Influence of effective electron interaction on the critical current of Josephson weak links

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The influence of the magnitude and sign of the effective electron interaction on the stationary properties of Josephson weak links was investigated within the framework of the microscopic theory. In the case of large and small lengths L of the weak links, expressions were obtained for the coordinate dependence of the order parameter, as well as for the critical field I_c as a function of the temperature T. At intermediate lengths L the $I_c(T)$ dependences were calculated numerically for a number of positive and negative values of λ. A comparison is made with the known experimental data.

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1. INTRODUCTION

The microscopic theory of Josephson junctions with indirect conduction (weak links), valid at arbitrary temperatures, was developed previously only for the case when the weak-link material, i.e., the material of the bridge neck or of the sandwich filler, is "genuinely" a normal metal with an interaction constant $\lambda = N(0)V$ equal to zero.[1] In real situations, however, the weak-link material can be a superconductor with $\lambda > 0$ and a critical temperature $T_c > 0$, as well as a normal metal with $\lambda < 0$. Metals with $\lambda < 0$ do not become by themselves superconducting at any temperature, but in electric contact with a superconductor they can carry a supercurrent on account of the proximity effect, i.e., can exhibit superconducting properties. These properties can be described[2] within the framework of the microscopic theory of superconductivity, provided that we replace in them formally T_c by

$$T^* = 1.14\omega_D e^{-\lambda^{-1}} \gg \omega_D, \quad \lambda<0 \quad (k_B=\hbar=1).$$

Unlike T_c, the quantity T^* has no clear physical meaning but is only a convenient parameter that describes the effective repulsion of the electrons.

The question of the influence of the magnitude and sign of λ on the critical current of a Josephson weak link was up to now investigated only at temperatures T close to the critical temperature T_{cs} of the superconducting electrods, in the following particular case: 1) large junction length $L \gg \xi$ (Ref. 3); 2) close temperatures T_{cs}, T and T_c with $T_{cs} \gtrsim T \approx T_c$ (Ref. 4).

The purpose of the present paper is to study the influence of the magnitude and sign of the effective electron-electron interaction constant on the critical currents of Josephson weak links at arbitrary temperatures $0 \leqslant T \leqslant T_{cs}$ and arbitrary junction lengths L.

2. GENERAL RELATIONS

We assume that the materials that make up the weak links satisfy the "dirty" limit conditions of the usual quasi-one-dimensional model which is valid if the transverse dimension w of the junction is much less than the Josephson penetration length λ_J. Introducing, as in Refs. 5 and 6, the functions $\Phi(\omega, x)$, we write down the Usadel equations[7] in the form

$$\Phi=\Delta+(\xi^*)^2\frac{\pi T_{cs}}{\omega G}[G^2\Phi']', \quad G=\omega(\omega^2+\Phi\Phi^*)^{-1/2}, \tag{1a}$$

$$\Delta\ln\frac{T}{T_c}+2\pi T\sum_{\omega>0}\left(\frac{\Delta}{\omega}-\frac{\Phi G}{\omega}\right)=0, \quad \lambda>0, \tag{1b}$$

$$\Delta\ln\frac{T}{T^*}+2\pi T\sum_{\omega>0}\left(\frac{\Delta}{\omega}-\frac{\Phi G}{\omega}\right)=0, \quad \lambda<0, \tag{1c}$$

$$I_S R_N=\frac{2\pi TL}{e}\sum_{\omega>0}\frac{1}{\omega^2}G^2(RI'-IR'). \tag{1d}$$

We have introduced here the quantity

$$\xi^*=(D/2\pi T_{cs})^{1/2}, \tag{2}$$

which is the coherence length of the filler material at a temperature equal to the critical temperature of the electrode materials; the prime denotes differentiation with respect to the coordinate along the current direction, D is the diffusion coefficient, R_N is the resistance of the junction in the normal state, L is its length, and R and I are respectively the real and imaginary parts of the functions Φ.

The system (1) must be supplemented with boundary conditions. We shall assume that on the boundary of the bridge with the superconducting electrodes are satisfied the simple "rigid" boundary conditions

$$R(\pm L/2)=\Delta_s\cos(\varphi/2), \quad I(\pm L/2)=\pm\Delta_s\sin(\varphi/2), \tag{3a}$$

where Δ_s and φ are the modulus and the phase difference of the electrode order parameter. These conditions are realized on the weak-link edges either in variable-thickness bridges (VTB) (Ref. 8) because of the special geometry of the junction, or else in sandwich junctions on account of the substantial difference between the parameters of the materials of the electrodes and the filler.[9,10] Namely, the sandwich filler material must have a sufficiently high resistance:

$$(\sigma_s\xi^*)/(\sigma\xi_s)\gg\min\{1, L/\xi\},$$

where σ_s and σ are the normal-state conductivities of the electrode and filler materials, and ξ_s is the coherent length of the electrode materials.

By virtue of the obvious symmetry of the problem, it can be solved on the segment $[0, L/2]$, assuming that at the point $x=0$ (at the bridge center)

$$R'(0)=0, \quad I(0)=0. \tag{3b}$$

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The solution of the boundary-value problem (1), (3) becomes simpler in the limiting cases of small and large junction lengths.

3. SMALL LENGTH LIMIT

At $L \ll \xi^*$ we can neglect in (1a) the non-gradient terms and obtain for Φ the equation[11,12]:

$$\Phi=\Delta_s \cos\frac{\varphi}{2}+i\left(\omega^2+\Delta_s^2\cos^2\frac{\varphi}{2}\right)^{1/2}\operatorname{tg}\left[\frac{2x}{L}\operatorname{arctg}\frac{\Delta_s \sin(\varphi/2)}{[\omega^2+\Delta_s^2\cos^2(\varphi/2)]^{1/2}}\right]. \tag{4}$$

This solution is valid only at frequencies $\omega \leqslant \omega_0 \approx l^{-2}T_{cs}$, where $l = L/\xi^*$. Since the Eq. (1d) for the supercurrent contains a rapidly converging sum over ω, the sum can calculated using only the first few terms defined by Eq. (4). It follows from the self-consistency equation (1b, c) that to determine the order parameter Δ we must know the behavior of the functions $\Phi(\omega, x)$ also at higher frequencies. It becomes easier to determine $\Delta(x)$ because the system (1) is linearized at the frequencies $\omega \gtrsim \omega_1 \approx T_{cs}$. In this case $\omega_1 \approx T_{cs} \ll \omega_0 \approx l^{-2}T_{cs}$, i.e., there exists a wide range of frequencies in which the solution of the system of linear equations [i.e., of Eqs. (1) with $G=1$] and the solution of (4) overlap.

We seek the function $\Delta(x)$ in the form

$$\Delta=C+iBx, \tag{5}$$

where C and B are as yet unknown constants. Substituting the sought form of the solution (5) in (1a) and putting $G \equiv 1$ we obtain the solutions of the linearized equations

$$R=C+\left(\Delta_s\cos\frac{\varphi}{2}-C\right)\frac{\operatorname{ch}\beta x}{\operatorname{ch}(\beta L/2)}, \tag{6a}$$

$$I=Bx+\left(\Delta_s\sin\frac{\varphi}{2}-BL/2\right)\frac{\operatorname{sh}\beta x}{\operatorname{sh}(\beta L/2)}, \quad \beta=\left(\frac{\omega}{\pi T_{cs}}\right)^{1/2}\frac{1}{\xi^*}. \tag{6b}$$

If $\lambda > 0$ we obtain, substituting the obtained solutions in the self-consistency equation (2b),

$$\Delta\ln\frac{T_{cs}}{T_c}+2\pi T\sum_{\omega>0}\left(\frac{\Delta}{(\omega^2+\Delta_s^2)^{1/2}}-\frac{\Delta}{(\omega^2+\Phi\Phi^*)^{1/2}}\right)$$
$$+2\pi T\left\{\left(\Delta_s\cos\frac{\varphi}{2}-C\right)\sum_{\omega>\omega_1}\frac{1}{\omega}\frac{\operatorname{ch}\beta x}{\operatorname{ch}(\beta L/2)}\right.$$
$$\left.+i\left(\Delta_s\sin\frac{\varphi}{2}-\frac{BL}{2}\right)\sum_{\omega>\omega_1}\frac{1}{\omega}\frac{\operatorname{sh}\beta x}{\operatorname{sh}(\beta L/2)}\right\}=0. \tag{7}$$

The last two sums in (7) converge exponentially in the internal points of the junction ($x \neq L/2$) and are determined by the sum of the terms with frequencies $\omega \lesssim \omega_0$. At $\omega \lesssim \omega_0$ we can put approximately

$$\operatorname{ch}\beta x/\operatorname{ch}(\beta L/2)\approx 1 \text{ and } \operatorname{sh}\beta x/\operatorname{sh}(\beta L/2)\approx 2x/L.$$

As a result we find that the indicated sums are equal with logarithmic accuracy to $\ln l^{-2}$ and $2x/L \ln l^{-2}$. Neglecting the first sum in (7) compared with the logarithmically large remaining terms, we have for $\Delta(x)$

$$\Delta(x)=\Delta_s\frac{\ln l^{-2}}{\ln(T_{cs}/T_c)+\ln l^{-2}}\left(\cos\frac{\varphi}{2}+i2\frac{x}{L}\sin\frac{\varphi}{2}\right). \tag{8a}$$

By similar calculations for the case $\lambda < 0$, recognizing that $\ln(T^*/T_{cs}) \gg \ln l^{-2}$, we obtain for $\Delta(x)$

$$\Delta(x)=-\Delta_s[\ln l^{-2}/\ln(T^*/T_{cs})]\left(\cos\frac{\varphi}{2}+i2\frac{x}{L}\sin\frac{\varphi}{2}\right), \tag{8b}$$

where the real part is negative. It follows from Eqs. (8) that $\Delta(x)$ undergoes on the boundary a jump equal to Δ_s at $T_c=0$ or, equivalently, as $T^* \to \infty$. The size of this jump increases with decreasing T^*, i.e., with increasing repulsion of the electrons in the N-metal. If the filler material, however, is superconducting, Δ is finite even at small T_c and is of the order of Δ_s, while the jump is small. The derived relations (8) can accelerate considerably the numerical solution of the boundary-value problem (1), (3) at finite $L \sim \xi^*$.

4. LARGE LENGTH

In the limit of long junctions, $L \gg \xi^*$, the following cases differ greatly: 1) $T < T_c, \lambda > 0$ and 2) $\lambda < 0$ or $T > T_c, \lambda > 0$. At $T < T_c$ ($SS'S$ junction) a homogeneous current state is realized in the system, and corresponds to a solution of the type

$$\Phi=|\Phi|e^{iqx}, \quad \Delta=|\Delta|e^{iqx},$$

where $|\Phi|$ and $|\Delta|$ are independent of the coordinates. The indicated substitution enables us to reduce (1) to the system of algebraic equations in $|\Phi|$ and $|\Delta|$ which was solved in Ref. 5. It was shown there that the temperature dependence of the critical current, which coincides with the pair-breaking current, is subject to the Ginzburg-Landau (GL) law $I_P(T) \propto (T_c - T)^{3/2}$ only in a small vicinity of T_c and differs substantially from it at lower temperatures. Recent experiments[13] were in good agreement with this theory.

In the case of an extended SNS junction ($T > T_c$ or $\lambda < 0$), Eqs. (1) can be solved by the method proposed in Ref. 4, i.e., we can seek their solutions in the form

$$\Phi=\tilde{\Phi}(x-L/2)e^{-i\varphi/2}+\tilde{\Phi}(x+L/2)e^{i\varphi/2}, \tag{9}$$

where φ is the phase differences between the electrode order parameters. The functions $\tilde{\Phi}(x)$ describe the superconducting properties of a semi-infinite N-metal located in the region $x \leqslant L/2$ and bordering on a superconductor. Since the functions $\tilde{\Phi}$ decrease exponentially with increasing distance from the SN boundary, substitution of the solution (9) in the formula (1d) for the supercurrent leads to a sinusoidal dependence of $I_s(\varphi)$ and to an exponential dependence of I_c on the weak-link length L:

$$I_c=\frac{V_0}{R_N}\frac{L}{\xi}e^{-L/\xi}, \quad L\gg\xi. \tag{10}$$

The actual forms of the quantities V_0 and of the effective coherence length ξ in (10) depend on the relation between the temperature T and the critical temperatures T_{cs} and T_c of the electrodes and of the weak link. Thus, if $T_c=0$ we can obtain for the functions $\tilde{\Phi}(x)$ analytic expressions that lead to explicit dependences of V_0 and of ξ on the temperature[14]:

$$V_0=\frac{64\pi T}{e}\frac{\Delta_s^2}{\{\pi T+\Delta^*+[2\Delta^*(\pi T+\Delta^*)]^{1/2}\}^2}, \quad \Delta^*=[(\pi T)^2+\Delta_s^2]^{1/2},$$
$$\xi=\xi_T=(D/2\pi T)^{1/2}. \tag{11}$$

At $T_c \neq 0$ the determination of V_0 and ξ is greatly facilitated by the presence of two characteristic lengths over which the function $\tilde{\Phi}(x)$ and the order parameter $\Delta(x)$ fall off in the normal metal. In the immediate vicinity of the boundary this falloff is described by the nonlinear equations (1) and takes place at distances on the order of several times ξ^* from the boundary. Further decrease of the functions $\bar{\Phi}$ and Δ takes place with a characteristic length $\xi > \xi^*$ and obeys either a system of linear equations [i.e., Eqs. (1) with $G\equiv 1$] if $T \gg T_c$, or the GL equations if $T \approx T_c$.

In the former case, in the region where $\bar{\Phi}$ and Δ are small, it is convenient to seek the solution in the form of a sum of damped exponentials. At large distances from the boundary, only one exponential is significant in these sums

$$\Phi=\frac{\Delta}{1-\pi T k_N^2/\omega}=\frac{\Delta_1}{1-\pi T k_N^2/\omega}\exp\{k_N(x-L/2)/\xi_T\}, \tag{12}$$

where k_N is the smallest positive root of the equation

$$\psi\left(\frac{1}{2}\right)-\psi\left(\frac{1}{2}-\frac{k_N^2}{2}\right)=\begin{cases}\ln(T/T_c), & \text{if } \lambda>0,\\ \ln(T/T^*), & \text{if } \lambda<0,\end{cases} \tag{13}$$

and $\psi(z)$ is the logarithm of the gamma function. Equation (13) determines the temperature dependence of the effective coherence length ξ:

$$\xi=k_N^{-1}\xi_T. \tag{14}$$

At small values of T_c or at large values of T^* we have for ξ from (13) and (14)

$$\xi\approx\xi_1=[1+2/\ln(T/T_c)]^{1/2} \quad \text{if} \quad \lambda>0, \tag{15a}$$

$$\xi\approx\xi_1=[1+2/\ln(T/T^*)]^{1/2} \quad \text{if} \quad \lambda<0,\quad T\ll T^*, \tag{15b}$$

and as $T \to T_c$ we obtain an equation that coincides with the GL coherence length for a superconductor with a critical temperature T_c:

$$\xi=\xi_{GL}(T)=\xi_T\frac{\pi}{2}\left[\frac{T_c}{|T_c-T|}\right]^{1/2}. \tag{16}$$

We note that expression (15a) follows also from the so-called single-frequency approximation (15), when only the first terms of the sums in (1) are taken into account in the self-consistency equation (1b) and in the equation (1d) for the supercurrent.

The $\xi(T)$ dependence obtained by solving (15) is shown in Fig. 1 (curve 1). Curve 2 shows the $\xi_1(T)$ dependence that follows from the single-frequency approximation.[15] A better approximation to curve 1 (not worse than 3% at arbitrary T/T_c) at $\lambda > 0$ is given by the approximation formula

$$\xi=\xi_T[1+(\pi^2/4)/\ln(T/T_c)]^{1/2}. \tag{17}$$

At negative values of λ the quantity ξ depends relatively weakly on the absolute value of λ, and ranges from ξ_T at $\lambda=0$ to $0.6\xi_T$ in the limiting case of strong electron repulsion $\lambda \to -\infty$.

To find the critical current (10), it remains to obtain the value of V_0. Substituting the solution (12) in (9) and using the expression (1d) for the supercurrent, we obtain the connection between V_0 and Δ_1:

$$V_0=\frac{\pi T}{e}\left(\frac{\Delta_1}{\pi T}\right)^2\psi'\left(\frac{1}{2}-\frac{k_N^2}{2}\right), \tag{18}$$

where $\psi'(z)$ is the derivative of the psi function. The temperature dependence of V_0 is easiest to determine at temperatures $T \approx T_{cs}$ (Refs. 3, 4) when, owing to the smallness of Φ and Δ compared with T_c, the linearized equations (1) are valid in the entire N region. At $T - T_c \gg T_{cs} - T$ and $T \approx T_{cs}$ these equations lead in the case of rigid boundary conditions[3]:

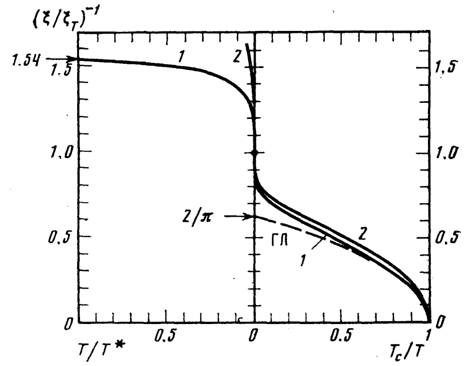

FIG. 1. Curve 1—temperature dependence of the effective coherence length ξ in Eq. (10) for cases of effective attraction ($\lambda > 0$, right half of the figure) and repulsion ($\lambda < 0$, left half) of the electrons. Curve 2—$\xi_1(T)$ dependence that follows from the single-frequency approximation,[15] $\xi_T = (D/2\pi T)^{1/2}$. The dashed line shows the relation (16) that follows from the GL theory.

$$V_0=\frac{32\pi}{7\zeta(3)}\frac{T_{cs}-T}{e}\left[\frac{\ln(T/T_c)}{k_N^2}\right]^2\left[\psi'\left(\frac{1}{2}-\frac{k_N^2}{2}\right)\right]^{-1}. \tag{19}$$

In the limit as $T_c \to 0$, Eq. (19) yields the result of Ref. 1:

$$V_0=\frac{32\pi}{7\zeta(3)}\frac{T_{cs}-T}{e}=\frac{4}{\pi}\frac{\Delta_s^2(T)}{eT}. \tag{20}$$

We determined V_0 numerically for arbitrary values of T. To this it was convenient to change over in (1) to the new functions

$$F=\bar{\Phi}(1-\pi T k_N^2/\omega)\exp\{-k_N(x-L/2)/\xi_T\}, \tag{21a}$$

$$\bar{\Delta}=\Delta\exp\{-k_N(x-L/2)/\xi_T\}, \tag{21b}$$

which have, at sufficiently large distances from the SN boundary, the meaning of the pre-exponential factor contained in (12) and determining the parameter V_0 (18). The system of equations for $\bar{F}$ and $\bar{\Delta}$, which appears when (21) is substituted in (1), was solved with the boundary conditions

$$\bar{F}'(-\infty)=0,\quad \bar{F}(L/2)=\Delta_s(1-\pi T k_N^2/\omega). \tag{22}$$

As T approaches T_c, the difference between ξ_T and ξ becomes according to (16) more and more significant, and to obtain numerical results it becomes necessary to decrease the intervals in the solution of the difference problem, so that several subdivision points are contained within the distance ξ^* from the boundary. This lengthens greatly the computation time.

We have therefore used in the region $T \approx T_c$ another method, more suitable for the problem, of calculating V_0; this method reduces to solving the system (1) on the segment $-x_0 \le x - L/2 \le 0$ with the following boundary condition at the point $x = x_0$:

$$\xi_T\Phi'=k_N\Delta[1+\Delta^2/2\Delta_\infty^2]^{1/2},\quad \Delta_\infty^2=\frac{8\pi^2 T_c}{7\zeta(3)}|T_c-T|,\quad \xi^*\ll x_0\ll\xi_{GL}(T). \tag{23}$$

This condition follows from the requirement that the function Φ become equal to the solution of the GL equations, which are valid in the weak-link region at a sufficient distance from the boundary.

Solution of Eqs. (1) with boundary condition (23) enables to determine the order parameter $\Delta(x_0)$ and its derivative $\Delta'(x_0)$. Using these values and the explicit form of the solution of the GL equations, which is given by Eq. (16) of Ref. 4, we easily obtained the value of Δ_b^*, which is an extrapolation of this dependence to the point $x = L/2$. If we introduce, as in Ref. 4, the quantity

$$A=\Delta_b^*/\Delta_\infty, \tag{24}$$

the coefficient V_0 is expressed in terms of A by the formula

$$V_0=\frac{64\pi}{7\zeta(3)}\frac{|T_c-T|}{e}\frac{A^2}{[1+(1+A^2/2)^{1/2}]^2}. \tag{25}$$

In the temperature region $T \approx T_c \approx T_{cs}$ it follows from (24) that

$$A=(T_{cs}-T)^{1/2}/|T_c-T|^{1/2},$$

and under the additional assumption $T - T_c \gg T_{cs} - T$ Eqs. (25) and (19) lead to identical results for V_0.

The calculated temperature dependences of the parameter V_0 are plotted in Fig. 2. It is seen that the plot of (11), which corresponds to $T_c = 0$, is actually the envelope of the family of $V_0(T)$ plots for different values of T_c. This means that an *SNS* junction with finite critical temperature of the weak-link material behaves in a sufficiently wide temperature range just as a junction with $T_c = 0$, i.e., the function $V_0(T)$ is determined by Eq. (11) but with a different coherence length ξ (14).

Thus, at $L \gg \xi$ the main influence of the effective-interaction constant λ of the electrons reduces to a change of the coherence length of the weak-link material.

5. ARBITRARY LENGTHS AND TEMPERATURES

In the case of arbitrary junction lengths, we determined the characteristic voltage $V_c = I_cR_N$ by the numerical methods described in detail in Ref. 6. The calculation results are given in Fig. 3, which shows the temperature dependence of I_c for different values of T_c/T_{cs} ($\lambda > 0$), T_{cs}/T^*($\lambda < 0$) and L/ξ^*. In the limit of

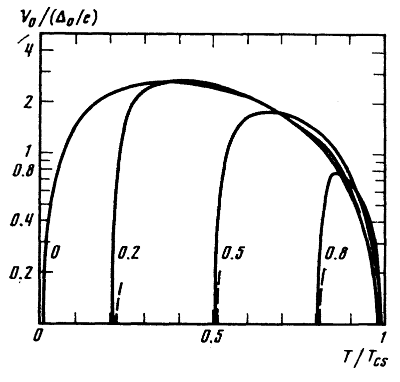

FIG. 2. Temperature dependence of the coefficient V_0 that determines the critical current (10) of long weak links, for different ratios T_c/T_{cs} marked on the curves. The dashed lines show the asymptotic behavior at $T \approx T_c$; V_0 near the critical temperature T_{cs} is determined by Eq. (19).

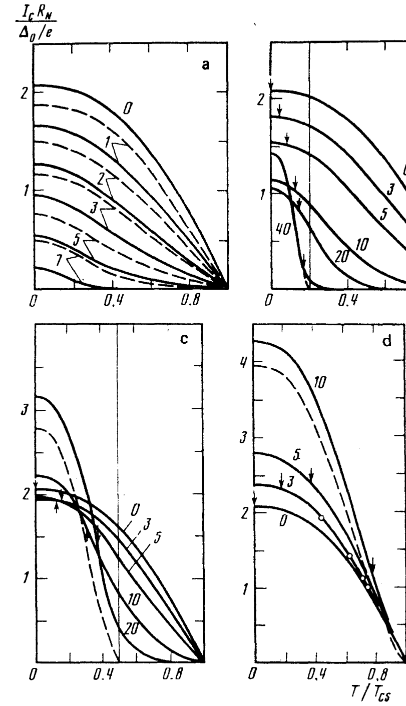

FIG. 3. Temperature dependence of critical current I_c for different values of L/ξ^* and λ: a) $\lambda < 0$, $T_{cs}/T^* = 10^{-3}$ (solid curves), $T_{cs}/T^* = 0$ (dashed), b) $\lambda > 0$, $T_c/T_{cs} = 0.2$; c) $T_c/T_{cs} = 0.5$; d) $T_c/T_{cs} = 1$, the circles show the experimental results for Ref. 21 for bridges. The arrows mark the points at which the $I_s(\varphi)$ become multiple valued; the dashed lines on Figs. b), c), and d) are plots of the pair-breaking currents,[5] and R_N was taken for the maximum values of L/ξ^*, which are marked on the curves.

short junctions ($L \ll \xi^*$) the value of I_cR_N is independent of the properties of the weak-link material and agrees with the one calculated earlier by Kulik and Omel'yanchuk.[11] At finite lengths L the value of I_c depends already substantially on the properties of the weak-link material. If $\lambda \leq 0$, then I_c decreases monotonically with increasing L. The curves calculated for the case of effective repulsion of the material in the weak-link material (solid curves of Fig. 3a) differ insignificantly in this case from the $V_c(T)$ plots for $\lambda = 0$ (dashed curves in the same figure). This proves once more that the stationary properties of the Josephson junctions depend little on the absolute value $|\lambda|$ at $\lambda < 0$.

If $\lambda > 0$ the weak-link material has a critical temperature $T_c > 0$ and the behavior of the characteristic voltage is substantially different at $T > T_c$ and $T < T_c$. In the former case the weak-link material is in the normal state and I_cR_N decreases with increasing L. In the latter case the weak-link material is superconducting and increase of the length L leads to a transition from the Josephson effect to the pair-breaking effect, and hence to an increase of the product I_cR_N. The aggregate of the parameters at which the function $I_s(\varphi)$ becomes multiple-valued can be taken to be the point of this transition; such points are marked by arrows in Fig. 3.

At $T = T_{cs}$ (Fig. 3d) I_cR_N increases monotonically with increasing L at all temperatures and (in the region where $I_s(\varphi)$ is single-valued) it reaches a maximum at $L=(4-6)\xi^*$ in the temperature region $T\approx T_{cs}/2$.

6. COMPARISON WITH EXPERIMENT

Almost all the experiments performed to date were aimed at determining the temperature dependence of the critical current only in structures with sufficiently large weak-links, $L\gg\xi$ (Refs. 16–20). It was observed in these studies, in accord with the theory developed above, that at $T<T_c$ the critical current is determined by pair-breaking effects in the bridge film and behaves like $I_c\propto(T_c-T)^{3/2}$ as T approaches T_c. Below the critical temperature the current falls off rapidly (exponentially).

Harris and Laibowitz[16] investigated ion-implanted molybdenum bridges. The circles in Fig. 4 show the experimental I_c for the sample Mo-15-B-C2 ($L=0.5$ μm). The solid curve is a plot of the $I_c(T)$ dependence that follows from the theory developed above for $L/\xi^*=47$, and agrees within approximately 10% with the independently measured value of this ratio. The thin lines show the asymptotic dependences in the limit of long junctions at $T<T_c$ and $T>T_c$. It is seen that the theoretical curve agrees well with the experimental results. The crosses mark the results of calculations by the Ginzburg Landau theory,[4] which is rigorously valid at temperatures $T\approx T_c\approx T_{cs}$ (the parameter L/ξ^* was assumed equal to 50). The relatively good agreement of even such a simple theory with experiment is in full accord with the results of Sec. 4 of the present paper, where it is shown that even a substantial difference between T_c and T_{cs} (at $T\approx T_c$) leads only to a small change of the pre-exponential factor in (10).

The remaining papers[17-20] dealt with the $I_c(T)$ dependence for *SNS* structures. Good agreement was observed with the theoretical calculations of Ref. 1 in the case $\lambda=0$ (see, e.g., Fig. 2 of Ref. 20). In the case of the variable-thickness Pb-Cu-Pb bridges investigated in Refs. 17 and 18, and in the case of Nb_3Sn-Cu-Nb_3Sn (Ref. 19), it was found (see Fig. 5) that the theory developed above, which takes into account the effective repulsion of the electron in the bridge film (solid curves), leads to better agreement with the experimental data than the results of Ref. 1 (dashed curves). The discrepancy between theory and experiment at $T\approx T_{cs}$ is due to the suppression of the superconductivity on the banks, i.e., with deviation from the rigid boundary conditions (3a) used by us.[12]

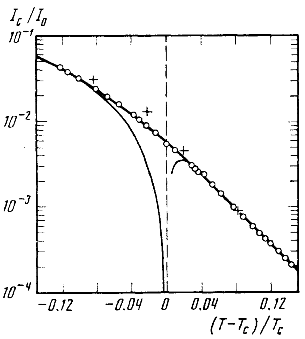

FIG. 4. Temperature dependence of the critical current. Circles—experimental data for the Mo-15-B-C2 sample of Ref. 16. Thick line—dependence calculated in accord with the present theory for $L/\xi^*=47$. The thin lines show the asymptotic dependences at $T>T_c$ and $T<T_c$ in the limit of long junctions. The crosses mark the points calculated[4] by the GL theory for $L/\xi^*=50$, I_0 is the proportionality coefficient in the temperature dependence of the pair-breaking current in the GL theory: $I_c=I_0(T_c-T)^{3/2}$.

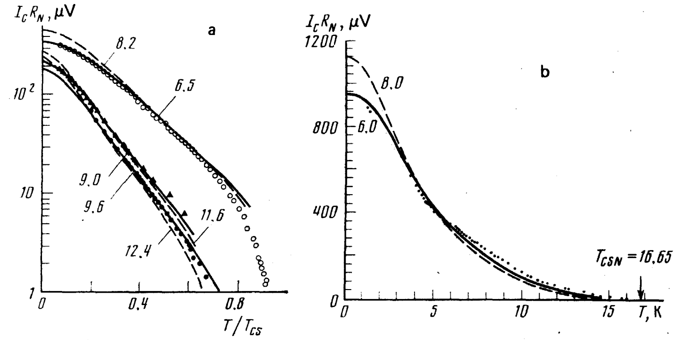

FIG. 5. Experimental temperature dependences of I_cR_N: a—for Pb-Cu-Pb bridges from Refs. 17 and 18; b—for Nb_3Sn–Cu–Nb_3Sn bridges from Ref. 19. Solid curves—theoretical plots for $\lambda<0$ ($T_{cs}/T^*=10^{-2}$) and $L/\xi^*=6.5$; 9.0; 9.6 (a), $L/\xi^*=6.0$ (b). Dashed curves—theoretical plots for $\lambda=0$ ($T_{cs}/T^*=0$) and $L/\xi^*=8.2$; 11.6; 12.4 (a), $L/\xi^*=8.0$ (b). The theoretical curves of Fig. 5(b) were calculated using for T_{cs} the value 17.5 K, which differs from the critical temperature $T_{csN}=16.65$ K of the composite SN electrode and corresponds to the critical temperature of only the superconducting film. In case (a) the difference between T_{cs} and T_{csN} was insignificant. The values of L/ξ^* indicated in Figs. 4 and 5 were chosen to obtain best agreement with experiment.

It can thus be concluded that the electron interaction in the investigated Cu films is repulsive. However, the low sensitivity of I_c to the value of λ at $\lambda<0$, as well as the additional decrease of I_c as a result of the suppression of the electrode superconductivity in these structures by the proximity effect do not make it possible to determine this parameter with sufficient accuracy.

Finally, Feuer and Prober[21] investigated bridges of variable thickness having banks and bridges made of the same material ($Pb_{0.9}In_{0.1}$ alloy) and having short lengths ($L\approx400$ Å) that are records for such structures. Their experimental data are marked by circles in Fig. 3d. It can be seen that they agree well quanitatively with the theoretical $I_c(T)$ dependence at $L/\xi^*\approx3$. The coherence length $\xi^*\approx130$ Å that follows from this comparison is close to the estimate made by the authors of the paper ($\xi^*\lesssim160$ Å) and is typical of such materials.[22]

7. CONCLUSION

An investigation of the influence of effective electron interaction in the weak-link material on the critical current has shown that the presence of effective repulsion ($\lambda<0$) does not lead to a substantial lowering of values of the critical current I_c of Josephson junctions. The reason is that the coherence length of the weak-link material depends relatively little on the absolute value of λ in this case.

At $\lambda > 0$ the stationary properties of the Josephson junctions differ substantially for the temperatures $T < T_c$ and $T > T_c$. In the former case ($SS'S$ structures) an increase of the length L is accompanied by a transition from the Josephson effect to the pair-breaking effect. In the second case (SNS structure) the values of $I_c R_N$ decrease monotonically with increasing L, and in the limit $L \gg \xi$ the properties of structures with finite T_c are practically the same as at $T_c = 0$, but with a somewhat different effective coherence length ξ (Fig. 1).

Comparison of the results of the theory with the latest experimental data has shown them to agree well, although the low sensitivity of the properties of the junction to the value of λ at $\lambda < 0$ does not make it possible as yet to determine $|\lambda|$ from the experimental data.

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