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Translated by J. G. Adashko

Transitions between levels of multiply charged ions in a strong external field

G. L. Klimchitskaya

Leningrad State University
(Submitted 3 September 1979)
Zh. Eksp. Teor. Fiz. 78, 924–928 (March 1980)

A scheme is proposed for a fully relativistic calculation of the transition probabilities between levels of multiply charged ions in a strong homogeneous electric field. The probability of the magnetic dipole transitions between the state $2s\,1/2$–$1s\,1/2$ of a single-electron ion is calculated. It is shown that interactions with a strong field can lead to an appreciable increase of the probability of a transition that is weak in the absence of a field.

PACS numbers: 33.55. + c, 33.70. – w

Much attention is being paid in recent years to experimental and theoretical investigations of the spectra of multiply charged ions. These spectra were observed both in the study of the solar corona and in special laboratory setups. The principal laboratory methods for the investigation of ions with high degree of ionization, with the aid of which most data on the spectra and transition probabilities were obtained, are passage of an ion beam through a foil and the action of laser radiation on matter. In the latter case, the multiply charged ions are situated in a strong electric field (whose intensity can reach 10^9 V/cm, comparable with intra-atomic fields). The theoretical investigation of the spectra of multiply charged ions in a strong electric field is therefore of considerable interest.

Labzovskiĭ and the author[1] have considered the influence of a homogeneous electric field on the energy levels in multiply charged ions. A relativistic calculation was made of the energy levels of two-electron ions in the configuration $1s2s+1s2p$ with ion charge $10\leqslant Z\leqslant 50$, situated in an external electric field that can be either weak or strong in comparison with the Coulomb interaction of the electrons. It was observed that in an external field new crossings of levels with different parity take place. As already shown,[2,3] these crossings can be used to check on the hypothesis that there are no weak neutral currents in atomic systems.

An external field, however, exerts a substantial influence not only on the energy levels, but also on the probability of the transitions between them. This change of the probabilities of the transitions by external electric field must be taken into account, in particular, when searching for situations that are most favorable for the observation of parity nonconservation effects. In the present study, which is a continuation of an investigation initiated in Ref. 1, we consider the influence of a homogeneous electric field on the transition probabilities in multiply charged single-electron ions. The transition probabilities are expressed in terms of the S-matrix element[4]

$$W_{AB}=2\pi|\langle B|M|A\rangle|^2,$$
$$\langle B|S|A\rangle=-2\pi i\langle B|M|A\rangle\delta(E_A{}^0-E_B{}^0). \tag{1}$$

To carry out a fully relativistic calculation, we shall use as the wave functions of the initial (A) and final (B) states the functions

$$\psi_{njm}=\sum_l a_l(nj)\psi_{njlm}, \tag{2}$$

where ψ_{njlm} are Dirac wave functions, and the coefficients $a_l(nj)$ are calculated in the course of diagonalization of the Hamiltonian that takes into account the interaction with the uniform electric field:

$$H(\mathbf{r})=h(\mathbf{r})+\alpha^{1/2}Fz. \tag{3}$$

Here $h(\mathbf{r})$ is the relativistic single-electron Dirac Hamiltonian for an electron in the field of a nucleus, z is the Cartesian coordinate of the electron in the field direction, F is the field intensity, and α is the fine-structure constant. We use a system of units in which $\hbar=c=m=1$ (m is the electron mass).

In the calculation of the probability of the transition, in first-order perturbation theory in the interaction with the external field, account must be taken of the contribution from the diagrams a–c of Fig. 1. A wavy line denotes here the emitted photon, a dashed line the photon absorbed from the external field, and a double line the electron propagator.

The matrix elements of the S matrix are determined using the exact relativistic operator of interaction with the electromagnetic field.[5] Summation over the virtual states is carried out using the Coulomb Green's func-

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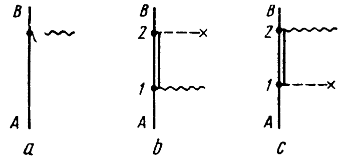

FIG. 1.

tion $G_E(\mathbf{r}_1, \mathbf{r}_2)$ of the Dirac equation.[6] To eliminate the intermediate state, which coincides with the initial one, we must use the reduced Green's function

$$G_{E_{nj}}(\mathbf{r}_1,\mathbf{r}_2)=\lim_{E\to E_{nj}}\left[G_E(\mathbf{r}_1,\mathbf{r}_2)-\sum_m\frac{|\psi_{njm}\rangle\langle\psi_{njm}|}{E-E_{nj}}\right], \quad (4)$$

where E_{nj} is the eigenvalue of the Dirac equation with principal quantum number n and with total angular momentum j. In this case, for the diagram of Fig. 1b we have $E_{nj}=E_B$, and in the case of diagram 1c we have $E_{nj}=E_A$.

The general expression for the transition probability in first-order perturbation theory in the interaction with the external field is of the form

$$W=W^{(0)}+F^2W^{(1)}. \quad (5)$$

Here $W^{(0)}$ takes into account the contribution of the diagram in Fig. 1a, while $W^{(1)}$ is connected with the S-matrix elements corresponding to diagrams of Fig. 1b and 1c in the following manner:

$$W^{(1)}=\alpha^2(2\pi)^2\frac{1}{2j_A+1}\sum_{m_A,m_B,\mu}|\langle\psi_B(\mathbf{r}_2)|z_2G_{E_B}(\mathbf{r}_2,\mathbf{r}_1)\gamma\mathbf{A}_{\lambda\mu}(\mathbf{r}_1)|\psi_A(\mathbf{r}_1)\rangle$$
$$+\langle\psi_B(\mathbf{r}_2)|\gamma\mathbf{A}_{\lambda\mu}(\mathbf{r}_2)G_{E_A}(\mathbf{r}_2,\mathbf{r}_1)\gamma_4z_1|\psi_A(\mathbf{r}_1)\rangle|^2. \quad (6)$$

In (6) we sum over the states of the polarization of the emitted photon and over the projections of the final angular momentum, and average over the projections of the initial angular momentum; λ is the multipolarity of the transition.

It should be noted that the transition amplitude contains terms proportional to the first power of F, but in the calculation of the total transition probability the summation is over the projections of the angular momenta, as a result of which the interference term proportional to F vanishes. To calculate $W^{(0)}$ and $W^{(1)}$ we use the wave functions (2) which take into account the degeneracy of the levels in the external field.

Using a partial expansion of the Green's function[6] and the standard methods of the algebra of the angular momenta,[7] we can write down the expression (6) in the form of a product of radial and angular parts. The radial part of the Green's function is an absolutely converging series, the integrals of which can be easily calculated with a computer. By way of example we have calculated the influence of the external field on the probability of the magnetic dipole transition $2s\frac{1}{2}-1s\frac{1}{2}$, which is considered in Ref. 8 in a discussion of parity-nonconservation effects in atomic systems. In the nonrelativistic approximation, this transition is forbidden. In a consistent relativistic analysis it turns out to be allowed, but in the absence of an external field its probability is low. At small Z it takes the form

$$W_{M1}^{(0)}=2^{-2}\cdot3^{-5}\alpha(\alpha Z)^{10}. \quad (7)$$

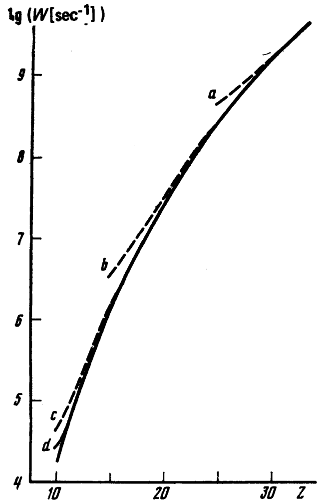

FIG. 2. Dependence of the transition probability on Z at various external-field intensities F (in V/cm): a—10^9, b—10^8, c—10^7, d—10^6.

The probabilities are given here in relativistic units. To change over to probabilities persecond, this expression must be multiplied by $mc^2/\hbar$. Fig. 2 shows the dependence of $W_{M1}^{(0)}$ (solid line) on the nuclear charge Z. The ordinates represent the quantity $\log(W[\sec^{-1}])$.

The considered level in an external electric field can be classified as $2s\frac{1}{2}$ if its field-induced admixture of the $2p\frac{1}{2}$ level is negligible, i. e., if the following condition is satisfied

$$eF\langle\psi_{2s^1/_2}|z|\psi_{2p^1/_2}\rangle\ll\Delta E_L(Z), \quad (8)$$

where $\Delta E_L(Z)$ is the Lamb shift.[9]

The maximum external-field intensity that still satisfies the condition (8) changes from 10^2 V/cm at $Z=1$ to 10^7 V/cm at $Z=10$. At $Z=30$ it amount to 10^9 V/cm. In sufficiently strong electric fields ($F\geqslant10^6$ V/cm) it is thus meaningful to consider multiply charged ions with $Z\geqslant10$. At these nuclear charges the relativistic effects are already significant.[10] At the same time, the tunnel ionization of such ions in fields whose intensity is limited by the condition (8) can be neglected.

Since the external electric field mixes the states $2s\frac{1}{2}$ and $2p\frac{1}{2}$, it is necessary to take into account both the magnetic and the electric dipole transitions in the calculation of the probability W of the $2s\frac{1}{2}-1s\frac{1}{2}$ transition, in accordance with (5). The total transition probability is therefore

$$W=W_{E1}+W_{M1}, \quad (9)$$

where W_{E1} and W_{M1} are obtained by calculating the matrix elements of the corresponding diagrams of Fig. 1 in accordance with formulas (5) and (6) with the respective operators of the electric dipole and magnetic dipole of the photons (there is no interference term in (9) because of the summation over the projections of the angular momenta).

It is known that the emission probability per second is[11]

$$W_{E1}\approx3.1\cdot10^3\left[\frac{FZ}{\Delta E_L(Z)}\right]^2, \quad (10)$$

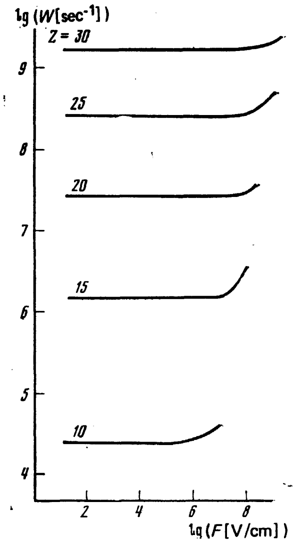

FIG. 3. Dependence of the transition probability on the external-field intensity at various values of the nuclear charge Z.

where the Lamb shift is measured in GHz. The dashed lines in Fig. 2 show the results of a calculation of the dependence of the probability of the magnetic dipole transitions W_{M1} in an electric field on the nuclear charge at various intensity values F satisfying the condition (8). As seen from the figure, the probability W_{M1} in an external field can be noticeably increased.

Figure 3 shows, in a logarithmic scale, the direct dependence of the probability W_{M1}, at several values of the nuclear charge Z, on the intensity of the field F that satisfies the condition (8). It is seen that for each Z there exists a certain limiting intensity F_0: at $F < F_0$ the interaction with the external field has practically no effect on the transition probability. We note that at $10 \leq Z \leq 30$ limiting field is much less than those fields at which the condition (8) is violated.

With the aid of (10) and the results of the calculation performed here, we can verify that at values of F and Z for which $W_{E1} \leq W_{M1}$ the influence of the external field on W_{M1} is negligible. Nonetheless, the obtained dependence of W_{M1} on F is of substantial interest for the determination of the influence of the external field on the angular anisotropy of the radiation, characterized by the ratio W_{M1}/W_{e1}, and can be used to study parity nonconservation effects in atomic systems.

In conclusion, I am grateful to L. N. Labzovskiĭ for numerous discussions and valuable remarks.

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