Reflection of fast electrons normally incident on a surface

I. S. Tilinin

Moscow Engineering Physics Institute
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A theory is developed of the reflection of fast electrons normally incident on a surface; the theory is valid in a wide range of initial energies from several keV to several MeV. The results are valid when the total range R_0 of the electrons in the substance exceeds greatly their transport length l_{tr}. The main idea of the solution of the problem is that an effective isotropic elastic-interaction cross section, equal in size to the transport cross section, is introduced for the description of the multiple scattering of the electrons by the atoms of the medium. Since the ratio R_0/l_{tr} exceeds unity for nonrelativistic electrons even in the case of beryllium, the theory developed is applicable at nonrelativistic energies to practically all the elements of the periodic table. The expressions obtained for the energy and angular spectra of the reflected particles, as well as for the total reflection coefficients, agree well with numerous experimental data.

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1. INTRODUCTION

Inelastic scattering of fast electrons by a solid surface is a constituent of the secondary electron emission and plays an important role in many problems of modern physical electronics. In addition, electron reflection from the surface of a material is of undisputed interest in problems of the radiation endurance of the surface and in flaw detection.

Inelastic reflection of fast electrons (with initial energies higher than several keV) has been the subject of rather prolonged and quite thorough investigation.[1-24] The main law governing the backscattering of normally incident beam electrons reduce to the following:

1. The total reflection coefficients and energies of the particles increase monotonically with increasing atomic number Z of the scatterer.

2. At nonrelativistic initial electron energies, the total reflection coefficients are practically independent of the particle energy, whereas those of relativistic electrons decrease with increase in initial energy.

3. The energy spectra of backscattered electrons take the shape of domes whose peaks are reached at energies close to the initial incident-particle energy.

4. The half-widths of the energy distributions decrease rapidly with increasing Z.

5. The angular distribution of the reflected particles is close to cosinusoidal.

Some of the noted features of the reflection process admit of a qualitative explanation.[24] However, the development of a unified theory capable of quantitatively describing the phenomenon has met the fundamental difficulties until most recently.

The theoretical research into electron backscattering followed mainly two directions. The first approach was based on a proposed model of single collision with deflection of the electron through a large angle.[25] Subsequently Dashen,[26] using the method of invariant switching, obtained for the differential backscattering coefficient a nonlinear integral equation whose solution he sought by iteration. Confining himself only to the first-order approximation, Dashen obtained in fact a solution corresponding to the single-collision model. Kalashnikov and Mashinin[27,28] analyzed the backscattering process on the basis of a quantum-mechanical formalism. The expression for the differential coefficient in Ref. 27 was represented in the form of an infinite series, each term of which corresponded to a definite number of large-angle scatterings. In contrast to Ref. 26, it was recognized in Ref. 27 that between two large-angle scattering events a particle undergoes multiple small-angle scattering. Taking the ratio of the second term of the iteration series to the first to be small, an assumption valid for sufficiently light media, Kalashnikov and Mashinin[28,29] likewise confined themselves to first-order approximation. The important question of the rate of convergence of the series was left thereby open. The expressions obtained in Refs. 25–28 for the total albedo of the electrons have a common shortcoming: they increase without limit with increasing Z.

A different approach was developed by Archard[29] and Tomlin[30] on the basis of the "diffuse" model of reflection. According to this model the electrons move in the medium along straight lines up to a certain depth l_D, after which they diffuse in all directions, i.e., they are multiply scattered through arbitrary angles. Unfortunately, in view of the far-fetched simplifying assumptions, the results of Refs. 29 and 30 can be regarded only as approximate.

It must be emphasized that the foregoing does not pertain to the case of grazing incidence of a beam on a target, when the angles, reckoned from the surface, at which the particles enter the medium, are small. The solution of the reflection of fast charged particles at grazing incidence was obtained in Refs. 31–33, whose results are equally valid for relativistic electrons provided that the scattering angles of the reflected particles are small.

The purpose of the present paper is to construct for backscattering of normally incident electrons a theory free of restrictions on the number of collisions in which the electron is reflected through an angle of the order

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of unity, and valid in a wide range of incident-particle energy and nuclear charge of the target atoms.

2. TRANSPORT EQUATION. REFLECTION

We consider a monoenergetic beam of electrons incident normally on the surface and having an initial energy E_0. We assume that the particle initial energy satisfies the condition

$$1\,\text{keV} \ll E_0 \ll Z^{-1}\cdot 10^7\ \text{keV}. \tag{1}$$

In this case the deceleration of the electrons is due mainly to inelastic collisions with the target atoms. On the contrary, scattering of particles with deviation of their trajectory from the initial direction is determined mainly by the elastic collisions.[24,34]

We choose a Cartesian coordinate frame with the z axis directed into the interior of the medium and the xy plane on the surface. We denote by $N=N(z,\mu,E)$ the flux density, at a depth z, of electrons of energy E moving at an angle $\vartheta=\arccos\mu$ to the z axis. The transport equation for the function $N(z,\mu,E)$ with allowance for condition (1), is written in the form

$$\mu\frac{\partial N}{\partial z}=-n_0[\sigma_{el}(E)+\sigma_{inel}(E)]N+n_0\int d\Omega'\frac{d\sigma_{el}(\Omega'\to\Omega)}{d\Omega'}N(z,\mu',E)$$
$$+n_0\int_0^{\varepsilon_{max}}d\varepsilon\frac{d\sigma_{inel}(E+\varepsilon|\varepsilon)}{d\varepsilon}N(z,\mu,E+\varepsilon), \tag{2}$$

where n_0 is the density of the atoms of the medium, $\sigma_{el}(E)$ and $\sigma_{inel}(E)$ are the total cross section of the elastic and inelastic interactions, respectively, $\Omega=\mathbf{v}/v$ (v is the particle velocity), and ε_{max} is the maximum possible energy lost by the electron in one collision with an atom of the material. Equation (2) must be supplemented by the boundary condition

$$N(z=0,\mu,E)=\begin{cases}N_0\delta(\mu-1)\delta(E-E_0), & \mu>0\\ N_0S(\mu,E), & \mu<0\end{cases}. \tag{3}$$

Here N_0 is the flux density in the incident beam, $S(\mu,E)$ is the reflection function and characterizes the distributions of the reflected electrons in energy and in emission angle from the target. The explicit form of the functions $S(\mu,E)$ must be found in the course of the solution.

At an arbitrary dependence of the cross sections of the inelastic and elastic scatterings on the energy E, it is impossible to solve Eq. (2) with boundary condition (3), To change to a simpler form of the transport equation, we consider when the total range R_0 of the electrons in the medium greatly exceeds their transport length $l_{tr}=2/\langle\theta_s^2(E_0)\rangle$ ($\langle\theta_s^2(E_0)\rangle$ is the mean square of the multiple scattering angle per unit path of a particle with energy E_0):

$$R_0\gg l_{tr}. \tag{4}$$

The inequality (4) means that rapid isotropization of the electrons over the directions takes place against the background of slow degradation of the particle energy. We add that condition (4) is well satisfied over the entire energy range (1) for heavy media, and in the nonrelativistic case the ratio R_0/l_{tr} exceeds unity even for beryllium.

Taking the condition (4) into account, we assume that most particles are backscattered by the medium without a noticeable loss of velocity, so that the average energy of the reflected particles is close to E_0. This allows us to confine ourselves to the approximation

$$\sigma_{el}(E)\approx\sigma_{el}(E_0)=\text{const},\quad \sigma_{inel}(E)\approx\sigma_{inel}(E_0)=\text{const}. \tag{5}$$

All the electrons can be arbitrarily broken up into two groups. We include in the first the particles that have traversed in the medium a path $s\leqslant l_{tr}$, while in the second the path is $s>l_{tr}$. Quantitative estimates show that in the case of normal incidence the contribution of the electrons of the first group to the total number of reflected particles is small and does not exceed 1% of the total number of those incidenton the target. This is easily understood, for to be backscattered from the medium a particle must be deflected by an angle $\vartheta\gtrsim\pi/2$, i.e., traverse a path

$$s\geqslant\pi^2/4\langle\theta_s^2\rangle\sim l_{tr}.$$

The total reflection coefficients of fast normally incident electrons range from 4% for beryllium[24] to 50% for lead and uranium.[10] This leads to the important conclusion that the reflected beam is made up practically entirely of electrons of the second group. By virtue of the fast isotropization of the particles over the directions, the passage of the second group of electrons through the medium can be described by a transport equation with an effective isotropic elastic-scattering cross section equal in magnitude to the transport cross section $\sigma_{tr}=(n_0l_{tr})^{-1}$.

Thus with the approximation (5) taken into account, the flux density of the electrons of the second group satisfies the equation

$$\mu\frac{\partial N}{\partial z}=-n_0[\sigma_{tr}(E_0)+\sigma_{inel}(E_0)]N+\frac{1}{2}n_0\sigma_{tr}\int_{-1}^{1}N(\mu',z,E)d\mu'$$
$$+n_0\int_0^{\varepsilon_{max}}d\varepsilon\frac{d\sigma_{inel}(E_0|\varepsilon)}{d\varepsilon}N(z,\mu,E+\varepsilon). \tag{6}$$

[For details of the derivation of (6) see the Appendix.]

When formulating the boundary conditions for (6) we shall neglect the fact that the electrons of the first group go over, with increasing traversed path, into the second group in a layer of thickness l_{tr}. The last assumption corresponds to the inequality (4). Taking the foregoing into account we can write

$$N(z=0,\mu,E)=\begin{cases}N_0\Phi(l_{tr},\mu,E), & \mu>0\\ N_0S(\mu,E), & \mu<0,\end{cases} \tag{7}$$

where $\Phi(l_{tr},\mu,E)$ stands for the angular and energy scatter of the electrons that negotiate a path $s=l_{tr}$ in the target. Since the reflection of the electrons of the first group is small, the function $\Phi(l_{tr},\mu,E)$ can in principle be obtained by solving exactly the transport equation (2) in an infinite medium.

For convenience in the calculations that follow, we introduce the dimensionless variables

$$\xi=z/l_{tr}=z\langle\theta_s^2(E_0)\rangle/2;\quad \Delta=(E_0-E)/E_0. \tag{8}$$

In the variables (8), Eq. (6) and the boundary conditions (7) take the form

$$\mu\frac{\partial N}{\partial\xi}=-N+\frac{1}{2}\int_{-1}^{1}N(\xi,\mu',E)\,d\mu'+\frac{1}{\sigma}\int_{0}^{\infty}d\Delta' w_{inel}(E_0|\Delta')\times[N(\xi,\mu,\Delta-\Delta')-N(\xi,\mu,\Delta)],\qquad(9)$$

$$N(\xi=0,\mu,\Delta)=\begin{cases}N_0\Phi(l_{tr},\mu,\Delta), & \mu>0\\ N_0S(\mu,\Delta), & \mu<0,\end{cases}\qquad(10)$$

where

$$w_{inel}(E_0|\Delta')=n_0R_0\frac{d\sigma_{inel}(E_0|\Delta')}{d\Delta'}.\qquad(11)$$

The function $w_{inel}(E_0|\Delta')$ is proportional to the probability that the relative energy function lost by the energy per unit path amounts to Δ'. The upper limit of integration with respect to the variable Δ' in (9) is taken to be infinity, so that the following condition is satisfied.

$$N(\xi,\mu,\Delta)=0,\quad \Delta<0.\qquad(12)$$

Equation (9) contains only one dimensionless parameter σ, equal to the ratio of the total range of the particles to their transport length:

$$\sigma=R_0/l_{tr}=R_0\langle\theta_s^2(E_0)\rangle/2.\qquad(13)$$

The quantity σ can be interpreted as the mean number of collisions between the electron and the atoms of the medium over the entire range, wherein each collision deflects the particle trajectory from the initial direction by an angle of the order of unity. The larger σ, the more intense the elastic scattering and the higher the probability of backward emission of the particle from the target. As seen from (13), the parameter σ plays here the same role as the quantity $\langle\theta_s^2\rangle R_0/4\zeta_0^2$ in the theory of reflection of charged particles at grazing incidence (ζ_0 is the angle of entry of the particle into the medium, reckoned from the surface, $\zeta_0\ll 1$).[32] Moreover, the indicated quantities practically coincide in the transition region at $\zeta_0\sim\pi/4$.

To solve Eq. (9) we take the Laplace transform with respect to the variable Δ. Multiplying both sides of this equation by $\exp(-p\Delta)$ and integrating with respect to Δ from 0 to ∞, we obtain for the function

$$N(\xi,\mu,p)=\int_0^\infty e^{-p\Delta}N(\xi,\mu,\Delta)\,d\Delta\qquad(14)$$

the equation

$$\mu\frac{\partial N}{\partial\tau}=-N+\frac{\omega(p)}{2}\int_{-1}^{1}N\,d\mu'\qquad(15)$$

with the boundary condition

$$N(\tau=0,\mu,p)=\begin{cases}N_0\Phi(l_{tr},\mu,p), & \mu>0\\ N_0S(\mu,p), & \mu<0\end{cases},\qquad(16)$$

where

$$\omega(p)=\sigma/[\sigma+W(p)],\quad \tau=\xi/\omega(p),\qquad(17)$$

and the function $W(p)$ is defined with the aid of the relation

$$W(p)=\int_0^\infty d\Delta' w_{inel}(E_0|\Delta')[1-e^{-p\Delta'}].\qquad(18)$$

Equation (15) is the transport equation with an isotropic complex scattering indicatrix. The role of the single albedo is played here by the quantity $\omega(p)$. The solution of Eq. (15), as well as the corresponding albedo problems, has been well investigated.[35] According to this reference, the reflection function $S(\mu,p)$ can be represented in the form

$$S(\mu,p)=\frac{\omega(p)}{2}\int_0^1 d\mu'\Phi(l_{tr},\mu',p)\frac{\mu' H[|\mu|,\omega(p)]H[\mu',\omega(p)]}{|\mu|+\mu'},\qquad(19)$$

where $H(\mu,\omega)$ is the Chandrasekhar H function.[36]

Taking the inverse Laplace transform with respect to the variable p, we obtain a final expression for the distribution of the reflected particles:

$$S(\mu,\Delta)=\frac{1}{4\pi i}\int_C dp\,e^{\Delta p}\omega(p)\times\int_0^1 d\mu'\Phi(l_{tr},\mu',p)\frac{\mu'}{\mu'+|\mu|}H[\mu',\omega(p)]H[|\mu|,\omega(p)].\qquad(20)$$

We emphasize that the integration contour C in the complex p plane is chosen such that it does not intersect the curve segment on which $W(p)$ takes on real values in the interval from $-\sigma$ to 0.

3. TOTAL REFLECTION COEFFICIENTS. ENERGY SPECTRUM AND ANGULAR DISTRIBUTION OF THE REFLECTED ELECTRONS

The spectrum of the backscattered electrons is best described by the differential backscattering coefficient $R(\mu,\Delta)$. The quantity $R(\mu,\Delta)$ is equal by definition to the ratio of the number of particles emitted from a unit surface of the target in the direction $\vartheta=\arccos(-|\mu|)$ and losing an energy Δ, to the number of particles incident on a unit surface. The differential backscattering coefficient is thus proportional to the reflection function $S(\mu,\Delta)$, calculation of which calls for knowledge of the concrete form of $W(p)$, as well as of the function $\Phi(l_{tr},\mu,p)$. In the case of fast electrons the probability that the particle will lose an energy Δ in one collision is determined by the Rutherford formula

$$w_{inel}(E_0|\Delta')=(2L_{ion}\Delta'^2)^{-1},\qquad(21)$$

where L_{ion} is the ionization logarithm. Substituting (21) in (18) and applying a procedure similar to that used in Ref. 37 to find the energy spectrum of electrons in a thin layer of matter, we write $W(p)$ in the form

$$W(p)=p+p[1-\beta-\ln p]/2L_{ion}.\qquad(22)$$

In the last expression $\beta=0.577$ is the Euler constant.

In the calculations that follow we neglect for simplicity the angle scatter of the electrons that have traversed a path $s=l_{tr}$ (the ensuing error in the final results does not exceed several percent). We put accordingly

$$\Phi(l_{tr},\mu,p)=\delta(1-\mu)\exp[-W(p)/\sigma],\qquad(23)$$

where $W(p)$ is defined via (22). In the case considered, obviously,

$$R(\mu,\Delta)=|\mu|S(\mu,\Delta).\qquad(24)$$

We consequently obtain from (20) and (24) for the differential backscattering coefficient

$$R(\mu,\Delta)=\frac{|\mu|}{4\pi i}\int_C dp\exp\left[\Delta p-\frac{W(p)}{\sigma}\right]\frac{\omega(p)}{1+|\mu|}H[1,\omega(p)]H[|\mu|,\omega(p)].\qquad(25)$$

Integrating (25) over the energies and emission angles of the electrons from the target, we obtain the total electron reflection coefficient r:

$$r=\int_{-1}^{0}d\mu\int_{0}^{1}d\Delta R(\mu,\Delta). \tag{26}$$

The quantity $1-r$ is equal to the relative number of electrons stoppped in the target. Substituting (25) in (26) and recognizing that in the case of fast electrons the fluctuations of the energy losses are small ($2L_{ion}\gg 1$), we have for the total reflection coefficient r

$$r=\{1-(1+\sigma)^{-1/2}H[1,\ \sigma(\sigma+1)^{-1}]\}e^{-\rho(\sigma)}, \tag{27}$$

$$\rho(\sigma)=1/\sigma+h(2\sqrt{2}-1-\sqrt{3})/2\sigma^{1/2},\quad h=H(1,1). \tag{28}$$

It follows from (27) that in the approximation (5) the total reflection coefficient is a universal function of the parameter σ. At $\sigma\ll 1$ the value of r is exponentially small, since the probability of particle scattering through an angle $\pi/2$ over the entire range is small. In the case $\sigma\gg 1$, on the other hand, the probability of particle emission from the target increases and approaches unity like

$$r\approx 1-h/\sigma^{1/2}. \tag{29}$$

A plot of $r(\sigma)$ is shown in Fig. 1.

Let us analyze in greater detail the behavior of the total reflection coefficient r as a function of the initial electron energy E_0 and of the charge Z of the target atoms. We substitute in (13) for this purpose the explicit expressions for the mean sqaured multiple-scattering angle per unit path of an electron of energy E_0 (Ref. 38):

$$\langle\theta_s^2(E_0)\rangle=8\pi n_0 r_e^2 Z(Z+1)(E_0+1)^2L_n(Z,\ E_0)/E_0^2(E_0+2)^2, \tag{30}$$

$$L_n(Z,\ E_0)=\ln\{241Z^{-1/3}(E_0+1)[E_0(E_0+2)]^{1/2}\}-1-[E_0(E_0+2)/2(E_0+1)^2]^2 \tag{31}$$

and for their total path[34]:

$$R_0=E_0^2/4\pi n_0 r_e^2 Z(E_0+1)L_{ion}. \tag{32}$$

In (30) and (32) r_e is the classical electron radius, and the energy E_0 is expressed in units of m_ec^2. Making this substitution, we arrive at the following expression for the parameter σ:

$$\sigma=\frac{2(Z+1)(E_0+1)}{(E_0+2)^2}\frac{L_n}{L_{ion}}. \tag{33}$$

The parameter σ in the energy range (1) is thus practically proportional to Z, so that it increases with increasing atomic number of the scatterer atoms. This leads in turn to a monotonic increase of the relative number of reflected particles r with increasing Z.

The ratio L_n/L_{ion} depends little on E_0 in a wide energy range and is approximately equal to 0.5. In the nonrelativistic energy region ($E_0\ll 1$) we consequently have $\sigma\approx(Z+1)/4$, so that the total albedo of the electrons is only a function of Z. On the contrary, in the relativistic case ($E_0\gg 1$) the parameters σ decreases with increasing E_0, and $\sigma\sim(Z+1)/E_0$; this lowers the yield of reflected particles with increase of their initial energy.

A quantitative comparison of the theory with the experimental data[1-18,20-24] is illustrated on Fig. 2. This figure shows the measured reflection coefficients r of beryllium, aluminum, copper, silver, and uranium in a wide range of initial energies E_0. The solid theoretical curves were calculated from Eq. (27) with account taken of (28) and (33). Good agreement between theory and experiment is observed for almost all elements. The only exception is aluminum, for which the difference between the theoretical and experimental data reaches in some cases 25%. With increasing atomic number Z of the target the agreement between the theory and experiment improves, and the difference between the corresponding values of r does not exceed several percent. This is understandable, for large $Z\gg 1$ correspond to heavy media, where condition (4) is known to be satisfied.

With the aid of the expression for the differential backscattering coefficient (24) we can easily calculate the total energy reflection coefficient γ:

$$\gamma=\int_{-1}^{0}d\mu\int_{0}^{1}(1-\Delta)R(\mu,\Delta)d\Delta, \tag{34}$$

as well as the average relative energy of the electrons emitted from the target

$$\langle E/E_0\rangle=\gamma/r. \tag{35}$$

The quantity $1-\gamma$ represents the fraction of the primary-electron energy absorbed in the target material, and plays an important role in the calculation of the truly secondary δ-electrons.[24]

The actual value of the average relative energy $\langle E/E_0\rangle$ of the reflected electrons at a given value of the parameter σ is most important from the point of view of confirming the theory developed, since all the foregoing results were obtained under the assumption $\langle E/E_0\rangle\approx 1$. At large values $\sigma\gg 1$ the following relation is valid

$$\left\langle\frac{E}{E_0}\right\rangle=\frac{1-\sqrt{2}(\sigma+2)^{-1/2}H[1,\sigma(\sigma+2)^{-1}]}{1-(\sigma+1)^{-1/2}H[1,\sigma(\sigma+1)^{-1}]}e^{-\rho(\sigma)}, \tag{36}$$

where $\rho(\sigma)$ is defined as before with the aid of (28). It follows from (36) that $\langle E/E_0\rangle$ varies from 0.6 to 1 when the parameter σ is varied from 5 to ∞. This means that the approximation (5) is perfectly justified in this range of variation. Plots of γ and $\langle E/E_0\rangle$ against σ, obtained by numerically integrating (34), are shown in Fig. 1.

The value of $\langle E/E_0\rangle$ was measured in Refs. 3, 6, 10, 39, and 40. The results of these measurements and theoretical curves calculated from (36) with allowance

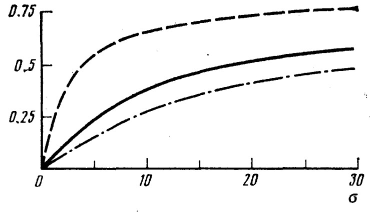

FIG. 1. Total reflection coefficient r (solid curve, average relative energy $\langle E/E_0\rangle$ (dashed curve) and total energy reflection coefficient γ (dash-dot curve) vs the parameter σ.

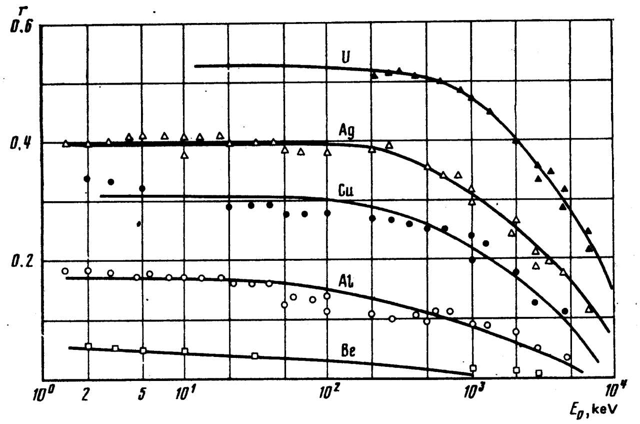

FIG. 2. Total fast-electron reflection coefficient r in a wide range of variation of the initial energy E_0, for various substances. Solid curves—calculated from Eq. (27), points—experiment[1–18,29–24]: □—Be, ○—Al, ●—Cu, △—Ag, ▲—U.

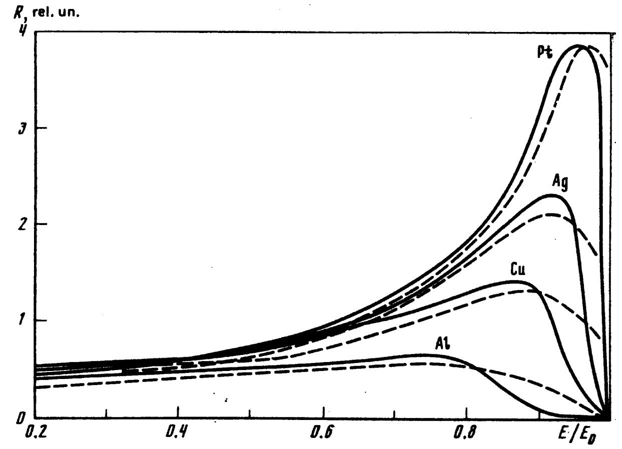

FIG. 4. Energy distributions of nonrelativistic electrons (E_0=40 keV) reflected from various targets. Solid curves—calculated from Eq. (27), dashed—experiment.[6]

for (28) and (33) are shown in Fig. 3. As expected, the results of the theory agree best with the experimental data at large values of the parameter σ.

The energy spectrum $R(\Delta)$ of the reflected electrons, regardless of the direction of particle emission from the target, can be obtained by integrating (25) with respect to μ from 0 to -1:

$$R(\Delta)=\int_{-1}^{0} d\mu R(\Delta,\mu)$$
$$=\frac{1}{2\pi i}\int_c dp\exp\left[p\Delta-\frac{W(p)}{\sigma}\right]\{1-[1-\omega(p)]^{1/2}H[1,\omega(p)]\}. \quad (37)$$

Although no simple expression can be obtained for $R(\Delta)$, the behavior of the energy spectrum of the electrons can be easily investigated in limiting cases.

An examination of (31) shows that at low energy losses, $\Delta \ll 1/\sigma$, the character of the spectrum is determined to a considerable degree by the fluctuations of the energy losses. In this spectral energy-loss region the rapidly oscillating exponential factor of the integrand in the right-hand side of (37) has a stationary point in the complex p plane. This enables us to estimate the integral (37) by the stationary-phase method. We have thus for $R(\Delta)$ in this case

$$R(\Delta)\approx\frac{\sigma}{(8\pi)^{1/2}h}\exp\left\{-\frac{|\lambda|-1}{2}-\exp(|\lambda|-1)\right\},\quad \Delta\ll\frac{1}{\sigma}, \quad (38)$$

$$\lambda=\{\Delta\sigma-(1-[\beta+\ln(2L_{ion}\sigma)-1]/2L_{ion})\}2L_{ion}. \quad (39)$$

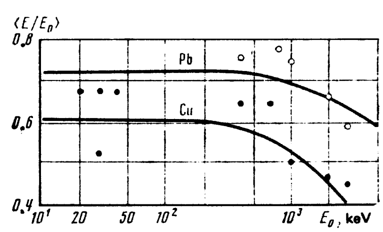

FIG. 3. $\langle E/E_0\rangle$ vs the initial energy E_0. Solid curves—calculated from Eq. (36), points—experiment, ○—Pb, ●—Cu; E_0=20–40 keV—data of Ref. 6, 32 keV—Ref. 2, 380–680 keV—Refs. 39 and 40, 1–3 MeV—Ref. 10.

It follows from (38) that the contribution to the energy spectrum of the particles that lose a very negligible fraction of their initial energy is exponentially small.

The function $R(\Delta)$ reaches a maximum at the point $\Delta=\Delta_{ne}\approx 1/\sigma$, and the maximum, accurate to small terms of order $(2L_{ion})^{-1}$ is equal to

$$R(\Delta_{ne})\approx\sigma/2h. \quad (40)$$

At large energy losses, $\Delta \gg 1/\sigma$, the energy scatter of the electrons is due entirely to the stochastic character of the paths traversed by the particles prior to leaving the target. At $\Delta\gg 1/\sigma$ the $R(\Delta)$ distribution decreases smoothly like

$$R(\Delta)\approx\frac{\sigma h}{2\pi^{1/2}}(\Delta\sigma+2)^{-3/2}\sim\Delta^{-3/2}. \quad (41)$$

To verify the validity of the last statement, we expand $1-\{1-\omega(p)\}^{1/2}H[1,\omega(p)]$ in powers of the single-collision albedo $\omega(p)$. The function $R(\Delta)$ can then be represented in the form

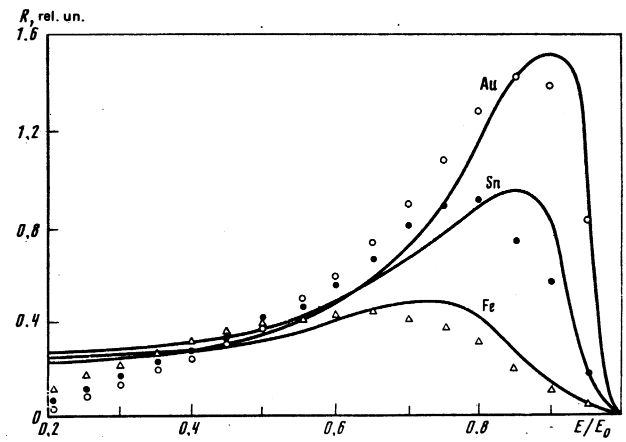

FIG. 5. Energy distributions of relativistic electrons (E_0 =1 MeV) reflected from various targets. Solid curves—calculated form Eq. (37), points—experiment[19]: ○—Au, ●—Sn, △—Fe.

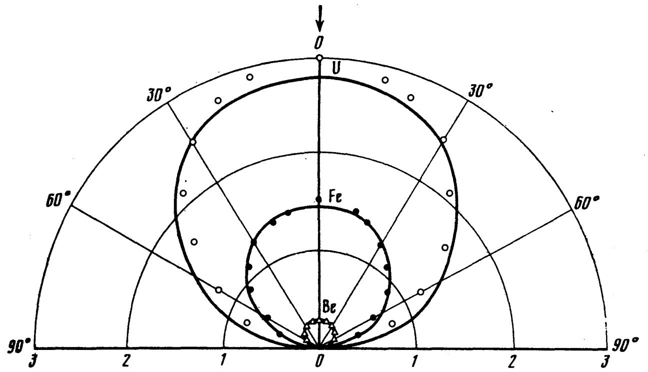

FIG. 6. Polar diagrams of angular distribution of electrons ($E_0=10$ keV) reflected from various targets. Solid curves—calculated from Eq. (45), points—experiment[22]: ○—U, ●—Fe, △—Be. The arrow marks the direction of incidence of the primary beam.

$$R(\Delta)=\frac{1}{2\pi i}\int_{C} dp \exp\left[\Delta p-\frac{W(p)}{\sigma}\right]\sum_{n=1}^{\infty} b_n \omega^n(p). \qquad (42)$$

The expansion coefficients b_n in (42) are defined as

$$b_n=\frac{1}{2\pi i}\int_{\Gamma}\left[1-(1-\omega)^{1/2}H(1,\omega)\right]\frac{d\omega}{\omega^{n+1}}. \qquad (43)$$

The integration contour Γ in (43) is a circle of radius $a_0<1$ that encompasses the origin of the complex ω plane.

The representation (42) is useful in many cases for the understanding of the backscattering process as a whole. Indeed, writing the transport equation with the effective elastic-scattering indicatrix (6), we have replaced by the same token the real substance by a medium filled with certain effective particles that scatter the electrons isotropically. Allowance for the deceleration has led to the appearance of a complex single-scattering albedo $\omega(p)$. This makes clear the physical meaning of the coefficients b_n: they are proportional to the probability of the electron being emitted from the medium as a result of n-fold scattering by the effective scattering centers. Electrons losing an energy $\Delta \gg \sigma^{-1}$ obviously leave the target after experiencing a large number of collisions. The low-energy part of the spectrum receives therefore the main contribution from those terms of the series (42) which have large numbers $n\gg 1$. Inasmuch as at $n\gg 1$

$$b_n=\frac{h}{2\pi^{1/2}}(n+3)^{-3/2}, \quad \omega^n(p)\approx \exp\left\{-\frac{nW(p)}{\sigma}\right\}, \qquad (44)$$

it follows that by substituting (44) in (42), integrating with respect to p, and summing over n we arrive at (41). It is of interest to note that in the case of grazing incidence the reflected-particle energy distribution calculated in the approximation (5) decreases like $\Delta^{-5/4}$ (Ref. 32), which is very close to (41).

The electron energy spectra calculated from (37) for various targets are shown in Figs. 4 and 5. Figure 4 shows the energy distributions of the backscattered electrons with initial energy $E_0=40$ keV bombarding aluminum, copper, silver, and platinum targets. Figure 5 shows the analogous distributions for relativistic electrons ($E_0=1$ MeV) reflected from gold, tin, and iron targets. The same figures show the spectra measured in Refs. 6 and 19, respectively. For convenience in comparison, just as in Refs. 6 and 19, the abscissas represent the relative energy E/E_0 of the emitted electrons. We note that in this case the "tie-in" between the theoretical and experimental curves is no problem, since the total reflection coefficients r (equal to the areas under the corresponding curves) have approximately equal measured and theoretically calculated values. It follows from Figs. 4 and 5 that Eq. (37) describes correctly both the position of the maximum in the energy spectrum and the decrease of the intensity peak of the reflected particles on going from heavy to lighter targets.

A disparity between the experimental and theoretical spectra is observed in the low-energy part of the distributions, where the approximation (5) is generally speaking inapplicable.

To find the angular distribution of the reflected particles, we integrate the differential backscattering coefficient (25) with respect to the variable Δ:

$$R(\mu)=\int_0^1 d\Delta R(\mu,\Delta)=\frac{\sigma}{2(1+\sigma)}\frac{|\mu|}{1+|\mu|}H\left(1,\frac{\sigma}{1+\sigma}\right)H\left(|\mu|,\frac{\sigma}{1+\sigma}\right)e^{-\rho(\sigma)}. \qquad (45)$$

In the derivation of (45) we have neglected the fluctuations of the electron energy losses, since their influence on the angular spectrum is negligible. Taking into account the weak dependence of the product $(1+|\mu|)^{-1}H(|\mu|,\sigma(1+\sigma)^{-1}$ on the variable $|\mu|$ in the interval from 0 to 1, we arrive at the conclusion that actually $R(\mu)\sim|\mu|$. The angular distribution of the reflected follows thus approximately the well known cosine law.[24] Figure 6 shows typical angular spectra of the reflected electrons (initial energy $E_0=10$ keV), calculated theoretically from Eq. (45) and measured in Ref. 22 for the case of uranium, iron, and beryllium and targets. The good agreement between the theoretical and experimental data is due in this case both to the correct functional dependence of (45) on the variable μ and to the fact that the experimental and theoretical values of the total reflection coefficients are approximately equal.

4. CONCLUSION

The theory expounded above is based on the idea that at sufficiently long paths the scattering of an electron beam in a medium can be described with the aid of an effective isotropic elastic-scattering cross section. As applied to the reflection problem, this approach is justified if the contribution to the emerging flux of particles that have experienced only small-angle multiple scattering in a thin surface layer of the medium is small. This is precisely the case for normal or near-normal incidence of a beam of fast electrons on a target.

The physical justification for the introduction of an effective isotropic elastic-interaction indicatrix for the description of the passage of the electrons is that for scattering of a beam of particles, starting with a certain path length, the important role is assumed not so much by the characteristic electron deflection angle in a single collision with the target electron, as by the size of the characteristic deflection angle acquired by the particle on a path segment of the order of l_{tr}.

The idea of using the methods of linear transport theory[36] to solve particle-backscattering problems is not new. Kornyushkin,[41] for example, attempted to describe the passage and reflection of electrons with the aid of an equation of type (6) in the single-velocity approximation, i.e., without allowance for deceleration. Instead of correctly allowing for the energy losses, Kornyushkin invokes a mechanism of "particle absorption along the surface" (Ref. 41a). The transport equation cited in Ref. 41a without proof differs from our Eq. (6) in that the transport cross section σ_{tr} is replaced in it by the quantity σ_{el} (w_s in the notation of Refs. 41a). It should be noted that this last circumstances contradicts the physics of multiple scattering of fast electrons. In addition, a particular solution of an integral transport equation [Eqs. (2) and (4) in Refs. 41a and 41b] is unjustifiably identified in Ref. 41 with the solutions of the corresponding boundary-value problem. The expression obtained in this manner for the electron flux density does not satisfy the boundary conditions posed by the author and can be regarded only as an asymptotic part of the sought solution.[35,36] Taking the foregoing into account, the results of Ref. 41 cannot be regarded as correct.

In conclusion, the author is sincerely grateful to M. I. Ryazanov for a helpful discussion of the results obtained in the present paper.

APPENDIX

Derivation of Eq. (6). The assumption of isotropization of the primary electron beam in direction is equivalent to assuming that the function

$$N(z,\mathbf{\Omega},k,E)=\int d\mathbf{\Omega}'\exp[-ik(1-\mathbf{\Omega\Omega}')]N(z,\mathbf{\Omega}',E) \tag{A1}$$

differs substantially from zero, starting with certain depths z and at arbitrary $\mathbf{\Omega}$, in the region $0\leq|k|\leq a<1$. Here $N(z,\mathbf{\Omega},E)$ is the flux density at the depth z of the particles moving in the direction $\mathbf{\Omega}$ with energy E. The function $N(z,\mu,E)$ is obtained from $N(z,\mathbf{\Omega},E)$ by averaging the latter over the azimuth:

$$N(z,\mu,E)=\frac{1}{2\pi}\int_0^{2\pi}N(z,\mathbf{\Omega},E)\,d\varphi. \tag{A2}$$

We write down the sum of the two transport-equation terms that contain the elastic-interaction cross section in the form

$$\begin{aligned}&-\sigma_{el}N(z,\mathbf{\Omega},E)+\int\frac{d\sigma_{el}(\mathbf{\Omega}'\to\mathbf{\Omega})}{d\mathbf{\Omega}'}N(z,\mathbf{\Omega}',E)\,d\mathbf{\Omega}'\\&=-\sigma_{tr}N(z,\mathbf{\Omega},E)+\frac{\sigma_{tr}}{4\pi}\int[1+f(1-\mathbf{\Omega\Omega}')]N(z,\mathbf{\Omega}',E)\,d\mathbf{\Omega}',\end{aligned} \tag{A3}$$

where

$$f(1-\mathbf{\Omega\Omega}')=\frac{2(\eta+1)}{\ln(\eta^{-1})[2\eta+1-\mathbf{\Omega\Omega}']^2}-\left[\frac{1}{\eta\ln(\eta^{-1})}-2\right]\delta(1-\mathbf{\Omega\Omega}')-1 \tag{A4}$$

and account is taken of the form of the dependence of the differential cross section of the fast-electron elastic scattering on the scattering angles[34]:

$$\frac{d\sigma_{el}(\mathbf{\Omega}'\to\mathbf{\Omega})}{d\mathbf{\Omega}'}=\frac{\sigma_{el}(\eta+1)\eta}{\pi[2\eta+1-\mathbf{\Omega\Omega}']^2}=\frac{\sigma_{tr}(\eta+1)}{2\pi\ln(\eta^{-1})[2\eta+1-\mathbf{\Omega\Omega}']^2}. \tag{A5}$$

In (A4) and (A5), $\eta\ll1$ is the screening parameter.

To check on the validity of the change from Eq. (2) to Eq. (6) we must show that the following condition is satisfied at the depths considered:

$$I=\left|\int f(1-\mathbf{\Omega\Omega}')N(z,\mathbf{\Omega}',E)\,d\mathbf{\Omega}'\right|\ll\int N(z,\mathbf{\Omega}',E)\,d\mathbf{\Omega}'. \tag{A6}$$

We rewrite the integral I in the form

$$I=\frac{1}{2\pi}\left|\int_{-\infty}^{\infty}f(k)N(z,\mathbf{\Omega},k,E)\,dk\right|, \tag{A7}$$

where the function $N(z,\mathbf{\Omega},k,E)$ is defined with the aid of (A1) and

$$f(k)=\int_0^2 f(x)e^{ikx}\,dx. \tag{A8}$$

The function $f(k)$ is bounded as $|k|\to\infty$, and its value at $|k|\leq1$, accurate to small terms of order η, is

$$\begin{aligned}f(k)&=[\ln(\eta^{-1})]^{-1}[2ik\{\mathrm{Ei}(2ik)-\ln(2ik)-\beta\}+1+2ik-e^{2ik}]\\&+(ik)^{-1}[e^{2ik}-1-2ik]-2ik\approx2ik[\ln(\eta^{-1})]^{-1}-k^2\{{}^4/_3+3[\ln(\eta^{-1})]^{-1}\}.\end{aligned} \tag{A9}$$

In the last expression, $\mathrm{Ei}(x)$ is the integral exponential function and β is the Euler constant. Since, in accord with the assumptions made, the main contribution to the integral I is made by values $|k|\leq a$, we obtain, taking (A1), (A7), and (A8) into account the following estimate for I:

$$\begin{aligned}I&\approx\frac{1}{2\pi}N(z,\mathbf{\Omega},0,E)\left|\int_{-a}^{a}f(k)\,dk\right|\\&\approx\frac{a^3}{9\pi}\left[4+\frac{9}{\ln(\eta^{-1})}\right]\int N(z,\mathbf{\Omega}',E)\,d\mathbf{\Omega}'\ll\int N(z,\mathbf{\Omega}',E)\,d\mathbf{\Omega}'.\end{aligned} \tag{A10}$$

Thus,

$$I\ll\int N(z,\mathbf{\Omega}',E)\,d\mathbf{\Omega}',$$

q.e.d.

Taking (A2) into account, we can write

$$\begin{aligned}&-\sigma_{el}N(z,\mu,E)+\int\frac{d\sigma_{el}(\mathbf{\Omega}'\to\mathbf{\Omega})}{d\mathbf{\Omega}'}N(z,\mu',E)\,d\mathbf{\Omega}'\\&\approx-\sigma_{tr}N(z,\mu,E)+\frac{\sigma_{tr}}{2}\int_{-1}^{1}N(z,\mu',E)\,d\mu'.\end{aligned} \tag{A11}$$

Substituting now (A11) in (2), we arrive at Eq. (6).

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