Manifestation of intrinsic electromagnetic radiation in tunneling in normal metals

E. V. Kochergin

Donetsk Physicotechnical Institute, Ukrainian Academy of Sciences
(Submitted 1 August 1979)
Zh. Eksp. Teor. Fiz. **78**, 773–781 (February 1980)

The effect of the inelastic tunneling of electrons with emission of photons on the current-voltage characteristic of a symmetrical *N–I–N* junction is considered. In the low-temperature limit ($T \to 0$ K) in the case of high-grade junctions (the facing and the layer contain no impurities) it is shown that the contribution made to the derivative of the conductivity $G(eV)$ by excitation of surface and volume modes of electromagnetic oscillations in the insulating layer leads to different results. The surface collective excitations cause a decrease of the derivative of the conductivity, while the volume excitations cause an increase. The decrease of $G(eV)$ is due to the transformation of the tunneling electron into surface plasmon oscillations. On the other hand, when the volume modes are taken into account, the increase of $G(eV)$ is steplike, owing to the appearance of singularities in the photon distribution function.

PACS numbers: 73.40.Rw

Lambe and McCarthy[1] have recently observed emission of light from a tunnel junction made up of aluminum films of thickness $d \approx 500$ Å and films of Ag, Au, Pb, In with $d \approx 200$ Å. The causes of this emission are the relaxation of the elastically tunneling electrons with excess energy $\Delta E \leq eV$ (V is the voltage on the junction), and the inelastic tunneling processes with emission of photons. Each of these radiation mechanisms has a threshold frequency $\omega_{max} = eV$. However, their manifestations are substantially different. When the electrons relax to the Fermi energy of the metal at $eV < \omega_p$ (ω_p is the plasma frequency of the electrons in the metal) the excess energy becomes redistributed rapidly enough among uncorrelated electron-hole excitations, which give up energy to the thermostat mainly via inelastic scattering with photon excitation. On the other hand, if $eV > \omega_p$, the energy of the electrons can become transformed into a collective plasma excitation, which in electrodes made of sufficiently narrow films can propagate to the boundary with the vacuum, where, by virtue of the corresponding boundary conditions, the longitudinal excitations become transformed into a transverse electromagnetic field (see Ref. 2). Radiation of this type was observed by Hwang *et al.*[3] in a tunnel junction made up of bulky aluminum and a thin silver film. This radiation, according to calculations by Fuse and Ichimaru,[2] is predominantly p-polarized, as was in fact observed in Ref. 3. The light emission observed by Lambe and McCarthy[1] is diffuse and essentially unpolarized. This circumstance gives grounds for assuming that the principal mechanism that leads to emission of light in the experiment of Ref. 1 is inelastic tunneling with participation of transverse electromagnetic excitations in the oxide, which forms a tunnel barrier, and of the junction electrons adjacent to the barrier. It is interesting in this connection to investigate the influence of a transverse electromagnetic field on the character of the tunneling. For simplicity we confine ourselves to a symmetrical tunnel junction whose electrodes are semi-infinite pieces of metal. Naturally, for such a junction the entire electromagnetic field will be concentrated in the vicinity of the barrier and cannot be taken to the outside. Nonetheless, an indirect manifestation of this electromagnetic field can be analyzed with the aid of the form of the current-voltage characteristic (CVC) of the junction.

On the basis of the approach developed by Ivanchenko,[4] we write down the Hamiltonian of the system that makes up the contact, with account taken of the quantized electromagnetic field

$$\begin{aligned} H_T &= T_0 + T_1, \\ T_0 &= \int d\mathbf{r}\, \Psi_I^+(\mathbf{r}) \left[-\frac{\nabla^2}{2m} + v(\mathbf{r}) \right] \Psi_{II}(\mathbf{r}) + \text{H.c.}, \\ T_1 &= -\frac{1}{c} \int dr \left\{ A_j(\mathbf{r}) \lim_{\mathbf{r}' \to \mathbf{r}} \left[\frac{ie}{2m} (\nabla_{r'} - \nabla_r)_j \right. \right. \\ &\left. \left. + \frac{e^2}{mc} A_j(\mathbf{r}') \right] + e^2 c \varphi(\mathbf{r}) \right\} \Psi_I^+(\mathbf{r}) \Psi_{II}(\mathbf{r}') + \text{H.c.} \end{aligned} \tag{1}$$

where $\Psi_I^+(\mathbf{r})$, $\Psi_{II}(\mathbf{r})$ are the operators of creation and annihilation of electrons in the first and second electrodes, respectively, $\mathbf{A}(\mathbf{r})$, $\varphi(\mathbf{r})$ are the operators of the potentials of the electromagnetic field, and $v(\mathbf{r})$ is the potential of the barrier.

With the aid of nonstationary perturbation theory in powers of $\sqrt{D}$ (D is the transparency of the barrier) we obtain the following expression for the tunnel current:

$$\begin{aligned} I(eV) &= -2e \operatorname{Im} \int_{-\infty}^{\infty} dt\, e^{-ieVt} \theta(t)\, i \langle [T_0^+(t) + T_1^+(t), T_0(0) + T_1(0)]_- \rangle \\ &= -2e \operatorname{Im} \int_0^{\infty} dt\, e^{-ieVt} \{K_{00}(t) + [K_{10}(t) + K_{01}(t)] + K_{11}(t)\} = -2e \operatorname{Im} K(eV). \end{aligned} \tag{2}$$

The averaging is carried out here over an equilibrium ensemble of the non-interacting subsystems of the first and second electrodes; $T_0(t)$, $T_1(t)$ are the operators of T_0 and T_1 in the interaction representation.

The meaning of (2) is quite obvious. The term with K_{00} leads to the usual elastic tunnel current. The increments containing K_{10} and K_0 are due to the "interference" of the elastic and inelastic tunneling mechanisms. They describe the process of deformation, in the vicinity of the barrier, of the virtual-photon cloud surrounding the electron. In fact, the first three terms contain only information on the tunneling-electron self-energy due to its interaction with the electromagnetic field, and are therefore of no importance in what fol-

0038-5646/80/020389-05$02.40

lows. The last term in (2) leads to the purely inelastic contribution with excitation (absorption) of electromagnetic-field quanta. After a number of transformations, using the analytic properties of the function $K(eV)$ we obtain in first order in the fine-structure constant e/c in T_1 an expression for the current:

$$I_{11}(eV)=\frac{4\pi e^2}{c^2}\sum_{pqk}\int\frac{d\omega\,d\omega_1}{(2\pi)^5}A^{(1)}(\mathbf{p}_+,\omega_1)$$

$$\times A^{(2)}(\mathbf{q}_-,\omega+\omega_1)B(\mathbf{p},\mathbf{q},\mathbf{k};\omega_-)F(\omega_1,\omega_1+\omega)[\nu(\omega_-)-\nu(\omega)],$$

$$B(\mathbf{p},\mathbf{q},\mathbf{k};\omega)=-\frac{1}{4m^2(2\pi)^4}\operatorname{Im}\int dr_1\,dr\,D_{ii}(x,x';\mathbf{k},\omega)$$

$$\times e^{i\mathbf{k}(\mathbf{r}-\mathbf{r}_1)}\lim_{\substack{\mathbf{r}'\to\mathbf{r}\\ \mathbf{r}_1'\to\mathbf{r}_1}}(\nabla_{r'}-\nabla_r)_i(\nabla_{r_1'}-\nabla_{r_1})_i f_p^{(1)*}(\mathbf{r}_1)f_p^{(1)}(\mathbf{r})f_q^{(2)*}(\mathbf{r}')f_q^{(2)}(\mathbf{r}_1'), \quad (3)$$

$$F(\omega,\omega_1)=f(\omega)-f(\omega_1),$$

where $A^{(i)}(\mathrm{p},\omega)$ is the spectral intensity of the electronic excitations of the ith ($i=1,2$) electrode, $f(\omega)$, $v(\omega)$ are respectively Bose and Fermi distribution functions, $\mathbf{p}_\pm=\mathbf{p}\pm\mathbf{k}/2$ (k is a two-dimensional operator lying in the plane of the barrier), $\omega_\pm=\omega\pm eV$, $f_p^{(i)}(\mathbf{r})$ are the single-particle electron wave functions with the aid of which the Hamiltonian of the junction is subdivided into Hamiltonians of the left-hand and right-hand electrons and the tunnel Hamiltonian, and $D_{ij}(x,x';\mathbf{k},\omega)$ is the Fourier transform of the retarded Green's function of the photon, defined in the usual manner:

$$D_{\alpha\beta}(\mathbf{r},t;\mathbf{r}',t')=\frac{\theta(t-t')}{i}\langle[A_\alpha(\mathbf{r},t),A_\beta(\mathbf{r}',t)]_-\rangle\quad(\alpha,\beta=0,1,2,3).$$

To simplify the calculations we assume henceforth that $A^{(i)}(\mathrm{p},\omega)$ is the spectran intensity of the free electrons. Neglecting also the terms of order v_0/c (v_0 is the velocity on the Fermi surface), we obtain for the second derivative of the current with respect to the voltage the expression

$$G(eV)=\frac{d^2I_{11}}{d(eV)^2}=\frac{2e^2}{c^2}N^2(0)\sum_{\mathbf{k}}\int\frac{d\omega}{2\pi}\bar{B}(\mathbf{k},\omega)\mathcal{F}(\omega_+),\quad (4)$$

where $N(0)$ is the state density of the electrons in the metal, taken on the Fermi surface, $\bar{B}(\mathrm{k},\omega)$ is the value of $B(\mathrm{p},\mathrm{q},\mathrm{k};\omega)$ averaged over the angles of the vectors p and q on the Fermi surface, the function $\mathcal{F}(\omega)$ is porportional to a δ function as $T\to 0K$ and is a dome-shaped function of width $\sim T$ at finite temperature $\mathcal{F}(\omega)$:

$$\mathcal{F}(\omega)=\beta\left(\frac{\beta\omega}{2}\operatorname{cth}\frac{\beta\omega}{2}-1\right)\Big/4\operatorname{sh}^2\frac{\beta\omega}{2};\quad \beta^{-1}=kT.$$

From (4) it is seen that the structure of the second derivative of the current with respect to the voltage is determined by the form of the function

$$\Phi(\omega)=\sum_{\mathbf{k}}\bar{B}(\mathbf{k},\omega).$$

To calculate $\Phi(\omega)$, as follows from (3), it is necessary to know the greatest function of the electromagnetic field $D_{\alpha\beta}$ for a system consisting of two metals separated by an insulator layer. The temperature Green's function of systems of this type was obtained by Dzyaloshinskiĭ, Lifshitz, and Pitaevskiĭ[5] in an analysis of the forces of the interaction between solids. We shall therefore use here their results to calculate $D_{\alpha\beta}$ (see also Ref. 6). It should be noted that if we use for the vector potential a gauge with $\varphi=0$, then the main contribution to the tunnel current in terms of the parameter v_0/c will be made by the xx component of the tensor function $D_{\alpha\beta}$, which takes in the region of the barrier the form

$$D(\bar{x},\bar{x}';\mathbf{q},\Omega)=D_1(\bar{x},\bar{x}';q,\Omega)+D_2(\bar{x},\bar{x}';q,\Omega)+D_3(\bar{x},\bar{x}';q,\Omega),\quad (5)$$

$$D_1(\bar{x},\bar{x}',\mathbf{q},\Omega)=-\frac{2\pi q^2}{\omega_p\Omega^2\varepsilon_i w_i}\operatorname{ch}w_i(\bar{x}-\bar{x}'),\quad (6)$$

$$D_2(\bar{x},\bar{x}';\mathbf{q},\Omega)=-\frac{2\pi q^2}{\omega_p\Omega^2\varepsilon_i w_i}\operatorname{sh}w_i\bar{x}\operatorname{sh}w_i\bar{x}'\operatorname{cth}z,\quad (7)$$

$$D_3(\bar{x},\bar{x}';\mathbf{q},\Omega)=\frac{2\pi q^2}{\omega_p\Omega^2\varepsilon_i w_i}\operatorname{ch}w_i\bar{x}\operatorname{ch}w_i\bar{x}'\operatorname{th}z,\quad (8)$$

where

$$2z=\xi w_i+\ln\frac{w_i\varepsilon_m+w_m\varepsilon_i}{w_i\varepsilon_m-w_m\varepsilon_i},$$

$$w_{i,m}=(q^2+\Omega^2\varepsilon_{i,m}(\Omega))^{1/2}.$$

We have introduced here the following dimensionless variables: $\mathbf{q}=\mathbf{k}\lambda_p$, $\lambda_p=c/\omega_p$, $\Omega=\omega/\omega_p$, $\bar{x}=x/\lambda_p$, $\zeta=d/\lambda_p$ (d is the thickness of the insulating layer).

For simplicity we assume that the dielectric constant of the insulator is $\varepsilon_i(\Omega)=1$. The dielectric function of metals is chosen in the form

$$\varepsilon_m(\Omega)=1+(\Omega^2+\delta|\Omega|)^{-1},\quad (9)$$

where δ is the dimensionless relaxation time of the electrons in the electrodes.

A few remarks must be made concerning this approximation. The local approximation for $\varepsilon_m(\Omega)$, as shown by Englman and Sondheimer,[7] is suitable in the case when the quantity $l\delta/|\delta+i\Omega|$ (the frequency is real throughout) is much less than in any other characteristic length. Here $l=v_0\tau$ is the mean free path of the electron, and the quantity $l\delta/|\delta+i\Omega|$ has the meaning of the effective dynamic length traversed by the electron without any collisions. For high frequencies $\omega\tau=\Omega/\delta\gg 1$ this length is equal to v_0/ω and constitutes the distance traversed by the electron during one period of the oscillations of the electromagnetic field. Other characteristic lengths in our problem are the photon wavelength $\lambda=2\pi c/\omega$ and the depth of penetration of the electromagnetic field $\lambda_0(\omega)$ in the electrodes that make up the junction:

$$\lambda_0(\Omega)=\lambda_p\left|\frac{\Omega^2+\delta^2}{\Omega^2}\right|^{1/4}.$$

In the case of a "dirty" metal ($\Omega\ll\delta$) the typical singularities on the tunnel characteristic are weakly pronounced because of the voltage smearing in the region $\Delta V\sim\hbar/e\tau$, so that one can hardly extract any information concerning the photon excitation in the region of the barrier. It is therefore of interest to investigate the case of a pure metal, when $\Omega\gg\delta$. In this limit the condition for the applicability of the approximation (6) takes the form $v_0/\omega_p\ll\min(\lambda_p,\lambda_p\Omega)$.

It is easily seen that at small Ω this condition can be violated even in very pure metals, for which furthermore the condition $\Omega\gg\delta$ is satisfied. We thus obtain a lower bound on the frequency, and consequently on the region of the minimal voltages ($eV=\hbar\omega$), in the form $\Omega\gg\max[\delta, qv_0/c]$.

In expressions (5)–(9) the functions $D(\Omega)$ and $\varepsilon_m(\Omega)$ are given, to abbreviate the notation, on the imaginary frequency axis. To obtain the retarded Green's function it is necessary to make the substitution $\Omega\to -i\Omega+\delta$, and furthermore in such a way as to obtain in the upper half-plane of the new variable Ω an analytic function that coincides on the imaginary axis with that given in (5). It is easiest to carry out the continuation for the function $D_1(\Omega)$, since the hyperbolic cosine is an analytic function, and w_i has two branch points $\pm iq$. For a unique determination of w_i on the real Ω axis it is necessary to make on this axis cuts at $|\Omega|>|q|$, with the signs on the edges of the cuts determined from the general relations that connect the temperature and the retarded Green's functions. At zero temperature we then have

$$D_1=\frac{2\pi q^2}{\omega_p\Omega^2|\varkappa|}[\theta(q^2-\Omega^2)\operatorname{ch}\varkappa(\bar{x}-\bar{x}')+i\theta(\Omega^2-q^2)\cos|\varkappa|(\bar{x}-\bar{x}')], \quad (10)$$

where $\varkappa=(q^2-Q^2)^{1/2}$ is an analytic continuation of the function w_i. On the other hand, the continuation of D_2 and D_3 is more complicated because they depend on the multiply valued function $\ln[\varphi_+(\Omega)/\varphi_-(\Omega)]$, which has two branch points $\Omega_\pm$ and $[(\Omega_\pm^2=q^2+\frac{1}{2}\pm(q^4+\frac{1}{4})^{1/2})]$ of infinite order:

$$\ln\varphi_+(\Omega)/\varphi_-(\Omega)=\ln|\varphi_+(\Omega)/\varphi_-(\Omega)|+i(\arg\varphi_+(\Omega)-\arg\varphi_-(\Omega))+2\pi ki \quad (k=0,\pm1,\ldots),$$

where $\varphi_\pm(\Omega)$ is the analytic continuation of $w_i\varepsilon_m\pm w_m$.

We investigate the behavior of $\arg\varphi_\pm(\Omega)$ on the entire real-frequency axis. From the analyticity of the causal function in the upper complex Ω half-plane follows the continuity of $\arg\varphi_\pm(\Omega)$ on the real frequency axis, and knowledge of the limits of $\arg\varphi_\pm(\Omega)$ as $\Omega\to\infty$ makes it possible to determine uniquely the region of its variation:

$$\arg\varphi_+=\theta(1+\varkappa^2)\theta(-\varkappa^2)\operatorname{arctg}\frac{|\varkappa|\varepsilon}{(1+\varkappa^2)^{1/2}}+\frac{\pi}{2}\theta(-1-\varkappa^2)\operatorname{sign}(\Omega_+^2-\Omega^2),$$
$$\arg\varphi_-=-\theta(1+\varkappa^2)\theta(-\varkappa^2)\operatorname{arctg}\frac{|\varkappa|\varepsilon}{(1+\varkappa^2)^{1/2}}-\pi\theta(\Omega^2-\Omega_-^2)-\frac{\pi}{2}\theta(-1-\varkappa^2). \quad (11)$$

The frequency dependence of $\arg\varphi_\pm(\Omega)$ at fixed q is shown in Fig. 1. We shall dwell in detail on the fact that in the limit of a "pure" metal $(\delta\to 0)$ the functions $\arg\varphi_+(\Omega)$ and $\arg\varphi_-(\Omega)$ have discontinuities at the frequencies Ω_+ and Ω_- respectively. The reason is that in these frequency regions the linear approximation for $\varepsilon(\Omega)$ is suitable only in the "dirty" limit. If the facing of the junction is made of "pure" metal, then in these frequency regions it is necessary to use the exact expression for $\varepsilon(q,\Omega)$,[8] since the approximate expressions for the dielectric function of the metal can lead to violation of the principal of relativistic causality (see, e.g., Ref. 9).

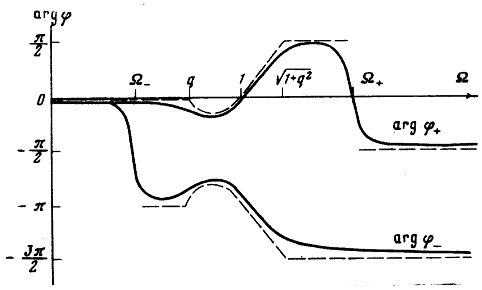

FIG. 1. Plots of the functions $\arg\varphi_\pm(\Omega+i\delta)$. The dashed curves show the functions in the limit as $\delta\to 0$.

The poles of the function $D_2(\Omega)$ are obtained from the solution of the equation $\sinh z=0$, the roots of which, obviously, are $z_n=i\pi n$ $(n=0,\pm1,\ldots,N)$. Since $z(\Omega)$ is a complex function of the frequency Ω, this condition breaks up into two conditions, for the real part $z'(\Omega,q)=0$ and for the imaginary part $z''(\Omega,q)=\pi n$. In the frequency region where $z'(\Omega q)\neq 0$, a band of electromagnetic oscillations exists in the layer $2N+1$ and the equation $z'(\Omega,q)=0$ determines the dispersion relation $\Omega_n(q)$. It is seen from Fig. 1 that at $\Omega^2<\Omega_-^2$ there exists one band $(n=0)$, and $\operatorname{Im}D_2(\Omega+i\delta)$ takes the form

$$\operatorname{Im}D_2(q,\Omega+i\delta)=\frac{2\pi^2}{\omega_p\Omega^2}\frac{q^2}{\varkappa}\operatorname{sh}\varkappa\bar{x}\operatorname{sh}\varkappa\bar{x}'\delta\left(\frac{\zeta\varkappa}{2}+\frac{1}{2}\ln\left|\frac{\varphi_+}{\varphi_-}\right|\right). \quad (12)$$

The dispersion equation is transcendental, and we consider therefore some limiting cases.

1. In the limit $|\varepsilon|\varkappa\gg 1$, $\varkappa\to 0$, which corresponds to the start of the band, we have $\Omega^2(q)=\zeta q^2/(\zeta+2)$.

2. The limit of large $q(q\gg 1)$ corresponds to surface plasma oscillations $2\Omega^2(q)=1-\exp(-\zeta q)$. Consequently, this dispersion equation describes a longitudinal surface mode that is adequately described by the longitudinal electrostatic theory.

We obtain analogously the poles of the function $D_3(q,\Omega)$, the imaginary part of which at $q^2>\Omega^2$ is obtained from (12) by replacing the hyperbolic sine with a hyperbolic cosine. The dispersion equation in the limit of large q describes the excitation of surface plasma oscillations $[2\Omega^2(q)=1+\exp(-\zeta q)]$.

On the other hand, in the frequency region $1+q^2>\Omega^2>q^2$, $z'(q,\Omega)\equiv 0$ the sum of the imaginary parts $D_2(q,\Omega)$ and $D_3(q,\Omega)$ takes the form

$$\operatorname{Im}D'(q,\Omega+i\delta)=-\frac{2\pi^2q^2}{\omega_p{}^2\Omega^2\tilde{\omega}}\sum_{n=0}^{N}\cos\left(\frac{\pi n}{2}+\bar{x}\tilde{\omega}\right)\times\cos\left(\frac{\pi n}{2}+\bar{x}'\tilde{\omega}\right)\delta\left(\frac{\pi n}{2}-\frac{\zeta\tilde{\omega}}{2}-\operatorname{arctg}\frac{\varepsilon\tilde{\omega}}{(1-\tilde{\omega}^2)^{1/2}}\right), \quad (13)$$

where $\tilde{\omega}(q,\Omega)=(\Omega^2-q^2)^{1/2}$.

The dispersion relation $\Omega_n(q)$ is determined from the solution of the following transcendental equation:

$$\frac{\pi}{2}n-\operatorname{arctg}\frac{\varepsilon\tilde{\omega}}{(1-\tilde{\omega}^2)^{1/2}}=\frac{\zeta\tilde{\omega}}{2}. \quad (14)$$

Putting $\Omega=1$ and $q=0$ in (14) we get $N=[\zeta/\pi]$ ($[X]$ is the integer part of X).

It should be noted that besides the N purely transverse modes of volume electromagnetic oscillations there exists a mode that is transformed at $\Omega^2=q^2=2/(\zeta+2)$ into a longitudinal surface mode. The form of the dispersion curves is shown in Fig. 2. In the limit of a small width of the dielectric layer $(\zeta<1)$ the results agree with those obtained by Economou.[10] The function obtained in this manner makes it possible to calculate the value of $\Phi(\Omega)$.

According to the results (10) and (12), the imaginary part of the photon Green's function of an $N-I-N$ junction is positive in the frequency region $\Omega^2<q^2$ and negative at $\Omega^2>q^2$, whereas by definition the imaginary part

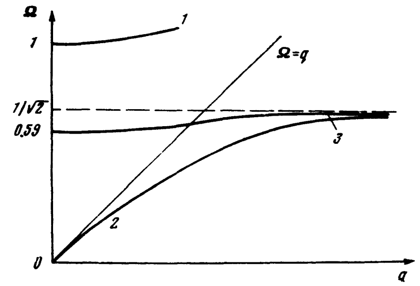

FIG. 2. Dispersion equation for the photon in an N–I–N junction at $\zeta = \pi + 0$: curve 1 corresponds to $\Omega^2 = 1 + q^2$ at $q^2 \ll 1$; curve 2 is described by the expression $\Omega = q[\zeta/(\zeta+2)]^{1/2}$ at $\Omega \ll 1$, $q \ll 1$ and by the expression $\Omega^2 = [1 + (1+q^{-2})^{1/2}(1+e^{-\zeta q})]^{-1}$ at $q \gg 1$. In the limit of large q curve 3 takes the form $\Omega^2 = [1 + (1+q^{-2})^{1/2}(1-e^{-\zeta q})]^{-1}$.

of the causal Green's function should be only negative in the entire frequency region. This paradox is due to the fact that the calculated function (5)–(8) is the difference between the Green's functions of an inhomogeneous ($N-I-N$ junction) and a homogeneous infinite medium. This procedure was used to be able to disregard the infinite contribution made to the tunnel current by electromagnetic oscillations with small wavelengths. Consequently, what should be negative in the entire frequency region is the imaginary part of the complete Green's function (the sum of the homogeneous and inhomogeneous parts).

The physical picture of the fact that in the frequency region $\Omega^2 < q^2$ the opening of an additional (inelastic) tunneling channel decreases the tunnel current rather than increasing it is quite clear. The point is that the electrons whose quasimomentum projection on the plane of the junction section exceeds their frequency ($q^2 > \Omega^2$) stimulate longitudinal surface collective oscillations of electron-hole pairs, as a result of which the current in the $N-I-N$ junction decreases.

We examine now the contribution of the surface oscillations to the tunnel current. At low voltages ($\Omega^2 \ll 1$) the derivative of the conductivity takes in the limit of low temperatures ($T \to 0K$) the form

$$G(\Omega_0) = \frac{8\pi e^3}{c^2\omega_p{}^2\zeta^3}(\zeta+2)\,|T|^2 N^2(0)\,\Omega_0{}^2\,\mathrm{sign}\,\Omega_0, \tag{15}$$

where $\Omega_0 = eV/\omega_p$, $|\tilde{T}|^2$ is the square of the modulus of the modified matrix element[1)] averaged over the Fermi surface

$$\tilde{T}_{pq} = \int_{-\zeta/2}^{\zeta/2} d\bar{x}\,\bar{x}\,T_{pq}(\bar{x}). \tag{16}$$

This expression is valid so long as both dimensions in the cross section of the junction (L_y and L_z) are of the same order. The case when one of the linear dimensions is much less than the other (for example $L_y = L \ll L_z$) was investigated in Ref. 11, where it was shown that in such junctions inelastic tunneling of electrons with emission of photons leads to oscillations of the derivative of the conductivity.

At high voltages ($\Omega_0^2 \sim \frac{1}{2}$) we obtain for the derivative of the conductivity the expression

$$G(\Omega_0) = \frac{4\pi e^3}{c^2\omega_p{}^2\zeta}|T|^2\theta(2\Omega_0{}^2-1)\,\theta(y^2-1)\,\theta(c^2-v_0{}^2y^2)\,y^2\,\mathrm{sign}\,\Omega_0, \tag{17}$$

where $y = [\ln|2\Omega_0^2 - 1|]/\zeta\Omega_0$. Equation (17) is clearly only qualitative in character, since neglect of the spatial dispersion in $\varepsilon(q, \Omega)$ in this frequency region leads to violation of the principle of relativistic causality. Nonetheless, notice must be taken of the good qualitative agreement between the result and the experimental data,[12] where the decrease of $G(\Omega_0)$ due to generation of surface plasmons was observed for the first time ever in this frequency region.

The contribution of the volume modes of the electromagnetic oscillations to the derivative of the conductivity of high-grade $N-I-N$ junctions (the facing and the insulating layer contained no impurities) can be represented in the form

$$\Phi(\Omega) = \sum_{n=0}^{N}\Phi_n(\Omega)$$

$$= -\frac{4\pi^2}{\omega_p{}^2\Omega^2}\sum_{n=0}^{N}\frac{\theta(\omega_n)\,\theta(\min\{1,\Omega\}-\omega_n)\,|T_n|^2}{|\zeta+2\varepsilon/(1-\omega_n{}^2)^{1/2}(1+(\varepsilon^2-1)\,\omega_n{}^2)|}(\Omega^2-\omega_n{}^2), \tag{18}$$

where $\omega_n(\Omega)$ are the roots of the dispersion equation (14), and $|T_n|^2$ is the square, averaged over the Fermi surface, of the modulus of the matrix element

$$T_{pq}{}^n(\zeta) = \int_{-\zeta/2}^{\zeta/2} d\bar{x}\,\cos(\pi n/2+\omega_n\bar{x})\,T_{pq}(\bar{x}).$$

Let us dwell in detail on our result. As follows from (18), when the frequency Ω is equal to the natural frequency ω_n of the n-th photon mode, the function $\Phi(\Omega)$ undergoes a jump equal to $\Delta\Phi_n = \Phi_n - \Phi_{n-1}$. The physical cause of these jumps is quite obvious—they are manifestations of singularities in the state density of the photons of the insulating layer. It must be borne in mind that in the low-temperature limit $\mathscr{F}(\Omega)$ has a smearing of the order of $4\beta^{-1}$. Therefore a clear-cut manifestation of the singularities in the state density of the photons on $G(eV)$ is possible when the condition $\beta\Delta\Phi_n \gg 1$ is satisfied.

We note that the effects considered here depend substantially on the width of the insulating layer. Thus, for sufficiently low but broad barriers ($d \sim 10^3$ Å) (using a semiconductor or a semimetal as the insulator), the state density of the photons has only two singularities.

In conclusion, the author thanks Yu. N. Ivanchenko for suggesting the problem and for a helpful discussion of the results.

[1)]The matrix element describing the inelastic tunneling with emission of photons is of the form

$$T_{pq}(x) = -\frac{\delta(\mathbf{p}_\parallel-\mathbf{q}_\parallel)}{2m}\left[f_{p_\perp}^{(1)*}(x)\,\frac{\partial f_{q_\perp}^{(2)}(x)}{\partial x} - f_{q_\perp}^{(2)}(x)\,\frac{\partial f_{p_\perp}^{(1)*}(x)}{\partial x}\right].$$

[1]J. Lambe and S. L. McCarthy, Phys. Rev. Lett. **37**, 923 (1976).
[2]M. Fuse and S. Ichimaru, J. Phys. Soc. Jpn. **40**, 830 (1976).
[3]T. L. Hwang, S. E. Schwarz, and R. K. Jain, Phys. Rev. Lett. **36**, 379 (1976).
[4]Yu. M. Ivanchenko and Yu. V. Medvedev, Fiz. Nizk. Temp. **2**, 141 (1976) [Sov. J. Low Temp. Phys. **2**, 67 (1976)].
[5]I. E. Dzyaloshinskiĭ, E. M. Lifshitz, and L. P. Pitaevskiĭ, Zh.

Eksp. Teor. Fiz. **37**, 229 (1959) [Sov. Phys. JETP **10**, 161 (1959)].
[6]A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, Metody kvantovoi teorii polya v statisticheskoi fizike (Quantum Field-Theoretical Methods in Statistical Physics), Fizmatgiz, 1962.
[7]R. Englman and E. H. Sondheimer, Proc. Phys. Soc. London Sect. B **69**, 449 (1956).
[8]J. Lindhard and J. Kgl, K. Dan. Vidensk. Selsk. Mat.-Fys. Medd. **28**, 8 (1954).
[9]A. A. Borgardt and K. B. Tolpygo, Ukr. Fiz. Zh. **23**, 60 (1978).
[10]E. N. Economou, Phys. Rev. **182**, 539 (1969).
[11]Yu. M. Ivanchenko and E. V. Kochergin, Pis'ma Zh. Eksp. Teor. Fiz. **28**, 305 (1978) [JETP Lett. **28**, 280 (1978)].
[12]D. C. Tsui, Phys. Rev. Lett. **22**, 293 (1969).

Translated by J. G. Adashko