Effect of vacancies on NMR is solid He

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1. INTRODUCTION

The principal characteristic feature of a quantum crystal is the large amplitude of the zero-point vibrations of its atoms and their delocalization as a result of the overlap of the atomic wave functions. The quantum-mechanical tunneling ability manifests itself in the existence of an excitation system that is absent from non-quantum crystals. The mobility of these quasiparticles and their interaction is easy to investigate in solid He, since the presence of spin-1/2 particles in it permits the use of the sensitive methods of nuclear magnetic resonance (NMR).

The available experimental data on NMR in solid He are summarized mainly in the review of Guver et al. The system of nuclear spins (I) in a constant external magnetic field $H_0$, described by a Zeeman Hamiltonian, arrives at thermal equilibrium with a definite temperature (spin-spin relaxation) via a dipole-dipole interaction that is modulated by the motion of one of the types of quasiparticles. The same mechanism is responsible also for the spin-lattice relaxation.

The first part of the paper deals with the temperature region $T \sim 1.5\, \text{K}$, where the dipole-dipole interaction is modulated by delocalized vacancies—vacancies (V).

In Ref. 1, the vacancy is regarded as a "bare" quasiparticle that moves through a homogeneous medium. Yet a vacancy in solid He produces around itself a ferromagnetically polarized region (FPR) of nuclear spins, and this region influences substantially the vacancy mobility. The character of the modulation of the dipole-dipole interaction is thereby altered.

We derive below the dependences of the spin-lattice and spin-spin relaxation times, as well as of the diffusion coefficient $D_v$ of the nuclear magnetization, on the size and mobility of the FPR. Reduction of the experimental data by means of these formulas makes it possible to determine the number of spins contained in the FPR and the diffusion coefficient $D_v$ of a vacancy "dressed" in this manner, and to estimate its band width.

At $T \leq 10^{-3}\, \text{K}$ the FPR can be regarded as almost immobile, so that their influence on a nonstationary process such as relaxation is very small even for large (nonequilibrium) FPR densities. On the other hand, the strong exchange interaction between the He nuclei leads to a relation between the direction of the FPR and the diffusion coefficient and the polarization of the nuclear magneton. By the same token, the existence of the FPR influences the absorption line shape in the case of stationary detection of the NMR signal. In the second part of the paper, this dependence is derived for weak and strong constant magnetic fields.

The conclusion deals with the conditions under which the results are valid, and contains some numerical estimates.

2. RELAXATION AND DIFFUSION

Relaxation times

At $T \sim 1.5\, \text{K}$ the $V$ system is closely coupled to the lattice and they have the same constant temperature. The temperature of the $Z$ system, which is "heated" by the RF field, approaches asymptotically the temperature of the $V$ system. The large mobility of the nuclear spins in solid He leads to a rapid change of the Hamiltonian of the dipole-dipole interaction, so that it is possible to use for the calculations the short-correlation times approximation (see Ref. 7, Chap. VIII). We ob-
tain the correlation function that ensues from this approach and ascertain the dependence of the correlation time on the size and mobility of the FPR.

The dipole-dipole interaction in the $Z$ system is described by the Hamiltonian $\mathfrak{H}_Z$,

$$\mathfrak{H}_Z = \sum_{i<j} B_{ij} S_i S_j$$

(2.1)

where $S_{ij}$ is an operator acting on the spins of the particles $i$ and $j$; $B_{ij}$ is a random function of the relative locations of the particles $i$ and $j$.

Standard calculations lead to the following expression for the spin-lattice relaxation times 1:

$$\tau_{\text{lattice}} = \frac{1}{J_{\text{eff}}(\omega)}$$

(2.2)

where the spectral densities $J_{\text{eff}}(\omega)$ of the correlation functions are in the case of polycrystalline samples

$$J_{\text{eff}}(\omega) = \sum_{i,j} J_{ij}(\omega)$$

(2.3)

where $M_j$ is the Van Vleck second moment for the Hamiltonian $\mathfrak{H}_Z$, and $g(t)$ is the reduced correlation function of the random function $B_{ij}(t)$ remaining constant during the time $t$.

Changing, for simplicity, to a coordinate system with center at the point $r_0$, we obtain for the considered case

$$P(t_0) = 1 - \frac{B_v}{B_v + B_0} \text{erfc} \left( \frac{r_0^2}{2B_v} \right)$$

(2.4)

where

$$\text{erfc} \left( \frac{r_0^2}{2B_v} \right) = \int_{r_0^2/2B_v}^{\infty} e^{-y^2} dy$$

is the complementary probability integral.

The sought correlation function $g(t)$ is equal to

$$g(t) = \text{erfc} \left( \frac{r_0^2}{2B_v} \right)$$

(2.5)

Changing over in $\text{erfc}(t)$ to the "gas" approximation

$$g(t) = e^{-t/\gamma}$$

(2.6)

and the correlation time is

$$\tau = \frac{\gamma}{\gamma_k}$$

(2.7)

where $\gamma$ is the gyromagnetic ratio.

Since the dipole-dipole interaction radius is small, it can be assumed that the change of the local magnetic field at the point $r_0$ is connected with the change in its immediate spin surrounding. In the considered temperature region, the mobility and density of the FPR are large enough to assume the highest-frequency changes of the local field to be the result of passages of the FPR through the point $r_0$. Let us find the probability that an immobile $k$-th spin located at the instant $t=0$ in a paramagnetic region "survives" a time $t$, i.e., does not land in any of the diffusing FPR. A spin that lands in an FPR is excluded from the "survivors," so that the sought probability is obtained by solving the equivalent problem of diffusion of a spin with diffusion coefficient $D_0$ in a medium with randomly distributed immobile and absolutely absorbing spheres of radius $R_0$, $R_0$ is the radius of the FPR and $D_0$ is its diffusion coefficient. The corresponding aggregate of boundary conditions for the probability density $P(r, r_0, t) = 0$ at $r = r_0$ takes the form

$$P(r_0, R_0, t=0) = 0$$

(2.8)

where $R_0$ are the radius-vectors of the vacancies. For times

$$t > r_0^2/2D_0$$

(2.9)

the probability density $P(r_0, R_0, t)$ becomes

$$P(r_0, R_0, t) = \frac{1}{(2\pi D_0 t)^{3/2}} \exp \left( -\frac{r_0^2}{4D_0 t} \right)$$

(2.10)

The total probability that the $k$-th spin will not land in a single FPR is

$$P_{\text{total}} = \frac{1}{(2\pi D_0 t)^{3/2}} \exp \left( -\frac{r_0^2}{4D_0 t} \right)$$

(2.11)

where $n$ is the vacancy density, we find that for times satisfying the condition (2.6), the correlation function is exponential

$$g(t) = \exp \left( -t/\tau \right)$$

(2.12)

and the correlation time is

$$\tau = \frac{\gamma}{\gamma_k}$$

(2.13)

where $\gamma$ is the gyromagnetic ratio of the vacancy. We note that $\gamma_k/\gamma_0 < 1$, and since we are considering integrals of time for which the short-correlation-time approximation is valid,

$$\Delta t > \tau$$

(2.14)

the condition (2.6) is satisfied automatically. Substituting (2.13) in (2.9) we obtain for (2.2)

$$\tau_{\text{eff}} = \frac{2}{\gamma_k} \left[ \frac{R_0^2}{\gamma_0} + \frac{R_0^2}{\gamma_0} \right]$$

(2.15)

The distinctive character of the modulation of the local
magnetic field is reflected in (2.15) and (2.17) via $v_x$.

(2.13).

The diffusion coefficient. Comparison with experiment

The diffuse motion of the longitudinal and transverse components of the nuclear magnetization $m$ are independent, so that a distinction must be made between the diffusion coefficients $D_x$ and $D_z$, which determine the transport of the Zeeman energy connected with the coupling between the $Z$ system and the lattice. \textsuperscript{1,4} The coefficient $D_z$ calculated by us describes the magnetization transport connected with the spatial motion of the paramagnetic spins in the FPR.

The macroscopic Bloch equations with the diffusion term are of the form

$$\frac{\partial m_x}{\partial t} = -\gamma H m_x - D_x \frac{\partial^2 m_x}{\partial x^2},$$

(2.18)

where $H$ is the magnetization of the paramagnet, $D$ is the spin self-diffusion coefficient, $\gamma$ is the magnetization of the FPR ensemble, and $T_1$ is the time of transverse relaxation of this ensemble. Assuming that the deflecting RF pulse rotates the vectors $m$ and $\mathbf{A}$ without changing their moduli, and recognizing that $\mu = \mu_0$ where $\mu$ and $\mu_0$ are the magnetic moments of the FPR and of the He$^3$ nucleus, respectively, and $\mu_0$ is the number of spins in the FPR, we obtain from the Curie law

$$\frac{\partial m_x}{\partial t} = -\frac{\gamma H m_x}{T_1} + D_x \frac{\partial^2 m_x}{\partial x^2},$$

(2.19)

Adding Eqs. (2.18) and substituting (2.19), we neglect the term with $T_1$ (see the Conclusion) and recognize that $\gamma \propto m$. As a result we have for the measurable quantities

$$\frac{\partial m_x}{\partial t} = -\frac{\gamma H m_x}{T_1} + D_x \frac{\partial^2 m_x}{\partial x^2},$$

(2.20)

where

$$D_x = D + D_x x.$$

(2.21)

The results call for a new look at the experimental data of Refs. 4-6. We note by way of introduction that the accuracy of the experiments considered by us is $\approx 1\%$. From (2.17) we have [cf. (2.23)]

$$T_1 = \frac{3}{\gamma H m_x} = \frac{1}{\gamma H m_x},$$

(2.24)

A temperature dependence of the spin-spin relaxation time $T_1$ is observed at $T > 1 \text{K}$, which corresponds for the employed fields $50 - 200 \text{MHz}$ to the case $\omega / \gamma H m_x \ll 1$. From (2.17) we obtain from (2.21)

$$D_x = D + D_x x.$$
Owing to the large vacuum exchange interaction, which couples the nuclear spins inside the FPR, we can neglect the internal structure of the region when it is acted upon by an external field of frequency up to \( \omega \sim 10^9 \text{Hz} \), when the natural modes of the FPR begin to be excited.\(^{3}\) It is necessary, however, to take into account the participation of the boundary spins in the diffusion outside the FPR, due to direct exchange between the He\(^3\) nuclei (which is weaker than the vacancy exchange). For the considered alternating fields are weak enough to disregard all the cases described below as far from saturation.

**Weak constant field**

We consider an He\(^3\) crystal with an FPR, placed in constant magnetic field directed along the \( z \) axis, such that \( \mu_0 H \ll T \), and a magnetic field \( H_l \ll H_0 \) perpendicular to the constant field and rotating with frequency \( \omega \).

The imaginary part of the magnetic susceptibility introduced above is expressed in this case by the formula (see Ref. 7, Chap. III)

\[
\chi' = \left. \frac{1}{4 d_0} \right\} \frac{\text{d}(0, \phi) \text{d}(0, \psi)}{\text{d} \phi \text{d} \psi} - \frac{1}{4 d_0} \} \frac{\text{d}(0, \phi) \text{d}(0, \psi)}{\text{d} \phi \text{d} \psi} 
\]

(3.1)

where \( \mu \) is the \( y \) component of the FPR magnetic moment in the coordinate system that rotates with frequency \( \omega \); \( \phi(t) \) is the distribution, meaning the number of FPR per unit volume with angle \( \theta \) between the vector of the magnetic moment of the FPR in the constant field \( H_0 \), and a magnetic-moment precession phase \( \varphi \) relative to the rotating field \( H_0 \).

The FPR magnetization change due to the diffusion flux through its surface consists of rotation of a vector \( M \) of constant magnitude. The continuity equation expressed in terms of the components is

\[
\frac{\partial n(r)}{\partial t} + \nabla \cdot \mathbf{j} = 0, \quad \text{where} \quad \mathbf{j} = \mathbf{v} \times \mathbf{B}.
\]

(3.4)

The frequency of tunneling in self-diffusion, \( \omega \sim 10^8 \text{ Hz} \), is much larger than the reciprocal relaxation time of the medium with the FPR. Changing over to the \( n(r, \omega) \) and neglecting the left-hand side of (3.4), we obtain

\[
\frac{\partial n}{\partial t} = 0.
\]

(3.5)

Expressed in components, the spherically symmetrical solution of (3.5) is

\[
m(r) = n(r) \sin \theta.
\]

(3.6)

where \( n(R) \) and \( \theta(R) \), specified on the FPR surface, constitute the boundary conditions.

The frequency dependence of the magnetic susceptibility in He\(^3\) (weak constant field) is expressed in this case by the formula

\[
\chi' = \chi_{\text{vac}, \omega} + \chi_{\text{nc}, \omega}, \quad \chi_{\text{nc}, \omega} = \frac{\mu_0}{2} \frac{1}{\omega^2} \left( \frac{1}{1 - \omega^2/4} \right)\chi_{\text{nc}, \omega},
\]

(3.7)

where \( n(r) \) is a function of the spin \( \mathbf{d} \) and is based on simple geometric considerations. The free precession around an angle \( \omega t \) of the vector \( \mathbf{d} \) is a function of the spin coordinates and must be calculated; the term with \( f \) describes the thermal motion of \( M \) in space.

In a spherical coordinate system that rotates with frequency \( \omega \) of the SCS(\( \omega \)), Eq. (3.3) takes the form

\[
\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) \left( n(R, \theta) \sin \theta \right) \frac{\partial}{\partial \theta} \left( n(R, \theta) \sin \theta \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( n(R, \theta) \sin \theta \right) = 0,
\]

(3.3)

where \( \dot{\theta} \) and \( \dot{\phi} \) are the rates of precession around \( H_0 \), in the SCS(\( \omega \)); \( \dot{e} \) is the relaxation rate (\( \dot{e} = 0 \)). To solve (3.3) we must express \( \dot{\theta} \), \( \dot{\phi} \), and \( \dot{\theta} \) in terms of the angles \( \theta \) and \( \phi \). We neglect in this case the action of the alternating field on the FPR via the paramagnetic medium, the error being of the order of \( \omega \chi_{\text{nc}}/\chi_{\text{nc}} \) (where \( \chi \) is the static nuclear magnetic susceptibility of He\(^3\)).

To calculate \( \dot{\theta} \), we consider the relaxation of the magnetization of the paramagnetic medium in a constant field \( H_0 \). The change of the magnetization vector is the sum of the precession and of the diffusion motion of the nuclear spins:

\[
\frac{1}{\omega} = \left( x + \frac{1}{\omega} \right), \quad \dot{\omega} = 0.
\]

(3.4)

The considered alternating fields are weak enough to regard all the cases described below as far from saturation.

**Weaker constant field**

We consider an He\(^3\) crystal with an FPR, placed in constant magnetic field directed along the \( z \) axis, such that \( \mu_0 H_0 \ll T \), and a magnetic field \( H_l \ll H_0 \) perpendicular to the constant field and rotating with frequency \( \omega \).

The imaginary part of the magnetic susceptibility introduced above is expressed in this case by the formula (see Ref. 7, Chap. III)

\[
\chi' = \chi_{\text{vac}, \omega} + \chi_{\text{nc}, \omega}, \quad \chi_{\text{nc}, \omega} = \frac{\mu_0}{2} \frac{1}{\omega^2} \left( \frac{1}{1 - \omega^2/4} \right)\chi_{\text{nc}, \omega},
\]

(3.7)

where \( n(r) \) is a function of the spin \( \mathbf{d} \) and is based on simple geometric considerations. The free precession around an angle \( \omega t \) of the vector \( \mathbf{d} \) is a function of the spin coordinates and must be calculated; the term with \( f \) describes the thermal motion of \( M \) in space.

In a spherical coordinate system that rotates with frequency \( \omega \) of the SCS(\( \omega \)), Eq. (3.3) takes the form

\[
\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) \left( n(R, \theta) \sin \theta \right) \frac{\partial}{\partial \theta} \left( n(R, \theta) \sin \theta \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( n(R, \theta) \sin \theta \right) = 0,
\]

(3.3)

where \( \dot{\theta} \) and \( \dot{\phi} \) are the rates of precession around \( H_0 \), in the SCS(\( \omega \)); \( \dot{e} \) is the relaxation rate (\( \dot{e} = 0 \)). To solve (3.3) we must express \( \dot{\theta} \), \( \dot{\phi} \), and \( \dot{\theta} \) in terms of the angles \( \theta \) and \( \phi \). We neglect in this case the action of the alternating field on the FPR via the paramagnetic medium, the error being of the order of \( \omega \chi_{\text{nc}}/\chi_{\text{nc}} \) (where \( \chi \) is the static nuclear magnetic susceptibility of He\(^3\)).

To calculate \( \dot{\theta} \), we consider the relaxation of the magnetization of the paramagnetic medium in a constant field \( H_0 \). The change of the magnetization vector is the sum of the precession and of the diffusion motion of the nuclear spins:

\[
\frac{1}{\omega} = \left( x + \frac{1}{\omega} \right), \quad \dot{\omega} = 0.
\]

(3.4)
Denoting the cone apex angle by \( \theta \), we get

\[
\begin{align*}
\cos \theta &= \cos a \cos \beta, \\
\sin \theta &= \sin a \cos \beta.
\end{align*}
\]

where \( a \) is the initial phase of the precession. Differentiating (3.11), we can express after simple transformations the sought quantities in terms of \( a \) and \( \theta \):

\[
\sin^2 \theta = \frac{\sin^2 \theta}{\sin^2 \theta - \cos^2 \theta}.
\]

To determine \( \beta, \theta \), we note that at equilibrium in a constant field \( H_0 \), the distribution-function flux density is zero in the laboratory frame. We have consequently in the FCS(\( \theta \))

\[
\begin{align*}
I_{m}(\theta) &= \frac{1}{2} I_0 (\theta), \\
I_{m}(\theta) &= \frac{1}{4} I_0 (\theta).
\end{align*}
\]

where

\[
\begin{align*}
\sin \theta &= \sin \theta \sin \theta, \\
\cos \theta &= \cos \theta \cos \theta.
\end{align*}
\]

is the normalized equilibrium Boltzmann distribution function. From (3.13) we get, taking (3.12) and (3.10) into account,

\[
\begin{align*}
\frac{\partial}{\partial \theta} = \frac{\sin \theta}{\sin \theta - \cos \theta}, \\
\frac{\partial}{\partial \theta} = \frac{\sin \theta}{\sin \theta - \cos \theta}.
\end{align*}
\]

where

\[
\begin{align*}
\sin \theta &= \sin \theta \sin \theta, \\
\cos \theta &= \cos \theta \cos \theta.
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\[
\begin{align*}
\sin \theta &= \sin \theta \sin \theta, \\
\cos \theta &= \cos \theta \cos \theta.
\end{align*}
\]

where

\[
\begin{align*}
\sin \theta &= \sin \theta \sin \theta, \\
\cos \theta &= \cos \theta \cos \theta.
\end{align*}
\]
where $\omega$ is a constant determined from the boundary conditions;

$$\Omega = \left( \frac{1 + \text{i}(T_2^0 T_1^0 \omega)^{-1}}{2\Omega} \right)^{-1}$$

is the wave vector of the standing spherical wave;

$$m^0 = \mu_0 r_0 (T_1^0 T_1^0 + 1)^{-1}$$

is the equilibrium value of the transverse component of the magnetization vector of the homogeneous paramagnet.

We consider the case $\lambda \ll 1$. We can confine ourselves then to an isolated FPR in a paramagnetic medium. The imaginary part of the susceptibility is then given by

$$\chi' = \chi' + i \chi''$$

where

$$\chi' = \mu_0 T_1^0 (1 + \text{i}(T_1^0 T_1^0 \omega)^{-1})$$

$\chi^0$ is the effective field in the homogeneous paramagnet.

We determine the constant $C$ by recognizing that the change of the magnetic moment of the FPR in the space-averaged (and by the same token, coordinate-independent) magnetization of the paramagnet is described by the Bloch equation, and the change of the FPR magnetization is described by the free-precession equation. The connection between the FPR and the medium is given by

$$\omega = \gamma_N \delta B / M$$

where $\delta B$ is the reciprocal length over which the influence of the phase-inhomogeneity center (the influence of the FPR on the paramagnet attenuates);

$$m^0 = \mu_0 r_0 (T_1^0 T_1^0 + 1)^{-1}$$

is the wave vector of the standing spherical wave;

$$m^0 = \mu_0 r_0 (T_1^0 T_1^0 + 1)^{-1}$$

is the reciprocal length over which the influence of the phase-inhomogeneity center (the influence of the FPR on the paramagnet attenuates);

$$m^0 = \mu_0 r_0 (T_1^0 T_1^0 + 1)^{-1}$$

is the wave vector of the standing spherical wave;

$$m^0 = \mu_0 r_0 (T_1^0 T_1^0 + 1)^{-1}$$

is the wave vector of the standing spherical wave.

The correlation time (2.13) for the vacancy relaxation processes at $T > 1.5 K$ is $\tau_2 = 10^{-8} \text{sec}$, thus justifying the use of the short-relaxation-time approximation (the conditions $T_1 = c_1, T_2 = c_2$). With decreasing temperature and with exponential decrease of the absorption, the vacuum regime "freezes out" and a transition takes place, within a time $\tau_2 = 10^{-8} \text{sec}$, to a regime in which the relaxation is determined by the tunneling of the nuclear spins themselves, thereby modulating the dipole-dipole interaction.

In principle, relaxation is also possible on account of dipole-dipole interaction of the magnetic moments of the FPR and the He nuclei in a paramagnetic region. An estimate (Ref. 7, Chap. VIII) shows, however, yields $T_2 = 10^{-8} \text{sec}$. Thus, an FPR ensemble relaxes very slowly: $T_2 = 10^{-8} \text{sec}$. Actually the line of the FPR sub-system should broaden just the same, on account of collisions with phonons, but at $T > 1 K$, in view of the smallness of $s = 1$, this line cannot be discerned at all against the background of the broad line of the paramagnetic medium with $T_2$ from Eq. (1.17).

Absorption that might be separable in an FPR system from the homogeneous absorption (3.23) in a paramagnet, would be observed if vacancy density $x \times 10^{20}$ were produced at low temperatures $T < 10^8 K$. The thermal change of the orientation of the FPR magnetic moment, which is possible in a field $H > 10^8 G$, manifests itself in the temperature dependence of the line shape (3.19). The resonance maximum is in this case (assuming $\chi_0 (T > 10^8 K)^{-1} \ll 1$, Ref. 10) to be $\chi'^{-1} \times 10^{-8}$.

More convenient from the point of view of the analysis of the experimental data is the case of a strong magnetic field ($H > 10^8 G$ at $T > 10^8 K$). On approaching resonance, $\lambda$ tends to its maximum value $\lambda_0 = 10^{-4} \text{cm}$. Near resonance, the transition from the regime $\lambda \ll 1$ to $\lambda \gg 1$ (formulas (3.29) and (3.31)) occurs at densities $x \times 10^{-10}$. The lowest densities needed to observe the renormalization of $T_2$ (see (3.29)) and of the amplitude of the resonant absorption (3.29) and (3.31) depend on the concrete experimental condition. The $\chi'(\omega)$ dependence makes it possible to monitor the vacancy density in the crystal, a factor that can be of use in the corresponding nonequilibrium experiments.
High temperature NMR experiments lend themselves easily to treatment with participation of FPR (Sec. 2) and yield quite reasonable values for their characteristics. Much attention is currently paid to NMR in the low-temperature region. It would be of interest to extend the scope of this research by performing experiments in the vacancy regime, i.e., accompanied by production of nonequilibrium vacancy densities. The results of Sec. 3 provide for this regime a theoretical description that can be used also in the search for zero fields.

I am deeply grateful to A. F. Andreev for attentive guidance and valuable advice, and to V. I. Marchenko for a helpful discussion.

APPENDIX

Taking into account the discussion, in Sec. 3, of the solutions of Eqs. (3.17), we write down in explicit form only those functions $F_2, F_4,$ and $F_6,$ which are needed for the calculation of the imaginary part of the susceptibility (3.19):

\begin{equation}
F_2(0) = \frac{1}{2} \left( 3 \sin \theta - \sqrt{3} \sin 2 \theta \right) - \frac{1}{2} \left( 1 + \frac{1}{\beta} \ln 2 \right) \sin \theta - \frac{1}{2} \ln 2 \right),
\end{equation}

\begin{equation}
F_4(0) = \frac{1}{4} \left( 9 \sin 2 \theta - 3 \sin 4 \theta \right) - \frac{1}{2} \left( 1 + \frac{1}{\beta} \ln 2 \right) \sin 2 \theta - \frac{1}{2} \ln 2 \right),
\end{equation}

\begin{equation}
F_6(0) = \frac{1}{16} \left( 27 \sin 3 \theta - 9 \sin 6 \theta \right) - \frac{1}{2} \left( 1 + \frac{1}{\beta} \ln 2 \right) \sin 3 \theta - \frac{1}{2} \ln 2 \right),
\end{equation}

\begin{equation}
\Phi_0(0) = \frac{1}{2} \sin \theta \left( 3 \sin \theta - \sqrt{3} \sin 2 \theta \right) - \frac{1}{2} \left( 1 + \frac{1}{\beta} \ln 2 \right) \sin \theta - \frac{1}{2} \ln 2 \right),
\end{equation}

\begin{equation}
-\frac{1}{8} \sin \theta \left( 9 \sin 2 \theta - 3 \sin 4 \theta \right) - \frac{1}{2} \left( 1 + \frac{1}{\beta} \ln 2 \right) \sin 2 \theta - \frac{1}{2} \ln 2 \right),
\end{equation}

\begin{equation}
-\frac{1}{16} \sin \theta \left( 27 \sin 3 \theta - 9 \sin 6 \theta \right) - \frac{1}{2} \left( 1 + \frac{1}{\beta} \ln 2 \right) \sin 3 \theta - \frac{1}{2} \ln 2 \right),
\end{equation}

\begin{equation}
-\frac{1}{16} \left( 27 \sin 3 \theta - 9 \sin 6 \theta \right) - \frac{1}{2} \left( 1 + \frac{1}{\beta} \ln 2 \right) \sin 3 \theta - \frac{1}{2} \ln 2 \right),
\end{equation}

\begin{equation}
\Phi_2(0) = \frac{1}{2} \sin 2 \theta \left( 3 \sin 2 \theta - \sqrt{3} \sin 4 \theta \right) - \frac{1}{2} \left( 1 + \frac{1}{\beta} \ln 2 \right) \sin 2 \theta - \frac{1}{2} \ln 2 \right),
\end{equation}

\begin{equation}
-\frac{1}{8} \sin 2 \theta \left( 9 \sin 4 \theta - 3 \sin 6 \theta \right) - \frac{1}{2} \left( 1 + \frac{1}{\beta} \ln 2 \right) \sin 4 \theta - \frac{1}{2} \ln 2 \right),
\end{equation}

\begin{equation}
-\frac{1}{16} \sin 2 \theta \left( 27 \sin 6 \theta - 9 \sin 8 \theta \right) - \frac{1}{2} \left( 1 + \frac{1}{\beta} \ln 2 \right) \sin 6 \theta - \frac{1}{2} \ln 2 \right),
\end{equation}

\begin{equation}
-\frac{1}{16} \left( 27 \sin 6 \theta - 9 \sin 8 \theta \right) - \frac{1}{2} \left( 1 + \frac{1}{\beta} \ln 2 \right) \sin 6 \theta - \frac{1}{2} \ln 2 \right),
\end{equation}

\begin{equation}
\Phi_4(0) = \frac{1}{2} \sin 4 \theta \left( 3 \sin 4 \theta - \sqrt{3} \sin 8 \theta \right) - \frac{1}{2} \left( 1 + \frac{1}{\beta} \ln 2 \right) \sin 4 \theta - \frac{1}{2} \ln 2 \right),
\end{equation}

\begin{equation}
-\frac{1}{8} \sin 4 \theta \left( 9 \sin 8 \theta - 3 \sin 10 \theta \right) - \frac{1}{2} \left( 1 + \frac{1}{\beta} \ln 2 \right) \sin 8 \theta - \frac{1}{2} \ln 2 \right),
\end{equation}

\begin{equation}
-\frac{1}{16} \sin 4 \theta \left( 27 \sin 10 \theta - 9 \sin 12 \theta \right) - \frac{1}{2} \left( 1 + \frac{1}{\beta} \ln 2 \right) \sin 10 \theta - \frac{1}{2} \ln 2 \right),
\end{equation}

\begin{equation}
-\frac{1}{16} \left( 27 \sin 10 \theta - 9 \sin 12 \theta \right) - \frac{1}{2} \left( 1 + \frac{1}{\beta} \ln 2 \right) \sin 10 \theta - \frac{1}{2} \ln 2 \right),
\end{equation}

\begin{equation}
\Phi_6(0) = \frac{1}{2} \sin 6 \theta \left( 3 \sin 6 \theta - \sqrt{3} \sin 12 \theta \right) - \frac{1}{2} \left( 1 + \frac{1}{\beta} \ln 2 \right) \sin 6 \theta - \frac{1}{2} \ln 2 \right),
\end{equation}

\begin{equation}
-\frac{1}{8} \sin 6 \theta \left( 9 \sin 12 \theta - 3 \sin 14 \theta \right) - \frac{1}{2} \left( 1 + \frac{1}{\beta} \ln 2 \right) \sin 12 \theta - \frac{1}{2} \ln 2 \right),
\end{equation}

\begin{equation}
-\frac{1}{16} \sin 6 \theta \left( 27 \sin 14 \theta - 9 \sin 16 \theta \right) - \frac{1}{2} \left( 1 + \frac{1}{\beta} \ln 2 \right) \sin 14 \theta - \frac{1}{2} \ln 2 \right),
\end{equation}

\begin{equation}
-\frac{1}{16} \left( 27 \sin 14 \theta - 9 \sin 16 \theta \right) - \frac{1}{2} \left( 1 + \frac{1}{\beta} \ln 2 \right) \sin 14 \theta - \frac{1}{2} \ln 2 \right),
\end{equation}

Translated, by J. G. Ashikho

Heat capacity of Pt$_3$Mn$_x$Fe$_{1-x}$ alloys

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Submitted 4 May 1980


The adiabatic-calorimeter method is used to investigate experimentally the heat capacity of the atomically ordered alloys Pt$_3$Mn$_x$Fe$_{1-x}$, $x = 0, 0.2, 0.3, 0.4, 0.5, 0.8, 1.0$ in the temperature interval 13–300 K. The alloys are antiferromagnetic at $x = 0$ and ferromagnetic at $x = 1$. The temperature and concentration dependences of the magnetic part of the heat capacity are discussed within the framework of the model of the nucleation mechanism of a concentration phase transition.

PACS numbers: 65.40.Em

It was shown by a number of workers that a ferromagnetic-antiferromagnetic phase transition is produced in quasibinary solid solutions based on the atomically ordered Pt$_3$Mn–Pt$_3$Fe alloys when the concentration of the magnetic components is varied at low temperatures. The initial alloys have at room temperature an fcc lattice with close parameters: $a = 3.89 \AA$, for the Pt$_3$Mn alloy and $a = 3.97 \AA$, for the Pt$_3$Fe alloy. In the antiferromagnetic state at $T = T_a = 164$ K the lattice of the Pt$_3$Fe alloy becomes weakly tetragonal ($c/a = 0.9986$). The Pt$_3$Mn alloy is a collinear ferromagnet at $T < T_a$ ($T_a = 390$ K).

The solid solutions Pt$_3$Mn$_x$Fe$_{1-x}$ have at manganese