Birefringence and gyrotrope due to nearly Bragglike processes in the x-ray region

N. B. Baranova and B. Ya. Zel'dovich

P. N. Lebedev Physics Institute, USSR Academy of Sciences

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Propagation of electromagnetic waves is considered in scalar spatially periodic media at frequencies that almost satisfy the Bragg condition. The process of virtual rescattering into other waves and back leads to corrections to the phase velocity of the initial wave. The dependence of the amplitude of the scalar scattering on the polarization causes these corrections to produce birefringence in the region of the two-wave Bragg resonance. Near three-wave and multioave resonances, subject to definite conditions on the symmetry of the medium, these corrections can lead also to gyrotropy, i.e., to rotation of the plane of polarization of the wave. The possibility of observing the effects in crystal at x-ray frequencies is considered.

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1. INTRODUCTION

It is known that the dielectric constant of a condensed medium differs from unity in the x-ray band (λ ~ 1 Å) only in the fourth or even fifth decimal point, and is furthermore a pure scalar:

\[ \varepsilon_\omega (\omega) = \varepsilon_\infty \left[ 1 - \frac{\omega_0^2}{\omega^2} \right] , \]

where \( \omega \) and \( \varepsilon \) are the mass and charge of the electron, \( N(\omega) \) is their density, and \( \omega = 2\pi c/\lambda \) is the radiation frequency. The propagation of x-ray photons through a medium is therefore not accompanied as a rule by a change in their polarization state.

Exceptions are cases when the conditions of Bragg rescattering from a given wave into another at the corresponding Fourier component \( \mathbf{q} \) are satisfied in the crystal. Since the amplitude of the scattering from a wave \( \mathbf{A} \) into a wave \( B \) on scalar perturbations is proportional to \( |f_{\mathbf{q}}| \), where \( \mathbf{q} \) and \( \mathbf{e} \) are the unit vectors of the polarization, it follows, as is well known, that the Bragg interaction is different for the s- and p-polarizations in both the kinematic and the dynamic theory (see, e.g., Refs. 1–3). More complicated are the polarization effects in the case of multilayer refraction (see Refs. 1–4). In all these cases, however, apart from the change in the polarization state of the incident wave itself, diffracted waves are excited in fact.

We wish to discuss in this paper the possibility of observing birefringence and gyrotropy in pure form, i.e., without real excitation of other waves. As the mechanism for producing these effects we propose the process of virtual rescattering into other waves and back. The smallness of the amplitude of the elementary rescattering act can be offset to a considerable degree by the proximity of the Bragg resonances.

2. BIREFRINGENCE IN PROPAGATION NEAR A SOLITARY BRAGG RESONANCE

We consider wave propagation in a direction close to the satisfaction of the Bragg condition for the Fourier component of the dielectric constant

\[ \Delta(\omega) \approx \frac{\omega_0^2}{\omega^2} \cos \left[ (\mathbf{q} \cdot \mathbf{e}) + (\mathbf{q} \cdot \mathbf{e}) \right] . \]

(1)
We assume here that, owing to the inexact satisfaction of the Bragg condition, the wave \( B \) is weakly excited.

We seek the field in the crystal, with allowance for two-wave refraction at the frequency \( \omega \), in the form

\[
E(t) = A(r) \exp(i k_A r) + B(r) \exp(i k_B r),
\]

where \( k_A, k_B \) are solutions of the propagation condition.

Maxwell’s equations then lead to the system

\[
\begin{align*}
(a) \quad & A(t) - i \frac{\partial A}{\partial t} - \nabla \times \mathbf{a} \times \mathbf{A}, \\
(b) \quad & B(t) - i \frac{\partial B}{\partial t} - \nabla \times \mathbf{a} \times \mathbf{B}.
\end{align*}
\]

Here \( \mathbf{a} \) is the operator of projection on a plane perpendicular to the wave direction.

In the right-hand side of Eq. (4a) we have retained, besides the operator \( \mathbf{a} \), a small perturbation \( h(r) = h_0 \exp(i k_0 r) \).

It is easy to verify that in the absence of rescattering, \( \Delta t \) is excited by a plane wave in the form

\[
\exp[i k_0 (\mathbf{a} + \mathbf{b})],
\]

where \( k_0, k_0 = 0 \).

It is convenient to work with the Fourier variables.

We consider the propagation of the wave \( A \) near a Bragg condition.

In this case \( B(\mathbf{r}) \) is excited only virtually:

\[
b(\mathbf{r}) = \frac{-\nabla \times \mathbf{a} \times \mathbf{B}(\mathbf{r})}{k} e^{i(k_0 \cdot \mathbf{r})},
\]

and substitution of this expression in (4a) yields an equation for the change of the amplitude \( \Delta B \) along the ray \( \mathbf{a} \):

\[
\left( a \right) \Delta B(t) = i \frac{\partial B}{\partial t} - \nabla \times \mathbf{a} \times \mathbf{B}(t).
\]

In this case the wave \( A \) is subject to the condition of the Bragg detuning \( \lambda_B \), which has the dimension of reciprocal centimeters.

In this case \( B(\mathbf{r}) \) is excited only virtually:

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\[
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\]

In the next section we discuss the dynamic resonance and the energy fraction contained in the wave \( B \) is negligible.

3. BIREFRACTING AND GYROTROPY NEAR THREE-WAVE BRAGG RESONANCES

We consider the propagation of the wave \( A \) near a Bragg condition.

In this case \( B(\mathbf{r}) \) is excited only virtually:

\[
b(\mathbf{r}) = \frac{-\nabla \times \mathbf{a} \times \mathbf{B}(\mathbf{r})}{k} e^{i(k_0 \cdot \mathbf{r})},
\]

and substitution of this expression in (4a) yields an equation for the change of the amplitude \( \Delta B \) along the ray \( \mathbf{a} \):

\[
\left( a \right) \Delta B(t) = i \frac{\partial B}{\partial t} - \nabla \times \mathbf{a} \times \mathbf{B}(t).
\]
from this point a line $00'$ perpendicular to the plane of triangle $ABC$ and find in its plane the point $E$. It is easy to show that the waves for the circumscribed circle of the triangle satisfy the section of the perpendicular bisectors (the center of the circle $E$ can, for example, the frequency $\omega_0$, so as to have at $h_0=\omega_0(c/w)^{1/2}$:

$$h_0=\omega_0(c/w)^{1/2}.$$  

(16)

In our problem we take into account three Fourier components of the dielectric constant (in the absence of absorption, the zeroth component can be included in $h_0=\omega_0(c/w)^{1/2}$:

$$h_0=\omega_0(c/w)^{1/2}.$$  

(17)

The phase shifts $\varphi_{AB} = \varphi_{BC}$ of the Fourier components depend on the choice of the origin, and a shift $\Delta \varphi$ causes them to change in accord with the law

$$\Delta \varphi = \Delta \varphi,$$  

(18)

By the same token, at least two out of three phases in (17) can be made to vanish (the two components $\Delta \varphi$ and $\Delta \varphi$ in the $ABC$ plane). By virtue of the condition $\varphi_{AB} + \varphi_{BC} = 0$ the sum $\varphi_{ABC}$ of these phases, taken in accordance with the rule for going around the triangle in some definite direction (e.g., $ABC$), turns out to be independent of the choice of the origin

$$\varphi_{ABC} = \varphi_{ABC} - \varphi_{ABC},$$  

(19)

with $\varphi_{ABC} = \varphi_{ABC} - \varphi_{ABC}$.

In the general case $\varphi_{ABC} = \varphi_{ABC}$ is not equal to zero, although in some concrete cases the equality $\varphi_{ABC} = 0$ can follow from the symmetry of the crystal (for more details see, e.g., Refs. 1–3).

The gyrotricity effect of interest to us, as will be seen from the sequel, is proportional to $\sin \varphi_{ABC}$. In analogy with the two-wave case, at the frequency $\omega = \omega_0 + \delta \omega$ the field in the crystal, which differs slightly from the exact Bragg frequency, will be represented, given $\varphi_{AB}$, $\varphi_{AC}$, and $\varphi_{BC}$, in the form

$$E(t) = A(t) \exp(i \omega_0 t + B(t) \exp(i \omega_0 t + C(t) \cos(k_0 t)),$$  

(20)

where $A$, $B$, and $C$ are slowly varying amplitudes that satisfy the equations

$$\begin{align}
(\omega_0 - \omega_0 + B(t) + C(t)) = 0,
(\omega_0 - \omega_0 + B(t) + C(t)) = 0,
(\omega_0 - \omega_0 + B(t) + C(t)) = 0.
\end{align}$$

(21a)

(21b)

(21c)

Assume that incidence on the crystal of a wave $A(x)$ at certain values of the angle $\phi_0$ and of the frequency $\omega_0 = \omega_0 + \delta \omega$, which led to a detuning from the exact Bragg condition. It is convenient then to seek the solution of Eqs. (21) in the form

$$A(x, t) \exp(i \omega_0 t + A(x, t) \exp(i \omega_0 t + \phi_0 || x ||)),$$  

(23)

where $\omega_0$, $\phi_0 = 0$, while $\omega_0$, $\phi_0$, and $\phi_0$ are even slower functions of the coordinates. It follows for them from (21) that

$$\begin{align}
(\omega_0 - \omega_0 + B(t) + C(t)) = 0,
(\omega_0 - \omega_0 + B(t) + C(t)) = 0,
(\omega_0 - \omega_0 + B(t) + C(t)) = 0.
\end{align}$$

(24a)

(24b)

(24c)

The expression for $\Delta \varphi$ differs from (25) by the change of subscripts $C - B$, and is contained in formula (7).

If we stipulate the vanishing of both Bragg detunings

$$\Delta \varphi = \Delta \varphi - \Delta \varphi = 0,$$  

then we can obtain the dependence of the admissible slopes $\varphi_0 = \varphi_0$ that preserve the exact Bragg condition and correspond to a shift of the center of the Ewald sphere along the line $00'$ in Fig. 1. The vector $m \cdot \mathbf{a}$ lies in this case in the plane containing the vectors $\mathbf{a}$ and the line $00'$, and its value can be easily determined by equating (23) to zero.

We shall be interested in the situation wherein the Bragg condition is not satisfied exactly for both the pair $AB$ and the pair $AC$, the waves $B$ and $C$ being ad-

mined only virtually, with a small amplitude of the order of $f_0^2/\lambda_0 - \alpha_0^2$ and of higher powers of this parameter.

We represent the amplitudes of the waves $B$ and $C$ in the form of a series in powers of the small parameter $f_0/\lambda_0$, i.e., $B = B_0 + B_1 + \ldots$ and $C = C_0 + C_1 + \ldots$; we assume here that $\mathbf{C}$ is of zeroth order of smallness. In addition we make the assumption (whose validity will be confirmed by the calculation results) that differentiation with respect to the coordinates in (24) leads to an additional smallness of order $f_0/\lambda_0$ or of a higher power of this parameter.

We are interested in the variation of the amplitude $\mathbf{a}$ of the signal $\mathbf{a}_0$ along the ray $\mathbf{a}_0$, assuming, for example, that the crystal boundary is perpendicular to the direction $\mathbf{a}_0$ of $\omega_0$, and then $\omega_0 = \omega_0$, $B = B_0$, and $C = C_0$. We divide the right and left sides of (24b) and (24c) by $\lambda_0$ and $\lambda_0$, respectively. Substituting $B_0 = B_0 + B_0 + B_0 + \ldots$ and $C_0 = C_0 + C_0 + \ldots$, and a similar expression for $C_0$, and equating terms of like powers of $\lambda_0^2$, we obtain

$$\Delta \varphi = \Delta \varphi - \Delta \varphi = 0,$$  

(25)

FIG. 1. Orientation of wave vectors $k^e, k^f$, and $k^i$ in three-wave Bragg resonance; $k^i$, $k^f$, and $k^i$ are the crystal reciprocal-lattice vectors.
We have introduced here the notation $AB$ and the $y$-axis in the plane $(n_1, n_2)$ perpendicular to the $z$-axis. Separating this relation, we obtain for the birefringence index

\[
\frac{\partial n}{\partial z} = i \left[ N \frac{\partial n_1}{\partial z} + N \frac{\partial n_2}{\partial z} \right] + i N \epsilon (n_1, n_2) \delta (n_1, n_2) \delta (n_1, n_2).
\]

The terms proportional to $\alpha_1^2$ and $\alpha_2^2$ in (27) describe the contributions of the virtual admittance of the waves $B$ and $C$ to the average refractive index and to the birefringence of the wave $A$; each of these contributions coincides with the one considered in Sec. 2 above. The gyrotropy of interest to us is due to the term $\alpha_1^2/\alpha_2$. To be able to observe this small gyration we must stipulate that there be no stronger birefringence of order $\alpha_1^2/\alpha_2$. In other words, conditions must be found such that the matrix acting in the $xy$ plane

\[
T = n_1 A_1 (n_1 A_1 - n_2 A_2) + n_2 A_2 (n_2 A_2 - n_1 A_1)
\]

is a multiple of the unit matrix, i.e., that it yield only a correction to the average refractive index of the wave $A$.

It is easy to verify that this is possible only if the following two conditions are simultaneously satisfied:

1) the planes made up by the vector pairs $(n_1, n_2)$ and $(n_2, n_1)$ are perpendicular to each other;

2) the differential $\alpha_1$ and $\alpha_2$ satisfy the relation

\[
\frac{\partial n_1}{\partial z} \left[ 1 - (\alpha_1 n_1 + \alpha_2 n_2)^2 \right] = \frac{\partial n_2}{\partial z} \left[ 1 - (\alpha_2 n_2 + \alpha_1 n_1)^2 \right].
\]

We assume these two conditions to be satisfied. The action of the terms $\alpha_1^2/\alpha_2$ then yields the relation $a(x) = \delta(x)^{\text{even}} (\mu(x))$, where

\[
a(x) = \frac{\int a(x)^{\text{even}} d\mu(x)}{\int a(x)^{\text{even}} d^2 \mu(x)}.
\]

Separating this relation, we obtain for $\delta(x)$

\[
\frac{\partial \delta}{\partial z} = i \delta(x).
\]

We shall assume that in the $xy$ plane, which is perpendicular to $n_1 - n_2$, the $x$ axis lies in the plane $(n_1, n_2)$, and the $y$ axis in the plane $(n_2, n_1)$; we recall that we assume that the last two planes are perpendicular to each other, so as to set equal to zero the birefringence of first order in $f/\lambda$. The matrix $T_1'$ contains a symmetrical real part that describes the average refractive index $T_1'$ and the birefringence $T_2'$, and an antisymmetrical pure imaginary part that describes the gyration or the optical activity $T_2''$.

\[
T_1' = \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) \left[ \frac{\partial n_1}{\partial z} (n_1, n_2) + \frac{\partial n_2}{\partial z} (n_2, n_1) \right],
\]

\[
T_2' = \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) \left[ \frac{\partial n_1}{\partial z} (n_2, n_1) - \frac{\partial n_2}{\partial z} (n_1, n_2) \right],
\]

\[
T_2'' = \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) \left[ \frac{\partial n_1}{\partial z} (n_1, n_2) - \frac{\partial n_2}{\partial z} (n_2, n_1) \right].
\]

We have introduced here the notation $AB = (n_1, n_2)$.

We consider first the case most favorable for the observation of gyration: $\varphi_{ABC} = \pi/2$. Then there is no birefringence in second order, and the rate $d\delta/\partial z$ of the polarization plane (if $\beta$ is the angle) is

\[
\frac{d\delta}{\partial z} = \frac{\delta c}{2} \left( \frac{\partial a}{\partial z} \right)^2 \frac{\sin 2\beta}{\sin 2\beta - \sin 2\beta \cos 2\beta}.
\]

We assume for numerical estimate a tentative value $a = 10^{-3} - 10^{-4}$ cm for $\lambda$ in (12) (see Ref. 2, Chap. 9). A deviation of 10 widths of the dynamic-diffraction curve from the Bragg condition, $a - 10$ [where $a$ is defined by Eq. (13)] will then yield, in the two-wave case, a birefringence of the order of

\[
\frac{a}{\lambda} (n_1, n_2) \approx 10^{-7} - 10^{-6} \text{ cm}^2.
\]

On the other hand, the absorption coefficient $\mu$ can amount under typical conditions to $10^{-3} - 5 \times 10^{-4}$ cm$^{-1}$. Thus, a phase difference $\varphi_{ABC} = \pi - 3 - 100$ rad can accumulate over an absorption length $l_\mu = \mu^{-1}$. This means that the production of strong birefringent elements based on almost-Bragg two-wave resonances is really feasible. We note that it is possible to use non-ideal (mosaic) crystals, since we need not satisfy exactly the conditions of dynamic diffraction.

As for gyrotropy (rotation of the polarization plane), we get for it from (33) the estimate

\[
\frac{d\varphi}{d\lambda} = \frac{1}{2} \frac{\sin 2\beta \cos 2\beta}{\sin 2\beta - \sin 2\beta}.
\]

Assuming $\sin \varphi_{ABC} = 1$, $l_\mu = 10^{-4}$, and $a = 10^{-4}$, we obtain $d\varphi/d\lambda = 10^{-4}$ cm$^{-1}$. Over an absorption thickness $l_\mu = 10^{-3} \text{ cm}$, the gyration angle can amount to $\beta = 1$ rad, i.e., a perfectly observable value.

We have thus predicted in this paper strong x-ray birefringence and gyration effects resulting from two- and multwave almost-Bragg interactions. Observation of these effects uncover wide possibilities for the application of various methods of polarization optics of the visible band in the x-ray region of the spectrum.

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903 Sov. Phys. JETP 52(5), Nov. 1980

N. S. Barnouse and B. Ya. Zel'dovich 903
Kinetic phenomena in the flow of a strongly rarefied molecular gas in an external field


Moscow Engineering Physics Institute
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A kinetic theory is developed for the effects that arise when a free-molecular polyatomic gas flows between two surfaces in an external field. The influence of the field on the transport processes is due to the nonequilibrium polarization of the gas molecules when they are nonspherically scattered from the surface of a solid, and to the destruction of this polarization in the field. The change of the gas flow velocity in a channel in a magnetic field, and the onset of a transverse heat flux between the surfaces whose temperatures are equal is examined in detail. In contrast to the previously investigated thermomagnetic phenomena, the considered effects in a gas stream can occur when the molecules are scattered from the surface not only inelastically but also elastically. At the same time, these effects occur only if the interaction with the surface is such that the states of the molecule before and after the collision are correlated.

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1. INTRODUCTION

The influence of a magnetic field on heat flow in a strongly rarefied (\(l > L\), where \(l\) is the molecule mean free path and \(L\) is the characteristic dimension) polyatomic gas (the thermomagnetic effect) has already been observed and investigated earlier.\(^1\)

Other possible effects in an inhomogeneously heating gas in a magnetic field were also analyzed, such as the appearance of transverse heat and mass fluxes in a gas contained between two surfaces having different temperatures, or of thermomagnetic forces acting on the walls.\(^3\) The physical causes common to the changes in the transport processes in a magnetic field are the polarization of the molecules (inelastically scattered from the solid surface and the precession of the magnetic moment of the molecule about the field direction. A distinguishing feature of the foregoing effects is the oscillatory character of the dependence of the macroscopic fluxes in the gas on the intensity of the constant external field at a fixed geometry of the problem. The concrete dependence of the macroscopic quantities on the intensity and orientation of the field is determined entirely by the law of nonspherical scattering of molecules by walls. Therefore the kinetic effects in a strongly rarefied gas in an external field serve as a unique source of information on the physical mechanism of the orientation-dependent interaction between molecules and the surface of a solid, and on the properties of the surface itself.\(^3\)

By virtue of the isotropy of the distribution of the molecules of the equilibrium gas with respect to their orientations and directions of motion, polarization of molecules deflected from the surface can occur only in a nonequilibrium gas. The effects listed above are due to the temperature inhomogeneity of the system. It can be assumed that the molecules reflected (elastically and inelastically) from the walls become polarized also in the case of gas flow. The presence of a predominant direction of the velocity of the molecules incident on the surface and the dependence of the probability of the scattering on the mutual orientation of the velocity \(v\) and of the angular momentum \(\mathbf{M}\) of the molecule should make the distribution function dependent also on the orientation of the vector \(\mathbf{M}\), i.e. should lead to polarization of the molecules. The molecule precession produced when the external field is turned-on changes this dependence (it destroys partially the polarization). As a result, the kinetic properties of the system are altered in an external field; in particular, the scalar transport coefficients acquire a tensor character.

In this paper we construct a theory of the phenomena connected with the influence of an external field on the transport processes in a stream of strongly rarefied polyatomic gas. We solve the problem of the flow of collisionless gas in a channel made up of two infinite surfaces in a magnetic field. We investigate the change of the channel resistance in the field and the onset of heat flow between the surfaces (which have equal temperatures). These effects are the Knudsen analogs of the known viscomagnetic effect\(^8\) and of the effect of viscomagnetic heat flow,\(^7\) which take place if \(l < L\). They are produced, however, by another physical mechanism, namely polarization of the molecules by nonspherical scattering from the surface.