Interaction of electromagnetic waves of different frequencies on reflection from the surface of a normal metal

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A change of the surface impedance of metal plates at frequency $\tilde{\omega}$ (100 to $3\cdot10^4$ Hz) is detected under the influence of radio frequency irradiation with a frequency $\omega \gg \tilde{\omega}$. The change of impedance is observed in zero external magnetic field and is not small: under irradiation, both the real and the imaginary parts of the impedance may change by a factor of several. The observed changes of surface impedance depend strongly on temperature.

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The physical properties of conducting media may change under the influence of powerful radiofrequency irradiation. For example, nonlinear effects of self-action and mutual interaction of radio waves in a plasma are known.[1] This effect is due to a breakdown of the superposition principle: by changing the properties of the medium in which they are propagated, electromagnetic waves interact in nonlinear fashion.

An advanced nonlinearity is observed also on irradiation of pure perfect metallic crystals with radio waves at helium temperatures. Under these conditions, experimenters have observed generation of a second harmonic,[2,3] appearance of a constant radioelectromotive force,[4-6] "current" states,[7] and a self-oscillatory mode.[8] One may have confidence that under simultaneous irradiation of a metal with radio waves of different frequencies, effects of mutual interaction will appear in this case as well.

The present paper is devoted to a study of nonlinear effects that occur under simultaneous irradiation of a metal by radio waves of different frequencies. The measurements were made by means of an alternating-current bridge operating at frequencies $\tilde{\omega}/2\pi \sim 100$ to 3×10^4 Hz. A schematic diagram of the measurement setup is given in Fig. 1. The specimen was located inside an inductance coil that was connected into one of the arms of the bridge. To the same inductance was fed simultaneously a voltage of frequency $\omega \gg \tilde{\omega}$. In our experiments, the frequency ω lay within the range 0.7 to 3.5 MHz.

The arms of the alternating-current bridge were made symmetric. The resistance R_1 and R_2 were so chosen that the current of frequency $\tilde{\omega}$ through the measurement coil was fixed ($R_1 = R_2 \gg \tilde{\omega}L, r$). The condenser C served to match the measurement cell with the output of the high-frequency generator.

The unbalance signal of the bridge entered the symmetric input of a low-frequency amplifier and, after amplification, a synchronous detector. A reference voltage was taken from the resistance R_2 and amplified by a wide-band low-frequency amplifier. There was no phase shifter in the synchronous detector; therefore the measurement scheme recorded a signal proportional to the real part of the low-frequency voltage across the measurement coil or, equivalently, to the power absorbed at frequency $\tilde{\omega}$ by the coil containing the specimen. The change of the absorbed power under linear conditions is proportional to the change of the real part of the surface impedance of the specimen. The description in terms of surface impedance is often used also under nonlinear conditions.[9] The criterion for applicability of such a description consists[10] in smallness of the surface impedance $|Z| = |E(0)/H(0)| \ll 1$. Since the relation $|Z| \sim |\tilde{\omega}\delta_{\tilde{\omega}}/c| \ll 1$ is certainly fulfilled at frequencies $\tilde{\omega}$, it may be supposed that what is observed experimentally is the variation of the real part $R_{\tilde{\omega}}$, of the surface impedance at frequency $\tilde{\omega}$ with the amplitude of the high-frequency field and with other parameters. In separate experiments, a signal proportional to the imaginary part of the surface impedance was recorded.

The high-frequency field considerably exceeded the low-frequency in amplitude. The amplitude H_1 of the high-frequency field was bounded from above by the critical value[7,8] above which current states occured in the specimen and the dependence of the magnetic moment of the specimen on a constant magnetic field became nonunique.

Most of the experiments were done on single crystals of bismuth, which were disks of diameter 17.8 mm and

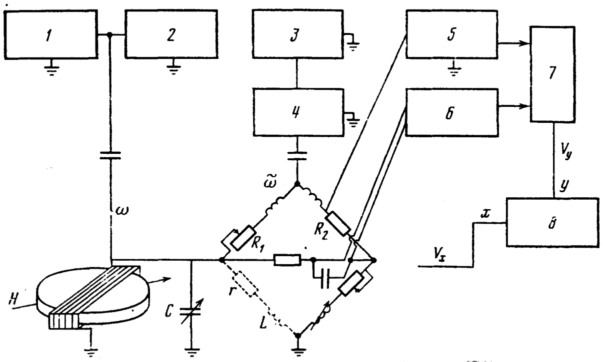

FIG. 1. Block diagram of the experimental setup. 1, high-frequency generator; 2, voltmeter; 3, low-frequency generator; 4, emitter follower; 5, wide-band amplifier; 6, wide-band amplifier with symmetric input; 7, synchronous detector; 8, xy recording potentiometer, $V_y \sim \mathrm{Re}\,Z(\omega)$ and $V_x \sim H$.

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thicknesses 1 to 0.6 mm. The trigonal axis of the specimens was directed along the normal to the disk surface. Some of the measurements were made on a monocrystalline tin disk of the same diameter. The thickness of the tin specimen was $d=0.6$ mm. The fourfold axis was directed along the normal to the disk surface. Separate measurements were made on a copper plate of thickness 0.55 mm with the [110] axis along the normal. By means of a Helmholtz coil, it was possible to produce a constant magnetic field H parallel to the surface of the specimen. The earth's magnetic field was compensated.

EXPERIMENTAL RESULTS

In the presence of high-frequency irradiation, an anomaly was measured on the $R_{\tilde{\omega}}(H)$ relation at a weak magnetic field H (Fig. 2). The anomaly was observed on all the specimens investigated. The form of the curve and its characteristic dimensions depended on the frequency $\tilde{\omega}$: at low frequencies the $R_{\tilde{\omega}}(H)$ curve was double-humped; at higher frequencies, there was a maximum at $H=0$. As is seen from the figure, the position of the maxima of the double-humped curve occurs at precisely that value of the field H where, on further increase of the amplitude of the high-frequency field, loops of current states are generated.[7]

The anomaly corresponded to an increase of the absorption of energy at frequency $\tilde{\omega}$ when the high-frequency electromagnetic field was switched on. The sign of the unbalance signal of the bridge that corresponded to an increase of absorption was easily established by introducing an active resistance in series with the measurement coil. For an additional test of the sign, we used the change of the signal on transition of the tin to the superconducting state. Comparison of the bridge unbalance that occurred in the latter case with the unbalance that appeared upon introduction of irradiation of the specimen with frequency ω enabled us to estimate the value of the change of the real part of the surface impedance under the influence of the radiation. It was found that the changes of the real part of the surface impedance were large. In tin, the maximum change exceeds by more than a factor two the initial value of the real part of the surface impedance. In bismuth, this ratio was even larger.

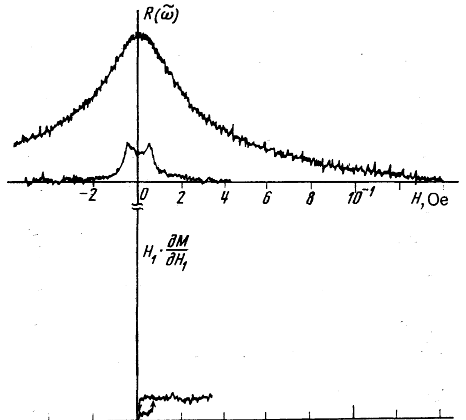

FIG. 2. The upper part of the figure shows an example of a record of $R_{\tilde{\omega}}(H)$ curves at two frequencies: $6 \cdot 10^3$ Hz (upper curve) and 600 Hz (lower curve). Amplitude of high-frequency field $H_1=1.8$ Oe, $H_{lf}=0.008$ Oe. In the lower part of the figure is a record showing the generation of hysteresis loops of "current" states at $H_1=1.9$ Oe. Bismuth specimen; $C_3 \parallel \mathbf{n}$, $d=0.76$ mm, $\mathbf{H} \perp \mathbf{j}_1 \parallel C_1$, $T=1.4$ K.

The results of measurements of the variation of the amplitude of the curve,

$$A=H_{lf}[R(H=0, H_1)-R(H=0, H_1=0)]$$

with the intensity of the low-frequency alternating field are shown in Fig. 3. Over a quite wide range, A is proportional to H_{lf}. Within this range, the change of the surface impedance is independent of the amplitude of the low-frequency alternating field. The influence of the low-frequency field on the change of impedance becomes appreciable only when H_{lf} is comparable with the width of the line. On further increase of the amplitude of H_{lf}, the line broadens.

Figure 3 also enables us to judge the relation between the amplitudes of the curves in bismuth and in tin. It must be remembered, however, that at equal frequencies, under anomalous skin-effect conditions, the impedance of bismuth exceeds the impedance of tin in modmodulus by an order of magnitude.

The variation of the amplitude of the curve with the intensity fo the high-frequency field is shown in Fig. 4. At all frequencies $\tilde{\omega}$ there was a monotonic increase of the amplitude with increase of the field H_1, but the form of the $A(H_1)$ curve was at different frequencies ω. Lowering of the temperature also led to increase of the amplitude of the curve (Fig. 5). The form of the temperature dependence also changed with change of $\tilde{\omega}$. The width of the line increased with increase of the frequency $\tilde{\omega}$. The dependence of the amplitude on the low frequency was investigated in detail in bismuth. The amplitude increased at small frequencies, reached a maximum value at frequency ~$6 \cdot 10^3$ Hz, and dropped on further increase of frequency. Change of the high frequency over the range 0.7 to 3.5 MHz did not affect the shape and dimensions of the curve.

On one of the bismuth specimens, the surface was etched. After etching, the amplitude of the curve increased at frequencies below that where the maximum

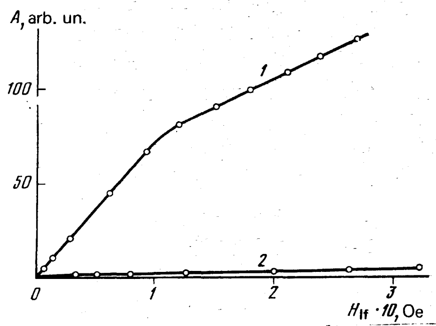

FIG. 3. Amplitude of the curve as a function of the intensity of the low-frequency field H_{lf}: 1, bismuth, $\tilde{\omega}/2\pi=6 \cdot 10^3$ Hz $H_1=1.6$ Oe, $\omega/2\pi=3 \cdot 10^6$ Hz, $T=1.4$ K; 2, tin, $H_1=11$ Oe, $\tilde{\omega}/2\pi=1.5 \cdot 10^3$ Hz, $\omega/2\pi=2.3 \cdot 10^6$ Hz, $T=3.78$ K.

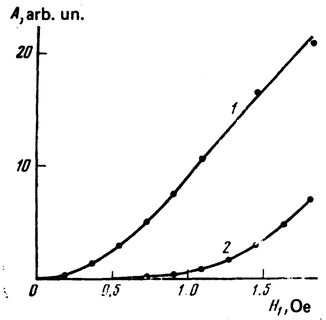

FIG. 4. Variation of the amplitude of the curve with the intensity of the high-frequency electromagnetic field for a bismuth specimen: 1, for $\tilde{\omega}/2\pi = 6\cdot10^3$ Hz; 2, for $\tilde{\omega}/2\pi = 600$ Hz; $T = 1.4$ K, $H_{\text{lf}} = 2.5\cdot10^{-2}$ Oe.

of $A(\tilde{\omega})$ is attained but did not change noticeably at frequencies above the maximum.

Most of the measurements on bismuth were made under the conditions $\mathbf{H}\|\mathbf{H}_{\text{lf}}\|\mathbf{H}_1\|C_2$; and on tin, with $\mathbf{H}\|\mathbf{H}_{\text{lf}}\|\mathbf{H}_1\|[100]$. Rotation of the external field in the plane of the specimen led to an increase of the linewidth, as is shown in Fig. 6, where the linewidth B at half height (see Fig. 5) is plotted as a function of the direction of the magnetic field. As is seen from the figure, the linewidth varies in proportion to $\cos^{-1}\varphi$. It is to be expected that in bismuth, rotation of the specimen with respect to the inductance coil would not change the dependence of the linewidth on the direction of the constant magnetic field. In rotations of the tin specimen through small angles $\varphi_0 < \pi/4$, the dependence of the linewidth changed in proportionality to $\cos^{-1}(\varphi + \varphi_0)$. But if the angle of rotation exceeded $\pi/4$ $(\pi/2 > \varphi_0 > \pi/4)$, the linewidth was proportional to $\cos^{-1}(-\pi/2 + \varphi + \varphi_0)$.

DISCUSSION

The effect of high-frequency irradiation on the impedance of a conducting medium, as has already been mentioned, may be due to a change of the properties of the medium, for example to heating of the electronic system by the high-frequency field. In this case the conductivity changes, and with it the depth of penetra-

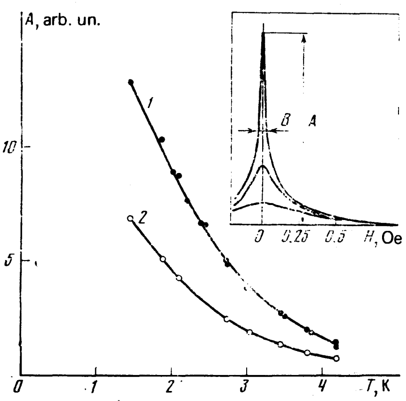

FIG. 5. Temperature dependence in bismuth for two different amplitudes of the high-frequency field ($\omega/2\pi = 3\cdot10^6$ Hz, $H_{\text{lf}} = 1.45\cdot10^{-2}$ Oe, $\tilde{\omega}/2\pi = 6\cdot10^3$ Hz): 1, for $H_1 = 1.6$ Oe; 2, for $H_1 = 0.77$ Oe. The inset shows experimental records for three temperatures (from top to bottom): 1.42, 3.0, 4.2 K; $H_1 = 1.6$ Oe.

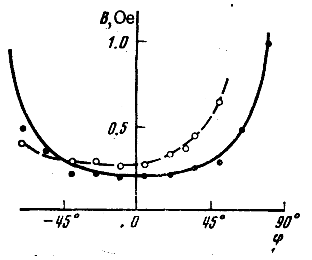

FIG. 6. Dependence of linewidth on direction of constant magnetic field for tin specimens: $T = 3.815$ K, $\omega/2\pi = 2.3\cdot10^6$ Hz, $\tilde{\omega}/2\pi = 3\cdot10^3$ Hz, $H_1 = 11$ Oe. The solid curve corresponds to the relation $B \sim \cos^{-1}\varphi$, where the angle φ is measured from the direction of H_1. Experiment: the dark points correspond to the orientation $\mathbf{H}_1\|\mathbf{H}_{\text{lf}}\|[100]$; the light points show the relation obtained after rotation of the specimen inside the measurement coil through an angle 20°.

tion of the low-frequency electromagnetic field. But one can hardly expect an appreciable change of conductivity in metals; in any case, one cannot expect such a large change of conductivity that the impedance would change by a factor of several.

Another possibility is enhancement of the low-frequency field inside the metal as a result of nonlinearity. As is well known,[7,8,11] in metals, thanks to rectification of high-frequency current, even a constant magnetic field can be enhanced. When $\omega\tau \ll 1$, under the conditions for an anomalous skin effect, nonlinearity is caused by the effect of the magnetic field of the wave on the electron trajectories. A relation to trajectory effects is also indicated by the sharp temperature dependence of the observed signal.

In a constant magnetic field H, the high-frequency current varies with the magnetic field in the following manner[12]:

$$j \sim E_1\sigma\frac{\delta_1}{l}\left(1 - \frac{l^2}{6R^2}\right), \quad \delta_1 \ll l \ll (R\delta_1)^{1/2}, \qquad j \sim E_1\sigma\delta_1/l, \quad l \gg (R\delta_1)^{1/2}, \tag{1}$$

where $\sigma = ne^2\tau/m$ is the static conductivity, l is the length of the free path of the electrons, $R = pc/eH$ is the radius of the electron orbit, and δ_1 is the thickness of the skin layer at frequency ω.

In an external low-frequency field H_{lf}, a similar relation leads to the appearance in the low-frequency skin layer of a current of frequency $\tilde{\omega}$. As a result, the magnetic field at a depth exceeding δ_1 is $H_{\text{lf}}^0 + \mathscr{H}$, where

$$\mathscr{H} \sim H_{\text{lf}}^0\delta_1^2/l^2, \quad H_{\text{lf}}^0 \ll h = mc\delta_1/e\tau l \ll H_1, \qquad \mathscr{H} = h\delta_1^2/l^2, \quad H_1 \gg H_{\text{lf}}^0 \gg h. \tag{2}$$

The correction to the low-frequency alternating field obtained in this way, $\mathscr{H} \ll H_{\text{lf}}$, is too small to explain the experimentally observed changes of impedance.

In the nonuniform magnetic field made up of H_1 and H_{lf}^0, there may occur trajectories that twist at the surface, such that electrons moving along them return repeatedly to the high-frequency skin layer.[8,11] Such trajectories originate because an electron ejected from the skin layer by the magnetic field H_1 of the wave returns back as a result of bending of the orbit by the field H_{lf}. The number of returns of effective electrons,

i. e., of ones that move into the skin layer parallel to the surface, increases during that half-period of the alternating field when H_1 and H^0_{lf} are antiparallel and these trajectories exist. Because of the breakdown of the equivalence of the two half-periods of the high-frequency current, an additional low-frequency current appears at the surface of the specimen.

The effective electrons fly out of the high-frequency skin layer at angles $\alpha \lesssim (\delta_1 eH_1/pc)^{1/2}$. Only those return back for which $\alpha < \alpha_0 = (\delta_{lf} eH^0_{lf}/pc)^{1/2}$. For electrons that fly out at angles larger than α_0, trajectories that twist at the surface are impossible. In other words, in a field H_{lf} that attenuates over a depth δ_{lf}, in contrast to a uniform constant magnetic field, trajectories that twist at the surface do not exist for all the effective electrons.

It is easy to estimate the correction to the low-frequency field because of this nonlinearity mechanism when $H_1 \gg H^0_{lf}$:

$$\mathscr{H} \sim H^0_{lf} \frac{(\delta\ \ l\alpha_0)^{1/2}}{\delta_1} \exp\left(-\frac{2\delta_{lf}}{l\alpha_0}\right). \tag{3}$$

When $H^0_{lf} \sim 10^{-1}$ Oe and with the parameters for bismuth, $\delta_{lf} \sim 10^{-2}$ cm for $\bar{\omega}/2\pi \sim 10^3$ Hz, $l \sim 10^{-1}$ cm, $\delta_1 \sim 10^{-3}$ cm, $\alpha_0 \sim 10^{-1}$, the correction to the low-frequency field coincides in order of magnitude with the field H^0_{lf}.

But in this case one must expect an exponential dependence of the impedance on the amplitude of the low-frequency field, whereas in the experiment such a dependence is completely absent. Furthermore, according to the estimate the correction to the low-frequency field is independent of the amplitude of the high-frequency field, and this also contradicts experiment. Consequently, the presence of a signal in zero external magnetic field can also not be explained by this mechanism of interaction of radio waves. Nevertheless its contribution is appreciable, and in our opinion it explains the presence of double-humped curves. In fact, at low frequencies $\bar{\omega}$, where $\delta_{lf} \gg d$, the received signal is proportional to $\partial M/\partial H$ (M is the static magnetic moment of the specimen). The curve $\partial M(H)/\partial H$ according to calculation[8,11] should have two maxima, symmetrically located with respect to the point $H = 0$. At higher frequencies $\bar{\omega}$, the experimentally observed curve is no longer proportional to $\partial M/\partial H$ but qualitatively retains the same form.

Under the conditions for the anomalous skin effect, there is yet another possibility for interaction of radio waves. The intrinsic magnetic field of the wave, ejecting effective electrons from the skin layer, produces a current of these electrons, directed into the interior of the specimen. The current density q normal to the surface is in order of magnitude

$$q = e^{-1}\frac{H_1}{\delta_1}\frac{c}{4\pi}\alpha e^{-z/\alpha l}.$$

Here z is the distance from the surface of the specimen to the point under consideration. Sinee there is no component of constant current normal to the surface, this current is compensated by a current of ineffective electrons. In the magnetic field of the low-frequency wave, the currents under consideration have components parallel to the surface. Since the relation between the fields and the currents is nonlocal ($\delta_1 \ll \delta_{lf} \lesssim l$), the components of the currents of the effective and of the ineffective electrons parallel to the surface are spatially separated and do not compensate each other. As a result, at the boundaries of the low-frequency skin layer there appears an additional current of frequency $\bar{\omega}$, dependent both on the value of the low-frequency field H^0_{lf} and on the amplitude of the high-frequency field. The magnetic field of this current is, in order of magnitude,

$$\mathscr{H} \sim H_1 \frac{\delta^2_{lf} eH^0_{lf}}{\delta_1 pc} \exp\left(-\frac{\delta_{lf}}{\alpha l}\right) = kH^0_{lf}, \tag{4}$$

here H^0_{lf} is the internal low-frequency field. It is $H^0_{lf} = \mathscr{H} + H_{lf}$; therefore

$$\mathscr{H} = \frac{k}{1-k} H_{lf}.$$

In bismuth, the momentum of the electrons is $p \sim 10^{-21}$ g cm/sec, so that $\mathscr{H}$ is of the order of magnitude of H_{lf} when $H_1 \sim 1$ Oe. In tin, $p \sim 10^{-21}$ g cm/sec and the depth of the skin layer is almost an order of magnitude smaller than in bismuth; therefore $\mathscr{H} \sim H_{lf}$ when $H_1 \gtrsim 10^2$ Oe.

The relation (4) agrees with most of the experimentally observed results: for example, with the sharp dependence of the amplitude of the curve on temperature and on the intensity of the high-frequency field, with the change of character of these dependences on increase of $\bar{\omega}$, and with the linear increase of the amplitude of the curve on increase of H_{lf}. But a detailed comparison of the results with the estimation formula (4) hardly makes sense. For this purpose, an accurate calculation of the interaction of electromagnetic waves is necessary.

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