Effect of impurities on superconductors with helical ordering of localized spins

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Cooper pairing of electrons in a crystal with regularly distributed localized spins and conduction electrons with an exchange interaction of the spins and electrons, and a Cooper pairing of the conducting electrons. It was assumed that in the absence of superconductive pairing the indirect exchange interaction of the localized spins via the conduction electrons (RKKY interaction) would lead to ferromagnetic ordering below a Curie temperature \( T_c \). The critical temperature for superconductive pairing is \( T_c \), where \( T_c \) is the critical temperature for superconductive pairing and \( T_c \) is the superconducting wave vector. It was shown that scattering of conducting electrons by nonmagnetic impurities narrows the region of existence of the superconducting phase with spin ordering (HS phase). The narrowing, however, is not very pronounced even in dirty crystals. Thus the requirement of the purity of the crystals in which the HS phase may be observed is not very rigid.

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1. INTRODUCTION

In connection with experimental work on the compound \( \text{ErRh}_5\text{B}_2\text{H}_2\), Kulic, Rusinov and one of the present authors \( ^1 \) considered a system of regularly distributed localized spins and conduction electrons with an exchange interaction of the spins and electrons, and a Cooper pairing of the conducting electrons. It was assumed that in the absence of superconductive pairing the indirect exchange interaction of the localized spins via the conduction electrons (RKKY interaction) would lead to ferromagnetic ordering below a Curie temperature \( T_c \). The critical temperature for superconductive pairing is \( T_c \), where \( T_c \) is the critical temperature for superconductive pairing and \( T_c \) is the superconducting wave vector. It was shown that scattering of conducting electrons by nonmagnetic impurities narrows the region of existence of the superconducting phase with spin ordering (HS phase). The narrowing, however, is not very pronounced even in dirty crystals. Thus the requirement of the purity of the crystals in which the HS phase may be observed is not very rigid.

Both impurity scattering and the effects of magnetic anisotropy have a substantial effect on the HS-phase and lead to a narrowing of its region of existence. In the present paper we investigate helicoidal ordering of the spins in a superconductor in the presence of nonmagnetic impurities and find the region of existence of the HS phase and the character of the quasiparticle spectrum in this phase as a function of the electron mean free path.

2. BASIC EQUATIONS FOR A SUPERCONDUCTOR WITH A HELICOIDAL EXCHANGE FIELD IN THE PRESENCE OF IMPURITIES

The Hamiltonian of a system of electrons in the presence of an exchange interaction with localized spins, Cooper pairing, and scattering by impurities has the form

\[
\mathcal{H} = \mathcal{H}_0 + \sum_{\mathbf{K}} \mathcal{H}_{\mathbf{K}},
\]

where \( \mathcal{H}_0 \) is the Hamiltonian of the free electron gas, \( \mathcal{H}_{\mathbf{K}} \) is the exchange interaction of the localized spins with conduction electrons, and \( \mathcal{H}_{\mathbf{K}} \) is the superconducting wave vector. It was shown that scattering of conducting electrons by nonmagnetic impurities narrows the region of existence of the superconducting phase with spin ordering (HS phase). The narrowing, however, is not very pronounced even in dirty crystals. Thus the requirement of the purity of the crystals in which the HS phase may be observed is not very rigid.

The localized spins are assumed to be ordered in a helicoidal structure: the HS (Helmholtz-spin) phase persists down to zero temperature, while for systems with \( T < T_c \) lowering of the temperature leads to a first-order transition from the HS phase to a nonsuperconducting phase with ferromagnetic ordering of the spins (F phase). Since Ref. 1 used the simple self-consistent field approximation, which neglects the scattering of the electrons by spin excitations, the results obtained there are valid only for a system with \( T < T_c \) and at temperatures \( T \) which are not close to the critical magnetic point \( T_c \). Thus, Ref. 1 found the region of existence of the HS-phase in an isotropic system without impurities in the region of the variables \( (T, \theta) \) where \( T < \theta < T_c \) and \( (\theta)_{2D} \approx T_c \).

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localized spins. The superconducting order parameter in this field is independent of coordinate.

In the Gor'kov-Nambu representation \( \psi(\phi, \psi^\dagger, \phi^\dagger) \) the Hamiltonian takes the form

\[
\mathfrak{H} = -\frac{1}{2} \sum_{n=0}^{N-1} \left[ \frac{-i}{\hbar} \frac{\partial}{\partial \phi_n} - \frac{1}{2} \left( \Delta^{\dagger} \phi_n - \Delta \phi_n^{\dagger} \right) \right] + \frac{1}{2} \sum_{n=0}^{N-1} V_n \phi_n \phi_n^{\dagger},
\]

where \( V_n \) are unit matrices \( I \), and

\[
\begin{align*}
\Delta &= \frac{\alpha \Delta^{\dagger}}{\alpha} \Delta \frac{2 \Delta^{\dagger}}{\alpha} \frac{\Delta^{\dagger}}{\alpha} \Delta \frac{2 \Delta^{\dagger}}{\alpha} \frac{\Delta^{\dagger}}{\alpha}, \\
\Delta^{\dagger} &= \frac{2 \Delta^{\dagger}}{\alpha} \Delta \frac{2 \Delta^{\dagger}}{\alpha} \frac{\Delta^{\dagger}}{\alpha} \Delta \frac{2 \Delta^{\dagger}}{\alpha} \frac{\Delta^{\dagger}}{\alpha},
\end{align*}
\]

Below we shall be interested in the Green's function averaged over the impurity configuration:

\[
G(r, r') = \frac{1}{\alpha} \sum_{\alpha} G(r, r')^\alpha.
\]

Carrying out the average over the impurity configuration in the usual way, we get for the Fourier transform of the function \( G(r, r') \) the equation

\[
\frac{d^3 p}{2 \pi} \frac{1}{2 \epsilon(p)} = \frac{1}{2 \epsilon(p)} \left[ \frac{1}{2 \epsilon(p)} - \frac{1}{2 \epsilon(p)} \right] \frac{d^3 p}{2 \pi} \left[ \frac{1}{2 \epsilon(p)} - \frac{1}{2 \epsilon(p)} \right].
\]

Inverting Eq. (6), we find the electron Green's function:

\[
G \left( p-K, p+K \right) = \frac{1}{2 \epsilon(p)} \left[ \frac{1}{2 \epsilon(p)} - \frac{1}{2 \epsilon(p)} \right] \frac{d^3 p}{2 \pi} \left[ \frac{1}{2 \epsilon(p)} - \frac{1}{2 \epsilon(p)} \right].
\]

In the opposite limit \( 2 \Delta^V \ll 1 \) we get

\[
\frac{d^3 p}{2 \pi} \left( \frac{1}{2 \epsilon(p)} - \frac{1}{2 \epsilon(p)} \right) = \frac{1}{2 \epsilon(p)} \left[ \frac{1}{2 \epsilon(p)} - \frac{1}{2 \epsilon(p)} \right] \frac{d^3 p}{2 \pi} \left[ \frac{1}{2 \epsilon(p)} - \frac{1}{2 \epsilon(p)} \right]
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\]
3. DETERMINATION OF THE FREE ENERGY
FUNCTIONAL FOR THE HS PHASE

Since we are interested in temperatures \( T < \theta \ll T_{m} \),

in calculating the right-hand side of Eq. (12) we can replace the

sum over \( n \) by an integration. As a result we get for the case

\( \theta >> 1 \)

\[
\frac{1}{\lambda} \int_{0}^{\infty} \frac{d\kappa}{(1+\kappa^{2})^{5/2}} \int_{0}^{\infty} \frac{d\kappa'}{(1+\kappa'^{2})^{5/2}} \eta_{K} \eta_{K'} \frac{1}{(\omega_{K}^{2}+\omega_{K'}^{2})^{2}}
\]

\[
\frac{\pi}{2\lambda^{2}} \int_{2\lambda}^{\infty} \frac{da}{(1+a^{2})^{5/2}} \eta_{a} \eta_{a'} \frac{1}{(\omega_{a}^{2}+\omega_{a'}^{2})^{2}}
\]

(15)

Here the upper limit of integration is set at infinity,

provided the integral converges. The quantity \( \eta_{a} \) is
determined by the condition that the expression in curly

brackets on the right-hand side of (11) tends to zero:

\( \eta_{a} \) satisfies the equation

\[
\eta_{a}/(1+a^{2}) = \eta_{a}(1+a^{2})^{2}.
\]

(16)

Going over in (15) to the new integration variable \( \kappa \rightarrow (1+a^{2})^{-1/2} \), \( \kappa' \rightarrow (1+a^{2})^{-1/2} \)

and using the integral representation for \( \kappa \) and \( \kappa' \), we get from (15)

\[
\frac{1}{\lambda} \int_{0}^{\infty} \frac{d\kappa}{(1+\kappa^{2})^{5/2}} \int_{0}^{\infty} \frac{d\kappa'}{(1+\kappa'^{2})^{5/2}} \eta_{K} \eta_{K'} \frac{1}{(\omega_{K}^{2}+\omega_{K'}^{2})^{2}}
\]

\[
\frac{\pi}{2\lambda^{2}} \int_{2\lambda}^{\infty} \frac{da}{(1+a^{2})^{5/2}} \eta_{a} \eta_{a'} \frac{1}{(\omega_{a}^{2}+\omega_{a'}^{2})^{2}}
\]

(17)

Below we shall show that \( \kappa_{2} \) is very close to unity, so we neglect the first and last terms on the right-hand side of (17) and replace \( \sin k_{2} \) by \( k_{2}/2 \). In the case

\( 2\lambda k_{2} \ll 1 \) similar but simpler calculations lead to the expression

\[
\eta_{a} = \frac{\pi}{4\lambda^{2}} \frac{k_{2}}{2} \frac{\omega_{a}^{2}+\omega_{a'}^{2}}{(\omega_{a}^{2}+\omega_{a'}^{2})^{2}}
\]

(18)

under the condition that \( \pi^{2}/22 \ll 1 \). This condition guarantees the positivity of the expression in square brackets on the right-hand side of (14), and we shall see below that it is satisfied in the region of existence of the HS phase.

The self-consistency equations (17) and (18) allow us to find the free energy functional \( F_{\alpha}(Q_{a}, A_{a}) \) (see Ref. 1). This functional is determined by the condition that minimization of it with respect to \( A \) gives the self-consistency equation for the magnetic order parameter. The equilibrium value of \( Q_{a} \) is also determined from the condition that the functional \( F \) should be a minimum, and the minimum value of \( F_{\alpha}(Q_{a}, A_{a}) \) gives the free energy of the HS phase.

According to previous work, for \( \theta \ll 1 \) helicoidal or

ordering of the spins with wave vector \( Q_{a} \) and \( Q_{a} \) appears at the point \( \Delta = 1 - 3/4 \). Here \( \Delta \) is a quantity of the dimensions of length, of the order of magnitude \( \kappa_{2} \). In Ref. 1 it was shown that impurity

scattering does not change the values of \( Q_{a} \) and \( T_{a} \) it is necessary to take into account critical magnetic fluctuations,

and the effects of these can change the results of the self-consistent field approximation, which was used to describe the magnetic system. In this paper we consider the behavior of the system far from \( T_{a} \), where these fluctuations are small. For this purpose it is
small provided $\theta_s \approx T_{c0}$, where $\theta_s = \partial \Delta(0)/\partial \epsilon$ is the characteristic energy parameter of the RKKY interaction.

The functional (19) was found for the case of scattering of the electrons by point impurities. In the Appendix it is shown that all the expressions in (19) are preserved also the case of symmetric scattering of the electrons by the impurities, if we take $\tau$ to be given by

$$c^{-1} = \sqrt{\int P(p=0)\varphi_0(p)\varphi(p)dp,}$$

Knowledge of the free energy functional allows us to determine the equilibrium values of $\Delta$ and $Q$ as functions of temperatures and to find the phase boundary between the HS phase and the normal ferromagnetic (F) phase.

4. TRANSITION FROM THE HELICOIDAL SUPERCONDUCTING PHASE TO THE NORMAL FERROMAGNETIC F PHASE

Along the first-order transition line from the HS superconducting phase to the F phase the equilibrium values of the free energy in the two phases are equal, while up to terms of order $\Delta f/\Delta^2$ the equilibrium value of $\epsilon$ in the HS phase is given by the same expression as in the ferromagnetic phase. Thus, at the first-order phase transition only the direction of the spins is changed. From Eq. (19) we find an equation for the equilibrium parameters of the HS phase on the phase boundary. For $\overline{2}\Delta f/\Delta^2 \approx 1$ we have

$$\ln \Delta \leq \text{ln} \Delta - \sqrt{\frac{1}{2}} \left( 1 + 2\Delta - \text{ln} \Delta - 2\Delta e^{-\Delta} \right),$$

$$\epsilon_{\text{eq}} = \frac{\text{ln} \Delta}{\epsilon_{\text{eq}}} = \frac{\text{ln} \Delta}{\epsilon_{\text{eq}}} - \frac{2\Delta e^{-\Delta}}{3\Delta - 1},$$

$$\Delta_{\text{eq}} = \frac{\text{ln} \Delta}{\epsilon_{\text{eq}}} = \frac{\text{ln} \Delta}{\epsilon_{\text{eq}}} - \frac{2\Delta e^{-\Delta}}{3\Delta - 1}.$$

In the case $I \ll \xi_0$ these equations give $\Delta_{\text{eq}} = \Delta e^{-\Delta}$. For $\overline{2}\Delta f/\Delta^2 \approx 1$ the parameters $\Delta_{\text{eq}}$, $Q_{\text{eq}}$ and $Q_{\text{eq}}$ are determined on the phase boundary by the equations

$$\Delta_{\text{eq}} = \Delta e^{-\Delta}.$$

Using Eqs. (20) and (21), we find the first-order phase transition line in explicit form

$$T_F - T_{c0} = \frac{3\text{ln} \Delta}{\text{ln} \Delta - \epsilon_{\text{eq}}} = \frac{3\Delta}{\text{ln} \Delta - \epsilon_{\text{eq}}} - \frac{2\Delta e^{-\Delta}}{3\Delta - 1},$$

$$T_F - T_{c0} = \frac{3\Delta}{\text{ln} \Delta - \epsilon_{\text{eq}}} - \frac{2\Delta e^{-\Delta}}{3\Delta - 1}.$$

where the parameter $\alpha$ is of order unity. The curve $T_F(0)$ intersects the axis $T = 0$ at the point $\theta_0$ defined by the relation $\Delta(\theta_0) = 0$. If the parameter $\theta_0$ decreases with decreasing $\theta$, but not very strongly, in fact when $\theta$ decreases from values such that $\theta \gg \theta_0$, to values such that $\theta \approx \theta_0$, the parameter $\theta_0$ decreases by no more than a factor of 2 or 3 [from $T_{c0}/\text{ln}(\epsilon_{\text{eq}}/\theta_0)$ to $T_{c0}/(\Delta_{\text{eq}}/\epsilon_{\text{eq}})^{1/2}$].

For $\theta < \theta_0$, the HS phase survives right down to zero temperature, whereas for $\theta > \theta_0$ there is a first-order phase transition to the F phase. From (22) it is clear that decreasing $\theta$ leads to a shrinking of the interval of existence of the HS phase. In the region where it is small compared to $\theta$ we have

$$T_F - T_T = \frac{3\Delta}{\text{ln} \Delta - \epsilon_{\text{eq}}} - \frac{2\Delta e^{-\Delta}}{3\Delta - 1}.$$

On the HS-F phase boundary the superconducting order parameter, which is $\Delta_{\text{eq}}/\epsilon_{\text{eq}}$ for $I \ll \xi_0$ and $\Delta_{\text{eq}} e^{-\Delta}$ for $I \ll \xi_0$, vanishes discontinuously. With decreasing $\theta$ the magnitude of the wave vector $\Delta_{\text{eq}}$ changes slightly from the value

$$q_{\text{eq}} = \frac{3\Delta}{\text{ln} \Delta - \epsilon_{\text{eq}}} - \frac{2\Delta e^{-\Delta}}{3\Delta - 1}$$

for $I \ll \xi_0$ to $q_{\text{eq}} = \Delta_{\text{eq}} e^{-\Delta}$ for $Q^2 \ll \xi_0$. Thus, in the HS phase the parameters $\Delta$ and $Q$ change more weakly with decreasing temperature the greater the impurity concentration.

The limit of supercooling of the HS phase is obtained from Eq. (22) by multiplying the quantity $\alpha$ by $(27/\delta \epsilon)^{1/2} = 1.1$ for $\xi_0 \ll \delta < \xi_0$, by $3^{1/2}/\epsilon \approx 1.7$ for $\xi_0 \ll \delta < \xi_0$, and by $3^{1/2} \approx 1.4$ for $Q^2 \ll \xi_0$. For $\xi_0 \ll \delta$ the limit of supercooling of the F phase in the absence of domain-wall effects (i.e., for $L \gg \xi_0$, where $L$ is the domain width) and without taking account of the exchange scattering of electrons by critical magnetic fluctuations is determined by the condition

$$\alpha = \frac{2\Delta}{\text{ln} \Delta - \epsilon_{\text{eq}}} - \frac{2\Delta e^{-\Delta}}{3\Delta - 1} - 1,$$

where $M$ is the magnitude of the magnetic moment per unit volume at $T = 0$ and $H_{c2}(0)$ is the critical orbital magnetic field at $T = 0$, which increases as $\alpha^{-1}$ for $I \ll \xi_0$. Thus in general the limit of superheating $T_H^* \ll T_{c0}$ will depend on $\theta$; however, this dependence vanishes if the condition

$$Q^2 \gg \xi_0$$

is met. In real superconductors of the ErRh$_4$B$_4$ type the condition $4\xi_0 \gg Q^2(0)$ is satisfied. Then for $\xi_0 \approx \xi_0$, the limit of superheating of the F phase is determined by the exchange interaction and does not depend on $\theta$.

Knowing the equilibrium parameters of the HS phase, we can check that the conditions assumed in obtaining the functional (19) are indeed fulfilled. In the case $\overline{2}\Delta f/\Delta^2 \approx 1$ we assumed that $\Delta f/\Delta^2 < 1$; on the $T_F$ line we get from Eq. (21) the result $\Delta f/\Delta^2 \approx 1 - 1/\xi_0$, and in the rest of the HS phase the value of this parameter is less than on the $T_F$ line. In the region $\overline{2}\Delta f/\Delta^2 \gg 1$ we took $\Delta f/\Delta^2 \approx 1 - 1/\xi_0$, and in the rest of the $T_F$ line we obtain

$$\Delta f/\Delta^2 = \frac{1}{\Delta f/\Delta^2} - \frac{2\Delta e^{-\Delta}}{3\Delta - 1} - \frac{2\Delta e^{-\Delta}}{3\Delta - 1}.$$

It then follows from (16) that $\Delta f/\Delta^2 \approx 1$, which justifies the simplification made in the expression (17). It also follows from Eqs. (20) and (21) that the condition $\alpha < 1$ is satisfied. In the derivation of the functional (19) we neglected the exchange scattering of the electrons, taking $\theta \approx T_{c0}$. This condition is the better fulfilled in the HS phase the smaller $\theta$.
Q, in an angular interval of order ξ (see Ref. 1). This effect is connected with the fact that electrons traveling perpendicular to the vector \( Q \) feel a strong exchange field \( h \) which is constant in space. Impurity scattering changes the quasiparticle spectrum in the HS phase substantially, since such scattering changes the direction of motion of the electrons. In the case \( 2\Gamma h^3 \gg 1 \) we get from Eq. (8)

\[
\rho(E) = \frac{\delta(0)}{n(Q)} \int d\omega \delta(Q, -Q' - \omega) \left[ \frac{1}{1 + \Delta} \right]
\]

(25)

where \( \delta(E) \) is defined as the analytic continuation of the function \( u(E) \) defined by expression (11) to the imaginary half-axis \( \omega = iE \).

For \( E = 0 \) we get from (11) and (25)

\[
\rho(0) = (1 + \frac{1}{1 + \Delta})
\]

(26)

i.e., the density of quasiparticles for small \( \delta \ll \Delta \) is considerably increased in comparison with the pure HS phase, if \( \xi_0 = l \ll \xi_q \). This effect is explained by the fact that impurity scattering leads to all electrons on the Fermi surface spending a part of their time \( t = 1/h \) moving perpendicular to the vector \( Q \) and feeling the effect of the strong exchange field \( h \).

The situation is quite different in the case \( 2\Gamma h^3 \ll 1 \), where the duration of motion perpendicular to \( Q \) is already too small for the electrons to experience this field effectively. For \( 2\Gamma h^3 \ll 1 \) we get from (9) and (14) formula (25), where we must put \( \phi = 0 \). The density of states per unit energy is the same as in a superconductor with magnetic impurities and a reciprocal lifetime for magnetic scattering of \( t^{-1} \propto \Theta Q \).

Thus, in the case \( 2\Gamma h^3 \ll 1 \) impurity scattering leads to an energy gap in the quasiparticle spectrum of the HS phase, although this gap is less than the superconducting order parameter. In general the density of states for \( E = 0 \) in the HS phase first increases with decreasing \( l \) so long as \( l \geq \xi_q \), then decreases and finally vanishes in the limit \( l < \xi_q \). From the point of view of the investigation of gapless superconductivity most interest would attach to specimens with a mean free path which satisfies the inequalities \( \xi_q \gg l \gg \xi_0 \).

6. CONCLUSIONS AND QUALITATIVE INTERPRETATION OF THE EXPERIMENTAL DATA ON ErRh4B4

We now summarize our fundamental results on phase transitions in a superconductor with \( T_Q \gg \xi_0 \), \( \beta_q = 1 \) and weak anisotropy in the easy plane.

1. As the temperature is decreased, at the point \( T_Q = E \) in the superconducting phase there appears helicoidal ordering of the spins in a second-order transition. The magnitude of the wave vector \( Q \) at the transition point is practically independent of mean free path provided \( \Delta Q \gg 1 \).

2. On cooling of the superconducting phase with helicoidal spin ordering, this HS phase persists to zero temperature, if \( \delta < \xi_0 \), or goes over by a first-order transition into a normal ferromagnetic phase at a temperature \( T_{F1} \). The parameters \( \xi_0 \) and \( T_{F1} \) depend on \( l \), but not very strongly, and in practice \( T_{F1} \) is very small; even at this point only the direction of the spins changes and the latent heat evolved is of the order of the superconducting energy.

3. For the ferromagnetic phase is stable below \( T_{F1} \), but for \( \delta > \xi_0 \) it is metastable, and the HS phase may be converted into this phase by the application and subsequent removal of a magnetic field. When superheated the F phase stays metastable right up to a temperature \( T_{F2} = T_{F1} \) and in principle could be converted by superheating directly into the nonmagnetic S phase, avoiding the HS phase.

4. In the HS phase the wave vector \( Q \) is practically independent of temperature, the variation being smaller the smaller the quantity \( l \). The amplitude of helicoidal ordering in the HS phase varies exactly as it would vary in a ferromagnetic phase in the absence of superconductivity. The discontinuity of the amplitude at the point \( T_{F1} \) is very small; even at this point only the direction of the spins changes and the latent heat evolved is of the order of the superconducting energy.

At present there are no data on the magnitude of the magnetic anisotropy in the easy plane for crystals of ErRh4B4. For sufficiently strong anisotropy a domain type of structure will be realized in ErRh4B4. The presence of such a structure changes the value of \( F_L \) only weakly from that obtained from the exactly soluble model with helicoidal spin ordering, but the functional \( F_L \) of the magnetic subsystem is modified considerably more strongly. In this case our qualitative conclusions remain in force but the quantitative results are changed.

Within the framework of this picture we may understand all the peculiarities of the behavior of ErRh4B4 observed experimentally in Refs. 11 and 12.

APPENDIX

For a spherically symmetric impurity scattering potential we have \( \rho = \|P - p'\| \). We rewrite Eq. (5) by inserting on the right-hand side terms which cancel one another:

\[
\epsilon^{-}(p', p)' - \epsilon^{-}(p, p') - \frac{\delta(0)}{n(Q)} \int d\omega d\omega' (\|P - \omega\| - \|P - \omega'\|) \\
\times \left( \delta \left( \|P - \omega\| - \|P - \omega'\| \right) \right) \sum_{n} a_n (\|P - \omega\|) \int d\omega' (\|P - \omega'\| - \|P - \omega\|) \\
+ \frac{\delta(0)}{n(Q)} \int d\omega d\omega' (\|P - \omega\| - \|P - \omega'\|) \left( \sum_{n} a_n (\|P - \omega\|) \right) \\
\times \left( \delta \left( \|P - \omega\| - \|P - \omega'\| \right) \right) \sum_{n} a_n (\|P - \omega'\| - \|P - \omega\|) \int d\omega'.
\]

We shall seek the function \( \epsilon^{-}(p', p') \) in the form (6) with \( \delta = \delta(0) + \delta(0) \) when \( \delta(0) \) is replaced by \( \delta(0) \) and \( \delta(0) \) by \( \delta(0) \). The functional depen-
We linearize (A.1) in \( \omega_1(0) = \omega_1(0) - \frac{\partial \omega(0)}{\partial \delta} \) and \( \Delta(0) = \Delta(0) - \frac{\partial \Delta(0)}{\partial \delta} \). For \( \delta^2 \ll 1 \) we have

\[
\frac{\partial \omega(0)}{\partial \delta} = \frac{\omega_1(0)}{\omega_1(0)^2} \frac{\partial \omega(0)}{\partial \delta},
\]

\[
\frac{\partial \Delta(0)}{\partial \delta} = \frac{\omega_1(0)}{\omega_1(0)^2} \frac{\partial \Delta(0)}{\partial \delta}.
\]

The equations for \( \omega_1(0) \) and \( \Delta(0) \) have the form

\[
\omega_1(0) = \omega_1(0) \left( 1 - \frac{1}{\omega(0)} \right) \left( 1 - \frac{1}{\omega(0)} \right) \left( \frac{\partial \omega(0)}{\partial \delta} \right),
\]

\[
\Delta(0) = \Delta(0) \left( 1 - \frac{1}{\omega(0)} \right) \left( 1 - \frac{1}{\omega(0)} \right) \left( \frac{\partial \Delta(0)}{\partial \delta} \right),
\]

We seek the solution of (A.3) in the form

\[
\Delta(0) = \Delta(0) \left( 1 - \frac{1}{\omega(0)} \right) \left( 1 - \frac{1}{\omega(0)} \right) \left( \frac{\partial \Delta(0)}{\partial \delta} \right),
\]

\[
\omega_1(0) = \omega_1(0) \left( 1 - \frac{1}{\omega(0)} \right) \left( 1 - \frac{1}{\omega(0)} \right) \left( \frac{\partial \omega_1(0)}{\partial \delta} \right).
\]

Here \( \omega(0) \) satisfies the equation

\[
\omega(0) = \omega(0) \left( 1 - \frac{1}{\omega(0)} \right) \left( 1 - \frac{1}{\omega(0)} \right) \left( \frac{\partial \omega(0)}{\partial \delta} \right).
\]

Integrating (A.5) with respect to \( \delta \), we see that the following integral relation is valid:

\[
\int_0^\delta \omega(0) d\delta = \frac{1}{\omega(0)} \left( 1 - \frac{1}{\omega(0)} \right) \left( 1 - \frac{1}{\omega(0)} \right) \left( \frac{\partial \omega(0)}{\partial \delta} \right).
\]

Here we used the definition of \( \tau^1 \) from (A.3).

The self-consistency equation (12), linearized in \( \omega_1(0) \) and \( \Delta(0) \), has the form

\[
\frac{\partial \omega_1(0)}{\partial \delta} = \frac{\omega_1(0)}{\omega_1(0)^2} \left( 1 + \frac{1}{\omega_1(0)} \right) \left( 1 - \frac{1}{\omega_1(0)} \right) \left( \frac{\partial \omega_1(0)}{\partial \delta} \right) + \frac{\partial \Delta(0)}{\partial \delta}.
\]

From (A.2) we find

\[
\Delta(0) = \Delta(0) \left( 1 - \frac{1}{\omega(0)} \right) \left( 1 - \frac{1}{\omega(0)} \right) \left( \frac{\partial \Delta(0)}{\partial \delta} \right) + \frac{\partial \Delta(0)}{\partial \delta}.
\]

Substituting (A.6) and (A.7) in (A.8) and integrating with respect to \( \delta \), we again obtain Eq. (16), with \( \tau^1 \) defined by (A.3). Here \( \Delta \) depends on \( \omega(0) \) only through the quantity \( \tau^1(0) \) defined in (A.9).

In the case \( (2 \pi h)^2 \ll 1 \) similar calculations lead to expression (18) with the parameter \( \tau^1 \) defined in (A.3).

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References: