Resonance transition radiation of relativistic charged particles in an ordered system of vacancion pores

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The transition radiation emitted by relativistic charged particles on defects—vacancion pores—of the crystal lattice of a solid is considered. It is shown that in the case in which the pore dimension is smaller than the characteristic distance over which the field produced by the moving charge falls off the intensity of the transition radiation depends on the concentration and dimensions of those pores. The use of the transition radiation to investigate the vacancion porosity of solids is therefore suggested. The resonance transition radiation emitted in ordered one- and three-dimensional vacancion-pore structures, i.e., in pore chains and vacancion-pore lattices, is analyzed. It is shown that in the region of emitted-quantum frequencies \( \sim 5 \times 10^7 \) keV the transition radiation emitted in the experimentally observed pore lattices in such metals as Al and Ni is more intense than the bremsstrahlung. It is suggested that solids containing vacancion-pore lattices and possessing a developed vacancion porosity can be used in relativistic-charged-particle detectors and as x-ray sources.

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1. INTRODUCTION

Upon the passage of a charged particle through the boundary between two media with different dielectric properties, there is emitted transition radiation whose properties were first studied by Ginsburg and Frank.1 Resonance transition radiation is emitted during the passage of a uniformly moving charged particle through an inhomogeneous medium whose dielectric properties vary periodically.2-4

Besides the transition radiation, an additional emission channel is possible in a solid because of the periodic disposition of the atoms in the crystal lattice.1-5 This radiation arises as a result of the diffraction of the field of the particle at the Bragg angles in the ordered system of atoms.

The properties of resonance transition radiation have been investigated in detail in a number of experiments (see, for example, Refs. 2, 9-12) on the flight of relativistic charged particles through various layered media, which are sets of periodically alternating laminae possessing different dielectric properties. The spacing of such one-dimensional periodic structures that are usually used in these experiments ranges from \( \lambda_1 = 5 \times 10^{-4} \) to \( \lambda_1 = 1 \) cm. Similar layered media are used to construct detectors for high-energy charged particles. Resonance transition radiation possesses threshold properties. An important parameter here is the layered medium’s spacing, which characterizes the minimum particle energy starting from which the entire emission spectrum is regenerated. To broaden the energy range of the detectable particles in the region of lower energies, we need periodic structures with sufficiently small spacings.

Interesting structure formations, which arise under certain conditions in various solids, and whose dielectric properties are periodic with a sufficiently small periodicity dimension (of the order of hundreds of angstroms), are ordered structures of crystal-lattice defects. Foremost among these structures are the ordered structures of vacancion pores,6-11

It has now been experimentally established that the irradiation of various metals by fast particles (fast neutrons, ions, electrons) produces in them vacancion pores that in a number of cases form ordered structures of several types: one-dimensional (pore chains6-10), two-dimensional (pore walls11), and three-dimensional (vacancion-pore lattices12-13).

In each of these structures the size distribution of the pores is very narrow, and all the pores are located strictly at a definite distance from each other. The mean dimension of the pores and the order constant in such ordered structures depend on the conditions of the irradiation and the type of metal. Usually the mean pore dimension \( R \) ranges from about 10 to 100 Å, while the order constant \( L \) for the various metals ranges from about 10 to 1000 Å.

The most interesting ordered defect structure is the vacancion-pore lattice. It is produced in a definite temperature range starting from some radiation doses. All the pores in the pore lattice are practically of the same size. The mean dimension of the pores located at the nodal sites for the various metals ranges from 20 to 100 Å, while the pore-lattice constant ranges from 200 to 800 Å. The symmetry of the pore lattice coincides with that of the original crystal lattice of the metal. The pore lattice is a stable structure formation.14-16 At present pore lattices have already been found in such metals as Mo, Ni, Al, Nb, Ta, and W, in the alloys ZTM and Mo-35%Th, as well as in fluo-

Evidently, pore lattices can be formed in other solids.

Ordered defect structures can also be produced under conditions other than those produced by irradiation. Thus, the diffusional decay of solid solutions leads to the formation of an ordered lattice of the precipitates of the new phase.17 This lattice is also periodic in its...
dielectric properties.

Another interesting phenomenon, which leads to the formation of one-dimensional ordered structures (vacancion-pore chains), is the "vacancion breakdown" of solids.14 This phenomenon is observed in ionic crystals, in which the vacancion-pore chains are formed under conditions of high-temperature annealing of the pores by the action of a constant electric field. The pores in such chains may have a fairly extended cylindrical shape, and the axes of the pore chains always coincide with the direction of the electric field.

Ordered vacancion-pore structures differ essentially from the synthetic layered media usually used in experiments on transition radiation in a number of their properties. First, they possess fairly small periodicity dimensions $L_1/L_2 = 10^4$ and, secondly, the ordered pore structures, being three-dimensional structure formations, consist of pores of finite linear dimensions. And, which is particularly important, the pore dimensions of the pores.

The investigation of the transition radiation emitted in ordered defect structures may prove to be useful in the determination of the general laws governing the formation of such ordered structures. Furthermore, the investigation of the transition radiation emitted on vacancion pores is of interest by itself. This radiation may prove to be especially effective in the investigation of the transition radiation emitted on the vacancion-pore structures. We first investigate the transition radiation of the relativistic charged particle in a one-dimensional layered medium.

Let us suppose that the permittivity of the medium containing the vacancion pore has the following form:

$$\varepsilon(\omega) = \varepsilon_0(\omega) + \varepsilon_1(\omega),$$

(2.1)

Here $\varepsilon_1(\omega)$ describes the permittivity of the homogeneous medium in the absence of the pore ($\varepsilon(\omega) = \varepsilon_0$) and $\varepsilon_1(\omega, \mathbf{r})$ characterizes the deviation of the permittivity $\varepsilon_1$ of the vacancion pore from the permittivity $\varepsilon_0$ of the solid in which the pore is located.

Below we shall consider the transition radiation emitted only on the vacancion pores, the case in which the charged particle passes through an arbitrary inhomogeneity (e.g., the precipitate of a new phase) characterized by the permittivity $\varepsilon_1$ can be considered in similar fashion. In this case the quantity $\chi(\omega)$ in the expression (2.3) should be replaced by the quantity

$$\chi(\omega) = \frac{\alpha}{\omega^2} \frac{\omega^2}{\mu^2} \frac{4e^2}{m_e} N_e \delta \nu_0.$$

(2.4)

where $N_e \delta \nu_0$ is the mean electron density in the precipitate.

In the region of frequencies $\omega > \delta \nu_0$ we have $\varepsilon_1(\omega) \approx \varepsilon_1(\omega)$ and therefore in computing the transition radiation emitted on the inhomogeneities of the medium we can use perturbation theory,22 which reduces in this case to the determination of the intensity of the radiation generated by the electromagnetic wave scattered on the inhomogeneity of the medium of permittivity $\varepsilon_1(\omega)$.

Below we shall designate the quantities characterizing the incident electromagnetic wave by one prime (e.g., $E', H'$) and the corresponding quantities for the scattered wave by two primes (e.g., $E''$, $H''$).

Let us find the radiation emitted by a particle on a pore in the case in which the relativistic charged particle traverses a vacancion pore in the form of a cylinder of radius $R_1$ and length $L_2$, the particle moving with constant velocity $v$ along the axis of this cylinder. As has already been noted, pores of a similar shape are formed under conditions of a "vacancion breakdown." The electromagnetic field produced by a relativistic charged particle in a medium of permittivity $\varepsilon_1$ has the
where the components $E'_x$ and $E'_y$ of the electric field (2.3) assume the form

$$E'_x = \frac{\partial A(x', y', z, t)}{\partial t}, \quad E'_y = \frac{\partial A(x', y', z, t)}{\partial y},$$

where $A(x', y', z, t)$ is the retarded potential.

The field of the scattered electromagnetic wave is described with the aid of the induction current $D'_x = -\partial E'_y / \partial t$ and the magnetic field $B'_x = \mu_0 \mu_n (\partial^2 E'_x / \partial t^2)$. Notice that Eq. (2.7) was derived in the perturbation-theory approximation for $\delta > 1$, where $\delta$ is the order of $L, \lambda, \gamma$, and practically does not depend on the particle energy.

The intensity of the emission of quanta in the frequency and solid-angle intervals $d\omega$ and $d\Omega$ is equal to

$$dI = \frac{2\pi e^2 \omega^4}{\epsilon_0^2 c^2} \frac{\sin^2 \theta}{(1 - \cos^2 \theta)^2} d\Omega d\omega.$$
where the radiation intensity $I \propto 1 - \beta - \gamma$ depends on the particle energy, i.e., $\Delta L/d\omega = E^2$, while $\Delta L/d\omega = E^2$ at $1 - \beta = \gamma$.

The dependence of the intensity of the transition radiation emitted on large vacancion pores (i.e., on vacancion pores for which $aR_1 \approx 1$) on the particle energy indicates that materials containing vacancion pores can be used in relativistic-charged-particle detectors.

Let us compare the transition radiation emitted on a vacancion pore having a sufficiently small longitudinal dimension, i.e., for which $\kappa = 1$, with the diffraction radiation generated when a charged particle flies through an aperture in a thin (i.e., $\kappa = 1$) screen. In Ref. 2 the diffraction radiation is described with the aid of an approximate computational method based on the Huygens principle. In that analysis the field of the emission from the aperture is the result of the superposition of secondary fields emanating from the points of the aperture, and has at a sufficiently large distance from the aperture the form (cf., the expressions (2.8) and (2.9))

$$E_{\omega} = \frac{k_{\omega}^{\omega}}{2\sin\theta_{\omega}} \left( E_0 (\omega, \theta_{\omega}) e^{-i\omega \frac{\Delta L}{d\omega}} \right),$$

(2.14)

where $k_0 = \omega/c$.

In this expression, in contrast to (2.8), $E^0$ is the field produced by the moving charge in a vacuum, and not in a medium with permittivity $\epsilon$. It has the form of (2.6) with $\chi = -\omega \Delta L$. Moreover, in the analysis of the diffraction radiation emitted during the flight through the aperture the screen essentially differs in its optical properties from the medium in which the vacancion pore is located, and which is traversed by the charged particle. Thus, it is assumed in the analysis of the diffraction radiation that the screen is ideally conductive (i.e., $\chi = 0$) and absolutely opaque. On the other hand, the investigation of the transition radiation emitted on a pore located in a medium has been performed for a transparent medium, with allowance for the polarization of the material of which it is made.

Let us compare the intensity of the transition radiation emitted on a vacancion pore having a sufficiently small longitudinal dimension $R = a R_1 \approx 1$ with the intensity of the diffraction radiation emitted on an aperture $E_{\omega} = \frac{k_{\omega}^{\omega}}{2\sin\theta_{\omega}} \left( E_0 (\omega, \theta_{\omega}) e^{-i\omega \frac{\Delta L}{d\omega}} \right)$.

For pores and apertures having small transverse dimensions $a R_1 \approx 1$ (i.e., $R = a R_1$), according to the expressions (2.8) and (2.14), the field of the transition radiation emitted on a pore $E_{\omega}^{\omega}$ is weaker than the field of the diffraction radiation emitted on an aperture $E_{\omega}^{\omega}$ by a factor of $\gamma$ (i.e., $E_{\omega}^{\omega} \approx E_{\omega}^{\omega}$) and, accordingly, the emission intensities are connected by the relation $I_{\omega} \approx I_{\omega}^{\omega}$. This difference in the intensities is due to the difference in the optical properties of the medium and the screen in the two investigations. For pores and apertures having large transverse dimensions $R = \gamma$, the deviation from each other of the transition-and-diffraction-radiation fields is due not only to the difference in the optical properties of the screens in the two approaches, but also to the fact that in (2.8) the field $E^0$ produced by a charged particle in a medium with permittivity $\epsilon$ differs from the field $E^0$ in (2.14), which is the field produced by the charged particle in a vacuum. For the radiation emitted at small angles $\beta \approx \gamma \approx 1$ in the frequency band $\omega \approx \omega_0$, the transition-radiation field is weaker than the diffraction-radiation field by a factor of $\gamma/4$, i.e., $E_{\omega}^{\omega} \approx \gamma E_{\omega}^{\omega}$, so that the transition radiation intensity $I_{\omega} \approx \gamma I_{\omega}^{\omega}$.

To conclude this section, let us formulate the conditions of applicability of the expression (2.12) for the intensity of the transition radiation emitted on a pore. The transition radiation emitted on a vacancion pore was considered above with the use of perturbation theory, and, besides the requirement that $c(\omega, \gamma, \epsilon) = c(\omega, \gamma, \epsilon_0)$ for the results obtained here to be applicable, it is also necessary that the next perturbation-theory approximation be smaller than the preceding one. For pencil radiation emitted on pores of small dimension, i.e., on pores for which $a R_1 \approx 1$, $\Delta L_1 \approx 1$, $R_1 \approx R_2 \approx R$, we obtain by comparing the expressions for $E_{\omega}^{\omega}$ and $E_{\omega}^{\omega}$ the following condition for the applicability of the expression (2.12):

$$\omega - (\omega_0 - \gamma \omega) \approx \epsilon w \approx \epsilon_0 w \approx \epsilon w_0 \approx \epsilon w_0.$$

It can be seen from this relation that the main results obtained here for the transition radiation emitted on vacancion pores are well applicable in the description of the pencil radiation.

3. TRANSITION RADIATION EMITTED ON A CHAIN OF VACANCION PORES

Let us now proceed to consider the radiation that arises during the flight of a relativistic charged particle through a chain of vacancion pores. As has already been noted, ordered vacancion-pore chains can be produced artificially in various solids as a result of the irradiation of them by fast-particle fluxes and also as a result of a "vacancion breakdown." The vacancion breakdown effect occurs in ionic crystals, in which the ordered pore chains form under the action of an electric field. The formation of the pore chains occurs as a result of the motion of the pores under the action of the external electric field; therefore, the axes of the pore chains coincide with the direction of the electric field. This, apparently, may significantly facilitate the performance of experiments in which it is necessary to produce ordered charged-particle motion along the axis of the pore chain.

Let us consider an ordered vacancion-pore chain consisting of $N$ pores, and having a spacing equal to $L$. All the pores in the chain have a cylindrical shape and linear dimensions $R_1$ and $R_2$. Let a relativistic charged particle transverse the pore chain, moving along the axis of the chain with velocity $v$. Let us orient the $z$ axis along the pore chain. Then, for $\omega = \omega_0$, the permittivity $\epsilon(\omega, \gamma)$ of the medium containing the vacancion pore chain can be written in a form similar to (2.1), in which the quantity $c'(\omega, \gamma)$ is, in the cylindrical system of coordinates $(\rho, \varphi, z)$, equal to

$$c'(\omega, \gamma) = c(\omega, \gamma) \sum_{n=0}^{N} \frac{1}{(R_2 - R_1)(n+\frac{1}{2})},$$

(23)

where A. I. Rukhadze 23
The transition radiation that arises during the flight of a relativistic charged particle through a vacuum pore chain is due to the scattering of the electromagnetic field produced by the moving charged particle not by one pore, but by the netic field \( G''(x) \) produced by the moving charged particle of a relativistic charged particle through a vacuum pore chain consisting of \( N \) pores, is denoted below by \( G''(x) \). The calculations lead to the following result for this quantity:

\[
G''(x) = \sum_{i=1}^{N} G(x_i)
\]

The general expression for the total transition intensity has quite an unwieldy form; therefore, let us consider the case—the most interesting one—in which the radiation emitted on the pore chain has a high-directional \((\lambda = 1)\) character, and all the pores in the pore chain have a small transverse dimension \( d_1 \approx 1 \).

According to (3.3), the emission of quanta at small angles will occur when the following condition is fulfilled:

\[
\sin^2 \left( \frac{\pi d}{2d_1} \right) = \sin^2 \left( \frac{\pi d}{2d_1} \right)
\]

It can be seen from this expression that the maximum emission angle being then equal to \( \theta_{\text{max}} = 2\pi d_1 / d \).

The frequency distribution of the radiation intensity in this case has the form

\[
\frac{d\nu}{d\theta} = \frac{d\nu}{d\theta} \left( \frac{d}{d_1} \right) \sum_{i=1}^{N} \left[ (\pi d_1 / d)^2 - 1 \right] \sin^2 \left( \frac{\pi d}{2d_1} \right)
\]

The resonance condition (3.3) determine the emission angle of the resonance quanta.

An increase in the intensity of the radiation emitted during the flight of a relativistic particle through a pore chain in comparison with the same quantity for one pore can also be achieved upon the fulfillment of the following condition:

\[
N = \frac{L}{d} - 1,
\]

which is fulfilled in the case of ultrasuperradiative particles \((\psi = 0)\), which emit pencil radiation \((\lambda = 1)\). In this case the radiation intensity is, as in (3.4), proportional to the square of the number of pores.

Let us now analyze a few limiting cases.

Let a relativistic charged particle traverse a chain of pores having an extended shape with a small transverse dimension, i.e., for which \( R_0 \gg R_1 \), \( \alpha R_1 \ll 1 \), the expression (3.2) obtained within the framework of perturbation theory for the intensity of the radiation emitted on a pore chain goes over into the expression obtained earlier (see Ref. 2) for the intensity of the transition radiation emitted on a pile of plates each of which is of thickness \( R_1 \).

Let us now determine the total radiation emitted by the relativistic particle during its flight through the pore chain. To do this, let us integrate the expression (3.2) over the angles with the aid of the well-known relation

\[
\sin^2 \left( \frac{\pi d}{2d_1} \right) = \frac{2\pi d}{d_1}
\]

The maximum emission angle being then equal to \( \theta_{\text{max}} = 2\pi d_1 / d \).

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value of the intensity for each harmonic increases with increasing particle energy \( E \), and that it approaches a limiting value at high energies (i.e., when \( \gamma \gg \gamma_e \)).

An important characteristic of resonance transition radiation is the threshold energy \( E_t \), starting from which the harmonic with the number \( n \) appears in the emission spectrum. This quantity is determined from the condition (3.6), and is equal to

\[
E_t = \frac{1}{2n} \frac{c}{\gamma_e}. \quad (3.6)
\]

The fact that \( E_t = \frac{1}{2} \frac{c}{\gamma_e} \), is very important in connection with the performance of experiments on resonance emission on a pore chain. The value of \( E_t \) for a pore chain is fairly small (\( L = 10^{-10} \) Å), and therefore the entire radiation spectrum is regenerated starting from energies \( E_t \). For example, for \( L = 5 \times 10^9 \) Å and \( \gamma_e = 20 \) eV, the quantity \( E_t = 1 \) MeV.

Let us now find the total number of quanta emitted by a relativistic charged particle as it passes through a unit length of a pore chain. For this purpose, let us divide the quantity (3.5) by the energy \( \hbar \omega \) of a quantum and the total length \( NL \) of the chain and integrate the resulting expression over the frequencies. The maximum value of the total number of quanta emitted from a unit length of the pore chain is approximately equal to

\[
m_{\text{total}} = \frac{1}{2n} \frac{\omega}{\gamma_e} \frac{1}{L} \int_0^\infty \frac{1}{2n} \frac{1}{\omega} \frac{d\omega}{\gamma_e} = 2nR/\ell. \quad (3.7)
\]

where \( n_{\text{max}} = \frac{\omega}{\gamma_e} L/2n \).

Let us, for comparison, point out that in the case of the transition radiation emitted on a pile of plates with the harmonic with the number \( n \), and the total length \( NL \) of the chain and integrate the resulting expression over the frequencies. The maximum value of the total number of quanta emitted from a unit length of the pore chain is approximately equal to

\[
m_{\text{total}} = \frac{1}{2n} \frac{\omega}{\gamma_e} \frac{1}{L} \int_0^\infty \frac{1}{2n} \frac{1}{\omega} \frac{d\omega}{\gamma_e} = 2nR/\ell. \quad (3.7)
\]

where \( n_{\text{max}} = \frac{\omega}{\gamma_e} L/2n \).

The ratio of the quantities (3.7) and (3.8) is equal to

\[
m_{\text{total}} = \frac{1}{2n} \frac{\omega}{\gamma_e} \frac{1}{L} \int_0^\infty \frac{1}{2n} \frac{1}{\omega} \frac{d\omega}{\gamma_e} = 2nR/\ell. \quad (3.9)
\]

For the characteristic values \( L_c = 5 \times 10^9 \) Å, \( R_1 = 50 \) Å, and \( \gamma_e = 60 \) eV of the parameters of the ordered vacancy-pore chain that forms in a solid, and for the spacing \( L_s = 5 \times 10^9 \) Å of the layered medium, the gamma-quantum yield ratio (3.9) is equal to \( m_{\text{total}}/m_0 = 10 \)

In the case of pores of a large dimension \( nR_1 \gg 1 \) the expression (3.3) obtained for the radiation intensity within the framework of perturbation theory goes over into the corresponding expression for the intensity of the radiation emitted by a relativistic charged particle in a layered medium. The quantities characterizing the transition radiation in this case are considered in detail in Refs. 2 and 3.

To conclude this section, let us make a few remarks about the effect of a deviation from the ordered disposition of the pores in the chain and the smearing of the narrow size distribution of the pores in the pore chain on the intensity of the resonance radiation emitted in the ordered structure. As is well known, the characteristic distance over which the transition radiation is produced in a medium with permittivity \( \varepsilon \) is the coherence length \( L_0 = \frac{c}{\omega} (1 - \gamma_0^2 + \gamma_0^2)^{-1/2} \). It can be shown in much the same way as is done in Refs. 2 and 3 that, if there are distortions in the periodicity of the ordered pore chain, so that the pores are shifted from site positions in the ordered structure through distances \( \pm \frac{L_0}{2} \), and if the size distribution of the pores has a width \( \frac{L_0}{2} \), then all the results obtained for the resonance transition radiation will not change provided the following conditions are fulfilled:

\[
\begin{align*}
(\varepsilon')^2 &< L_0^2, \\
(\varepsilon')^2 &< L_0^2.
\end{align*}
\]

In the case of fairly large shifts of the pores from the nodal sites, i.e., for \( (\varepsilon')^2 > L_0^2 \), the transition radiation is emitted on \( N \) independent pores, and the intensity of the radiation emitted on a pore chain is the sum of the radiation intensities for the individual pores each considered separately (cf. (3.5));

\[
d\Omega = d\alpha - Nd\Omega, \quad (4.1)
\]

The occurrence in a system of randomly disposed pores of transition radiation with intensity proportional to the pore concentration, and dependent on the pore dimensions, can be used to investigate the vacancy porosity of solids.

4. TRANSITION RADIATION PRODUCED BY FLIGHT OF A RELATIVISTIC CHARGED PARTICLE THROUGH A VACANCION-PORE LATTICE

Let us determine the radiation that arises during the flight of relativistic charged particles through a vacanion-pore lattice.

Let us represent the permittivity \( \varepsilon(\omega, r) \) of the solid containing the pore lattice in the frequency region \( \omega \gg \omega_0 \) in the form of a sum of the two quantities: \( \varepsilon^{(1)}(\omega) \) and \( \varepsilon^{(2)}(\omega, r) \) [see the expression (2.1)]. Here, as before, the quantity \( \varepsilon^{(1)}(\omega) \) characterizes the permittivity of the solid in the absence of pores, i.e., is given by (2.2), while the quantity \( \varepsilon^{(2)}(\omega, r) \) describes the permittivity of the ordered pore system, and is equal to

\[
\varepsilon^{(2)}(\omega, r) = \varepsilon(\omega, r) \cdot \rho(r),
\]

where \( \rho(r) \) is a function characterizing the position of the pores in the solid, being equal to one if the radius vector \( r \) lies inside any pore and zero otherwise.

The permittivity \( \varepsilon(\omega, r) \) of a solid containing a vacanion-pore lattice is a periodic function of the coordinates, i.e.,

\[
\varepsilon(\omega, r) = \varepsilon(\omega, r + \mathbf{r}_n), \quad (4.1)
\]

where \( \mathbf{r}_n \) is an arbitrary translation vector in the pore lattice.

Let us, using the periodicity property (4.1), expand the spatially inhomogeneous component \( \varepsilon^{(2)}(\omega, r) \) of the permittivity \( \varepsilon(\omega, r) \) in a Fourier series in the reciprocal pore lattice vectors \( \mathbf{q} \):

\[
\varepsilon^{(2)}(\omega, r) = \sum_{n} \varepsilon_n \rho(r) \cdot \mathbf{e}_n, \quad \varepsilon_n = \frac{1}{V} \int \varepsilon^{(2)}(\omega, r) e^{-i\mathbf{q} \cdot \mathbf{r}} dV.
\]

Here the reciprocal pore lattice vector \( \mathbf{q} \) has the com-
ponents $2\ell/L$, $2m/L$, and $2n/L$, where $l$, $m$, and $n$ are the arbitrary whole numbers, $L$ is the length of the unit-cube edge in the pore lattice, and $V$ is the volume of the unit cell of the pore lattice ($V=L^3$).

Let us find the intensity of the transition radiation emitted during the motion of a relativistic charged particle through the pore lattice. The calculation for $w\gg w_0$, $cL/\gamma^2 << cL/\gamma^2 \gamma$ yield the following expression for the radiation intensity per unit particle-path length in the pore lattice for the quanta emitted in the frequency range $dw$ into the solid angle $d\Omega$ (see also Ref. 2): 

\[
\frac{dI_{\text{TR}}}{d\Omega \, dw} = \frac{\epsilon_0^2}{2^{3/2}} \sum_{k=1}^{\infty} \left[ \left( \frac{w_{\text{in}}}{w} \right)^2 + 1 \right] \left[ \frac{4(w_{\text{in}} - \nu)\nu}{(k\gamma - q)^2 - \nu^2} \right].
\]

(4.2)

Here $k=w_{\text{in}}c^2/\gamma^2 \rho$ is the wave vector of the emitted quantum.

Notice that the transition radiation emitted in an ordered system of precipitates can be investigated with the aid of the expression (4.5). For this purpose the quantity $\chi(w)$ should be taken to be the difference between the permittivities of the matrix of the solid and the material of which the precipitate is made [see the expression (3.4)]. The maximum value of the intensity (4.2) will occur in a pore lattice for which the quantity $|\chi(w)|$ has its greatest value. It can be seen from the expression (4.2) that the transition radiation possesses resonance properties when the following condition is fulfilled:

\[
w_{\text{in}} = kv = qv.
\]

(4.3)

This condition determines the emission angle $\theta$ (i.e., the angle between $k$ and $v$) at which resonance transition radiation occurs in the pore lattice.

Below we shall consider in greater detail the most interesting case, when the charged particle traverses the vacancor pore lattice in a direction parallel to one of the axes of the unit cube (i.e., the case in which $V=1$).

The orientation of the axis of the pore lattice can easily be established in the experiments that can be performed on transition radiation emission in a pore lattice, using, for example, electron microscopy, or any other method that allows the investigation of the microscopic structure of the parent crystal lattice of the solid. Knowing the orientation of the pore lattice, we can direct the relativistic particle at any angle to the chosen axis in that lattice. Then, since for $w\gg w_0$, we have $\epsilon_1 \gg 1$, the relation (4.3) will be satisfied only in the case in which $q \cdot v = -2mc/L \ll 0$ ($w \gg 0$). The condition (4.3) in this case assumes the form

\[
\cos \theta = \frac{q \cdot v}{\sqrt{q \cdot q} \sqrt{v \cdot v}} = -\frac{2mc}{L}.
\]

(4.4)

It can be seen from this relation that, when the condition $2mc/\omega_0 L << 1$ is fulfilled, the intensity of the transition radiation emitted by ultrarelativistic particles ($\nu = 1$) in a pencil emission occurring in a narrow cone with maximum apex angle $\beta_0 = 2mc/\omega_0 L$.

To determine the radiation intensity, we must find the Fourier components $\epsilon_1$ of the spatially inhomogeneous component of the permittivity $\epsilon(\omega, \mathbf{r})$. As has already been noted, the symmetry of the vacancor pore lattice coincides with that of the parent crystal lattice of the solid in which the ordered pore structure is formed. Therefore, in such metals as Al and Ni, in which pore lattices have thus far been detected, the pores form the face-centered cubic (fcc) lattice, while in Mo, Nb, Ta, and W the pores form the body-centered cubic (bcc) lattice. Let us find the quantities $\epsilon_1$ for these two types of pore lattices. To do this, let us assume that each pore has the cubic shape with a cube-edge length equal to $2l$. The calculations yield for the bcc pore lattice the expression

\[
c_1 = \frac{\chi}{2\varepsilon_0} \left[ \frac{\cos(n(2m-1)) \sin(2l \sin(\theta L))}{L^3} \frac{2\sin L}{L} - \frac{2\sin L}{L} \right]
\]

(4.5)

For the fcc pore lattice the Fourier transform $\epsilon_1$ has the form

\[
c_1 = \frac{\chi}{2\varepsilon_0} \left[ \frac{\cos(n(2m-1)) \sin(2l \sin(\theta L))}{L^3} \frac{2\sin L}{L} - \frac{2\sin L}{L} \right]
\]

(4.6)

It can be seen from the expressions (4.2), (4.5), and (4.6) that the emission spectra of charged particles in the pore lattices are superpositions of various harmonics having the numbers $l$, $m$, and $n$, the radiation intensities being largely determined by the contributions of the first harmonics.

The condition $\cos \theta = q \cdot v = -2mc/L \ll 0$ ($w \gg 0$) determines the emitted-frequency range for each harmonic number considered separately. It should be noted that the emitted-frequency range in the case of the pencil radiation $(\beta = 1)$ is specified as

\[
\lambda \leq \frac{2mc(\nu + 1)}{\omega_0 L} \gamma^2 \gamma^2 \gamma^2
\]

(4.7)

The wavelength of the pencil radiation depends on the emission angle $\theta$ and the emitted-harmonic number $n$.

For $|\theta|^2 << 1$ the wavelength of the radiation is equal to

\[
\lambda = \frac{1}{2(1 - \beta^2)^{1/2}}
\]

(4.8)

For $2mc/\omega_0 L << 1$ the angular radiation is not a pencil beam, and is directed into the rear hemisphere. The emitted-frequency range in this case is specified as follows:

\[
\frac{2mc(\nu + 1)}{\omega_0 L} \gamma^2 \gamma^2 \gamma^2
\]

(4.9)

In the metals Ni and Nb the observed pore-lattice constants are respectively equal to $L = 660$ and $750$ Å (Ref. 17), and the first harmonics $(n = 1, 3)$ satisfy the relation $2mc/\omega_0 L = 1$, which indicates that the radiation emitted by charged particles in these metals should have a high-directional characteristic with an emitted-
frequency range given by (4.7). As an example, let us point out that, in the case of the motion of relativistic electrons with energy \( E = 500 \text{ MeV} \) through a pore lattice of lattice constant \( L = 100 \text{ Å} \) in a solid with \( \rho = 5.9 \text{ eV} \), the emitted-frequency range for the harmonics with number \( n \) is 35 eV \( \leq \nu \leq 99 \text{ eV} \).

In the other metals (Al, Mo, Ta, and W) in which pore lattices have found to occur, the observed pore-lattice constants are somewhat smaller [\( L \) ranges from about 200 to 300 Å (Ref. 17), and in these metals the relation \( 2mc/\omega L \approx 1 \) obtains even for the first harmonics. The radiation in these metals should be emitted into the rear hemisphere, and the emitted-frequency range is given by the relation (4.8). For example, for the radiation emitted by relativistic electrons with energy \( E = 500 \text{ MeV} \) in the case in which \( L = 200 \text{ Å} \) the emitted-frequency range is equal to 35 eV \( \leq \nu \leq 31 \text{ eV} \).

Let us now determine the intensity \( dI/d\nu \) of the radiation emitted by a relativistic particle moving in a pore lattice along one of the unit-cube edges. For this purpose, let us integrate the expression (4.2) over the solid angle \( \Omega \). As a result of the computations we obtain

\[
dI/d\nu = 2\pi \nu^2 \sum \omega^2 \left\{ (1 - 2\omega x_2)^2 + 1 + 2 \left( (1 - 2\omega x_2)^2 + 1 \right)^{1/2} \right\}^{1/2} \left( 1 - (1 - 4\omega^2 x_2^2) \right)^{-3/2}.
\]

Here \( \omega = \omega_0/\omega_0 \), \( \omega_0 = 2m\nu_0/L \), and \( x_2 = (q^2_2 + q^2_0)/q^2_0 \).

In the region of high quantum-emission frequencies where \( \omega \sim 1 \), \( |x_2| \gg 1 \), and for small pore dimensions \( R \ll L \), the radiation intensity (4.9) varies according to the following law:

\[
dI/d\nu = \omega^2 \pi^2 \nu^2 \omega_0^2 / \omega_0^2 \cdot \omega_0^2.
\]

In this frequency range the radiation intensity practically does not depend on the particle energy.

At higher frequencies, i.e., for \( \omega \omega_0 \ll 1 \) and \( |x_2| \ll 1 \), the radiation intensity depends weakly on the relativistic-particle energy:

\[
dI/d\nu = \omega^2 \pi^2 \nu^2 \omega_0^2 / \omega_0^2 \cdot \omega_0^2.
\]

As has already been noted, an important characteristic of the transition radiation emitted in an ordered vacancy-pore structure is the presence of a threshold effect for the energy of the moving particle, i.e., the existence of a particle energy starting from which there appear in the radiation spectrum (4.9) harmonics having the ordinal numbers \( n \). Starting from the energy \( E^{(1)} \) the harmonic with the number \( n = 1 \) appears in the radiation spectrum, and the entire spectrum is re-established. The radiation intensity reaches saturation, having then the highest possible value. The threshold energy \( E^{(1)} \) is equal to

\[
E^{(1)} = \frac{2mc}{\omega_0}\nu_0.
\]

The fact that \( E^{(1)} \ll \nu_0 \) is very important for the performance of experiments on transition-radiation emission in pore lattices. The point is that the value of the threshold energy \( E^{(1)} \) in a pore lattice having a sufficiently small spacing (i.e., with \( L = 10^4 - 10^5 \text{ Å} \)) is significantly lower than the value of the same quantity for the transition radiation emitted in the normally used layered media. Let us, for comparison, point out that, for layered media with \( L = 10^4 \text{ cm} \), \( E^{(1)} \geq 10^6 \text{ MeV} \). On the other hand, in a pore lattice (in, for example, Mo) with spacing \( L = 220 \text{ Å} \) (Ref. 17) the threshold energy for relativistic electrons has the value \( E^{(1)} = 0.54 \text{ MeV} \). In other experimentally observed pore lattices \( E^{(1)} \approx 1 \text{ MeV} \); therefore, resonance radiation can appear in a pore lattice at significantly lower electron energies: \( E \ll 1 \text{ MeV} \).

Let us now determine the number \( dw \) of quanta emitted by a relativistic particle as it travels a unit length in a vacancy-pore lattice. We perform the numerical computation for the radiation emitted by relativistic electrons in the experimentally observed\(^{17} \) vacancy-pore lattices of the two metals Al and Ni.

In Fig. 1 we present the results of the numerical calculation of the number \( dw \) of quanta emitted in the frequency interval \( d\nu \) for Al having a pore lattice constant \( L = 700 \text{ Å} \) and \( R = 250 \text{ Å} \) (the curve 1) and for Ni with \( L = 650 \text{ Å} \) and \( R = 125 \text{ Å} \) (the curve 2) in the case in which the electron energy \( E = 50 \text{ MeV} \).

The results of the calculations show that the number of quanta emitted over a unit length in a pore lattice practically does not depend on the electron energy when \( E = E^{(1)} \). At the same time, the number of emitted quanta essentially depends on the lattice constant \( L \) and the mean radius \( R \) of the pores. According to (4.9), the highest yield of emitted quanta should occur in a pore lattice in which the pores have fairly large dimensions, i.e., for which \( R \approx L \).

Such pore lattices can be produced with large radiation doses under conditions of continuous pore swelling that conform to the stability conditions for the pore lattice.\(^{18,19} \)

In Fig. 2 we present the results of the calculation of the number \( dw \) of quanta emitted in the pore lattice of Al for different pore radii \( R_0 = 100 \text{ Å} \), \( R_0 = 150 \text{ Å} \), and \( R_0 = 200 \text{ Å} \) and a fixed pore-lattice constant \( L = 700 \text{ Å} \). All the calculations were performed for an electron energy \( E = 50 \text{ MeV} \).

![FIG. 1. Resonance transition radiation spectrum \( dw \) (number of quanta/electron-keV) for the experimentally observed pore lattices of the two metals Al and Ni. The energy of the radiation quanta is plotted along the abscissa axis. The curve 1 pertains to the pore lattice in Al (Ref. 17) for which \( L = 200 \text{ Å} \) and \( R = 250 \text{ Å} \), the curve 2 corresponds to the results of the calculation for the pore lattice in Ni (Ref. 17) for which \( L = 650 \text{ Å} \) and \( R = 125 \text{ Å} \). The calculations were performed for an electron energy \( E = 50 \text{ MeV} \).](image-url)
energy of $E = 200 \text{ MeV}$. It can be seen that the yield of emitted quanta increases as the vacancy porosity and the mean pore dimension increase.

Let us compare the number of quanta emitted during resonance emission in the pore lattice with the yield obtained in the bremsstrahlung emitted in the crystal. For electron energies $E = E_0 = (\text{mc}^2 E_0)^{1/2}$, usually, for metals $E_0 = 1 \text{ GeV}$ and in the region of quantum-emission frequencies $\nu_0 \ll \omega$ [$= \omega_0/\gamma$], the number of emitted quanta per unit length as determined by the bremsstrahlung obtained with allowance for the polarization of the medium is equal to

$$\alpha \gamma \pi \omega_0^2 (\frac{1}{\gamma^2} - 3) \frac{d\nu}{d\nu_0} \approx \frac{d\nu}{d\nu_0} \left( \frac{\nu_0}{\gamma \omega_0} \right)^2 \frac{d\gamma}{d\nu_0} \left( \frac{\nu_0}{\gamma \omega_0} \right)$$

(4.10)

In Fig. 3 we compare the differential yield for the quanta emitted in the pore lattice and the yield (4.10) due to the bremsstrahlung of the relativistic particles in the crystal. The calculations were performed for electrons with energy $E = 200 \text{ MeV}$ moving in the pore lattice of Al with $L = 700 \text{ Å}$ for different pore dimensions: $R_1 = 100 \text{ Å}$, $R_2 = 150 \text{ Å}$, and $R_3 = 200 \text{ Å}$.

In conclusion, let us note that the present investigation of transition radiation emission on vacancy pores has been carried out for transparent media, and therefore the results obtained are valid for solids whose thickness $L$ is smaller than the characteristic absorption lengths $\lambda_0$. (For example, in Al, $\lambda_0 = 15 \text{ Å}$ for $\hbar \omega = 10 \text{ keV}$, while $\lambda_0 = 1 \text{ cm}$ for $\hbar \omega = 50 \text{ keV}$.)

We can conclude on the basis of the above-performed analysis and the data obtained on transition-radiation emission in ordered pore structures that materials containing pore lattices, and possessing a developed porosity can be used for the construction of charged-particle detectors and as x-ray sources. In this case it is desirable to use materials with small $L$ in order to decrease the quantum-absorption effects, since the absorption cross section in the x-ray frequency region, as determined by the photoelectric effect, $\alpha \gamma \pi \omega_0^2 (\frac{1}{\gamma^2} - 3)$.

Furthermore, the vacancy porosity of solids can be investigated with the aid of transition radiation emitted on pores. This method can apparently be especially effective in the study of the defect structure of the surface layers of solids.

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