Hyperfine structure of the energy levels of $\mu$-mesic molecules of the hydrogen isotopes

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In the first order of perturbation theory in $e^2$, a calculation accurate to $-10^{-4}$ eV is made of the hyperfine structure of the energy levels of all stationary states with quantum numbers $j$, $l$, and $v$ of the total orbital angular momentum and the vibrational motion, respectively, for mesic molecules of the hydrogen isotopes.

The solutions to the nonrelativistic problem of the bound states of a system of three particles with Coulomb interaction found in the adiabatic representation are chosen as the zeroth approximation. Expressions are given for the probability amplitudes of the different values of the total spin of the nuclei and the total spin of the $\mu$-mesic molecules in the stationary states of the hyperfine structure. Calculations are made of the populations of the stationary states of the hyperfine structure of the $\mu$-mesic molecules formed in collisions of the mesic atoms $\mu^+$, $\mu^-$ and $\mu_0$ in the paranorm or orthoform with the nuclei $p$, $d$, and $\mu$.

1. INTRODUCTION

Recent experiments on the resonance formation of $d\mu s$ and $d\mu u$ mesic molecules\(^1\) confirmed the theoretical predictions in Ref. 2. These mesic molecules should have excited weakly bound states with quantum number $j=1$ for the total orbital angular momentum and quantum number $v=1$ for the vibrational motion and

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\(^1\)There are arguments supporting the view that the real physical space must be orientable.\(^2\)

\(^2\)In the Lemaitre model with cosmological constant $\Lambda \neq 0$ and $\Lambda > 0$, the observation of the same object in two opposite directions is possible in the late stages of expansion, see, for example, Refs. 2 and 8.

\(^3\)Below, it is assumed that the cosmological constant vanishes, $\Lambda = 0$, and, in addition, the equation of state of the matter is $p = \rho$, i.e., we ignore radiation pressure. We recall that in the standard model of a hot Universe the total density of the background radiation at the contemporary epoch is $\rho_{\gamma, 0} \approx 3 \times 10^{-37}$ g/cm$^3$. Here, $\rho_{\gamma, 0}$ includes the density of photons, gravitons, all types of neutrinos, and also all as yet unknown particles with zero rest mass that have survived from the superdense phase. If, however, $\rho_{\gamma, 0} \approx 30$ eV, then today $\rho_{\gamma, 0} \approx 1$ eV, i.e., we still have $\rho_{\gamma, 0} = 0$.
binding energy $-\epsilon_p$, $\rho_{pj}(d\mu)$ = 1.91 eV and $-\epsilon_p$, $\rho_{pj}(d\mu)$ = 0.64 eV. The hyperfine structure of the energy levels of these stationary states was not taken into account in the evaluation of the experiment in Ref. 1 or in the theoretical calculations of Ref. 2. So far, the hyperfine structure is known only for the mesic molecules $pp\mu$ and $pd\mu$ in the states with $(J=1,\nu=0)$ and $(J=0,\nu=0)$, respectively, for which the hyperfine splitting of the energy levels is $\approx 0.1$ eV.3,4 The hyperfine splitting of the energy levels of $\mu$-mesic atoms of the hydrogen isotopes is also $\approx 0.1$ eV, which is comparable in magnitude with the binding energy of the $(J=1,\nu=1)$ states of the $d\mu$ and $dp\mu$ mesic molecules and the kinetic energy of the thermal motion at normal temperatures ($\approx 0.04$ eV). Thus, calculation of the hyperfine structure of the mesic molecules $d\mu$ and $dp\mu$ is of great interest and, as is noted in Ref. 4, is necessary for calculations of the kinetics of processes taking place in a mixture of hydrogen isotopes.5

The stationary states $(\psi(\,ff\,))$ of a mesic molecule, i.e., a system of three spin particles $a, b, c = \mu$ (with spins $s_1, s_2, s_3$) with electromagnetic interaction, are characterized by the value $J$ of the total angular momentum $\vec{J}$ = $\vec{S} + \vec{\pi}$ and its projection $\pi$ onto the $Z$ axis of the laboratory coordinate system and in the zeroth order of perturbation theory are represented by a linear combination of the states $(J\piff)$:

$$
\psi(\,ff\,)=\sum_{J=0}^{\infty} C_J \psi_{J\pi}(R, r, \theta, \phi)
$$

with definite values of the total spin $S = 1 + s$ of the mesic molecule ($s = \frac{1}{2}$ is the spin of the $\mu$ meson) and the total spin $S = s + b + c$ of the nucleus a and b. $N$ is the number of the state, $1 \leq N \leq N_{max}$ and $N_{max}$ is the degeneracy of the level with the given values of $(J\piff)$. The $\mu$-mesic molecules are formed in collisions of $\mu$-mesic atoms $(a, c)$ in the ground state and characterized by spin $F = a + s$ and $s$ with nuclei of the hydrogen isotopes b with spin $s$. The mass of nucleus a is greater than or equal to the mass of the nucleus $b$; for example, $a(2S+1)L_2 \neq a(2S+1)L_1$. In the present paper, we calculate, in the first order of perturbation theory in $\alpha$, the energy levels $\epsilon_{J\pi}$ of the hyperfine structure, the probability amplitudes $\psi_{J\pi}$ of the stationary states $(J\piff)$ of the mesic molecule; these depend on the values of $F$ and $S$. In the nonrelativistic approximation, the stationary states $(\psi(\,ff\,))$ of the mesic molecules are formed in the zeroth approximation in the calculations, we have used the solutions of the nonrelativistic problem with the Hamiltonian $H_{nrel}$ found in the adiabatic representation of the three-body problem with Coulomb interaction,10 which makes it possible to treat in a unified manner the ground $(J=0,\nu=0)$, excited $(J \neq 0,\nu \neq 0)$, and, particularly important, weakly bound stationary states of the $\mu$-mesic molecules.5

2. HYPERFINE STRUCTURE OF THE ENERGY LEVELS OF THE STATIONARY STATES OF $\mu$-MESIC MOLECULES

In the nonrelativistic approximation, the stationary states $|\psi(\,ff\,)=|\psi(\,ff\,)|_{nrel}$ of the $\mu$-mesic molecules are characterized by the quantum numbers of the motion of the $\mu$ meson $(a)$, the vibrational motion of the nuclei $(c)$, the total orbital angular momentum $(J)$, its projection $(\nu)$ onto the $Z$ axis of the laboratory coordinate system, and the total parity $(\lambda = (-1)^{F})$. The corresponding wave function $\psi(\nu; R, r) = \psi(\nu; R, r)$ is the solution of the nonrelativistic Schrödinger equation

$$
H_{nrel}(R, r)\psi(\nu; R, r) = E_{nrel}\psi(\nu; R, r),
$$

where $E_{nrel}$ is the total energy of the $\mu$-mesic molecule in the center-of-mass system, $H_{nrel}(R, r)$ is the Hamiltonian of the three particles $(a, c)$ with Coulomb interaction, R is the vector joining the nuclei $a$ and $b$, and $r$ is the vector joining the center of $R$ and the $\mu$-meson.

In the adiabatic representation, the wave function $\psi(\nu; R, r)$ can be written in the form of the expansion

$$
\psi(\nu; R, r) = \sum_{J=0}^{\infty} \sum_{\pi} \sum_{\lambda} C_{J\pi\lambda}(\nu; R, r) \psi_{J\pi\lambda}(R, r)
$$

Here, $C_{J\pi\lambda}(\nu; R, r) = \langle \psi_{J\pi\lambda}(R, r) | \psi(\nu; R, r) \rangle$ is the complete set of solutions to the problem of two fixed Coulomb centers,10 the dependence of these solutions on the angle $\phi$ (around the axis $z = R/|R|$) being included in the symmetric functions corresponding to total parity $\lambda = (-1)^{F}$.

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where $\Omega^p_n(\Phi, \Theta)$ are normalized $D$ functions of Wigner, $\Phi$ and $\Theta$ are the spherical angles of the vector $R$, and $m$ is the projection of the orbital angular momentum $J$ onto the $z$ axis. The symbol $\sum_j$, denotes summation over the discrete spectrum and integration over the continuum of the two-center problem, the limits of the present calculations being taken as follows:

$$
\sum_j \sum_{\pm} \left\{ \sum \int \left[ \frac{1}{r} \right] \right\}.
$$

Here, $p=g,u$ are the eigenvalues of the operator $P_\mu$ which inverts the coordinates (i.e., $\xi = -z_1 - z_2 - s - q$) of the $\mu$ meson; $\nu_1$ and $\nu_2$ are the parabolic quantum numbers of the "separated" atom $a,b$, and $\hbar$ is the momentum of the $\mu$ meson for motion in the field of the two fixed nuclei $a$ and $b$ with positive energy.

The binding energy $-\varepsilon_\mu$, of the $\mu$-mesic molecule,

$$
\varepsilon_\mu = E_{\mu} - E_{\mu}\text{atom},
$$

(in the ground state of the motion of the $\mu$ meson), is measured from the energy $E_{\mu\text{atom}}$ of the ground state $\mu$ of the isolated atom $a,b$, whose nucleus has mass greater than (or equal to) the mass of the nucleus $b$. In what follows, we shall omit some indices, for example, $n=[(0,0,0)],$ and $x=(\xi^2)$, writing $\left\{\left(\nu_1\nu_2\right)\right\}=\left\{\left(\nu_1\nu_2\right)\right\}$. The binding energy $-\varepsilon_\mu$ and the wave functions

$$
\phi_{vJ\mu}(R) = \langle mpR|J\nu]\nu,\mu\rangle
$$

were found by means of the algorithm\textsuperscript{19} for numerical solution of the Sturm-Liouville problem for the system of ordinary integro-differential equations\textsuperscript{50} obtained by averaging (3) with respect to the functions $\left\{\left(\nu_1\nu_2\right)|mpR|J\nu]\nu,\mu\rangle$:

$$
\sum_{\nu_1\nu_2} \left\{ \langle mpR|J\nu]\nu,\mu\rangle \phi_{vJ\mu}(R) \right\} = -\varepsilon_\mu \phi_{vJ\mu}(R),
$$

The values $-\varepsilon_\mu$ of the binding energy ($2J+1$-fold degenerate with respect to $\nu_2$) of all the stationary states $\phi_{vJ\mu}(R)$ were found by means of the hydrogen isotopes were taken from our other papers\textsuperscript{60} and are given in the third columns of Table II.

The addition to the Hamiltonian $H_{\mu\text{atom}}$ of the operator $V^{\mu\text{atom}}$ of the spin interaction leads to a hyperfine splitting of the energy level $E_\mu$, of the stationary state $\phi_{vJ\mu}(R)$. The correct functions of the zeroth approximation corresponding to the stationary state $(Jv)$ of the $\mu$-mesic molecule have the form\textsuperscript{14}

$$
\phi_{vJ\mu}(R) = \sum_{\nu} \phi_{\nuJ\mu}(R, \nu) \phi_{\nu}(R) - \langle R, v|Jv\rangle \\
\times \langle Jv|\phi_{vJ\mu}(R, \nu)\rangle \\
\times \langle Jv|\phi_{\nu}(R)\rangle.
$$

Here $\langle (Jv)|\phi_{vJ\mu}(R, \nu)\rangle$ are Clebsch-Gordan coefficients\textsuperscript{11}; $x, y, z, \xi, s$ are constant spinors; $\xi_1, \xi_2, s_1, s_2$ are the values of the spins $S_1, S_2, v_1, v_2$, and their third projections onto the $Z$ axis of the laboratory coordinate system.

For the given values of $J$ and $\nu$, the coefficients $g_{\nuJv} = \langle (Jv)|\phi_{vJ\mu}(R)\rangle$, i.e., the probability amplitudes of states with definite $S$ and $I$ in the stationary state $(Jv)$, are solutions of the system of linear algebraic equations

$$
\sum_{\nu} \langle (Jv)|\phi_{vJ\mu}(R, \nu)\rangle = 0,
$$

in which $\phi_{vJ\mu}(R, \nu)$ are found from the secular equation

$$
\det |(Jv)|\phi_{vJ\mu}(R, \nu)\rangle = 0
$$

and represent the required hyperfine splitting of the nonrelativistic energy level $E_\mu$ for fixed $J$ (the degeneracy with respect to $\nu$ remains).

In the case of identical nuclei, the only roots of the secular equation among all the $N_{\text{state}}$ roots that have physical meaning are those that correspond to states $(Jv)$ with definite symmetry $\nu = \nu_1, \nu_2$ for fixed $J$ and $\nu$.

3. EFFECTIVE SPIN HAMILTONIAN AND HYPERFINE STRUCTURE OF THE STATIONARY STATES OF $\mu$ MESIC MOLECULES

The spin operator $V^{\mu\text{atom}}$, whose explicit form is described in the previous papers of Ref. 7, was averaged

$$
\phi_{vJ\mu}(R) = \sum_{\nu} \sum_{\nu_1\nu_2} \langle (Jv)|\phi_{vJ\mu}(R, \nu)\rangle
$$

TABLE I. Coefficients $g_{\nuJv} = \langle (Jv)|\phi_{vJ\mu}(R)\rangle$ of the effective spin Hamiltonian (33) for the mesic molecules $pp, dd, dd$, $uu, uu, uu$ in the stationary states $(Jv)$ with total orbital angular momenta $J=1$ and vibrational quantum numbers $\nu=1$.

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<th>$v$</th>
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<th>$\nu_2$</th>
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*Mesic-atom energy unit $E_p = 5626.506$ eV.\textsuperscript{11}

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TABLE II. Hyperfine structure of μ-mesic molecules of the hydrogen isotopes.

A. Hyperfine structure of the levels of the mesic molecule \( ppp \).

<table>
<thead>
<tr>
<th>( j \times \mu \text{.eV} )</th>
<th>( \nu )</th>
<th>( \nu' )</th>
<th>( \nu'' )</th>
<th>( \nu''' )</th>
<th>( \nu'''' )</th>
<th>( \nu''''' )</th>
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B. Hyperfine structure of the levels of the mesic molecule \( ddp \).

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<th>( \nu' )</th>
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<th>( \nu''' )</th>
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C. Hyperfine structure of the levels of the mesic molecule \( ttp \).

<table>
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<tr>
<th>( j \times \mu \text{.eV} )</th>
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<th>( \nu' )</th>
<th>( \nu'' )</th>
<th>( \nu''' )</th>
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D. Hyperfine structure of the levels of the mesic molecule \( Pdp \).

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<th>( \nu' )</th>
<th>( \nu'' )</th>
<th>( \nu''' )</th>
<th>( \nu'''' )</th>
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E. Hyperfine structure of the levels of the mesic molecule \( ptp \).

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<th>( \nu'' )</th>
<th>( \nu''' )</th>
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TABLE I (continued).

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<th>( v_2 )</th>
<th>( E_{hf} ) (( \text{eV} ))</th>
<th>( N_{\text{deg}} )</th>
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</tr>
</tbody>
</table>

Note.

- \( g_1 \) and \( v \) are the orbital and vibrational quantum numbers, respectively;
- \( -E_J \) is the binding energy of the stationary state \( \psi_1 \) of the mesic molecule in the nonrelativistic approximation;
- \( f \) is the total angular momentum of the mesic molecule;
- \( E_N \) and \( N_{\text{deg}} \) are the populations of the level \( (J, v, f, N) \) of the mesic molecule formed as a result of collision of the mesic atom \( (J_1, v_1, f_1, N_1) \) with the \( p, d, \) or \( t \) nucleus (of the same kind or lighter);
- \( \Psi_{1,2} \) is the probability amplitude of the state \( \psi_1 \) with definite values of the total spin \( S \) and the total spin \( I \) of the nuclei of the mesic molecule in the stationary state \( \psi_1 \):
- \( N \) is the number of the state \( 1N_{\text{deg}} \), where \( N_{\text{deg}} \) is the degeneracy of the level with the given values of \( (J, v) \).

The values of the energy levels of the hyperfine structure measured from the nonrelativistic value \( E_{hf} \) (7) (see Table II) and the probability amplitudes \( \Psi_{1Jv} \) of states with definite \( J \) and \( v \) of the stationary states \( (J, v, f, N) \) with \( J < 1 \) and \( v < 1 \) are given in Table II for all the \( \mu \)-mesic molecules of the hydrogen isotopes.

It is well known that molecular ions of p-mesic molecules are formed in collisions of \( \mu \)-mesic atoms in the \( 1s \) state with...
spin $F = s_{\alpha} + s_{\beta}$ in the parastate ($F = + 1$) or orthostate ($F = + 1$) with nuclei of the hydrogen isotopes with spin $s_{\alpha}, s_{\beta}$, i.e., in accordance with the scheme of type (2). Under the assumption of an arbitrary orientation of the spins $F$ and $s_{\alpha}, s_{\beta}$ and the total angular momentum $J$ of the produced $\mu$-mesic molecule, its original spin state is represented by the density matrix

$$\rho = \sum_{l=0}^{J} \sum_{m=-J}^{J} \rho_{lm} |l, m\rangle \langle l, m|.$$  

Then the probability of finding the $\mu$-mesic molecule in the stationary state $|\psi_{s} (F_\mu)\rangle$ of the hyperfine structure is determined as a function of the two possible spin states, $F = + 1$ or $F = + 1$, of the mesic atom $a, c$ by the expression

$$W_{\mu}^{(F)}(F) = \sum_{s=0}^{+1} \langle \psi_{s} (F_\mu) | \rho_{s} | \psi_{s} (F_\mu) \rangle,$$  

where

$$\langle \psi_{s} (F_\mu) | \rho_{s} | \psi_{s} (F_\mu) \rangle = \frac{1}{2J + 1} \sum_{l=0}^{J} \sum_{m=-J}^{J} \rho_{lm} |l, m\rangle \langle l, m|.$$  

In Fig. 1, we show schematically the "transformation" of the hyperfine energy levels of the $\mu$-mesic atom into the hyperfine energy levels of the $d\alpha$ and $d\beta$ mesic molecules in accordance with the scheme for adding angular momenta in reactions of the type (2), i.e., the spins $F$ of the mesic atom $a, c$, the deuteron $b$ ($s_{\alpha}, s_{\beta}$), and the total orbital angular momentum $F = S + J$ of the mesic molecules. Thus, the complete set of energy levels of the states (1) of the hyperfine structure of the molecule, which are expanded naturally into two groups in accordance with the possible ways in which the $\mu$-mesic molecule can be formed from the parastate ($F = + 1$) or orthostate ($F = + 1$) of the $\mu$-mesic atom $a, c$.

In Table II we give the parastate, $W_{\mu}^{(F)}(F)$, and orthostate, $W_{\mu}^{(F)}(F)$, populations of the stationary states $|\psi_{s} (F_\mu)\rangle$ with $J, \nu = 1$ for all the $\mu$-mesic molecules of the hydrogen isotopes calculated in accordance with Eqs. (18) and (19). (We recall that the mass of nucleus $a$ is greater than or equal to the mass of nucleus $b$.)

4. CONCLUSIONS

In the present paper (see Table II) we have calculated in the first order of perturbation theory in $\Delta \delta$ to accuracy $10^{-6}$ eV the hyperfine structure of the energy levels of the stationary states $|\psi_{s} (F_\mu)\rangle$ of the $\mu$-mesic molecules of the hydrogen isotopes. In the calculation of the coefficients of the effective spin Hamiltonian (12) in the expansion (4)–(6) of the wave function of the $\mu$-
mesonic molecule we have retained the terms corresponding to the states of the first two shells in accordance with the classification of the “separated” atom of the discrete spectrum of the two-center problem. The terms corresponding to the continuum states of the two-center problem were not taken into account, since their contribution is less than the adopted accuracy ~10^{-3} eV of the calculations.

As can be seen from Table I, the coefficients of the effective spin Hamiltonian (13) that determine the spin-orbit \((E,E, ,E, )\) and spin-tensor \((E, , . . . , E, )\) interactions are appreciably smaller than the coefficients that determine the spin-spin \(\langle E, , E, \rangle\) interaction of the \(\mu\)-meson with the nuclei. This has the consequence that some of the energy levels \(E, , E,\) are degenerate to an accuracy \(-1\times 10^{-5}\ eV\); for example, see Fig. 1b and Table II E. At the same time, the contribution of the corrections of \(\sigma(E)\) is \(-1\times 10^{-5}\ eV\) according to estimates.

It follows from our calculations that the corrections for the internal electromagnetic structure of the nuclei to the coefficients of the effective spin Hamiltonian (13) are \(-0.5-1.5\%\) of the corresponding quantities calculated without allowance for them. The Foldy-Kraich corrections, which represent the motion of the center of mass of a pair of particles, significantly change the coefficients \((E, , E, , E, )\), which determine the spin-orbit interaction in the \(\mu\)-mesonic molecule. Allowance for both these effects changes the hyperfine splitting of the energy levels \(E, , E,\) of the \(\mu\)-mesonic molecule by \(-0.5\%\) (the corrections for the internal electromagnetic structure of the nuclei) and \(-0.05-0.1\%\) (Foldy-Kraich corrections).

Despite the smallness of the effects due to the internal electromagnetic structure of the nuclei, their inclusion is of fundamental importance, since in this case the operators of the spin interaction do not have \(\sigma(E)\) and \(\sigma(E)\) singularities.

The results of the present paper can be used to make more detailed calculations of the rates of resonance formation of \(d\mu\) and \(d\psi\) mesonic molecules and the kinetics of mesic-molecular processes in a mixture of hydrogen isotopes.

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15A. Messiah, Quantum Mechanics, North Holland, Amsterdam (1961) [Russian translation published by Nauka (1970)].
17A. Bethe and E. E. Salpeter, Quantum Mechanics of One and Two Electron Atoms, Springer, Berlin (1957) [Russian translation published by Mir (1960)].

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