{16} \frac{q^2}{(2\pi)^2} \left[ \frac{1}{M^2 - m_1^2 - \frac{q^2}{2}} \right]$$

$$\Pi_{\alpha\beta} = \frac{2}{3} \frac{q^2}{(2\pi)^2} \left[ \frac{1}{M^2 - m_1^2 - \frac{q^2}{2}} \right]$$

$$\Pi_{\alpha\beta} = \frac{10}{16} \frac{q^2}{(2\pi)^2} \left[ \frac{1}{M^2 - m_1^2 - \frac{q^2}{2}} \right]$$

The right-hand side of the sum rule (33b) is positive, whereas the first (principal) term in the left-hand side is negative. Several ways out of the resultant contradiction can be considered. The first, obvious and most likely, is that the sum rules (33) are of no great significance, since the contribution of the unaccounted-for corrections $\sim \alpha_0$ can exceed that of the accounted-for terms. We assume, however, that this is not the case, and attempt to satisfy the sum rules (33) in their present form. It is then clear that the second term in the left-hand side of (33b), which is proportional to $a_{\alpha_0}$, should be large enough, i.e., parameter $m_1^2 > 0.5$ GeV$^2$ is large. As will be shown below, this condition does indeed hold, so that we can attempt to satisfy the sum rule (33). The best fit leads to the following values of the parameters:

$$m = 0.9 \pm 0.2 \text{ GeV}, m_1 = 0.45 \pm 0.45 \text{ GeV}, m_0 = 0.65 \pm 0.15 \text{ GeV}$$

However, as seen from Fig. 4a, the left-hand side of the second of the sum rules, as a function of $M^2$, is in quite poor agreement with the right-hand side, as well as with the left-hand side of the first sum rule. The second sum rule can be brought into agreement only by assuming that besides the nucleon contribution an important role is played in the right-hand side of (33) also by the contribution of the negative-parity resonance which enters in (33b) with a minus sign.

If a negative-parity resonance with mass 1.5 GeV is taken into account explicitly, the agreement between the sum rules improves markedly. This is seen from Fig. 4b. The mass and residue of the nucleon do not change significantly in this case. If we now substitute the mass and residue, which are known from Secs. 3a and 3b, into the sum rules (33), we can obtain the mass of the resonance $N_{\gamma\gamma}$, since its contribution to the sum rules is not small. The mass and residue of this resonance are found to be

$$m_{N_{\gamma\gamma}} = 1.5 \text{ GeV}, \quad \Gamma_{N_{\gamma\gamma}} = 1.0 \text{ GeV}$$

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$$m_{N_{\gamma\gamma}} = 1.5 \text{ GeV}, \quad \Gamma_{N_{\gamma\gamma}} = 1.0 \text{ Gev}
\[ m_\pi = 1.5 \text{ GeV}, \quad \rho_\pi = 0.25 \text{ GeV}. \]  

(35)

The conclusion obtained in this variant should not, of course, be regarded as important. All that matters is that even in this case, which to some extent is extreme, the mass and the residue of the nucleon remain practically the same as before, and the mass of the resonance with \( J^P = \frac{3}{2}^- \) turns out to be large enough. If we consider seriously the variant with allowance for the resonance \( R^- \) with \( J^P = \frac{3}{2}^- \), the same resonance would have to be taken into account also in the sum rules for \( H_{\pi \pi}(\eta, \eta') \) and \( H_{\pi \pi}(\eta, \eta') \). The good agreement between these sum rules without allowance for the resonance \( R^- \) shows that its residue in the current \( \eta_1 \) should be small compared with \( \beta_1 \). This is not surprising, since it has a natural explanation within the framework of the constituent quark model, in which \( R^- \) consists of quarks situated in a p wave.

4. THE RESONANCE \( N_{3/2}^- \)

As seen from (11) and (12), only resonances with spins \( \frac{1}{2}^- \) contribute to the sum rules at the structures \( g_{1/2} \) and \( g_{3/2} \). From (32) and (12) we obtain the following sum rules for a resonance with spin \( \frac{1}{2}^- \):

\[ (\gamma \omega) = \frac{24}{5} \frac{M_\pi M_\omega}{M_\pi^2} \left( 1-e^{-\pi \omega \Lambda} \left( \frac{W_\omega}{M_\pi} + \frac{1}{M_\pi^2} \right) \right) \]

\[ - \frac{2}{3} \pi M_\pi M_\omega \left( 1-e^{-\pi \omega \Lambda} \right) - 16\omega L_\omega, \]

(36)

\[ (\gamma \omega) = 16\pi M_\omega \left( 1-e^{-\pi \omega \Lambda} \left( \frac{W_\omega}{M_\pi} + \frac{1}{M_\pi^2} \right) \right) \]

\[ - 8\pi M_\pi M_\omega \left( 1-e^{-\pi \omega \Lambda} \right) \]

where \( M_\pi \) and \( \Lambda_\pi = (2\pi)^2 \Lambda_\pi \) are the mass and residue of the resonance \( N_{3/2}^- \).

The fact that the sum rule (36) should be saturated with a resonance with negative parity follows from the sign of the second sum rule. The mass and residue of the resonance from (36) are found to be

\[ M_{N_{3/2}^-} = 1.75 \text{ GeV}, \quad \Lambda_\pi = 20-100 \text{ GeV}^4. \]  

(37)

Figure 5 shows the behavior of the sum rules for this resonance. The large scatter of the resonance \( N_{3/2}^- \) in mass and in resonance is due to the fact that the region \( D \) is very small.

5. THE \( \Delta \) ISOBAR

As shown in Ref. 2, the current \( q_\pi^2 \) with isospin \( T = \frac{1}{2} \) is the most suitable for the calculation of the \( \Delta \)-isobar mass. The polarization operator \( H_{\pi \pi}(q^2, p^2) \) is of the form

\[ N \]

(38)

[Figure 5: The mass and residue of the resonance \( N_{3/2}^- \) are chosen to be \( M_{N_{3/2}^-} = 1.6 \text{ GeV}, \quad \Lambda_{\pi\pi} = 0.1 \text{ GeV}^4, \quad W_{\pi\pi} = 1.6 \text{ GeV}, \quad W_{\pi\pi} = 2.2 \text{ GeV} \) are the thresholds of the continuums.]

From the signs of the polarization operator at the structure \( g_{1,2} \), it follows that a resonance with positive parity, i.e., a \( \Delta \) isobar, will predominate in the \( g_{1,2} \) and \( g_{1,2} \) sum rules. The sum rules for \( \Delta \) are

\[ \Pi_{\Delta} = \frac{4}{3} (2\pi)^3 \frac{\rho_\pi}{\pi} \ln \left( \frac{q^2}{\Lambda^2} \right) \left( \frac{M_\pi}{M_\Delta} \right)^2 \left( \frac{W_\pi}{M_\pi} + \frac{1}{M_\pi^2} \right) \]

\[ + \frac{1}{16} \left( \frac{M_\pi^2}{M_\Delta} \right) \left( \frac{W_\pi}{M_\pi} + \frac{1}{M_\pi^2} \right) \]

\[ + \frac{2}{3} \pi M_{\Delta} \left( 1-e^{-\pi \Lambda \Lambda} \right) - 16\omega L_\omega, \]

(38)
FIG. 7. The mass and residues of the resonances are the following:

$$m_1 = 1.0 \text{ GeV}, P_1 = 0.6 \text{ GeV}, m_2 = 1.7 \text{ GeV}, P_2 = 0.25 \text{ GeV}.$$  

The thresholds chosen for the continuums are

$$W_1 = 2.1 \text{ GeV} \quad \text{and} \quad W_2 = 1.6 \text{ GeV}.$$  

If we represent graphically the agreement with the sum rules (see Fig. 7), it can be seen that accurate to 15–30% the right-hand side agrees with the left. We note here that the substituted masses and residues are already those obtained, and are not adjusted for a best fit. We have thus shown that a large number of sum rules can be transformed into one another at a relatively small number of free parameters, thereby verifying convincingly the validity of our analysis.

7. ESTIMATE OF $m_3$

Even earlier, in Sec. 3c, we advanced argument favoring the assumption that $m_3$ is relatively large and positive. In this section we show that the value of $m_3$ can be obtained from other sum rules. To this end we shall use Eqs. (23) and (39), since these sum rules contain $m_3$ with a relatively large weight, and the region $f_2$ is not small.

We shall vary the quantity $m_3$ in the sum rules (23) and (39) and reconcile these sum rules in the best fashion at each fixed value of $m_3$. The mismatch parameter $Q$ will vary in this case. If we plot $Q(m_3)$ we obtain the picture shown in Fig. 8, from which it is seen that $m_3 = 0.6–1.1 \text{ GeV}$. In all the

FIG. 8. Plots obtained from the sum rules (23) (a) and from the sum rules (39) (b).

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preceding calculations we used the value $m_c^2 = 0.8 \text{ GeV}^2$, a good choice in accordance with the result of this section.

8. CONCLUSION

The present investigation of the QCD sum rules for baryons has shown that it of these sum rules, the masses of the nucleon, of the $\Delta$ isobar, and of the resonance with $J^P = \frac{1}{2}^-$, $T = \frac{1}{2}$ as well as their residues in the quark currents. Allowance for higher-power corrections hardly changes the results, which agree with those obtained earlier in Ref. 2 and, within the limits of the calculation accuracy, with experiment. At the same time, the calculated masses were found to be systematically 10–15% higher than the observed ones, possibly indicating that the quark condensate $(\bar{q}q)(0l\lbar{l}) = -(0.24 \text{ GeV})^3$ assumed in the calculation is somewhat (~20%) overestimated. It was shown further that a large number of independent sum rules are in good agreement with one another and lead to the same results for the masses and the residues. On the basis of the sum rules, we determined the value of the quark-gluon condensate

$$g_s(0)[\bar{q}_s(\lambda/2)G_{\mu\nu}\psi(0)] = m_s^2 [\bar{q}_s(0)]\psi(0)^{\prime},$$

and obtained the residues that determine the proton decay amplitude in the SU(5) grand unification theory, and the asymptotic value of the electric form factor of the neutron.

The mass and residue errors cited below correspond to the condition $\Delta - A_{\text{err}}$; $\Delta_{\text{err}}$ is obtained with account taken of the uncertainty of the contribution of the continuum; as a rule $\Delta_{\text{err}} \sim 5\%$.


Translated by J. G. Adashek

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